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Community energy projects in the context of generation and transmission expansion planning

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Abstract

We extend multi-level generation and transmission expansion planning optimisation methods to analyse the impact of the community energy sector emergence on power system expansion. Methodologically, we do this by incorporating biform games and linear production games into the classic power transmission and generation expansion planning problem. The model is formulated as a Mathematical Program with Equilibrium Constraints (MPEC). Using a stylised example based on Chilean power market data, we show that community energy projects can be an optimal choice for residential customers. Furthermore, we find that community energy projects can widely impact the power system as a whole. Importantly, unlike some large generation projects, they can have positive effects on electricity customers who do not participate in them. Nevertheless, the implementation of net billing schemes, potential competitors for community energy projects, may be preferred when net billing injection prices are comparatively high, there are potentially high levels of confidence about the final payoffs, and there exist comparatively lower investment costs. Yet, we show that, depending on the specific case analysed with our model, it is likely that under the above circumstances nodal prices increase, social welfare decreases, and capacity limitations need to be addressed. The methodology developed in this work can be a valuable tool to examine and better understand the complexities of community energy projects and their interactions with market incumbents.

JEL Classification codes: C6, C7, L1, Q4

Keywords: Community energy, Biform games, Linear production games, Mathematical program with equilibrium constraints (MPEC), Generation and transmission expansion planning

1. Introduction

The role of community energy projects in liberalised electricity markets is rapidly becoming more relevant. A variety of community-led projects are currently generating energy or being implemented in many countries, with emphasis on Europe. For example, projects have been developed in the United Kingdom (Seyfang et al., 2013), some Scandinavian countries and the Netherlands (Kooij et al., 2018), Spain (Heras-Saizarbitoria et al., 2018), Germany (Yildiz et al., 2015), Italy (Wirth, 2014), among others. Community energy projects are encouraging citizen participation in (renewable) energy production and therefore help tackle climate change and democratise energy markets.

Empirical and theoretical studies have shown the importance of community energy developments with a particular focus on their financial-economic characteristics. Kalkbrenner and Roosen (2016) note the existence of a willingness

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to devote money among people, though emphasising that such willingness is lower than the willingness to volunteer in community energy projects. Other authors have highlighted the importance of community-based investments (and ownership). Based on the Scottish experience, Haggett and Aitken (2015) note that a better and deeper access to capital, as well as guidance and information for communities is critical, as community-based investments and ownership have positive impacts. Holstenkamp and Kahla (2016) show that the return motive plays a part in community energy initiatives, but there are differences in its importance across particular legal entities and organisations based on different technologies. Bauwens (2019) highlights that the return on investment is the most significant driver in large communities of interest. However, the above literature only focuses on the economic viability of the projects themselves, without considering the wider energy system.

Community energy projects are expected to influence electricity markets in the near future, especially if a high citizen participation in energy production is genuinely desired. These impacts can be varied, but include changes to wholesale market prices, transmission line investments, line congestion, generation capacity investments, as well as cooperation and payoff distribution mechanisms among market participants. It is therefore crucial for electricity market incumbents, such as large generators, distributors or suppliers, transmission system operators, and system planners, among others, to understand potential systemic effects derived from community energy projects implementation. Regulators and policy makers need to understand such whole-system effects to correctly quantify related benefits, costs, and risks. Moreover, community energy project managers need to understand the interactions within energy markets and the consequences derived from strategic decisions made by incumbents and new entrants of such markets. This is challenging, as community energy initiatives are different from other investments in generation capacity. A community energy project firstly needs to assure stability to its members, in the sense of offering suitable incentives to belong to and remain in the project or coalition, and also needs to be competitive, in the sense of offering long-term viability and attractive benefits in comparison with other projects.

The above illustrates the dual behaviour of community energy projects. On the one side, these projects need cooperation mechanisms that help to secure a stable collaboration among their members. On the other side, community energy projects need to compete against other projects (Fuentes González et al., 2020). These behavioural dynamics need to be addressed to properly evaluate the merits of community energy projects and their impact on the wider electricity system. This might be done at the power system expansion planning stage, considering all the potential generation and transmission expansion investments, or when it is necessary to have an initial idea about the economic-strategic viability of such projects, before including them in the market explicitly.

Biform games (Brandenburger and Stuart, 2007) provide a useful alternative to modelling projects or initiatives with the dual behaviour exhibited by community energy projects. Biform games belong to the category of hybrid games, which can tackle the cooperative and non-cooperative worlds of game theory at the same time. This methodology therefore allows a more comprehensive analysis of community energy projects and their economic-strategic viability. Biform games take into account the Nash equilibrium (Nash, 1951) and the Core (Gillies, 1959) as solution mechanisms for solving both non-cooperative and cooperative stages within a particular game. Likewise, in this work we also consider a different Core-based model, called the linear production game (Owen, 1975), which will help us to address not only the economic-strategic projects feasibility perspective, but also the amount of new installed capacity might be invested in, given a limited amount of resources.

There is a research gap in terms of characterising, in generation and transmission expansion planning models, the role of community-based energy projects and resulting impacts on electricity markets derived from their implementation, through adequate game-theoretical tools that effectively address cooperation and competition in a simpler, more intuitive fashion.

We address this research gap here, by extending the generation and transmission expansion planning literature, through a novel framework that allows for the inclusion of community energy initiatives in expansion planning models. We include biform games and linear production games in a simple and intuitive multi-level generation and transmission expansion planning model. We illustrate the proposed methodology in a simple three-node case study that represents a stylised version of the Chilean electricity market, as a way of complementing and extending recent studies suggesting the feasibility of community energy projects implementation in Chile (Fuentes González et al., 2019, 2020). Using our framework, we aim at answering the following research questions:

a) Are there differences, especially in terms of model outcomes, between a conventional Stackelberg modelling approach (that does not consider citizen participation in energy production) and our methodology based on biform games?
b) In which cases are other ways of citizen participation in energy production, specifically net billing projects, preferred to community energy projects?
c) What are the impacts of community energy projects on electricity markets, particularly, in terms of social welfare, consumer surplus, nodal wholesale prices, demand and optimal power generation, generation and transmission expansion, and CO_2 (carbon dioxide) emissions?

Hence, this paper makes three contributions to the existing body of knowledge. First, we extend the transmission and generation expansion planning literature to allow for the inclusion of community energy initiatives, in addition to large generation investors which only maximise profits. Second, this work is the first attempt to demonstrate the usefulness of using biform games in power system expansion planning. The state-of-the-art literature on biform games has not addressed applications in this area, but instead focuses on management, telecommunications, etc; the present paper is a first attempt to show the relevance of these methods for power system expansion applications. We use the following metrics to gauge the potential added value of using biform games: social welfare, incumbents' profit, carbon dioxide emissions, and price and demand met at the node where communities are. Third, the proposed methodology allows us to formally characterise the impacts of community energy developments on the wider energy system for our case study, representing the first study to do so quantitatively.

The rest of this paper is organised as follows. Section 2 presents the theoretical background necessary to build up the proposed model. In section 3, we outline the main features and assumptions of our model. The results are presented in section 4. In section 5, a discussion about our model and its results is presented. Finally, section 6 concludes the paper.

2. Theoretical background

2.1. Optimisation and equilibrium problems: an overview

Energy and electricity markets have evolved significantly throughout the years, due to several factors: technological progress, a better knowledge of the economic drivers behind markets, more experience in managing electrical equipment, empowered customers, changes in regulation and legislation, new market participants, etc. As a consequence, modelling the full complexity of energy and electricity markets has become crucial to strengthen them by understanding the interactions among the incumbents and derived consequences. Such incumbents include large generators, retailers, residential customers, regulators, suppliers, system operators, among others.

When understanding complex markets, optimisation models provide an intuitive, flexible, and relatively complete way to deal with issues or situations that deserve attention. Long-term optimisation-based power system planning models can be roughly categorised in three groups: generation expansion models, transmission expansion models, and integrated generation and transmission expansion models (Gacitua et al., 2018). In these models, the aim is to chiefly minimise (maximise) costs (benefits or welfare), subject to a variety of constraints related to meeting demand, generation limits, budget constraints, power balance at each node, power flow constraints, security constraints, among others (Gacitua et al., 2018). These models require a significant amount of data about current and future prices, interest rates, demand structures or loads, emissions, taxes, technical capacity of power units and lines, etc. These parameters are often fundamentally uncertain, which is why many models consider uncertainty and risk (Koltsaklis and Dagoumas, 2018).

More generally, optimisation models can be categorised in different ways. For instance, Dagoumas and Koltsaklis (2019) classify models as optimisation models (simple and multiple objective functions), computable general or partial equilibrium models, and alternative models. Lumbreras and Ramos (2016) classify the optimisation models used in generation expansion problems as classical (linear and non-linear programming problems) and non-classical (meta-heuristic algorithms that iteratively improve a solution). Of course, simple optimisation models within the above-mentioned classifications cannot easily account for strategic interactions among market participants. Gabriel et al. (2012) note that mathematical problems with equilibrium constraints (MPEC), equilibrium problems with equilibrium constraints (EPEC), mixed complementarity problems (MCP), and variational inequality (VI), which draw on game theory to account for strategic interactions, are particularly useful to work on modern energy markets. These models are now increasingly being used by experts. For instance, Hobbs and Nelson (1992) present and analyse a Stackelberg-based bi-level optimisation problem which is used to quantify how customers react to subsidies and some distortions based on decisions and prices. Chao and Peck (1996) present a mechanism for establishing a competitive market for transmission services, mainly based on Coasian and Pigouvian principles and the Kirchhoff's law, and therefore deploying tradable transmission capacity rights. Using a variational inequality approach, Jing-Yuan and Smeers (1999) find the Nash equilibrium for an oligopolistic market with scattered generators and customers considering regulated transmission prices. Hobbs (2001) develops two models *"à la"* Cournot, formulated as mixed linear complementarity problems, highlighting the effects on prices, welfare, profit, among others, considering arbitrage and imperfect competition. Pineau and Murto (2003) address the effects of uncertainty on investments decisions by modelling an optimisation problem for players who might devote money in new thermal capacity, using variational inequality and mixed complementarity problem approaches. Sauma and Oren (2006) formulate MPEC and EPEC problems in order to evaluate the impacts of transmission investments on social welfare and investment locations, taking into account the generators' expansion response to transmission expansion and congestion protocols. Later, the same authors (Sauma and Oren, 2007) extend the concept of proactive transmission planning proposed in (Sauma and Oren, 2006) to show the impacts of different planning objectives on network optimal expansions. Ruiz and Conejo (2009) reveal a procedure to obtain an optimal offering strategy for a strategic power producer based on the development and solution of an MPEC. Pozo et al. (2013) propose a mixed integer linear programming optimisation problem which integrates transmission and generation investment planning, as well as market operation decisions, and characterises pure Nash equilibria related to generation expansion (EPEC) as a set of linear inequalities. Lo Prete and Hobbs (2016) develop a cooperative game theoretic approach that examines the interaction of an incumbent electric utility, an investor in microgrids, and residential customers (who may represent a microgrid load), in the context of microgrid introduction in a regulated electricity network. The authors utilise the Shapley Value and the Core for the analysis, showing that: inefficient market outcomes are obtained in the case of a misalignment among players' objectives (i.e., when comparing market efficiency versus profit/surplus maximisation); microgrid-based social benefits decrease and suboptimal investments are obtained due to potential regulator's incorrect anticipation of microgrid introduction and rate-making process; and there exists an important reaping of benefits from microgrid introduction by residential customers, in the case of lower rates for all customers. Munoz et al. (2017) examine the impacts derived from risk aversion on generation and transmission investments by modelling a proactive risk-averse transmission planner problem (Stackelberg equilibrium problem). More recently, Acuña et al. (2018) propose a Stackelberg game considering generators and intermediaries (marketers), which is implemented through a bi-level optimisation problem, in order to evaluate two cooperation schemes.

We note, however, that none of these models is completely appropriate for the analysis of community energy projects, as they do not seem to consider and capture the dual cooperative and non-cooperative nature of community energy initiatives properly. Biform game theory offers an alternative which, to our best knowledge, has not been applied to generation and transmission expansion planning models that focus on community energy promotion and deployment.

2.2. *State-of-the-art community energy planning optimisation models*

Optimisation problems applied to community energy matters have recently attracted some attention. For example, Huang et al. (2017) survey some of the available tools, identifying a variety of them that would be useful for community energy promotion and deployment. Ashok (2007), based on an optimisation model, finds that micro-hydro-wind systems are the optimal solution to electrify rural villages in India. Cai et al. (2009a,b) develop a method based on different optimisation techniques and show different community-scale renewable energy alternatives for a study system involving three communities. Mendes et al. (2011) highlight and recommend appropriate optimisation planning models and tools for integrated community energy systems, noting that a "Distributed Energy Resources Customer Adoption Model" (DERCAM) might be a good tool for designing such systems, given its three-level optimisation algorithm. Moret and Pinson (2018), based on optimisation models and communication-based fairness indicators, study the interaction between an energy collective and a market and system operator, involving other energy collectives too. Roy and Ni (2018) develop a game theoretical approach for a distributed power system and highlight that residential consumption is relevant to control prices. They find the market clearing price at the Nash equilibrium point and show that residential customers individually reduce costs. Olivella-Rosell et al. (2018) propose a novel optimisation problem for managing flexible energy resources throughout a Smart Energy Service Provider (SESP), considering (residential) prosumers and energy cooperatives. Yazdanie et al. (2016) study decentralised generation and storage benefits for rural places in Switzerland by developing and solving a least-cost optimisation model, noting that small hydro and solar PV improve self-sufficiency, storage improves results, and carbon pricing mitigates pollutant emissions.

The aforementioned studies show the contributions that mathematical programming can provide to community energy matters. Nevertheless, they are often unclear about what community energy really means and what the differences are with other concepts like microgrids or distributed generation projects. They also do not consider the hybrid (cooperative and non-cooperative) nature of community energy initiatives. For this reason, in the next section we provide our definition of community energy and distributed generation projects, which we will use to define our own modelling approach.

2.3. Key definitions

To develop our approach, it is important to first define the concepts of community energy and distributed generation. The fundamentals of the former can be found in (Walker and Devine-Wright, 2008; Walker et al., 2010). There are also other studies that take these fundamentals and add more elements that contribute to an updated definition, which can be seen, for example, in (Seyfang et al., 2013; Wirth, 2014; Haggett and Aitken, 2015; Fuentes González et al., 2019).

We consider the updated definition of community energy proposed by Fuentes González et al. (2019). Hence, a community energy project, in our context, is *"a project conceived, carried out, and implemented by people who are interested in generating energy, located in or close to the project, well-organised under a legal and organisational structure, the owner or have control of the project ownership, the main beneficiary, and interested in welfare maximisation as well as income generation"*. Accordingly, for the purposes of this work, community energy projects are front-of-the-meter and therefore not subject to net billing schemes.

Regarding distributed generation, we use the definition proposed by Ackermann et al. (2001), who define a distributed generation project *"an electric power source connected directly to the distribution network or on the customer site of the meter"*. Within this concept, it is possible to find specific citizen-oriented electricity provision schemes, such as net billing or net metering schemes. In this work, the former is considered. A net billing scheme (or distributed generation scheme), for the purposes of this work, can be simply defined as a scheme where the energy injection is valued at a rate that is different from the consumption rate, and the resulting value is therefore subtracted from the energy consumption expenses. We reflect this situation in this work by establishing a case where residential customers have an opportunity to install mainly residential rooftop solar photovoltaics (PV) panels for self-consumption and selling/injecting any disposable surplus to the grid. This is equivalent to the Chilean example of net billing scheme. In this country, the net billing injection rate is set by the government and considers only energy, whereas the consumption rate considers energy and capacity. The above applies for the main retail rate called *BT1*, which is paid by most of the residential customers in Chile. For other rates, the valuation for injected and consumed energy is the same (Fuentes González et al., 2019).

Having defined these concepts, we will develop a generation and transmission expansion planning model that can include both community energy and distributed generation projects. Since an understanding of biform games is fundamental for the rest of this paper, we first set out their main characteristics in the next section.

