# Axial-torsional dynamics of a drilling bit with non-uniform blade arrangement

Mohammad Amin Faghihi[1], Ehsan Azadi Yazdi[1], Hossein Mohammadi[1*], Shabnam Tashakori[2,3] and Vahid Vaziri[3]

[1*]Department of Mechanical Engineering, Shiraz University, 71956-15735, Iran.
[2]Department of Mechanical Engineering, Shiraz University of Technology, 71557-13876, Iran.
[3]Center of Applied Dynamics Research, School of Engineering, University of Aberdeen, AB24 3UE, UK.

*Corresponding author(s). E-mail(s): h_mohammadi@shirazu.ac.ir;
Contributing authors: amin.faghihi@hafez.shirazu.ac.ir; ehsanazadi@shirazu.ac.ir; tashakori@sutech.ac.ir; vahid.vaziri@abdn.ac.uk;

**Abstract**

In this paper, a model for the coupled axial-torsional dynamics of a drilling bit with a non-uniform blade arrangement is proposed. The neutral-type time-delay model is used to model the drill pipes, which transfer the actuation force and torque to the bit. The employed bit-rock interaction law is a rate-independent law including both cutting and frictional effects. A novel method for determining the depth of cut, which is the key component in estimating the cutting forces, is developed to capture different phenomena, including multiple-regenerative effects. In this method, a functional description of the well surface pattern is presented to determine the depth of cut. Unlike the previous studies, the well surface pattern evolution is represented by an algebraic equation rather than a partial differential equation (PDE). Illustrative simulation results are presented for a representative case study, which demonstrates the validity of the proposed model even in the presence of multiple-regenerative effects. The effect of the non-uniformly arrangement of the blades on the drilling vibration is also discussed. It is shown that the non-uniform arrangement of the cutting blades can reduce the vibration amplitude in some operating conditions. Moreover, disregarding the multiple-regenerative effects (which appear in the presence of severe oscillations) in the model, causes a considerable modeling error.

# 1 Introduction

Drill strings suffer from self-excited undesirable oscillations leading to detrimental phenomena such as stick-slip and the bit-bounce. A schematic view of a drill-string is illustrated in Fig. 1. Stick-slip is a cyclic rotational oscillation during which the bit sticks (the angular velocity becomes zero) in some time intervals. On the other hand, the angular velocity may become several times larger than the nominal angular velocity in other time intervals. Moreover, in extreme torsional vibrations, the bit is susceptible to rotating in the reverse direction. The other destructive phenomenon, bit-bounce, corresponds to the axial motion of the bit. In this case, intensive axial vibrations lead to bit-bounce and loss of contact with the formation. These undesired phenomena significantly reduce the drilling efficiency in different aspects, such as bit wear, rate of penetration reduction, and failure in drilling tools. Accordingly, mitigating such destructive vibrations is important, which necessitates understanding their root causes.

Self-excited coupled axial-torsional vibrations are (mainly) a result of the complex interactions between the bit and the formation. Two independent processes form the interactions (forces) between Poly-crystalline Diamond Compact (PDC) bits and the formation: (i) the (pure) cutting process occurring on the cutting face of the bit and (ii) the frictional process acting between the underside part of the bit and the formation [1]. Cutting forces are proportional to the depth of cut. The depth of cut is the summation of the thickness of the rock layers (the cutting chips) that the cutting blades face. The cutting

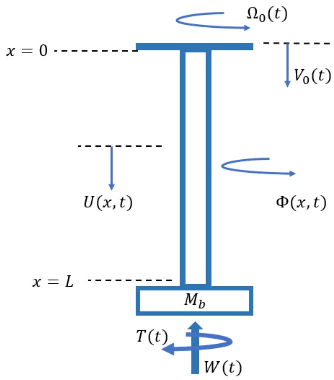



**Fig. 1**: A schematic of the drill-string.

chip thickness is a function of the current and the preceding (axial) position of the drill bit. This introduces delay terms, which are state-dependent, to the equations of motion. Using state-dependent delay terms for formulation of the cutting chip thickness was introduced in the modeling of milling dynamics [2]. For drilling systems, the state-dependent delay model was initially proposed in [3]. However, the significance of the state-dependent delay model was not realized until much later [4, 5]. The state-dependent delay model have been employed by many researchers to investigate the root cause of these unwanted vibrations [6–12]. However, this model assumes that the cutting operation is in progress whenever the bit rotates and hence, cannot capture the multiple-regenerative effects.

Multiple-regenerative effects are the result of losing contact between the cutting blades and the formation. More precisely, multiple-regenerative effects occur when the bit rotates without cutting the formation. There are two possibilities for this; bit-bounce, and bit reverse rotation. In the case of multiple-regenerative effects, the depth of cut is dependent on the multiple preceding positions of the blade rather than the earlier position. There exist two different approaches in the literature considering multiple-regenerative effects in the bit-rock interactions [13–15]. In [14] and [13], the depth of cut is determined by defining the rock surface pattern function, which is determined by a first-order Partial Differential Equation (PDE). In this model, since the cut pattern is defined by employing a PDE, nonsmooth and discontinuous cut patterns cannot be obtained. The discontinuous cut pattern can be generated while the torsional stick and the bit-bounce take place simultaneously. On the other hand, in the approach developed in [15], multiple preceding positions of the bit are compared to estimate the instantaneous depth of cut. Since comparing multiple preceding positions introduces multiple state-dependent delays to the system, this model is called the complex time delay model. In the complex time delay model, a history of the bit states should be saved conservatively, and in each solving step, the state-dependent complex delay should be estimated by comparing the current values of the states and the history of the states, which increases the computational burden of the model simulation.

Recently, a new trend has been created in [16] to improve drilling stability by rearranging the cutting blades. In this research, a non-uniform angular blade distribution on the bit is proposed rather than arranging the blades uniformly on the drill bit. It is shown in [17] that a non-uniform angular blade distribution can enhance the stable region for stationary drilling. Note that similar research can be found in the literature for milling operations with variable pitch or variable helix tools [18, 19]. In [20], a combination of full and partial blades is considered in addition to multiple angular offsets between the blades, and the stability for different bit designs is determined. Drilling bits with round cutters instead of radial flat cutters are studied in [21], which introduce distributed time delays to the equation of motion rather than several time delays. The PDE formulation presented in [14] is employed for PDC bits with a non-uniform cutter layout in [22, 23].

In this paper, a formulation for the regenerative effects caused by the bit-bounce or the reverse rotation is developed. Compared to the previous related paper [24], the formulation is developed generally assuming that the blades are non-uniformly arranged on the cutting bit. The regeneration of the rock surface pattern at the bottom of the well is employed to determine the instantaneous depth of cut. In the previous models which use the well surface pattern in order to calculate the depth of cut, the time evolution of the well surface pattern is governed by a PDE [14]. However, in the proposed model in this paper, the time evolution of the well surface pattern is governed by an algebraic equation, needless of defining a PDE, which enables the model to capture the discontinuous well surface patterns.

The main contributions of this paper are, firstly, presenting a computationally efficient model for the cutting operation of a bit equipped with non-uniform distributed cutting blades, and secondly, using the proposed model for studying the coupled axial-torsional vibrations of a drill-string represented by an infinite-dimensional Neutral-type Time Delay (NTD) model.

It is noteworthy that the goal of this paper is to propose a comprehensive model that can be used for parametric design or active controller design to reduce unwanted vibrations. For example, the proposed model (which is a more comprehensive model) can be used along with the approaches in [17, 21, 25] for dynamical analysis, parametric design, or design an anti-stall tool for vibration reduction.

The paper is organized as follows: Section 2 presents the NTD model for the coupled axial-torsional motion of a drill-string and the boundary conditions. Simulation results for a representative case study are presented in Section 3. Conclusions are presented in Section 4.