2.4. Biform games

Biform games (Brandenburger and Stuart, 2007), part of the wider game theory field [1], integrate and jointly solve cooperative and non-cooperative games, considering the stages explained next. Firstly, the Core for the cooperative (or second) stage of the game is determined. Secondly, a range of payoffs for each player i is calculated. Thirdly, a weighted average payoff is obtained for each player i, by applying confidence indices α_i and (1-α_i) to the largest and smallest potential payoffs that each player could receive. The above allows for evaluation and reduction of the cooperative stage to a non-cooperative stage. With regards to the confidence index, this parameter can be seen as the connection between the cooperative and non-cooperative stages of the game. Moreover, such an index can be understood as describing the expectations that the players have about the payoff or result of the game, in other words, about the fraction of the total coalitional value they could capture. A confidence index close to one reflects a high degree of optimism among players about the game and its outcome, and an index close to zero reflects the opposite. The confidence index can also be seen as players' perceived competition and negotiation opportunities (Brandenburger and Stuart, 2007; Fuentes González et al., 2020). Finally, in the non-cooperative (first) stage of the game, the Nash equilibrium is computed.

More formally, a biform game is defined as follows (Brandenburger and Stuart, 2007; Fuentes González et al., 2020):

$$(Z_i, ..., Z_n; V; \alpha_i, ..., \alpha_n) \tag{1}$$

The model involves the following components: Z_i is a finite strategy set for each $i = 1, ..., n$ player; z_i is player i's selected strategy; V is a map from $Z_i \times ... \times Z_n$ to the set of maps $P(I) \rightarrow \mathbb{R}$, with $V(z_i, ..., z_n)(\varnothing) = 0$ for every $z_i, ..., z_n \in Z_i \times ... \times Z_n$; and, α_i is player i's confidence index between 0 and 1. A transferable utility game, with characteristic function $V(z_i, ..., z_n) : P(I) \rightarrow \mathbb{R}$, where $P(I)$ is the set of all possible subsets of the set of players I, is defined by the set of strategies $z_i, ..., z_n \in Z_i \times ... \times Z_n$. Consequently, for each coalition $S \subseteq I$, $V(z_i, ..., z_n)(S)$ is the value created by coalition S, given players' selected strategies $z_i, ..., z_n$ (Brandenburger and Stuart, 2007; Fuentes González et al., 2020).

The above implies two solution mechanisms considered when solving biform games, namely the Core (for the cooperative or second stage of the biform game) and Nash equilibrium (for the non-cooperative or first stage). Regarding the Core (Gillies, 1959; Pérez et al., 2004; Gilles, 2010; Fuentes González et al., 2020), this solution mechanism can be formally defined as follows:

$$C(v) = \left\{ x \in \mathbb{R}^n \mid \sum_{i \in I} x_i = v(I), \sum_{i \in S} x_i \geq v(S) \ \forall \ S \in P(I) \right\} \tag{2}$$

Where:

- $\sum_{i \in I} x_i$ represents the sum of all payoffs of players that belong to set I (the grand coalition).
- $v(I)$ is the coalitional value of I.
- $\sum_{i \in S} x_i$ denotes all players' payoffs that belong to a certain coalition S.

[1] Game theory aims at determining the best possible result, as well as the corresponding strategy for obtaining it, for a set of rational decision-makers (or players) in a game (strategic interaction). Games can be cooperative (where players communicate and cooperate to find a jointly optimal solution), non-cooperative (where players do not cooperate), and hybrid.

- $v(S)$ represents the coalitional value of S.

To assure a payoff distribution where no player has the incentive to unilaterally abandon a particular coalition, the solution delivered by the Core has to meet the efficiency principle and coalitional rationality (Fuentes González et al., 2020). We note, however, that the Core could be either non-empty (involving a solution) or empty (with no solution). Based on the Bondareva-Shapley theorem (Bondareva, 1963; Shapley, 1965), in order to ensure a non-empty Core, the game has to be balanced. This means that a family $S_1, S_2, ..., S_m$ of subsets of I is balanced over I, if positive weights $w_1, w_2, ..., w_m$ exist, such that $\forall\ i = 1, 2, ..., n$, $\sum_{j:i \in S_j} w_j = 1$ holds. A cooperative game $\Gamma = (I, v)$ is then balanced, if for any balanced family $S_1, S_2, ..., S_m$ over I, with weights $w_1, w_2, ..., w_m$, the inequality $\sum_{j=1}^{m} w_j v(S_1) \leq v(I)$ holds. Such game $\Gamma = (I, v)$ is therefore balanced and has a non-empty Core.

In the Nash equilibrium (Nash, 1951), the strategies $z_i^*, ..., z_n^*$ are Nash equilibria if z_i^* is player i's best response to the other players' strategies $z_1^*, ..., z_{i-1}^*, z_{i+1}^*, ..., z_n^*$ that solve the following optimisation problem:

$$\text{Max}_{z_i \in Z_i} f_i(z_1^*, ..., z_{i-1}^*, z_i^*, z_{i+1}^*, ..., z_n^*) \tag{3}$$

The payoff function f_i is based on the cooperative part of the biform game, which can be solved by the Core.

In terms of specific applications of biform games, it is possible to find several studies with applications to microeconomics, particularly, the newsvendor problem (Stuart Jr, 2005), monopoly and market power (Stuart Jr, 2007), competitive advantages of intermediaries (Ryall and Sorenson, 2007); to management matters, namely manufacturing and supply chain economics (Hennet and Mahjoub, 2010, 2011; Feess and Thun, 2014), investment in relationship support assets (Jia, 2013), innovation networks (Li and Chen, 2015), strategic mental models (Menon, 2018); and even to telecommunications (Kim, 2012). There is no specific existing application of biform games to generation and transmission expansion planning optimisation problems, even without considering community energy projects. We note that Fuentes González et al. (2020) previously studied economic-strategic interactions in the context of community energy emergence in Scotland and Chile, by modelling and analysing simple three-player biform games. However, the scope of that work did not entail exploring the potential impacts on power system expansion. Compared to (Fuentes González et al., 2020), the main novelty of the present work is the investigation of such impacts by considering, utilising appropriate optimisation models and techniques that incorporate biform games (and linear production games), as well as accounting for the participation in the wholesale market, represented by a stylised test network, of community energy projects alongside other incumbents. Hence, given the expanding role of community-led energy projects in electricity markets, there exists a need to develop biform transmission and generation expansion models, as we do here.

2.5. *Linear production games main aspects*

Before formulating the proposed model and its main features, it is important to mention another type of game that is considered in this work: the linear production game. These games can be useful in situations where there are producers that want to produce goods maximising their profit, considering market prices, subject to limited available resources (bundles). Owen (Owen, 1975; Tijs et al., 2001; Pérez et al., 2004) demonstrates that these games are balanced and therefore have a non-empty Core, according to the Bondareva-Shapley theorem (Bondareva, 1963; Shapley, 1965) shown above. These games can be mathematically formulated as follows:

$$v(S) = \text{Max} \quad p_1\, \text{good}_1 + p_2\, \text{good}_2 + ... + p_r\, \text{good}_r \tag{4}$$

Subject to

$$R_{11}\, \text{good}_1 + R_{12}\, \text{good}_2 + ... + R_{1r}\, \text{good}_r \leq \sum_{i \in S} R_{i1}(S) \tag{5}$$

$$R_{21}\, \text{good}_1 + R_{22}\, \text{good}_2 + ... + R_{2r}\, \text{good}_r \leq \sum_{i \in S} R_{i2}(S) \tag{6}$$

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$$R_{q1}\,\text{good}_1 + R_{q2}\,\text{good}_2 + ... + R_{qr}\,\text{good}_r \leq \sum_{i \in S} R_{iq}(S) \tag{7}$$

In the above optimisation problem, $v(S)$ is the coalitional value of coalition $S \in I$; p_r is the market price for good r; R_{qr} is the amount of resource q necessary to linearly produce the good r; and $\sum_{i \in S} R_{iq}(S)$ is the sum of the amounts of resource q contributed by player i to coalition S, which player i belongs to. This type of game then considers how coalition S should produce/launch any good r and maximise the corresponding profit, taking into account the market price p_r for that good, subject to the amount/quantity of resource q that any player i, who belong to S, contribute to the coalition $\sum_{i \in S} R_{iq}(S)$. This can be used to model a cooperative energy project which is financed by its members. One of the most useful properties here is that the solution of this problem is within the Core, so there is no incentive for any player to unilaterally change its production level.

The specific use of the aforementioned elements in our model is explored in the next sections, where the main features, assumptions, and mathematical representations are shown in more detail.

3. Modelling methodology

3.1. Framework and main data

We present the main features of our model in this section. To make this easier to follow, we do this in the context of our case study, although the model formulation can be applied more generally.

In our stylised test network, shown in Figure 1, there are three nodes which have relatively inelastic (but not perfectly inelastic) demand functions and peak loads as shown in Table 1. Moreover, there are two large generators, located at nodes 1 and 2, which represent a municipal solid waste/landfill gas power plant (MSWLG) and a natural gas combined cycle power plant (NGCC), respectively. Their technical and economical features are shown in Table 2. As far as possible, these values have been derived from real-world data. Generation costs are calculated as the average full load variable costs registered for two power plants in Chile (Santa Marta and Nueva Renca) in January 2019 (Coordinador, 2019b), which correspond to MSWLG and NGCC technologies, respectively. Investment costs are taken from (U.S. EIA, 2019a), and represent the overnight cost for each technology. Concerning the CO_2 emissions of each power plant at nodes 1 and 2, these values are calculated considering a typical heat rate for a gas turbine and a combined cycle gas turbine, respectively, multiplied by the corresponding emissions factors taken from (U.S. EIA, 2019b).

Table 1: Demand function features and peak loads at each node

Node	Intercept	Slope	Peak load (MW)
1	110	-0.05	200
2	50	-0.1	35
3	70	-0.04	150

Table 2: Technical and economical features of large generators at nodes 1 and 2

Node	Generation costs ($/MWh)	Generation investment costs ($/MW)	Maximum capacity (MW)	CO_2 emissions (kgCO_2/MWh)
1	15.00	8,895,000	14	465.92744
2	47.11	999,000	330	405.93243

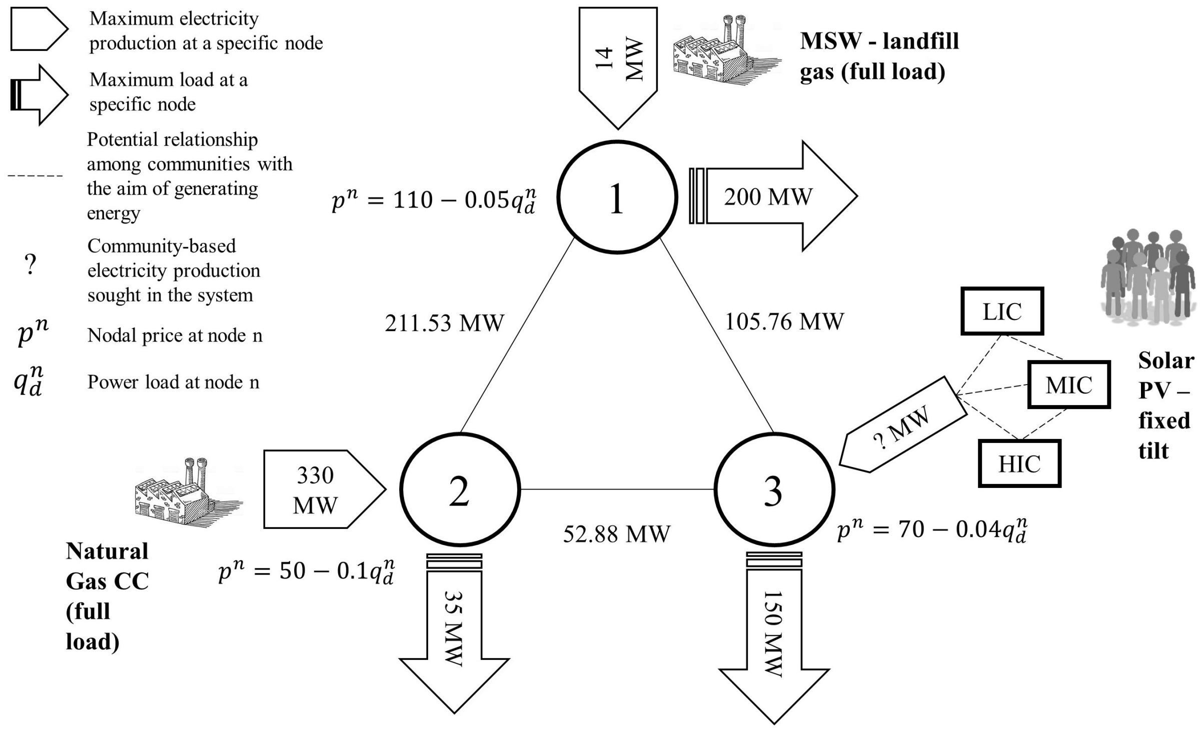

Figure 1: Market representation to be examined

At node 3, there are three groups of residential electricity consumers (or communities) with different annual disposable income and equal willingness to devote money to energy production. The rationale for considering a "willingness to devote money" of 0.1 is the conservative assumption that consumers, regardless of their social-economic class, would only devote a small portion of their disposable income to energy production initiatives, due to other vital expenditures (e.g. food, clothing, water, etc.) and potential financial, credit constraints that consumers could have as individual households because of other financial commitments (e.g. mortgages, student or car loans, etc.). This information is detailed in Table 3. To build up Table 3, we have gathered representative socio-economic information related to income characterisation of inhabitants in Chile, which can be found in (AIM Chile, 2018). Particularly, the customers' annual disposable income values in Table 3 consist of the upper bound for categories YPCE7, YPCE4, and YPCE3 of the equivalent autonomous per capita income intervals (in Spanish, *"Tramos de ingreso autónomo per cápita equivalente"*) multiplied by the number of customers, the model time horizon (12 months), the willingness to devote money factor, and the corresponding CLP/$ exchange rate [2].

Table 3: Communities located at node 3 and their characteristics

Community	Socio-economic classification	Number of customers	Total annual disposable income ($)	Willingness to devote money factor
1	Low Income	500	324,900	0.1
2	Medium Income	300	340,200	0.1
3	High Income	100	352,080	0.1

The resulting communities represent three groups of residential electricity customers who interact within the market: low income customers (LIC), medium income customers (MIC), and high income customers (HIC). These groups

[2]The exchange rate used in this work is CLP/$ 0.00150, which corresponds to the 22nd March 2019, taken from www.OANDA.com

of customers, or communities, may choose to participate in energy production by carrying out either a community energy project or a distributed generation/net billing project. The details about these projects are shown in Table 4.

As shown above, all communities may participate in energy production by using solar PV technologies. The generation cost for the community energy project is very low, as only some minor operations and maintenance costs are required. The investment cost for the projects is obtained from (NREL, 2015), and is the same for all communities. On the other hand, although the generation cost considered for the net billing scheme is also very low, investment costs are different for each community, given that each group of residential customers only has access to a particular type of solar PV equipment with a specific cost, which depends upon the specific capacity and vendor. Lower-income communities have a lower energy consumption level and lower access to capital. The investment costs therefore correspond to solar PV panels with an installed capacity of 1kWp, 3kWp, and 5kWp (ENEL, 2019), for low-income, medium-income and high-income communities, respectively. The final values are converted into USD through the corresponding exchange rate and expressed in per-MW.

Table 4: Details for citizen participation in energy production projects at node 3

	Community energy project		Net billing project	
Community	Generation costs ($/MWh)	Generation investment costs ($/MW)	Generation costs ($/MWh)	Generation investment costs ($/MW)
1	0.01	2,020,000	0.01	3,285,000
2	0.01	2,020,000	0.01	1,795,000
3	0.01	2,020,000	0.01	1,887,000

The details about the existing transmission lines in the network are shown in Table 5. Data for line 1 is based on real-world data about a line in the Chilean national transmission system (Chena 220 – Alto Jahuel 220) (Coordinador, 2019a; CNE, 2019). Data for lines 2 and 3 are scaled from line 1 to represent lines with a lower capacity. Additionally, the details about the candidate new lines portfolio, based on arbitrary modifications of the above table, are shown in Table 6.

Table 5: Existing transmission lines

Line	From	To	Circuits	Length (km)	Voltage (kV)	Reactance (p.u.)	Capacity (MW)	Investment cost ($)
1	1	2	1	34.85	220	0.08	211.53	12,591,500
2	1	3	1	23.24	220	0.11	105.76	6,295,750
3	2	3	1	17.43	220	0.15	52.88	3,147,880

Table 6: Candidate transmission lines portfolio

Line	From	To	Circuits	Length (km)	Voltage (kV)	Reactance (p.u.)	Capacity (MW)	Investment cost ($)
1	1	2	1	34.85	220	0.08	200	16,368,950
2	1	3	1	23.24	220	0.11	100	5,036,600
3	2	3	1	17.43	220	0.15	50	3,620,062

3.2. Characterisation of the incumbents' problems

Considering all the aforementioned elements and data, the relationships among the incumbents' problems are characterised as shown in Figure 2.

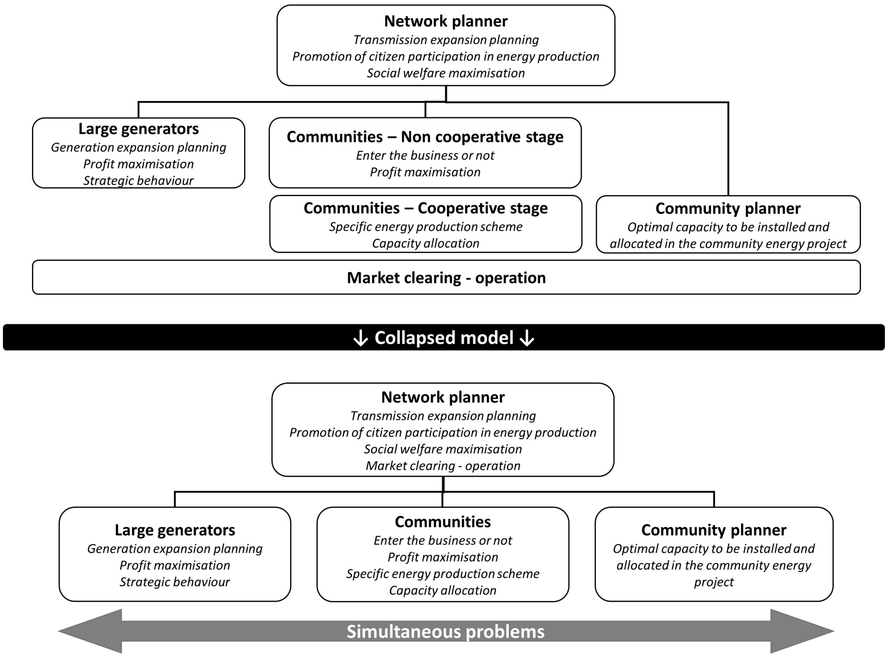

Figure 2: Model overview

As shown in Figure 2, the first stage in our model consists of a proactive network planner, who maximises social welfare. This network planner decides the transmission network expansion, taking into consideration a rational expectation of the expansion of the current two large generators' capacity and the inclusion of new citizen-led energy generation initiatives. Such citizen-led initiatives can be either a community energy project or a net billing scheme, which are in agreement with the definitions shown above in subsection 2.3. The market clearing (operation) occurs in a subsequent (lower) stage, with capacity expansions decided before this, as it can be seen on the top of Figure 2. However, as shown in our collapsed model on the bottom of the same Figure, we embed the market clearing stage into the network planner investment stage for mathematical convenience. This assumption, made in order to keep the model tractable, is possible because there is no uncertainty in our model, which implies the proactive network planner anticipates the generation capacity expansion while perfectly foreseeing the market operation (clearing).

As mentioned beforehand, there are two large generators that maximise their profits. We assume they have perfect information about the demand at each node of the system and they strategically act and exercise market power in a *Nash-Cournot* fashion. Moreover, these generators can expand their current capacities, making rational expectations of the market operation for profit calculation purposes.

There are three communities that consist of three groups of residential electricity customers with different disposable income. We assume that they choose to participate in energy production by forming a grand coalition $S = 1, 2, 3 = I$ and using solar PV technologies within a community energy project or net billing scheme. Although any coalition $S \subset I$ could be formed, customers are assumed to receive a payoff equal to zero if they acted alone or out of the grand coalition. They seek to maximise profit if they enter the business. These decisions can be seen on the top of Figure 2, where communities face the non-cooperative part of their problem (game) related to the potential competition they might have with other initiatives or projects. However, they also need to invest in new installed capacity and allocate it among the customers that are participating in energy production. This implies deciding which initiative (community energy and/or net billing) is profit-maximising as well as stable, in the sense that no one would leave (or change) the project or initiative. It is worth recalling that there is no previously installed capacity of such initiatives (in agreement with Figure 1). These decisions can also be seen on the top of Figure 2, where the cooperative part of the problem (game) related to the decisions about capacity/scheme (payoff) distribution is presented. These two stages, the non-cooperative and the cooperative one, are therefore collapsed and included within a same level (by modelling a biform game), as can be seen on the bottom of Figure 2.

In parallel, another problem arises as residential consumers have limited resources and do not have complete access to financial markets. To address this issue, we include a community planner which sets a rule that communities have to follow (solve) at the same time they evaluate the opportunity to participate in energy production. The rule establishes that communities need to take into account the costs, market prices, as well as their disposable income, in order to decide the optimal way to use their resources in energy generation (via a community energy project and/or net billing scheme). We note, however, that this community planner can be seen as a function, role or organisation either related to (inside) or independent from (outside) communities. We therefore model such a planner explicitly. Embedding the community planner's optimisation problem into the communities' optimisation problem would also be possible.

From a more game-theoretical point of view, we highlight that the community planner cooperates with communities, as it aims at supporting communities to make an optimal decision on capacity investment, based on available resources only. Furthermore, by modelling the above problem as a linear production game, which is proven to have a non-empty Core, we can find an optimal maximum capacity to be invested, which ensures, at the same time, stability in the sense that no one would deviate or change such investment decision. In addition, through our formulation, we can facilitate the calculation of a solution (under the Core) for the communities' problem and the maximum coalitional value to be distributed among the players.

Thus, conceptually speaking, the collapsed model (with just two levels) shown on the bottom of Figure 2 is our proposed bi-level three-node optimisation problem. From a more mathematical viewpoint, the upper-level (network planner) problem is a mixed-integer quadratic optimisation problem, which is solved considering the optimality conditions of the lower-level optimisation problems as constraints, together forming an MPEC. Since the upper-level objective is convex and the lower-level problems are also convex, it is possible to replace the lower level problems by their Karush–Kuhn–Tucker (KKT) conditions and include these optimality conditions as constraints in the upper level. The KKT conditions are necessary and sufficient for lower-level optimality. We linearise the complementarity conditions using the Fortuny-Amat linearisation method (Fortuny-Amat and McCarl, 1981), for which the corresponding equations are shown in the Appendix B.

3.3. Problem formulation: a bi-level three-node optimisation model

In this section, we outline the mathematical formulation of the problems described above. Based on Figure 2, we explore our bi-level three-node optimisation model following a bottom-up view of its collapsed version, showing the mathematical formulation of the problems for large generators, communities, community planner, and network planner, respectively, in the next subsections. However, we first begin by revealing some considerations followed when solving all problems. The nomenclature we use for the aforementioned mathematical formulation is shown in the Appendix A; still, variables and parameters are explained for each optimisation problem in the next subsections.

3.3.1. Units and time frame

As there are some differences in units for some terms in all problems, all variables and parameters are converted to an hourly basis. Particularly, in terms of investment costs we assume that the corresponding incumbent would pay them back via an ordinary annuity considering an interest rate compounded hourly during the life cycle of a typical solar PV panel, which is 25 years in this case (NREL, 2015). We are aware of the dissimilarities in terms of the lifetime for each component of the system considered in this work. Yet, we chose this time frame given that the focus is on the communities and their potential investment in solar energy technologies. The hourly-basis interest rate $r = 0.1/8760$ and the number of payments or capitalisation periods $t = 25 \cdot 8760$ are therefore included in the model as follows [3]:

[3] 8760 represents the number of hours in one year.

$$\left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]^{-1} = \left[\frac{1}{0.1/8760} - \frac{1}{(0.1/8760)(1+(0.1/8760))^{25\cdot 8760}} \right]^{-1} \quad (8)$$

The customers' annual disposable income, denoted by $\sum_{i\in S} R_{inv}(S)$ in the model, then corresponds to the sum of all customers annual disposable income divided by 8760.

3.3.2. Large generators' problem

Large generators chiefly face two problems. First, they need to determine how much capacity to expand based on rational expectations of the market operation. Second, they need to determine, in expectation, the operation of the power units. There are, therefore, two decision variables to be optimised, namely electricity produced (in MWh) by each large generator g located at node n represented by q_g^n, and new capacity installed by a large generator g located at node n (in MW) represented by $g_{g\,new}^n$. We recognise that these two decision variables have different time spans; our modelled power production can be interpreted as an estimation of the power plant operation, in expectation, over time. To determine these two variables, the large generators maximise profit (income minus expenses). They strategically act *à la* Cournot, as they are oligopolists having perfect information about the demand at each node of the system. We reflect this in our model by considering the parameters a_d^n, the demand curve intercept at node n, and β_d^n, the demand curve slope at node n.

In order to reflect generators' expenses, we consider the following parameters: GC_g^n, the variable generation costs for large generator g located at node n (in \$/MWh); GIC_g^n, generation investment costs for large generator g located at node n (in \$/MW); r, interest rate; and t, time. As generators' electricity production cannot exceed their existing capacity, leaving any potential additional (new) installed capacity aside, we also take into account the parameter $q_g^{n\,EX}$, which is the existing capacity of large generator g located at node n (in MW). Thus, the large generators' decision problem can be set out as follows:

$$\text{Max}_{\{q_g^n,\, g_{g\,new}^n\}} \sum_n \sum_g \left\{ \left[\left(a_d^n - \beta_d^n q_d^n \right) q_g^n \right] - GC_g^n q_g^n - GIC_g^n g_{g\,new}^n \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} \right\} \quad (9)$$

Subject to

$$g_{g\,new}^n \geq 0 \Rightarrow -g_{g\,new}^n \leq 0 \quad (\mu_{1g}^{\;n}) \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \quad (10)$$

$$q_g^n \geq 0 \Rightarrow -q_g^n \leq 0 \quad (\mu_{2g}^{\;n}) \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \quad (11)$$

$$q_g^n \leq q_g^{n\,EX} + g_{g\,new}^n \Rightarrow q_g^n - q_g^{n\,EX} - g_{g\,new}^n \leq 0 \quad (\mu_{3g}^{\;n}) \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \quad (12)$$

Constraints (10) and (11) enforce the non-negativity of new installed capacity and energy production, respectively. Constraint (12) implies that the power generation must be less than or equal to the existing installed power capacity plus any additional (new) capacity.

The KKT conditions for this problem are as follows:

$$\frac{\partial L}{\partial q_g^n} \Rightarrow a_d^n - \beta_d^n q_d^n - GC_g^n + \mu_{2g}^{\;n} - \mu_{3g}^{\;n} = 0 \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \quad (13)$$

$$\frac{\partial L}{\partial g_{g\,new}^n} \Rightarrow -GIC_g^n \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} + \mu_{1g}^{\;n} + \mu_{3g}^{\;n} = 0 \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \quad (14)$$

$$0 \leq \mu_{1}{}_{g}^{n} \perp g_{g\,new}^{n} \geq 0 \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \tag{15}$$

$$0 \leq \mu_{2}{}_{g}^{n} \perp q_{g}^{n} \geq 0 \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \tag{16}$$

$$0 \leq \mu_{3}{}_{g}^{n} \perp -q_{g}^{n} + q_{g}^{n\,EX} + g_{g\,new}^{n} \leq 0 \quad \forall\, g \in G, n \in N \wedge \exists g : g \in G \cap N \tag{17}$$

3.3.3. Community energy problem

Two groups of problems are faced by communities in our market representation, namely non-cooperative and cooperative ones. Within the former, communities need to decide whether they enter into the business or not. If they enter, they pursue profit maximisation. Within the latter, communities need to make a decision on the specific energy production scheme they will use, either community energy and/or distributed generation with net billing, as well as on the specific capacity allocation to these schemes (via the Core). For the above purposes, we assume that the best payoff that a community may get if it acts under another coalition $S \neq I \neq \{1, 2, 3\}$ is 0.

There are therefore four decision variables to be determined in this problem, namely electricity produced by a community energy project owned by community i located at node n (in MWh) - q_i^n -, electricity produced by a net billing scheme project owned by community i located at node n (in MWh) - $q_i^{n\,NB}$ -, community i's installed capacity allocation based on a community energy project (in MW) - CEP_i^n -, and community i's installed capacity allocation based on a net billing project (in MW) - NB_i^n -. To optimally determine all these variables, the community energy model is set out as a profit (income minus expenses) maximisation problem. However, we assume that communities, because of their size and nature, do not act as Cournot price-makers but consider the nodal prices p^n (in \$/MWh) and net billing injection price p^{NB} (in \$/MWh) as given instead. The particularity here, however, is that communities face the aforementioned non-cooperative and cooperative problems simultaneously. To link and collapse both groups of problems, and therefore determine the final payoff, we use the parameter α_i which represents the community i's confidence index (continuous parameter $[0, 1]$).

In order to reflect communities' expenses, we consider the following parameters: GCC_i^n, community energy project generation costs for community i located at node n (in \$/MWh); $GICC_i^n$, community energy project generation investment costs for community i located at node n (in \$/MW); $GCC_i^{n\,NB}$, net billing project generation costs for community i located at node n (in \$/MWh); $GICC_i^{n\,NB}$, net billing project generation investment costs for community i located at node n (in \$/MW); r, interest rate; and t, time. Following the criterion considered for the large generators' problem, leaving any potential additional (new) installed capacity aside, communities' electricity production cannot exceed their existing capacity, which is initially zero, non-existent. We then take into account the following parameters: $q_i^{n\,EX}$, existing capacity based on a community energy project for community i located at node n (in MW); and $q_i^{n\,NB\,EX}$, existing capacity based on net billing projects for community i located at node n (in MW). Likewise, as communities' disposable income plays a role when determining the (new) installed capacity to be implemented by communities, the parameters $CEPMAX_{new}^n$, maximum new installed capacity allocation based on a community energy project for communities located at node n (in MW), and $NBMAX_{new}^n$, maximum new installed capacity allocation based on a net billing project for communities located at node n (in MW), are also considered. More details on these two components are given in the following subsection.

Given that each (member of every) community may still obtain and set up a small amount of solar PV-based installed capacity, potentially without collaboration from other neighbours or without contracting a well-established solar PV technology provider and installer (which usually happens to be a distribution company, in Chile), we explicitly include the parameter $g_{i\,exp}^{n\,NB}$ in our model formulation. This parameter therefore represents the community i's expected individual installed capacity, based on a net billing scheme, if customers acted by themselves without any collaboration from other neighbours or without contracting a solar PV technology provider and installer (in MW)[4]. Thus, the community energy problem is formulated as follows:

$$\text{Max}_{\{q_i^n,\, q_i^{n\,NB},\, CEP_i^n,\, NB_i^n\}} \sum_n \sum_i \left\{ \alpha_i \left[p^n q_i^n - GCC_i^n q_i^n - GICC_i^n CEP_i^n \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} \right] + \left(1-\alpha_i\right) \left[p^{NB} q_i^{n\,NB} - GCC_i^{n\,NB} q_i^{n\,NB} - GICC_i^{n\,NB} NB_i^n \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} \right] \right\} \tag{18}$$

Subject to

$$q_i^n \geq 0 \Rightarrow -q_i^n \leq 0 \quad (\mu_{1i}^{cn}) \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{19}$$

$$q_i^{n\,NB} \geq 0 \Rightarrow -q_i^{n\,NB} \leq 0 \quad (\mu_{2i}^{cn}) \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{20}$$

$$q_i^n \leq q_i^{n\,EX} + CEP_i^n \Rightarrow q_i^n - q_i^{n\,EX} - CEP_i^n \leq 0 \quad (\mu_{3i}^{cn}) \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{21}$$

$$q_i^{n\,NB} \leq q_i^{n\,NB\,EX} + NB_i^n \Rightarrow q_i^{n\,NB} - q_i^{n\,NB\,EX} - NB_i^n \leq 0 \quad (\mu_{4i}^{cn}) \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{22}$$

$$\sum_n \sum_i CEP_i^n = CEPMAX_{new}^n \Rightarrow \sum_n \sum_i CEP_i^n - CEPMAX_{new}^n = 0 \quad (\lambda_{2i}^{cn}) \tag{23}$$

$$\sum_n \sum_i NB_i^n = NBMAX_{new}^n \Rightarrow \sum_n \sum_i NB_i^n - NBMAX_{new}^n = 0 \quad (\lambda_{3i}^{cn}) \tag{24}$$

$$CEP_i^n + CEP_j^n \geq 0 \Rightarrow -CEP_i^n - CEP_j^n \leq 0 \quad (\mu_{5i}^{cn}) \quad \forall\, i \neq j \in I, n \in N \wedge \exists i, j : i, j \in I \cap N \tag{25}$$

$$NB_i^n + NB_j^n \geq g_{i\,exp}^{n\,NB} + g_{j\,exp}^{n\,NB} \Rightarrow -NB_i^n - NB_j^n + g_{i\,exp}^{n\,NB} + g_{j\,exp}^{n\,NB} \leq 0 \quad (\mu_{6i}^{cn}) \quad \forall\, i \neq j \in I, n \in N \wedge \exists i, j : i, j \in I \cap N \tag{26}$$

$$CEP_i^n \geq 0 \Rightarrow -CEP_i^n \leq 0 \quad (\mu_{7i}^{cn}) \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{27}$$

$$NB_i^n \geq g_{i\,exp}^{n\,NB} \Rightarrow -NB_i^n + g_{i\,exp}^{n\,NB} \leq 0 \quad (\mu_{8i}^{cn}) \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{28}$$

Constraints (19) and (20) enforce the non-negativity of energy production variables for both the community energy and net billing schemes. Constraints (21) and (22) imply that the energy production, based on either community energy or net billing, must be less than or equal to the existing capacity plus any additional (new) capacity. The following constraints determine the Core: Constraints (23) and (24) enforce the efficiency principle; constraints (25) and (26) enforce the coalitional rationality; finally, constraints (27) and (28) force the non-negativity of variables sought to be within the Core.