# 2 Mathematical modelling

It is assumed that the drilling pipes are elastic and uniform structures with equally distributed mass. Accordingly, the drill-string is governed by the simple (undamped) wave equation in both axial and torsional directions [26, 27]. The boundary conditions of the wave equations are defined by the imposed velocities at the top and the BHA dynamics at the bottom of the drill-string, as depicted in Fig. 1. Then, the equations of motion are obtained by solving d'Alembert's solution and employing Riemann variables [28, 29]. Subsequently, the following NTD model is given to represent the coupled axial-torsional dynamics of a drill-string [11, 24, 28–30]:

$$\ddot{\Phi}_b(t) - \ddot{\Phi}_b(t-2\tau_t) = -\frac{GJc_T}{J_b}(\dot{\Phi}_b(t) + \dot{\Phi}_b(t-2\tau_t)) \tag{1}$$

$$+\frac{1}{J_b}(-T(t) + T(t-2\tau_t)) + \frac{2GJc_t}{J_b}\Omega_0(t-\tau_t), \tag{2}$$

**Table 1**: Parameter values [30].

| Parameter | Definition | Value |
|---|---|---|
| $M_b$ | BHA mass | 40000 kg |
| $J_b$ | BHA moment of inertia | 89 $\text{kgm}^2$ |
| $E$ | Young modulus | $200 \times 10^9$ $\text{N/m}^2$ |
| $G$ | Shear modulus | $79 \times 10^9$ $\text{N/m}^2$ |
| $A$ | Drill-string cross sectional area | $35 \times 10^{-4}$ $\text{m}^2$ |
| $J$ | Drill-string moment of area | $1.9 \times 10^{-5}$ $\text{m}^4$ |
| $L$ | Drill-string length | 2000 m |
| $c_t$ | Torsional wave constant | $3.2 \times 10^{-4}$ s/m |
| $c_a$ | Axial wave constant | $1.99 \times 10^{-4}$ s/m |
| $\tau_t$ | Torsional wave travel time along the drill-string | 0.64 s |
| $\tau_a$ | Axial wave travel time along the drill-string | 0.39 s |
| $\epsilon$ | Rock intrinsic specific energy | $60 \times 10^6$ $\text{N/m}^2$ |
| $a$ | Bit radius | $10.8 \times 10^{-2}$ m |
| $\zeta$ | Cutter face inclination number | 0.6 |
| $\sigma$ | Maximum constant pressure at the wearflat interface | $60 \times 10^6$ $\text{N/m}^2$ |
| $l$ | Length of the drill bit wearflat | $1.2 \times 10^{-3}$ m |
| $\mu$ | Friction coefficient at the wearflat-rock interface | 0.6 |
| $\gamma$ | Bit geometry number | 1 |
| $n$ | number of blades | 4 |

$$\ddot{U}_b(t) - \ddot{U}_b(t-2\tau_a) = -\frac{EAc_a}{M_b}(\dot{U}_b(t) + \dot{U}_b(t-2\tau_a)) \tag{3}$$

$$+\frac{1}{M_b}(-W(t) + W(t-2\tau_a)) + \frac{2EAc_a}{M_b}V_0(t-\tau_a), \tag{4}$$

where $\Phi_b(t) := \Phi(L,t)$ and $U_b(t) := U(L,t)$ are the angular and axial displacements of the bit, respectively (see Fig. 1), $T(t)$ represents the torque on bit, and $W(t)$ represents the weight on bit. The parameters in (1) and (3) are defined in Table 1. The delay term $\tau_t$ ($\tau_a$) is the time required for the torsional (axial) wave to travel along the string. The existence of the terms with $2\tau_t$ ($2\tau_a$) is due to the wave reflection at the top and returning back to the bit. Indeed, by employing the NTD model, the drill-string model is still infinite-dimensional with delayed terms in the equations of motion.

## 2.1 Bit-rock interactions

The following bit-rock interaction law is employed to represent the torque and the weight on the bit [3]:

$$T(t) = T_c(t) + T_f(t), \tag{5a}$$

$$W(t) = W_c(t) + W_f(t), \tag{5b}$$

with the following cutting and frictional components:

$$T_c(t) = \frac{1}{2}\epsilon a^2 \mathbf{R}\left(d(t)\right)\mathbf{H}\left(\dot{\Phi}_b(t)\right), \tag{6a}$$