The KKT conditions for this problem are as follows:

$$\frac{\partial L}{\partial q_i^n} \Rightarrow \alpha_i p^n - \alpha_i GCC_i^n + \mu_{1i}^{cn} - \mu_{3i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{29}$$

[4]Yet, according to our assumptions, this parameter is effectively equal to zero.

$$\frac{\partial L}{\partial q_i^{n\,NB}} \Rightarrow (1-\alpha_i)p^{NB} - (1-\alpha_i)GCC_i^{n\,NB} + \mu_{2i}^{cn} - \mu_{4i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{30}$$

$$\frac{\partial L}{\partial CEP_i^n} \Rightarrow -\alpha_i GICC_i^n \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} - \lambda_{2i}^{cn} \sum_n \sum_i \frac{\partial L}{\partial CEP_i^n} + \mu_{3i}^{cn} + \mu_{5i}^{cn} + \mu_{7i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{31}$$

$$\begin{aligned} \frac{\partial L}{\partial NB_i^n} \Rightarrow &-(1-\alpha_i)GICC_i^{n\,NB} \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} - \lambda_{3i}^{cn} \sum_n \sum_i \frac{\partial L}{\partial NB_i^n} + \\ &\mu_{4i}^{cn} + \mu_{6i}^{cn} + \mu_{8i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \end{aligned} \tag{32}$$

$$\frac{\partial L}{\partial \lambda_{2i}^{cn}} \Rightarrow -\sum_n \sum_i CEP_i^n + CEPMAX_{new}^n = 0 \tag{33}$$

$$\frac{\partial L}{\partial \lambda_{3i}^{cn}} \Rightarrow -\sum_n \sum_i NB_i^n + NBMAX_{new}^n = 0 \tag{34}$$

$$0 \leq \mu_{1i}^{cn} \perp q_i^n \geq 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{35}$$

$$0 \leq \mu_{2i}^{cn} \perp q_i^{n\,NB} \geq 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{36}$$

$$0 \leq \mu_{3i}^{cn} \perp -q_i^n + q_i^{n\,EX} + CEP_i^n \geq 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{37}$$

$$0 \leq \mu_{4i}^{cn} \perp -q_i^{n\,NB} + q_i^{n\,NB\,EX} + NB_i^n \geq 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{38}$$

$$0 \leq \mu_{5i}^{cn} \perp +CEP_i^n + CEP_j^n \geq 0 \quad \forall\, i \neq j \in I, n \in N \wedge \exists i, j : i, j \in I \cap N \tag{39}$$

$$0 \leq \mu_{6i}^{cn} \perp NB_i^n + NB_j^n - g_{i\,exp}^{n\,NB} - g_{j\,exp}^{n\,NB} \geq 0 \quad \forall\, i \neq j \in I, n \in N \wedge \exists i, j : i, j \in I \cap N \tag{40}$$

$$0 \leq \mu_{7i}^{cn} \perp CEP_i^n \geq 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{41}$$

$$0 \leq \mu_{8i}^{cn} \perp NB_i^n - g_{i\,exp}^{n\,NB} \geq 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{42}$$

3.3.4. Community energy planner problem

Communities can invest in community energy, net billing or both types of energy production initiatives. However, regardless of what energy generation scheme they choose, any decision is subject to their financial resources. As mentioned in section 3.2, we formulate this situation as a maximisation problem (or as a linear production game, strictly speaking), where an internal or external community energy planner seeks to maximise the following variables: $CEPMAX_{new}^n$, which is the maximum installed capacity allocation based on a community energy project for communities located at node n (in MW); and $NBMAX_{new}^n$, the maximum installed capacity allocation based on a net billing project for communities located at node n (in MW). In other words, communities want to maximise what they can get in terms of community energy and/or net billing-based installed capacity. Following the optimal allocation rule set out by the community planner, we notice that such installed capacity maximisation is affected by the following elements reflected in the parameters defined for this model: R^n_{inv}, which represents the mean of community energy project investment costs ($GICC^n_i$) or necessary resources to conceive 1 MW of a community energy project (in \$/MW); R^{NB}_{inv}, the mean of net billing project investment costs ($GICC^{n\,NB}_i$) or necessary resources to conceive 1 MW of a net billing project (in \$/MW) [5] [6]; p^n, which is the nodal price at node n (in \$/MWh); p^{NB}, the net billing injection price (in \$/MWh); $\sum_{i\in S} R_{inv}(S)$, which represents coalition S's resources available (disposable income) for covering installed capacity investment costs (in \$); r, interest rate; and finally, t, which is time.

As both community energy and net billing schemes are affected by communities' confidence indices, and all communities face the above problem at a coalitional level, we include the term $\alpha_{com} \in [0, 1]$. This parameter represents communities' average confidence index. It consists of the average of all communities' confidence indices. This is a basic way to combine each community's confidence index into just one global index. This facilitates the computation and solution of the community energy planner model. We encourage further research in terms of finding the most appropriate way to combine different confidence indices, which is beyond the scope of this work.

The community energy planner problem is therefore set out as follows[7]:

$$\text{Max}_{\{CEPMAX^n_{new},\, NBMAX^n_{new}\}}\ \alpha_{com} p^n CEPMAX^n_{new} + (1-\alpha_{com}) p^{NB} NBMAX^n_{new} \tag{43}$$

Subject to

$$\begin{aligned}&\left[\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}\left(\alpha_{com}R^n_{inv}CEPMAX^n_{new}+(1-\alpha_{com})R^{NB}_{inv}NBMAX^n_{new}\right)\right]-\\&\sum_{i\in S}R_{inv}(S)\le 0 \quad (\mu_1^{ccn}) \quad \forall\, n\in N\end{aligned} \tag{44}$$

$$NBMAX^n_{new}\ge 0 \Rightarrow -NBMAX^n_{new}\le 0 \quad (\mu_3^{ccn}) \quad \forall\, n\in N \tag{45}$$

$$CEPMAX^n_{new}\ge 0 \Rightarrow -CEPMAX^n_{new}\le 0 \quad (\mu_4^{ccn}) \quad \forall\, n\in N \tag{46}$$

Constraint (44) prevents the overuse of coalitional resources when investing in community energy projects and/or net billing schemes. Constraints (45) and (46) force the non-negativity of variables.

The KKT conditions for this problem are as follows:

$$\frac{\partial L}{\partial NBMAX^n_{new}} \Rightarrow (1-\alpha_{com})p^{NB}-(1-\alpha_{com})\mu_1^{ccn}R^{NB}_{inv}\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}+\mu_3^{ccn}=0 \quad \forall\, n\in N \tag{47}$$

$$\frac{\partial L}{\partial CEPMAX^n_{new}} \Rightarrow \alpha_{com}p^n-\alpha_{com}\mu_1^{ccn}R^n_{inv}\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}+\mu_4^{ccn}=0 \quad \forall\, n\in N \tag{48}$$

$$\begin{aligned}&0\le\mu_1^{ccn}\perp\left[\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}\left(-\alpha_{com}R^n_{inv}CEPMAX^n_{new}-(1-\alpha_{com})R^{NB}_{inv}NBMAX^n_{new}\right)\right]+\\&\sum_{i\in S}R_{inv}(S)\ge 0 \quad \forall\, n\in N\end{aligned} \tag{49}$$

[5] As can be noted, the first two parameters rely on the mean of other parameters set out above. This criterion was arbitrarily followed based on the information available.

[6] The expression *"... necessary resources to conceive 1 MW of..."* is aligned with the canonical formulation of linear production games.

[7] The equations related to this problem are only valid for those nodes where communities are located. In this case, these equations are referred to node 3, as the variables $CEPMAX^n_{new}$ and $NBMAX^n_{new}$ directly affect the community energy problem set out in the previous subsection.

$$0 \leq \mu_3^{ccn} \perp NBMAX_{new}^n \geq 0 \quad \forall\, n \in N \tag{50}$$

$$0 \leq \mu_4^{ccn} \perp CEPMAX_{new}^n \geq 0 \quad \forall\, n \in N \tag{51}$$

Apart from the above problems and KKT conditions, in order to include, to some extent, the concept of fairness at the moment of allocating the capacity among communities, we additionally consider a payoff allocation that is set out by the community planner based on how much each community could spend on new installed capacity for energy production, according to their available resources and the total resources that the coalition could have in total. Consequently, the community planner has the following two additional constraints:

$$CEP_i^n = \frac{R_{inv}(\{i\})}{\sum_{i \in S} R_{inv}(S)} CEPMAX_{new}^n \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{52}$$

$$NB_i^n = \frac{R_{inv}(\{i\})}{\sum_{i \in S} R_{inv}(S)} NBMAX_{new}^n \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{53}$$

The above constraints are reflected separately from the main problem because the Core does not necessarily address fairness, rather stability in the sense that no one would have the incentive to abandon a coalition; this might also involve fairness, which can be key in real-world coalitions. Fairness could take various forms and involve many challenges when dealing with the distribution of benefits or payoffs, so it deserves a dedicated study in the context of community energy projects, which is beyond the scope of this work.

3.3.5. Alternative representation for problems of communities and community planner

From equations (18) to (51), which represent the communities and community planner (maximum capacity to be invested) problems, it can be noticed that the parameter α is applied to both community energy projects and net billing scheme projects. Under this formulation, we assume that the best payoff could come from a community energy project and the worst one could come from a net billing scheme project. Based on our market representation, the rationale for the above is that a community energy project would imply a higher installed capacity and the corresponding payoff would then be higher.

In addition, we apply the parameter α to income and expenses, in order to reflect how confident the players are about the influence that income and/or expenses have on the final payoff, and then on a specific strategy. This alternative approach implies changes in some objective functions, constraints, and then KKT conditions. The alternative KKT conditions considered in this approach are shown as follows:

Alternative version of constraints (29) to (32):

$$\alpha_i p^n - (1-\alpha_i) GCC_i^n + \mu_{1i}^{cn} - \mu_{3i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{54}$$

$$\alpha_i p^{NB} - (1-\alpha_i) GCC_i^{n\,NB} + \mu_{2i}^{cn} - \mu_{4i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{55}$$

$$-(1-\alpha_i) GICC_i^n \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} - \lambda_{2i}^{cn} \sum_n \sum_i \frac{\partial L}{\partial CEP_i^n} + \mu_{3i}^{cn} + \mu_{5i}^{cn} + \mu_{7i}^{cn} = 0 \quad \forall\, i \in I, n \in N \wedge \exists i : i \in I \cap N \tag{56}$$

$$-(1-\alpha_i)GICC_i^{n\,NB}\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}-\lambda_{3i}^{cn}\sum_n\sum_i\frac{\partial L}{\partial NB_i^n}+\mu_{4i}^{cn}+\mu_{6i}^{cn}+\mu_{8i}^{cn}=0\quad\forall\, i\in I, n\in N\wedge\exists i: i\in I\cap N \quad (57)$$

Alternative version of constraints (47) to (49):

$$\alpha_{com}p^{NB}-(1-\alpha_{com})\mu_1^{ccn}R_{inv}^{NB}\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}+\mu_3^{ccn}=0\quad\forall\, n\in N \quad (58)$$

$$\alpha_{com}p^{n}-(1-\alpha_{com})\mu_1^{ccn}R_{inv}^{n}\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}+\mu_4^{ccn}=0\quad\forall\, n\in N \quad (59)$$

$$0\leq\mu_1^{ccn}\perp\left[\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}\left(-(1-\alpha_{com})R_{inv}^{n}CEPMAX_{new}^{n}-(1-\alpha_{com})R_{inv}^{NB}NBMAX_{new}^{n}\right)\right]+\sum_{i\in S}R_{inv}(S)\geq 0\quad\forall\, n\in N \quad (60)$$

These alternative KKT conditions are obtained by multiplying the parameter α by the parameters and variables associated with income and by multiplying the parameter $(1-\alpha)$ by the parameters and variables associated with expenses. The Lagrangian function is then determined and solved accordingly. To exemplify these changes, the corresponding objective functions are shown below.

Equation (18) becomes:

$$\text{Max}_{\{q_i^n,\, q_i^{n\,NB},\, CEP_i^n,\, NB_i^n\}}\sum_n\sum_i\left\{\alpha_i\left[p^nq_i^n+p^{NB}q_i^{n\,NB}\right]+(1-\alpha_i)\left[-GCC_i^nq_i^n-GICC_i^nCEP_i^n\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}\right.\right.$$
$$\left.\left.-GCC_i^{n\,NB}q_i^{n\,NB}-GICC_i^{n\,NB}NB_i^n\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)^{-1}\right]\right\}$$

Equation (43) becomes:

$$\text{Max}_{\{CEPMAX_{new}^n,\, NBMAX_{new}^n\}}\ \alpha_{com}p^nCEPMAX_{new}^n+\alpha_{com}p^{NB}NBMAX_{new}^n$$

3.3.6. Network planner problem

As shown in Figure 2, in our market representation, the first mover is a network planner who pursues social welfare maximisation while perfectly anticipating generation capacity expansion and market clearing. This implies that the variables q_d^n, power load at node n (in MWh), fl^n, power flow from/to node n (in MWh), and l_{new}, new transmission line expansion (binary variable $\{0,1\}$) are all determined to maximise social welfare. Naturally, these variables are affected not only by costs, but also by physical flow limitations. To reflect the corresponding constraints, we define the parameters TIC_l, which represent the investment costs of new transmission line l (in \$/line); $PTDF_{l,n}$, the swing factors on line l related to power injection/withdrawal at node n; th_l, which is the thermal capacity of existing line l (in

MW); $th_{l\,new}$, the thermal capacity of new line l (in MW); r, interest rate; and t, time. Again, as the power peak load cannot be limitless, we also include the parameter $q_d^{n\,MAX}$, maximum load at node n (in MW). The network planner model is therefore formulated as follows:

$$\text{Max}_{\{q_d^n,\, fl^n,\, l_{new}\}} \sum_n \sum_g \sum_i \Bigg\{ \int_0^{q_g^n+q_i^n+q_i^{n\,NB}+fl^n} \left(a_d^n - \beta_d^n(q_d^n) \right) dq_d^n - \left[GC_g^n q_g^n + \left(GCC_i^n q_i^n + GCC_i^{n\,NB} q_i^{n\,NB} \right) \right] -$$

$$\left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]^{-1} \left[GIC_g^n g_{exp}^n + \left(GICC_i^n CEP_i^n + GICC_i^{n\,NB} NB_i^n \right) + TIC_l l_{new} \right] \Bigg\} \tag{61}$$

Subject to

$$q_d^n \leq q_d^{n\,MAX} \quad \forall\, n \in N \tag{62}$$

$$p^n = a_d^n - \beta_d^n q_d^n \quad \forall\, n \in N \tag{63}$$

$$q_d^n - \sum_g q_g^n - \sum_i q_i^n - \sum_i q_i^{n\,NB} - fl^n = 0 \quad \forall\, n \in N \tag{64}$$

$$\sum_n fl^n = 0 \tag{65}$$

$$-\sum_n PTDF_{l,n} fl^n \leq th_l + th_l^{new} l_{new} \quad \forall\, l \in L \tag{66}$$

$$\sum_n PTDF_{l,n} fl^n \leq th_l + th_l^{new} l_{new} \quad \forall\, l \in L \tag{67}$$

$$\textit{Optimality conditions } (13) \textit{ to } (17) \tag{68}$$

$$\textit{Optimality conditions } (29) \textit{ to } (42) \tag{69}$$

$$\textit{Optimality conditions and fairness constraints } (47) \textit{ to } (53) \tag{70}$$

Constraint (62) limits the peak demand to the maximum load at each node of the system, so demand and price are always positive. Constraint (63) sets the nodal price as a function of the demand at each node. Constraints (64) and (65) ensure power balance within the network. Constraints (66) and (67) enforce Kirchhoff's law; we use a DC approximation of AC power flows. Constraints (68) to (70) are the KKT conditions (and constraints associated with fairness) determined in the above optimisation problems [8].

[8]When applying the parameter α (confidence index) to income and expenses, KKT conditions (54) to (60) are considered instead.

4. Results

4.1. Experimental design

We designed and carried out an experiment to solve our bi-level three-node optimisation problem considering various cases of interest. To do so, we used Julia© 1.5.0 and Gurobi© 9.1.1. In terms of hardware, we used a PC with Intel®Core™i5-7200U CPU @ 2.50 GHz and 8.00 GB of RAM. Likewise, to provide insights into the performance when carrying out this experiment, the average execution time per simulation and the total execution time for a complete run of simulations are listed in each figure's caption. The results for this experiment are shown in section 4.2. However, as a way of introducing this content, we detail the equations considered for analysis, as well as the structure of the results and its rationale, in the paragraphs below.

Table 7: Equations considered for experimentation

Application of α_i	Equations	Applicable to
Schemes	(61) to (70)	Figures 3 to 10, save Figure 4 which does not consider optimisation problems for communities and community planner
Income and expenses	For network planner and large generators: (61) to (68); for communities: (54) to (57) and (33) to (42); for community planner and fairness: (58) to (60) and (50) to (53)	Figures 3 to 10, save Figure 4 which does not consider optimisation problems for communities and community planner

Table 7 summarises the equations used for experimentation, which are detailed in the previous section. Both alpha applied to schemes and alpha applied to income and expenses are applicable to all figures, except Figure 4. Each one of these figures consists of the graphical representation of the aforementioned experiment and its (simulation) results, which aim at answering the research questions stated in the Introduction section; this is detailed below.

Figure 3 shows the results of the base case for both approaches (confidence index applied to schemes and income/expenses). In this base case, customers or communities are neither optimistic nor pessimistic about the game and its payoff. This implies that a confidence index of 0.5 is assumed for each community ($\alpha_1 = \alpha_2 = \alpha_3 = 0.5$). A net billing injection price of $p^{NB} = 95$ \$/MWh is also assumed for this base case. Figure 4 aims at contrasting the above base case with a conventional Stackelberg formulation that does not account for communities involved in energy production at all. Thus, through Figure 3 and Figure 4, and their results, we chiefly address research question a), which is related to the differences between a conventional Stackelberg modelling approach and our methodology.

Figures 5 and 6 mainly shed light on the cases where community energy is not the selected energy production scheme - research question b) -; yet these figures also provide insights into some key market indicators - research question c) -, such as nodal price, social welfare, demand, etc. In particular, Figure 5 details at what net billing injection prices communities would prefer to implement a net billing project, *ceteris paribus*. This simulation also provides a useful contrast to the above cases, where the net billing injection price is fixed. Futhermore, Figure 6 reveals the simulation results for different levels of net billing capacity costs (R^{NB}_{inv}). This figure aims at providing insights into the potential attractiveness (in terms of costs) that net billing should exhibit to be chosen as the preferred energy production scheme. Two levels for p^{NB}, namely 95 \$/MWh and 50 \$/MWh, are considered for analysis. Yet, based on the definitions set out in section 2.3, it may be unrealistic to think that each customer (or household) could have more than 5kWp of installed capacity derived from rooftop solar PV panels. Hence, we set a limit of 4.5 MW of installed net billing capacity for the grand coalition. This means that each customer (or household) cannot have more than 5kWp of installed capacity derived from rooftop solar PV panels. This limit is only applied when p^{NB} = 50 \$/MWh. The above represents a way of recognising that such a limitation in installed capacity, (potentially) due to physical and/or economic restrictions, may make net billing injection prices more competitive.

Figures 7 and 8 show the simulation results when low income residential customers have differing levels of confidence (α_1) and the rest of communities are either very optimistic ($\alpha_i = 1.0$) or very pessimistic ($\alpha_i = 0.1$). The net billing injection price considered is the same one used in the base case ($p^{NB} = 95$ \$/MWh). Figure 9 details the simulation outcome when the low income residential customers' confidence index (α_1) changes, while keeping the other communities' confidence indices at a neither optimistic nor pessimistic level; a simulation taking into account a $p^{NB} = 50$ \$/MWh is also revealed. These figures aim at showing not only potential changes in terms of the preferred energy production scheme - research question b) -, but also possible impacts on various market indicators - research question c) - resulting from having different confidence indices.

As the communities' (grand coalition) disposable income ($\sum_{i \in S} R_{inv}(S)$) may also significantly influence the specific project or scheme selected for energy production - research question b) -, as well as some key market indicators - research question c) -, Figure 10 shows the simulation results for different levels of $\sum_{i \in S} R_{inv}(S)$ and derived impacts on key market indicators. The communities' confidence is assumed to be *"neutral"* ($\alpha_1 = \alpha_2 = \alpha_3 = 0.5$) and two net billing injection prices are considered for analysis, particularly $p^{NB} = 95$ \$/MWh and $p^{NB} = 50$ \$/MWh.

We note that all simulations are interrelated, so they can effectively deliver insights into most research questions, other than Figure 4, which is entirely related to research question a). We considered the above when discussing the results in section 5. Again, the simulation results in regards to the aforementioned experiment are revealed in section 4.2. The details of the parameters altered in each simulation are shown in each figure's caption.

4.2. Numerical outcome

As can be seen on Figure 3, there is an opportunity for community energy implementation, even though a high net billing injection price ($p^{NB} = 95$ \$/MWh) is assumed to be available in the market and the investment costs for net billing projects are also comparatively higher. Furthermore, communities get a profit per hour when they enter the business, which is allocated according to the capacity allocation rule stated in section 3.3.4. Such rule enforces a *"fair"* capacity allocation among all groups of residential customers, as a way of preventing unfairness (in real-world coalitions) that may lead to an exit from the grand coalition. Again, the Core assures stability rather than fairness. Moreover, we note that the role of the community energy project is not enough to significantly deteriorate generator 2's importance and market power. The generator located at node 2 influences the market and electricity prices at each node, which can be seen, for example, in the new line that is built for delivering energy from node 2 to node 3, as well as in the flows across the network and wholesale prices. In addition, the highest profit goes to generator 2 and the overall CO_2 emissions are not significantly reduced.

Figure 4 offers a contrast to the above results through a conventional Stackelberg formulation that considers no involvement of communities in energy production. A decrease in the social welfare can be seen when communities are not involved in energy production, particularly by conceiving a community energy project (14,595.85 \$/h versus 14,221.89 \$/h). Furthermore, communities do not receive the profit that could be earned if they entered the business; generator 2 captures part of the communities' losses via an additional payoff of 39.11 \$/h. Likewise, the price at node 3 increases from $p^n = 65.40$ \$/MWh to $p^n = 65.77$ \$/MWh and the demand at that location drops from 115.03 MWh to 105.79 MWh. No effect on CO_2 emissions is observed when comparing Figures 3 and 4.

Figure 5 corroborates that a higher (net billing injection) price reduces social welfare. This particularly happens when the grand coalition (of residential customers) changes from a community energy project to a net billing scheme, when $p^{NB} = 98.2$ \$/MWh is reached, *ceteris paribus*. Unsurprisingly, the communities' profit increases due to a higher injection price. The above implies an increase of the spot price at node 3, as the production and demand decrease; no effects on CO_2 emissions are observed.

Figure 6 details the simulation results when we decrease the net billing installed capacity costs, still assuming that all customers are neither optimistic nor pessimistic ($\alpha_i = 0.5$). Again, two net billing injection prices are taken into account for analysis, namely $p^{NB} = 95$ \$/MWh and $p^{NB} = 50$ \$/MWh. The above-mentioned limitation in terms of the effective installed capacity that net billing projects might show in reality (4.5 MW) is considered when the latter p^{NB}

is used for analysis. The effects on demand at each node, price at node 3, generation at each node, and CO_2 emissions are then shown in this figure. Compared to a situation where no capacity restrictions are considered (Charts 1 and 2), it can be noticed that a capacity limitation (Charts 3 and 4) unsurprisingly affects the net billing scheme deployment, as well as the price at node 3 and demand requirements. CO_2 emissions are diminished as more expensive generation from generator 2 is replaced by more affordable generation and because of decreasing demand and production. Yet, there are still incentives to deploy both schemes at the same time (Chart 4), particularly from the point where the (net billing) investment costs reach 22.88 \$/h; by contrast, when there are no capacity constraints, a switch from community energy to net billing occurs when the investment costs reach 27.88 \$/h (Chart 2). Although having lower costs available helps increase competition, in the sense that more energy production schemes are operating in the market, there exists a clear barrier for delivering lower prices to the coalition members and outsiders, when a capacity restriction exists.

Figures 7 and 8 show the effects on social welfare, consumer surplus, power generation, and CO_2 emissions derived from extreme magnitudes of confidence for communities 2 and 3 ($\alpha_2 = \alpha_3 = 0.1$ and $\alpha_2 = \alpha_3 = 1$), while simulating different levels of α_1, namely the low income customers' confidence index. In addition, a net billing injection price of $p^{NB} = 95$ \$/MWh is considered. We note that the application of confidence indices to schemes and income and expenses give different results. As can be seen in Figure 7 (Chart 1), when the confidence index is applied to schemes, the higher the low income customers' confidence, the more stable the social welfare gets (circa 15,000.00 \$/h) and the lower the consumer surplus at node 3 is (around 250.00 \$/h), if communities 2 and 3 are pessimistic ($\alpha_2 = \alpha_3 = 0.1$). When the confidence index is applied to income and expenses (Chart 2), the consumer surplus at node 3 remains stable at circa \$ 200/h, again, if $\alpha_2 = \alpha_3 = 0.1$.

When examining Figure 7, Charts 3 and 4 this time, we notice that the more confident all customers are, the higher the social welfare is, save when α_1 is very close to 1. At these points, generator 2 stops power generation; generator 1 and communities (via a net billing scheme) then start supplying the demand at all nodes. The consumer surplus rises almost one third from initial values, regardless of how the confidence index is applied or used. The above can be explained as follows. As shown in Figure 8, there is no minimum level of confidence needed to carry out a community energy project (Charts 3 and 4). The difference then lies in the magnitudes. If the confidence index is applied to each scheme when we have $\alpha_2 = \alpha_3 = 0.1$ (Chart 1), then the communities' average confidence α_{com} is low and so are the costs associated with community energy projects. This implies having a higher installed capacity of community energy in the market. However, this stabilises when α_1 converges to 1. When the confidence index is applied to income and expenses (Chart 2), the results reveal that it is profitable to conceive a community energy project even though the confidence and therefore the final installed capacity are low (save for the points when $\alpha_1 = 0$ and $\alpha_1 \geq 0.94$, where net billing is preferred by communities). The above implies that there are limited options to counter the market power exerted by the large generators, especially generator 2, and therefore no significant effects on prices and emissions are observed. On the other hand, when the confidence is much higher and the net billing injection price is more attractive for generators, the grand coalition invests in such projects (Charts 3 and 4). Of course, because there will be more local generation at node 3 and then a lower demand for more expensive electricity from other nodes, the price at node 3 decreases, demand and production within the system (including node 3) increase, and CO_2 emissions from generator 2 exponentially drop. However, based on our definitions, having a significant installed capacity based on rooftop residential solar PV panels (net billing scheme) might be impractical in reality (this is addressed in Figure 6).

Figure 9 chiefly shows some differences in the trajectory of the social welfare curve, based on how the confidence indices are used (α_i applied to schemes or to income/expenses) [9]. The social welfare curve shown in Chart 4 does not have a kinked shape like the other curves in the figure, but instead increases for all values of α_1. In this case, there is no change from community energy to net billing, save for the point $\alpha_1 = 0$ where net billing is the preferred energy production scheme. The investment in community energy then steadily increases as long as α_1 rises. The main factor for this is the lower net billing injection price ($p^{NB} = 50$ \$/MWh), so there is no incentive for investing in

[9] Changes from one energy production scheme to another are highlighted in every chart in this figure for informative purposes.

net billing schemes that offer lower profits. Chart 3, however, shows a switch from community energy to net billing (when $\alpha_1 \geq 0.81$) because the confidence index is applied to schemes instead. Although the net billing injection price is lower, a change in the confidence index more than compensates the gap and then a switch happens. Nevertheless, when the confidence index is applied to income and expenses considering a lower net billing injection price (Chart 4), the social welfare is slightly higher than in the rest of the cases (14,781.55 \$/h versus 14,666.71 \$/h), when $\alpha_1 = 1$ and $\alpha_2 = \alpha_3 = 0.5$. At this point, the low income residential customers are totally confident whereas the rest of the customers are neither pessimistic nor optimistic. Moreover, when $\alpha_1 = 0$ and the confidence indices are applied to schemes (Charts 1 and 3), the social welfare reaches its maximum. Only one switch from community energy to net billing is revealed in Chart 1, specifically when $\alpha_1 \geq 0.68$. Conversely, if the confidence indices are applied to income and expenses (Charts 2 and 4), then the social welfare reaches its minimum. We also note that Chart 2 shows several changes between community energy projects and net billing schemes for different levels of confidence for community 1 (α_1).

Figure 10 shows the results for both approaches of confidence index application (schemes and income and expenses), considering again a case where all residential customers are neither pessimistic nor optimistic ($\alpha_i = 0.5$) about the game and its outcome, as well as two net billing injection prices, again, $p^{NB} = 95$ \$/MWh and $p^{NB} = 50$ \$/MWh. This figure specifically shows how a change in the customers' total disposable income affects the demand and electricity production at each node, price at node 3, and CO_2 emissions. As can be seen, the net billing injection price does not reach competitive levels, so the customers' (grand) coalition $S = \{1, 2, 3\} = I$ invests in more installed capacity based on community energy projects, given the investment costs and potential revenues that can be obtained in the spot market. Furthermore, the minimum price at node 3 that electricity consumers who are outside the grand coalition (outsiders, from now on) might obtain gets lower (from 65.77 \$/MWh to 64 \$/MWh), as long as the communities' disposable income increases; in other words, as long as the community energy installed capacity increases. Likewise, even when two levels of p^{NB} are considered for analysis, we highlight that more community energy installed capacity can have positive effects on those customers who do not take part in energy production. Outsiders can then save money when they consume electricity. We also note that such a deployment of community energy involves reasonable levels of disposable income and confidence, which may be more appropriate in real-life situations. In addition, when customers invest in community energy projects, as long as there is ample capital available, they can counter the market power exerted by generator 2 and (to some extent) decrease CO_2 emissions.