$$T_f(t) = \frac{1}{2}\mu\gamma a^2 \sigma l \mathbf{Sign}\left(\dot{\Phi}_b(t)\right)\mathbf{H}\left(d(t)\right)\mathbf{H}\left(\dot{U}_b(t)\right), \tag{6b}$$

$$W_c(t) = \epsilon a \zeta \mathbf{R}\left(d(t)\right)\mathbf{H}\left(\dot{\Phi}_b(t)\right), \tag{6c}$$

$$W_f(t) = \sigma a l \mathbf{H}\left(d(t)\right)\mathbf{H}\left(\dot{U}(t)\right), \tag{6d}$$

where $\epsilon$, $\mu$, $\zeta$, $l$ and $\sigma$ are constant parameters defined in Table 1, and $d$ is the depth of cut. Moreover, $\mathbf{R}(.)$, $\mathbf{H}(.)$, and $\mathbf{Sign}(.)$ are the Ramp, Heaviside, and Sign functions, respectively [31].

### 2.1.1 Depth of cut estimation

Depth of cut is the summation of the thickness of the rock layers which are being cut by the cutting blades. If the cutting blades are arranged non-uniformly, the depth of cuts corresponding to different cutting blades are not the same. Accordingly, the depth of cut is given by

$$d(t) = \sum_{i=1}^{n} d_i(t), \tag{7}$$

where $d_i(t)$ is the depth of cut corresponding to the $i$th cutting blade, and $n$ is the number of blades. One of the blades is considered the first blade (arbitrarily), and the other blades are numbered according to their angular distance from the first blade as follows (the angles are measured in the positive angular direction):

$$\alpha_1 = 0 < \alpha_2 < ... < \alpha_n < 2\pi. \tag{8}$$

Assuming that the first blade has zero angular position at time $t = 0$, the angular position of the $i$th cutting blade at time $t$ is given by

$$\Phi_i(t) = Rem(\Phi_b(t) + \alpha_i, 2\pi) \quad i = 1, 2, ..., n. \tag{9}$$

The function $Rem(a, b)$ is the remainder of $a$ divided by $b$. The remainder function is employed in (9) to define the angular position of the blades in the interval $[0, 2\pi)$. If the well surface pattern i.e., the depth of the well surface as a function of fixed polar coordinates, is known, the depth of cut for each cutting blade can be obtained by knowing the axial position of the bit.

As shown in Fig. 2, the $i$th blade depth of cut is obtained as follows:

$$d_i(t) = \lim_{\varepsilon \to 0^+} \Big(U_b(t) - P\big(\Phi_i(t) + \varepsilon\big)\Big)\mathbf{H}\Big(U_b(t) - P\big(\Phi_i(t) + \varepsilon\big)\Big), \tag{10}$$

where $P(\theta)$ represents the well surface depth at the azimuth angle $\theta$ in a cylindrical coordinate attached to the ground. The Heaviside function $\mathbf{H}(.)$ indicates that if the axial penetration of the blade is smaller than the well depth, the blade does not penetrate to the formation, and the depth of cut corresponding to such blade equals zero. In other words, when the $i$th blade crosses the angular position $\Phi_i$, there are two posibilities:

1. The blade axial position is less than the well depth at $\Phi_i$, the blade does not penetrate to the formation and the well pattern remains unchanged.
2. The blade is penetrated to the formation and imposes its axial position on the well surface at the radial plane $\theta = \Phi_i$.

Accordingly, the following implicit relation governs the well-trajectory time evolution:

$$P\big(\Phi_i(t)\big) = max\Big(U_b(t), P\big(\Phi_i(t)\big)\Big), \quad i = 1, 2, ..., n. \tag{11}$$

The general idea behind the proposed model is as follows. If the cutting is in progress and the bit penetrates the formation, the well surface depth behind the cutting blade is equal to the axial position of the blade. Otherwise, the depth of cut is zero, and the well surface pattern remains unchanged. Accordingly, the depth of cut can be obtained from the well surface function, like the approach in [14]. Note that, here, unlike [14], the formulation is not in the differential framework. This enables the model to capture the cases with discontinuous and non-differentiable well surface, e.g., when the torsional stick and bit-bounce occur simultaneously. For more insight, see Fig. 3, which illustrates the discontinuity of the well surface trajectory in this case. First, the bit sticks according to insufficient operating torque to overcome the Torque On Bit (TOB). Then, the bit bounces which reduces the depth of cut, and eventually, the TOB becomes less than the operating torque, and the bit torsionally slips. Summarizing, the total dynamics of the drill-string is governed

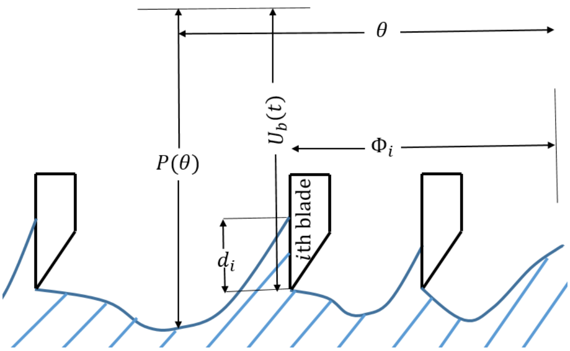
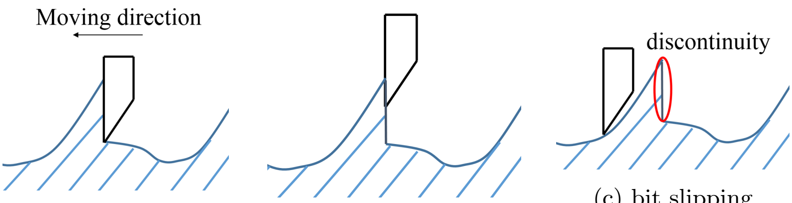



**Fig. 2**: A planar representation of a portion of the full cut surface by unfolding the cylinder.




(a) bit sticking (b) bit bouncing (c) bit slipping

**Fig. 3**: Discontinuous cut pattern.

by (1), (3) with the TOB and the Weight On Bit (WOB) obtained by (5) and (6) with the depth of cut (7), (10) obtained by the well depth function $P(\theta)$ which is evolving according to (11).

Compared to the model presented in [14], the proposed model in this paper has some advantages; first, discontinuous and non-differentiable well surface patterns can be modeled, second, bits with non-uniform blade arrangement are in the scope of this work, third, reverse rotation of the bit can be captured, fourth, the computation burden corresponding to the well surface pattern evolution is less since there is no need to solve a PDE. On the other hand, compared to the model presented in [20], multiple-regenerative effects are taken into account, and the presented model is more reliable in the presence of bit-bounce and reverse rotation of the bit.

# 3 Results and discussion

Simulation results for the proposed distributed model (1), (3), (5), (6) with the new depth of cut calculation method (7), (10) are presented. The parameter values used for the simulations are given in Table 1. In order to present the time-domain simulations of the drilling system with the proposed bit-rock interaction law, the continuous azimuth angle $\theta \in [0, 2\pi)$ should be discretized, as well as the operating time $t$. Accordingly, $n_\theta$ nodes representing the azimuth angle in the interval $[0, \frac{2\pi}{n})$ are defined. Subsequently, the rock surface depth is discretized and represented by $n_\theta$ variables corresponding to the discretized nodes as follows:

$$p_k := P(\theta), \quad k = [\frac{\theta}{2\pi} n_\theta] + 1, \quad \theta \in [0, 2\pi). \tag{12}$$

As a result, the discretized form of the cut pattern evolution equation (11) is given by

$$p_k = max(U_b(t), p_k), \quad k = [\frac{\Phi_i}{2\pi} n_\theta] + 1, \tag{13}$$

which indicates that although $n_\theta$ states represent the rock surface depth in the discretized coordinate, only $n$ operation is carried out according to these

$n_\theta$ states ($n << n_\theta$ since $n$ is the number of the blades and $n_\theta$ is the number of discretized nodes in the azimuth direction).