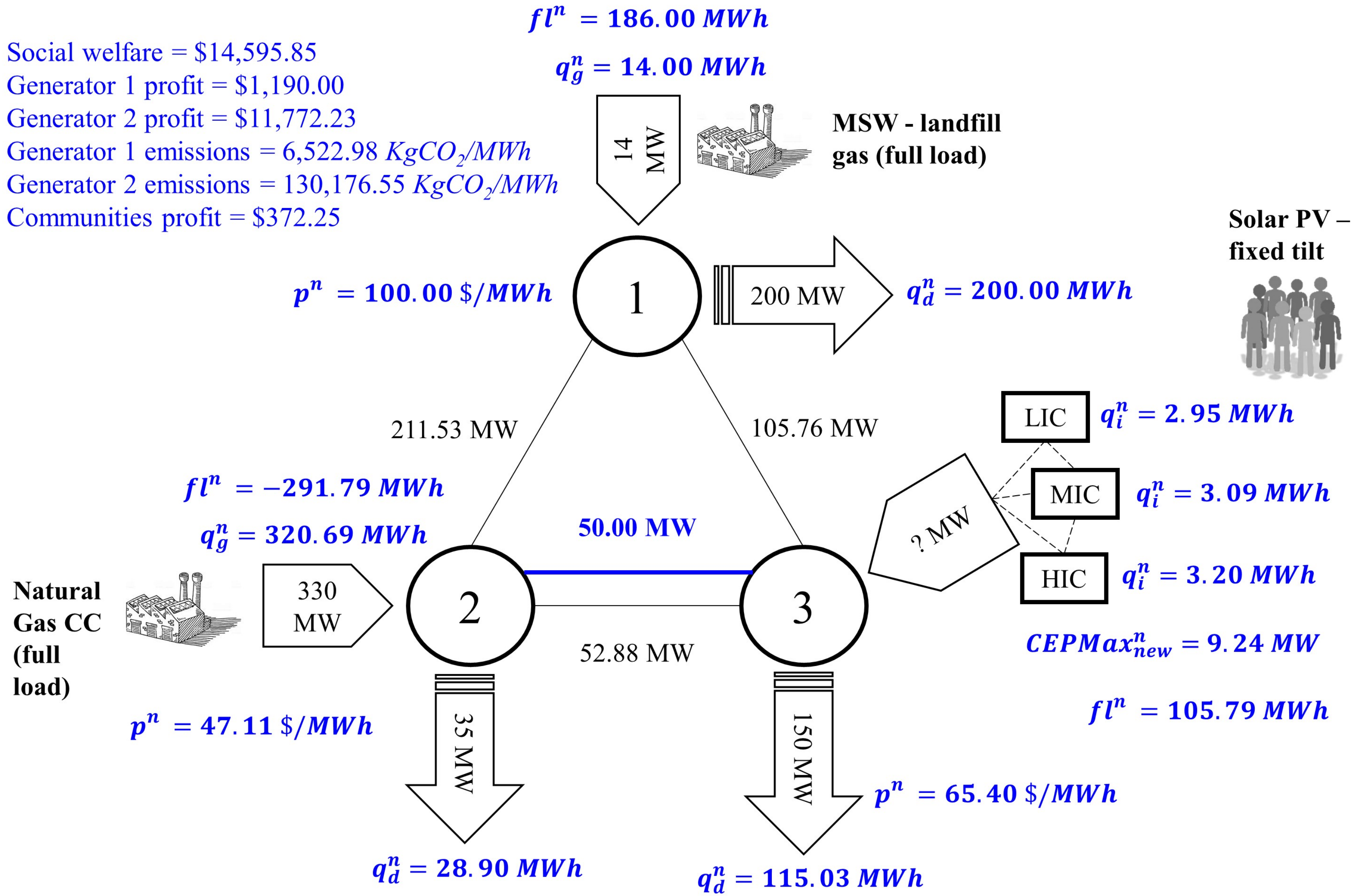

Figure 3: Results for the base case considering both approaches (confidence index applied to schemes and income/expenses) - Average execution time: 0.1263 seconds.

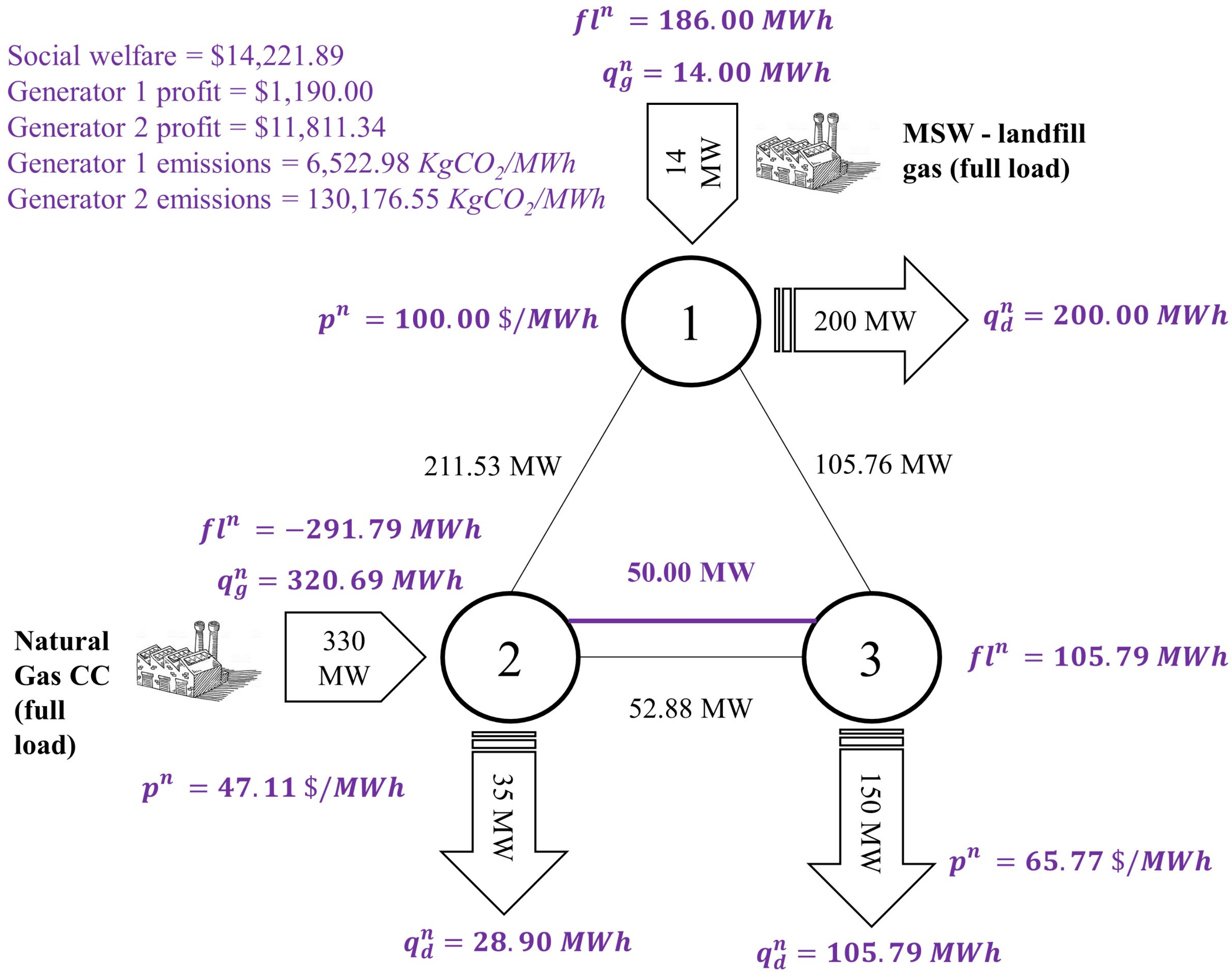

Figure 4: Results for the base case without considering communities in the market representation (conventional Stackelberg approach) - Average execution time: 0.1392 seconds.

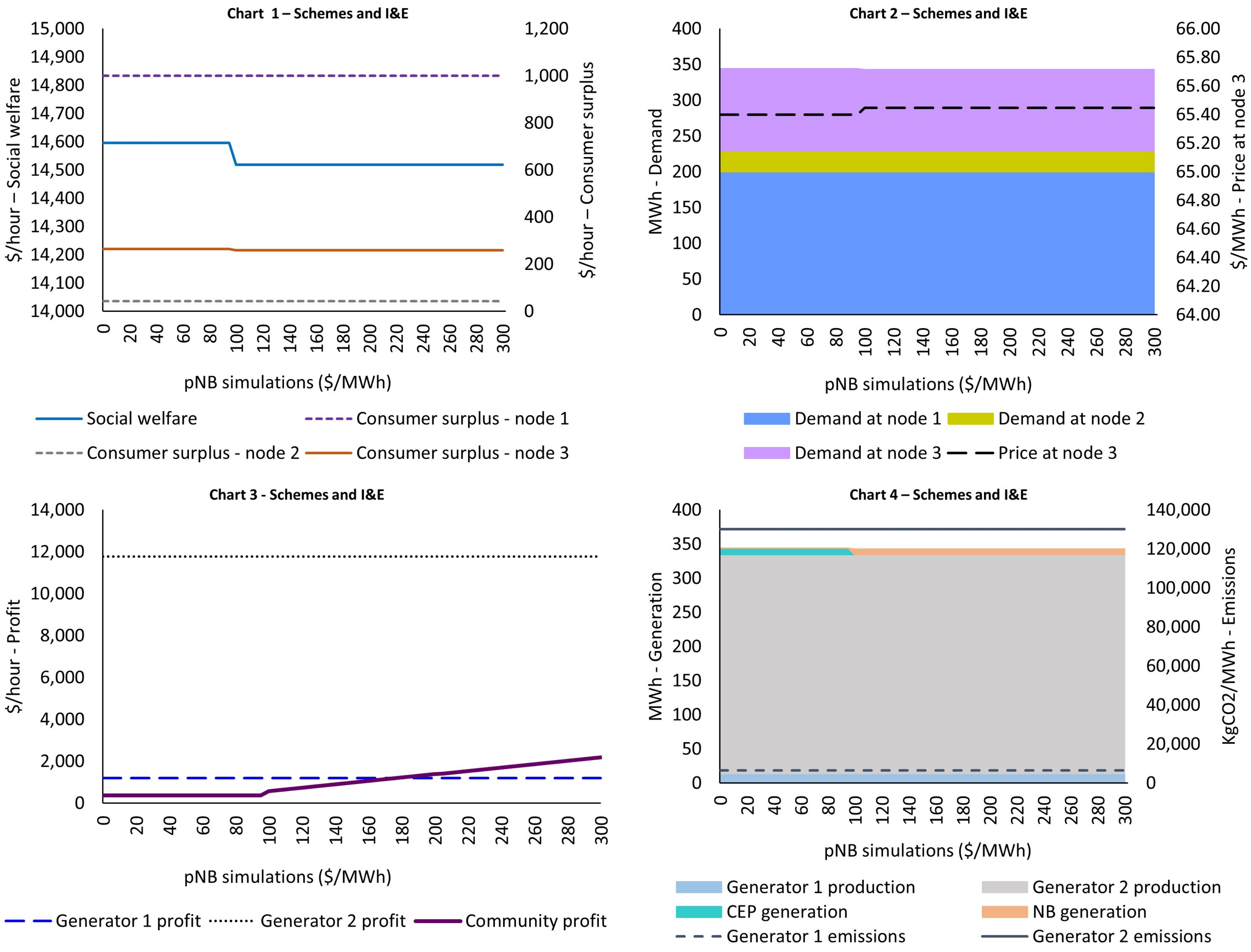

Figure 5: Simulation results for net billing injection price considering $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$. Confidence indices applied to schemes and income/expenses shown on each chart (same results for both approaches) - Average execution time: 0.0598 seconds - Total execution time: 7.2996 seconds.

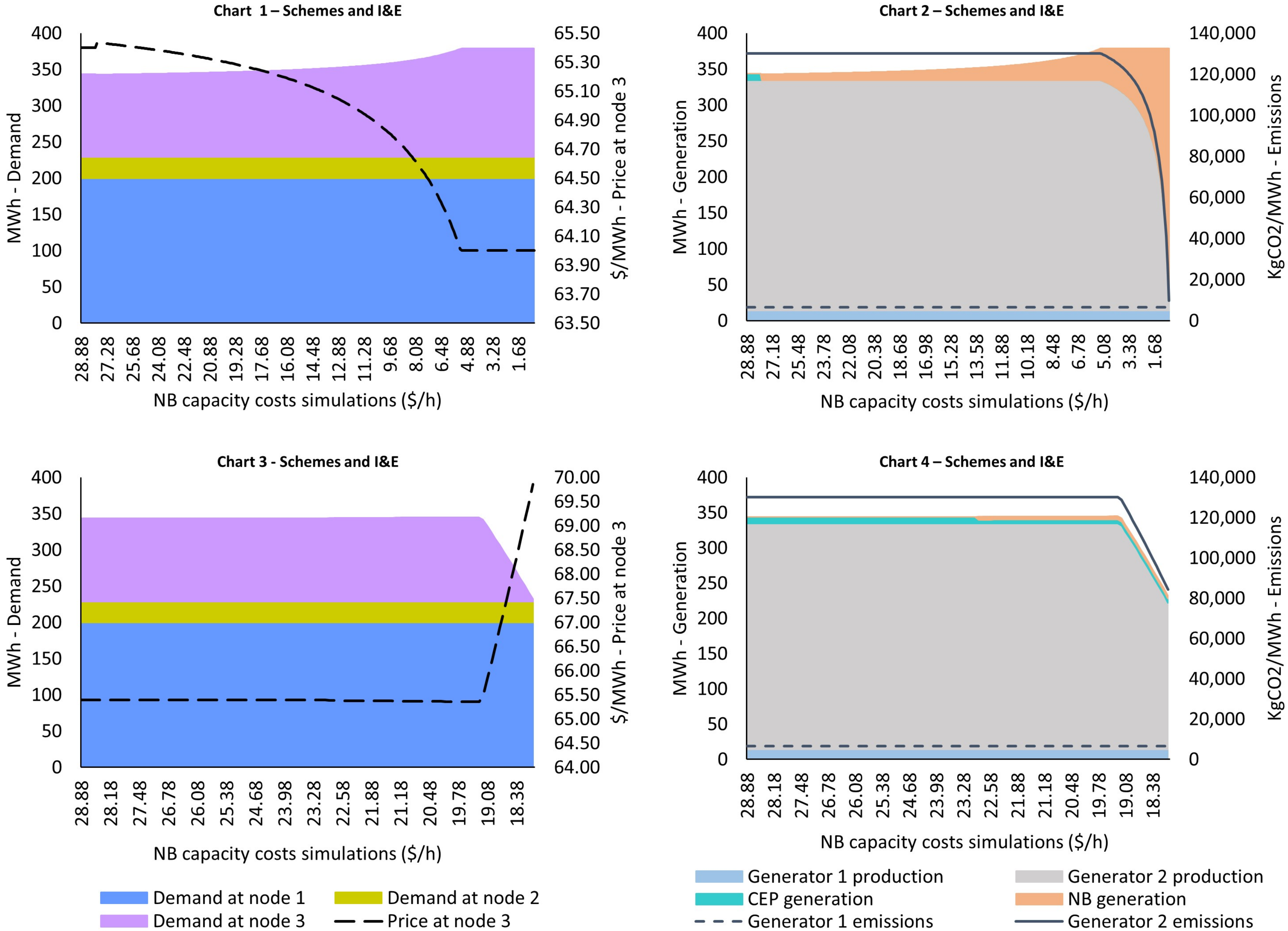

Figure 6: Simulation results for net billing installed capacity costs considering $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$ and $p^{NB} = 95$ \$/MWh (top), and $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$, $p^{NB} = 50$ \$/MWh, and a limit on net billing installed capacity of 4.5 MW for communities on the bottom. Confidence indices applied to schemes and income/expenses shown on each chart (same results for both approaches) - Average execution time: 0.0404 seconds - Total execution time: 31.7123 seconds.

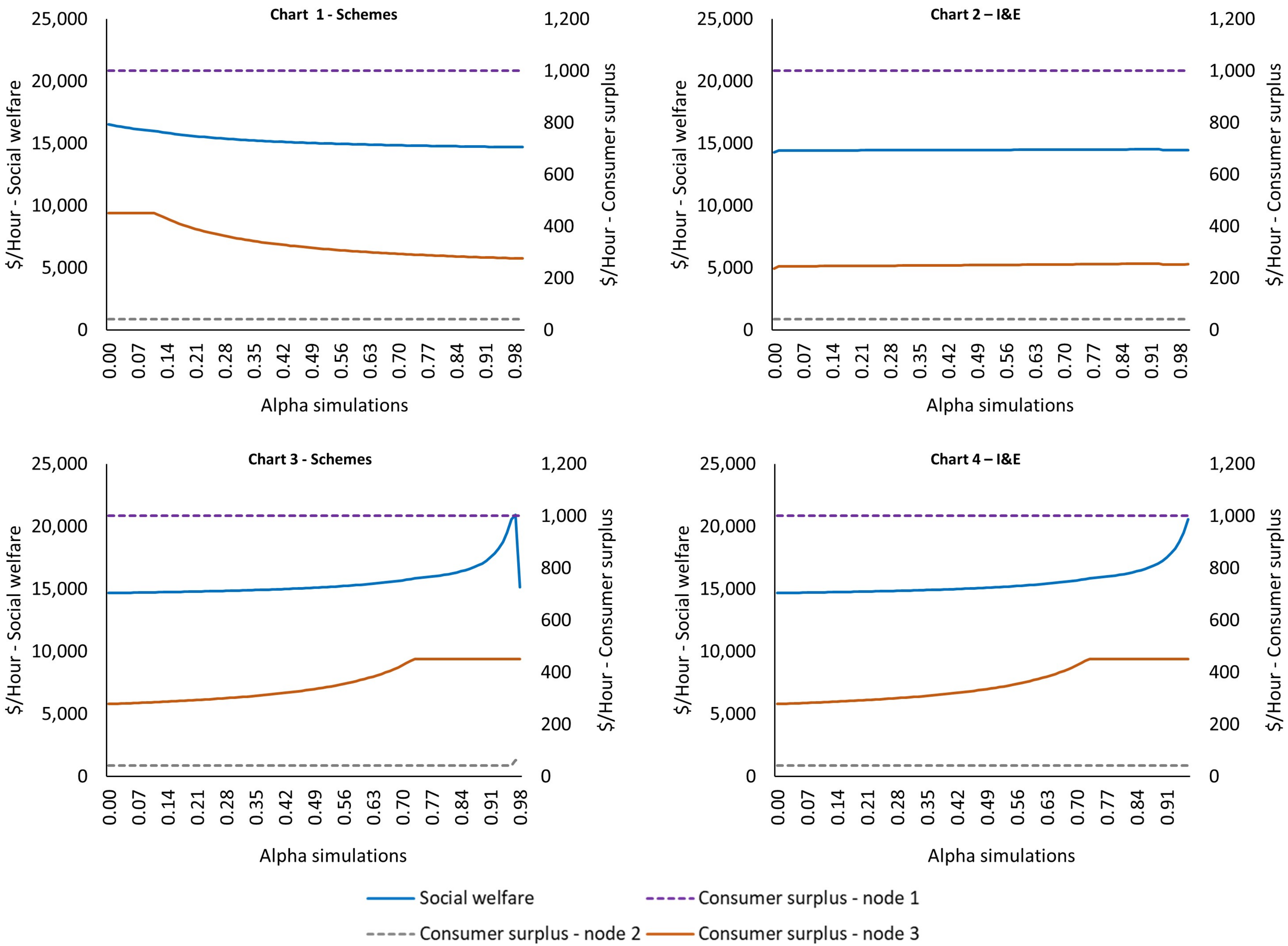

Figure 7: Simulation results for varying α_1 while $\alpha_2 = \alpha_3 = 0.1$, and $p^{NB} = 95$ \$/MWh (top), and $\alpha_2 = \alpha_3 = 1$ and $p^{NB} = 95$ \$/MWh (bottom). Confidence indices are applied to schemes (left-hand side) and income/expenses (right-hand side) - Average execution time: 0.0575 seconds - Total execution time: 22.8716 seconds.

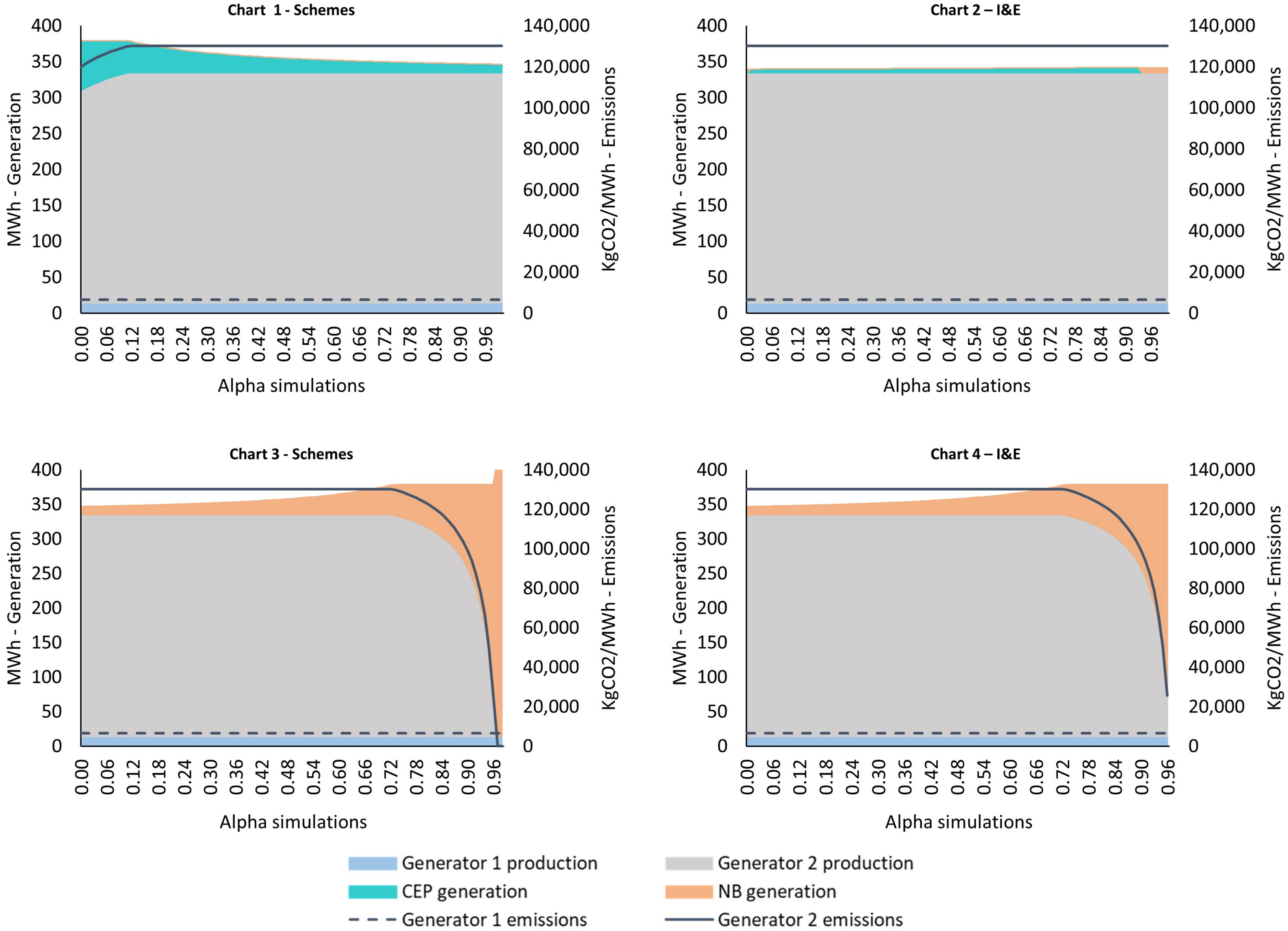

Figure 8: Simulation results for varying α_1 while $\alpha_2 = \alpha_3 = 0.1$, and $p^{NB} = 95$ \$/MWh (top), and $\alpha_2 = \alpha_3 = 1$ and $p^{NB} = 95$ \$/MWh (bottom). Confidence indices are applied to schemes (left-hand side) and income/expenses (right-hand side) - Average execution time: 0.0575 seconds - Total execution time: 22.8716 seconds.

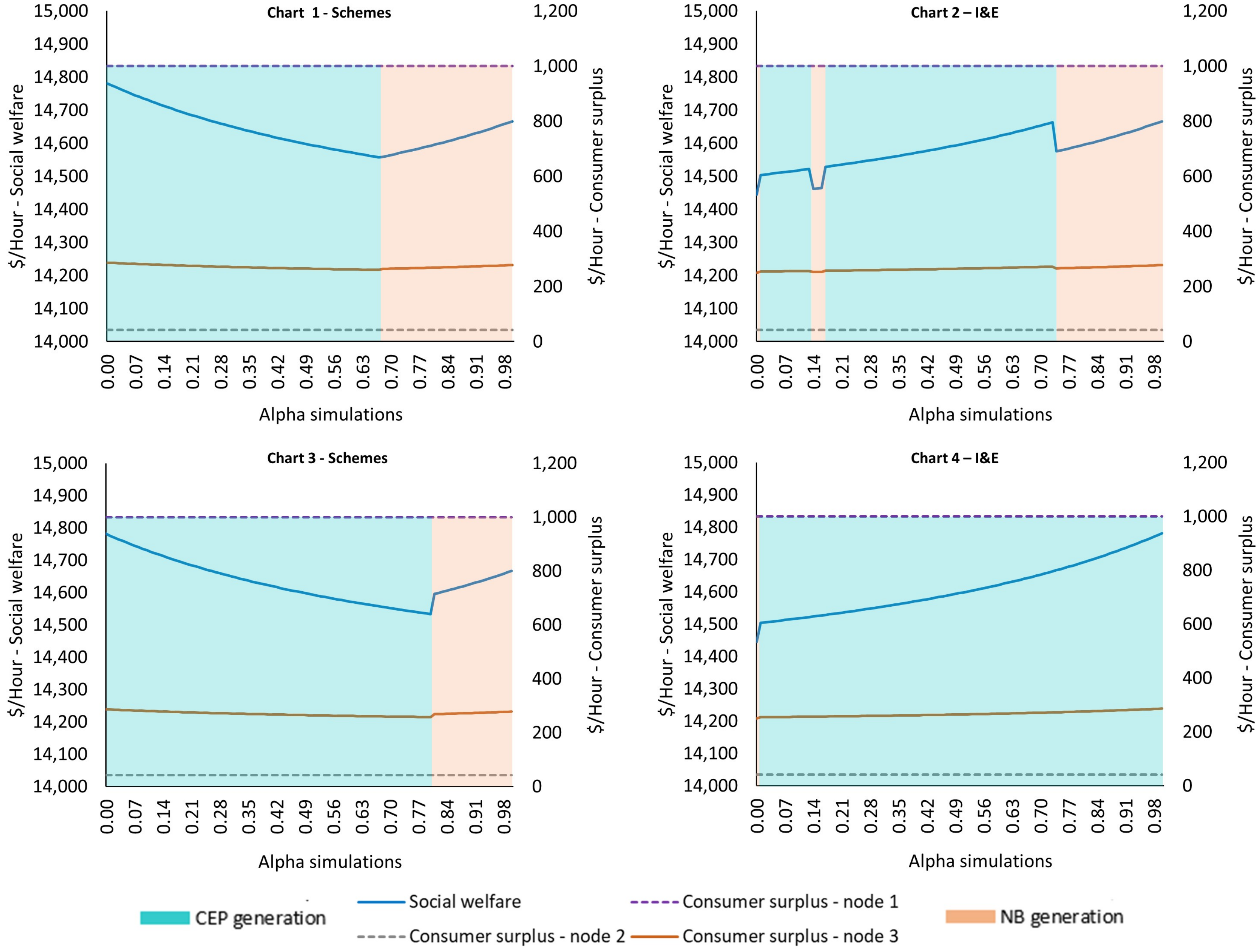

Figure 9: Simulation results varying α_1 while $\alpha_2 = \alpha_3 = 0.5$, and $p^{NB} = 95$ \$/MWh (top), and $\alpha_2 = \alpha_3 = 0.5$, and $p^{NB} = 50$ \$/MWh (bottom). Confidence indices are applied to schemes (left-hand side) and income/expenses (right-hand side) - Average execution time: 0.0727 seconds - Total execution time: 29.3514 seconds.

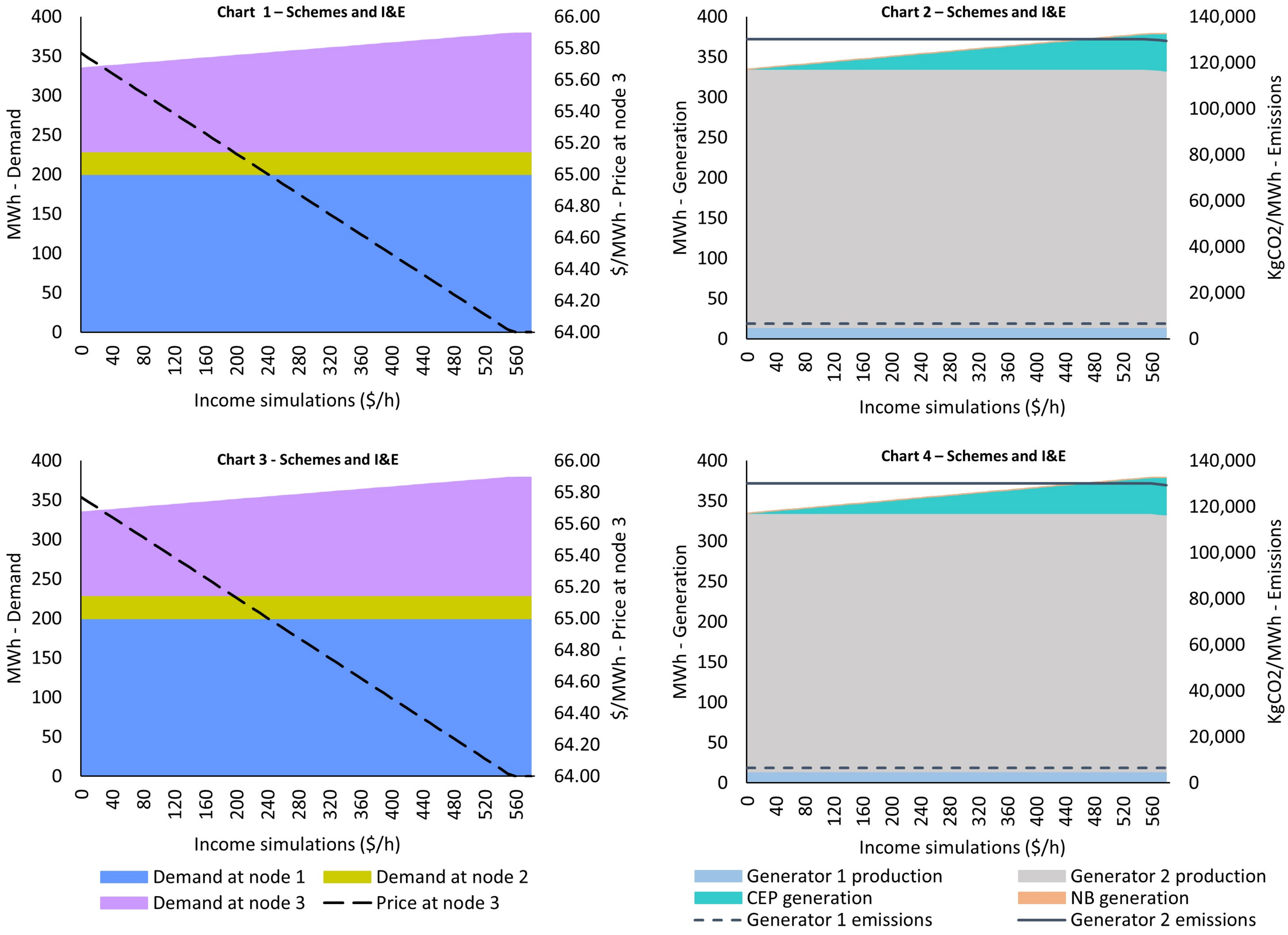

Figure 10: Simulation results for customers total disposable income considering $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$ and $p^{NB} = 95$ \$/MWh (top), and $\alpha_1 = \alpha_2 = \alpha_3 = 0.5$ and $p^{NB} = 50$ \$/MWh (bottom). Confidence indices applied to schemes and income/expenses shown on each chart (same results for both ways of applying confidence indices) - Average execution time: 0.0516 seconds - Total execution time: 12.1652 seconds.

5. Discussion and recommendations

In the previous section, various cases of interest were explored through an experiment that involved simulations in which we varied different parameters. We first outlined a base case where customers were neither optimistic nor pessimistic about the game and its outcome. Secondly, the above case was compared to a conventional Stackelberg formulation that considered no involvement of communities in energy production, but only investment by profit-maximising firms. Thirdly, the net billing injection price and net billing investment costs were then altered in order to shed light on the (minimum) level of attractiveness needed for net billing to be the chosen energy production scheme. Fourthly, we analysed the effects derived from having different confidence indices under analysis, focusing on altering the low income residential customers' confidence index; other customers' confidence indices were also modified to contrast results. Finally, the customers' disposable income was varied to examine the preferred energy production scheme, when more money is available, and derived impacts on key market indicators.

We recall that the confidence index represents players' *expectations* about the result of the game, i.e. the potential fraction of coalitional value they could capture. We applied α_i to community energy projects, as they would offer the best (highest) payoff due to potential access to a high installed capacity. Conversely, we applied (1-α_i) to net billing scheme projects, as they would comparatively offer players the worst payoff. We varied this formulation and applied α to income and (1-α_i) to expenses, so as to explore players' views on the influence that income and/or expenses would have on the outcome.

Regarding the first research question stated in Section 1, our results suggest that there are indeed differences between our biform-games-based approach and a conventional Stackelberg formulation that considers no participation of communities in energy production. The main effects are noted through a decrease in the social welfare in a Stackelberg formulation, a partial appropriation of communities' potential profit, and an increase in the price at node 3 due to a lower demand (and production). Of course, these results are partly a result of the way that we formulated each optimisation problem examined in this paper. Still, we think that they show the value of using biform games within expansion planning models. Biform games, compared to conventional Stackelberg models, deliver opportunities to explicitly distinguish and examine the effects on the different metrics used in electricity markets (e.g. social welfare, prices, demand, etc.) derived from implementing projects that require strategies and actions orientated towards simultaneously support coalitional stability and competition against other projects, i.e. citizen-led energy production initiatives. Depending upon the specific aim, biform games can also support decision-making processes for (network) planners, communities, and other interested parties in many ways. For instance, in terms of assessing impacts on the electricity market metrics (as mentioned before), establishing adequate levels of installed capacity and financial resources to be devoted to citizen-led energy production, exploring pathways to counterbalance oligopolistic incumbents or market power, shaping appropriate investment and benefits-sharing mechanisms, etc.

Concerning the research question about when net billing is preferred to community energy projects, the answer depends upon the specific case under analysis; notably, the variables, parameters, and their magnitudes that interact in a particular situation play a key role. Based on our analysis, we note that higher net billing injection prices, in the context of communities who are neither optimistic nor pessimistic about the game and its outcome, help implement net billing projects. Likewise, high levels of confidence in the payoffs to be received, alongside comparatively high net billing injection prices, usually support the implementation of net billing projects. This is also noted in (Fuentes González et al., 2020), particularly, the relevance of high levels of confidence (alternatively, the relevance of low levels of uncertainty) for such projects. In this sense, Fuentes González et al. (2020) claim that the confidence index can also have a component related to uncertainty. The complement of α, (1-α), can then be seen as players' uncertainty about getting a particular payoff. Accordingly, the higher the confidence, α, the lower the uncertainty, (1-α). Net billing projects may entail a comparatively low degree of uncertainty in some cases; such projects may be more accessible for (some) communities due to stratified investment costs, diverse installed capacities available for each household, low organisational costs, etc. However, as the results suggest, the implementation of net billing projects may involve a rise in nodal prices and a reduction in social welfare. Finally, and more obviously, comparatively lower investment costs may also help to deploy net billing schemes. Yet, capacity restrictions should be taken into consideration when planning their wider implementation in the system.

Concerning the research question about the impacts of implementing community energy projects, our simulation results suggest that there are impacts on social welfare, consumer surplus, nodal prices, demand and optimal power generation, generation and transmission expansion, and CO_2 emissions, derived from the implementation of community energy projects. Unsurprisingly, such impacts differ in terms of their magnitude or significance. According to our simulation results, the most noticeable (positive) impacts can be seen on social welfare, demand requirements and affordable generation, prices at the community energy location, and CO_2 emissions. There exist incentives to develop and deploy community energy projects, even considering the potential interactions that they may have with other market participants, such as large generators that exert market power, net billing projects, etc. Of course, the magnitude of these positive impacts and incentives that advocate for a wider implementation of community energy initiatives have to be understood in the context of the specific parameters chosen for the analysis. However, the qualitative conclusions are expected to be similar for other parameters.

In regards to the above, the way the confidence index is applied is crucial in finding the optimal solution and equilibrium. A dilemma therefore arises: how to apply the confidence index? Is the best option to apply it to schemes or to income and expenses? We observe more conservative values in those cases where the confidence index is applied to income and expenses, putting prices and costs of both energy production options within a competitive range. This means that prices of both types of projects can simultaneously be subject to the same confidence index. If customers have a high confidence, these prices take more relevance and become more important than costs for decision-making purposes. The opposite occurs when the confidence index is low, as costs become more relevant. Thus, the confidence index can be interpreted as the confidence about how prices (income) or costs (expenses) influence the final outcome or payoff, rather than a direct determinant of a weighted average payoff. Conversely, when the confidence index is applied to both schemes, the confidence index itself becomes more relevant and decision-making processes are mostly based on its specific value rather than predefined prices and costs. This is especially relevant when prices and costs do not have a significant gap or distance between them. In this case, it is important to define which options should be catalogued as the 'best' and 'worst'. This might be complex if a decision-making process involves choosing between two worthy options. To sum up, both approaches seem to be valid and therefore more research is necessary to clarify this issue. Beyond the confidence index, the customers' disposable income is clearly important, as are investments costs; changes to both can help increase citizen participation in energy production.

Nevertheless, community energy projects are an attractive option to be considered, as they offer stability and viability, from an economic-strategic perspective, as well as an interesting outcome, namely profits to be allocated. They also do not face, in principle, any significant limitation in terms of capacity in the same way net billing projects do, and do not need a high confidence and a very high (imposed) price (p^{NB} = 95 \$/MWh) to be implemented. Regulated prices or subsidies for renewable energy projects are becoming less popular in many countries, so projects with a direct involvement in the spot market could contribute to a deeper citizen participation in energy production. A key factor therefore needs to be addressed in a more detailed way, namely how people can fund or devote money to a community energy project. We encourage more research in order to design more advanced financial mechanisms that motivate people, under fair conditions, to get access to funding and then carry out community energy projects.

Community energy projects can have a positive impact even on those who do not participate in them. Again, this depends on the exact model parameters but, still, we find cases where this happens, and where net billing or distributed energy projects do not lead to the same result. This is an important outcome which should be taken into account by policy makers, as positive externalities provide a justification for more support for community energy projects. Still, there is currently some concern that local energy initiatives may have negative impacts on lower-income households that do not participate in energy production; as we show, this concern may be valid but, nevertheless, this effect can also go the other way.

This work considered a number of restrictive assumptions. Furthermore, there are situations that were not accounted for in this work. For instance, we did not consider the case where large, conventional generators are perfectly competitive. Based on economic theory, we conjecture a higher availability in the market of conventional and/or large generation (which usually present strong availability of financial and operational resources), more competitive or lower nodal prices, and less attractive incentives for community energy development. Yet, the situation for community energy development in this scenario could be similar to what we have modelled above, as long as communities have good access to finance and public policy does not impede or hamper free access to the grid. We leave the specific implications of this for future research. In addition, one key point in our model is the coalition formation, as only the grand coalition plays a role in our market representation. As mentioned before, communities could also form other coalitions and more groups could potentially join the game. In these situations, the number of players and coalitions can increase, and hence, so will the constraints that have to be met in the optimisation problem. When incorporating more players and coalitions, it is important to bear in mind the Bondareva-Shapley theorem and appropriate techniques for determining the final payoff. Scarf (1967) through his seminal paper offers insights into a proper algorithm to deal with the above situation. Given the nature and extent of our problem, we did not include n-person-based algorithms in our model. We encourage further investigation in the context of community energy emergence.

However, we believe that the assumptions considered in this work are valid and deliver reasonable, coherent results. Likewise, in order to model and solve the above models, only representative, real-world data was used, which contributes to strengthen the reliability of our findings. Yet, the above models need to consider other variables. For example, as our formulation uses information on hourly basis and is focused on the communities' strategic-economic decision-making process, future research should explicitly account for the maximum technical potential or seasonal variability for solar generation. We conjecture that considering these inputs may strengthen the role of large generators in meeting demand requests, if the availability of the solar resource is more limited. In addition, variables or parameters related to social sciences should also be accounted for, so as to produce more accurate outcomes by addressing people's intentions, feelings, expectations, etc. One path could involve measuring the confidence index - in the real world - by using, for example, surveys or other quantitative sociological/psychological tools. Additional market-based information, such as price volatility, market indices, among others, may also be considered in order to strengthen the above models and their results.

We formulate the following recommendations, based on the above content, for policy and further research:

1. Design and evaluate more complex benefit allocation mechanisms that assure stability as well as fairness, considering models that deal with cooperative and non-cooperative behaviours at the same time, such as biform games.
2. Explore more options to provide better access to funding with proper and fair payback mechanisms, interest rates, uncertainty, and risks assessments, among other elements.
3. Include social variables in this type of model in order to explicitly consider human behaviour in generation and transmission expansion planning optimisation problems.
4. Promote the use of this kind of model at a governmental level in order to better encourage citizen participation in energy production with a stronger financial and economic base.
5. Define clearer and more explicit goals and milestones related to the aforementioned points, in order to aid the community energy discipline, taking into account a multidisciplinary perspective.

6. Conclusions

This paper proposes a bi-level power generation and transmission expansion planning optimisation model, which attempts to evaluate the incentives for deploying community energy projects in real-world markets. This work addresses the interaction between community energy projects and other energy production initiatives, namely large generators and net billing projects. The methodology behind the above-mentioned model combines biform games and linear production games, with the aim of finding stable and feasible solutions for the incumbents, from an economic and strategic perspective. As mentioned before, to our best knowledge, such inclusion and combination of these game theoretical tools has not been implemented yet, so we see many opportunities for more research in community energy emergence and wider energy markets complexities. Through this work, we therefore extend the transmission and generation expansion planning literature to promote the inclusion of community energy initiatives, in addition to large profit-maximising generation investors.

Using real-world data, mainly considering the Chilean context, we have implemented and solved the above mentioned optimisation model through an experiment that involved various simulations. From our results, it is remarkable that community energy projects appear as an attractive option for residential customers willing to be involved in energy production, even when such an option would be economically disadvantaged compared to a net billing project with a higher injection price. Moreover, residential customers who do not participate in such initiatives, can also benefit from them. This finding may encourage policy makers to further explore a wider implementation of community energy projects. Community energy projects do not need a high participant confidence to be carried out, given their investment costs and the spot price they can obtain in the market. In any case, they might be jointly deployed with net billing projects, as the latter would face more stringent capacity limitations. We also find that the confidence index itself and its implementation is key when modelling community energy in a biform game theory application.

Our findings might help policy makers around the world who would like to explore the idea of promoting citizen participation in energy production. This is especially true in those regions where the community energy sector is still unknown or incipient. Of course, our results are strongly dependent on the parameters considered in this work. Nevertheless, the methodology proposed here is a useful and generic way to examine the complexities around community energy projects and their interaction within energy/electricity markets. This work also attempts to motivate further research to examine public policies that tackle climate change, promote a sustainable development, foster a more democratic access to free markets, reduce inequality, and transfer more power to communities.

Inclusion and Diversity

While citing references scientifically relevant for this work, we actively worked to promote gender balance in our reference list. The author list of this paper includes contributors from the location where the research was conducted who participated in the data collection, design, analysis, and/or interpretation of the work.

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Appendix A. Nomenclature

Sets

- G : Set of all large generators in the system.
- N : Set of all nodes considered in the system.
- I : Set of all communities considered in the system.
- L : Set of all (current and new) lines (to be) considered in the system.

Variables

- q_g^n : Electricity produced by each large generator g located at node n (in MWh).
- $g_{g\ new}^n$: New capacity installed by a large generator g located at node n (in MW).
- q_i^n : Electricity produced by a community energy project owned by community i located at node n (in MWh).
- $q_i^{n\ NB}$: Electricity produced by a net billing scheme project owned by community i located at node n (in MWh).
- CEP_i^n : Community i's installed capacity allocation based on a community energy project (in MW).
- NB_i^n : Community i's installed capacity allocation based on a net billing project (in MW).
- $CEPMAX_{new}^n$: Maximum new installed capacity allocation based on a community energy project for communities located at node n (in MW).
- $NBMAX_{new}^n$: Maximum new installed capacity allocation based on a net billing project for communities located at node n (in MW).
- p^n : Nodal price at node n (in \$/MWh).
- q_d^n : Power load at node n (in MWh).
- fl^n : Power flow from/to node n (in MWh).
- l_{new} : New transmission line expansion (binary variable $\{0, 1\}$).

Parameters

- GC_g^n : Variable generation costs for large generator g located at node n (in \$/MWh).
- GIC_g^n : Generation investment costs for large generator g located at node n (in \$/MW).
- $q_g^{n\ EX}$: Existing installed capacity of large generator g located at node n (in MW).
- p^{NB} : Net billing injection price (in \$/MWh).
- α_i : Community i's confidence index (continuous parameter $[0, 1]$).
- GCC_i^n : Community energy project generation costs for community i located at node n (in \$/MWh).
- $GICC_i^n$: Community energy project generation investment costs for community i located at node n (in \$/MW).
- $GCC_i^{n\ NB}$: Net billing project generation costs for community i located at node n (in \$/MWh).
- $GICC_i^{n\ NB}$: Net billing project generation investment costs for community i located at node n (in \$/MW).

- $q_i^{n\,EX}$: Existing installed capacity based on a community energy project for community i located at node n (in MW).
- $q_i^{n\,NB\,EX}$: Existing installed capacity based on net billing projects for community i located at node n (in MW).
- $g_{i\,exp}^{n\,NB}$: Community i's expected individual installed capacity based on a net billing scheme if they acted by themselves (in MW).
- α_{com} : Communities' average confidence index (continuous parameter [0, 1]).
- R_{inv}^{n} : Mean of community energy project investment costs ($GICC_i^n$) or necessary resources to conceive 1 MW of a community energy project (in \$/MW).
- R_{inv}^{NB} : Mean of net billing project investment costs ($GICC_i^{n\,NB}$) or necessary resources to conceive 1 MW of a net billing project (in \$/MW).
- $\sum_{i \in S} R_{inv}(S)$: Coalition S's resources available (disposable income) for covering installed capacity investment costs (in \$).
- TIC_l : Investment costs of new transmission line l (in \$/line).
- $PTDF_{l,n}$: Swing factors on line l related to power injection/withdrawal at node n.
- th_l : Thermal capacity of existing line l (in MW).
- $th_{l\,new}$: Thermal capacity of new line l (in MW).
- $q_d^{n\,MAX}$: Maximum load at node n (in MW).
- a_d^n : Demand curve intercept at node n.
- β_d^n : Demand curve slope at node n.
- r : Interest rate.
- t : Time.

Appendix B. Fortuny-Amat & McCarl complementarity inequalities linearisation

The complementarity inequalities (15) to (17) are linearised as follows (Fortuny-Amat and McCarl, 1981):

$$0 \le \mu_{1g}^{\,n} \le MU$$

$$0 \le g_{g\,new}^{n} \le M(1-U)$$

$$0 \le \mu_{2g}^{\,n} \le MU$$

$$0 \le q_g^n \le M(1-U)$$

$$0 \le \mu_{3g}^{\,n} \le MU$$

$$0 \le -q_g^n + q_g^{n\,EX} + g_{g\,new}^{n} \le M(1-U)$$

The complementarity inequalities (35) to (42) are linearised as follows (Fortuny-Amat and McCarl, 1981):

$$0 \le \mu_{1i}^{c\,n} \le MU$$

$$0 \le q_i^n \le M(1-U)$$

$$0 \le \mu_{2i}^{c\,n} \le MU$$

$$0 \le q_i^{n\,NB} \le M(1-U)$$

$$0 \le \mu_{3i}^{c\,n} \le MU$$

$$0 \le -q_i^n + q_i^{n\,EX} + CEP_i^n \le M(1-U)$$

$$0 \le \mu_{4i}^{c\,n} \le MU$$

$$0 \le -q_i^{n\,NB} + q_i^{n\,NB\,EX} + NB_i^n \le M(1-U)$$

$$0 \le \mu_{5i}^{c\,n} \le MU$$

$$0 \le CEP_i^n + CEP_j^n \le M(1-U)$$

$$0 \le \mu_{6i}^{c\,n} \le MU$$

$$0 \le NB_i^n + NB_j^n - g_{i\,exp}^{n\,NB} - g_{j\,exp}^{n\,NB} \le M(1-U)$$

$$0 \leq \mu_{7i}^{c\,n} \leq MU$$

$$0 \leq CEP_i^n \leq M(1-U)$$

$$0 \leq \mu_{8i}^{c\,n} \leq MU$$

$$0 \leq NB_i^n - g_{i\,exp}^{n\,NB} \leq M(1-U)$$

The complementarity inequalities (49) to (51) are linearised as follows (Fortuny-Amat and McCarl, 1981):

$$0 \leq \mu_1^{ccn} \leq MU$$

$$0 \leq \left[\left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right)^{-1} \left(-\alpha_{com} R_{inv}^n CEPMAX_{new}^n - (1-\alpha_{com}) R_{inv}^{NB} NBMAX_{new}^n \right) \right] + \sum_{i \in S} R_{inv}(S) \leq M(1-U)$$

$$0 \leq \mu_3^{ccn} \leq MU$$

$$0 \leq NBMAX_{new}^n \leq M(1-U)$$

$$0 \leq \mu_4^{ccn} \leq MU$$

$$0 \leq CEPMAX_{new}^n \leq M(1-U)$$

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