In comparison with the discretized form of a PDE, which demands $n_\theta$ operations in each solving step, the proposed method reduces the computational burden significantly. Now, consider a bit with four blades, as illustrated in Fig. 4, distributed non-uniformly with the following blade angles:

$$\alpha_2 = \frac{\pi}{3},\ \alpha_3 = \pi,\ \alpha_4 = \frac{4\pi}{3}. \tag{14}$$

In each step, the depth in four discretized nodes corresponding to the four cutting blades may be updated with the following time evolution relation:

$$p_{k_1} = max(U_b(t), p_{k_1}), \quad k_1 = [\frac{\Phi_1}{2\pi}100] + 1, \tag{15a}$$
$$p_{k_2} = max(U_b(t), p_{k_2}), \quad k_2 = [\frac{\Phi_2}{2\pi}100] + 1, \tag{15b}$$
$$p_{k_3} = max(U_b(t), p_{k_3}), \quad k_3 = [\frac{\Phi_3}{2\pi}100] + 1, \tag{15c}$$
$$p_{k_4} = max(U_b(t), p_{k_4}), \quad k_4 = [\frac{\Phi_4}{2\pi}100] + 1, \tag{15d}$$

with

$$\Phi_1 = Rem(\Phi_b(t), 2\pi), \tag{16a}$$
$$\Phi_2 = Rem(\Phi_b(t) + \alpha_2, 2\pi), \tag{16b}$$
$$\Phi_3 = Rem(\Phi_b(t) + \alpha_3, 2\pi), \tag{16c}$$
$$\Phi_4 = Rem(\Phi_b(t) + \alpha_4, 2\pi). \tag{16d}$$

The depth of cut corresponding to each blade is obtained as

$$d_1 = max(U_b(t) - p_{k_1+1})H(U_b(t) - p_{k_1+1}), \quad k_1 = [\frac{\Phi_1}{2\pi}100] + 1, \tag{17a}$$
$$d_2 = max(U_b(t) - p_{k_2+1})H(U_b(t) - p_{k_2+1}), \quad k_2 = [\frac{\Phi_2}{2\pi}100] + 1, \tag{17b}$$
$$d_3 = max(U_b(t) - p_{k_3+1})H(U_b(t) - p_{k_3+1}), \quad k_3 = [\frac{\Phi_3}{2\pi}100] + 1, \tag{17c}$$

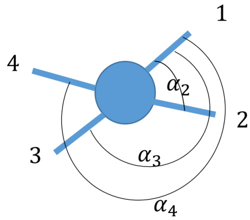



**Fig. 4**: Top view of the bit with four non-uniformly distributed blades.

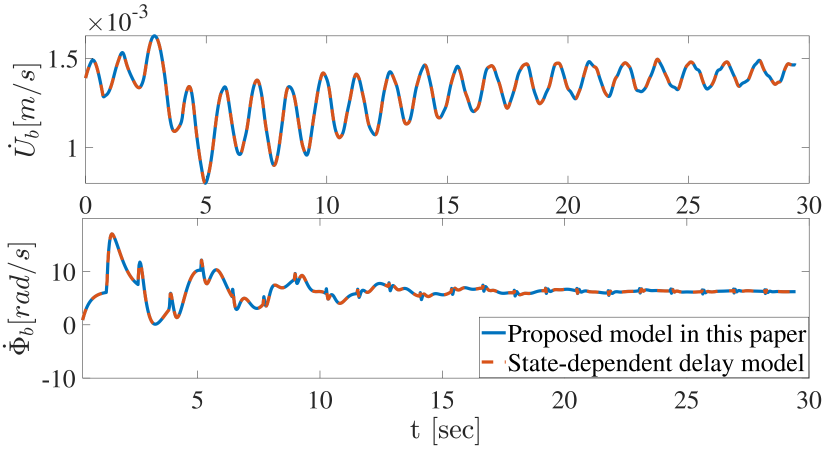



**Fig. 5**: A comparison between the dynamical behavior of a bit with four non-uniformly distributed blades predicted by our model and predicted by the state-dependent delay model presented in [20]. The nominal operating conditions are $\Omega_0 = 6.28\,rad/s$ ($60\,rpm$) and $V_0 = 0.0014\,m/s$ ($5\,m/h$), and the rock intrinsic specific energy is $\epsilon = 8Mpa$.

$$d_4 = max(U_b(t) - p_{k_4+1})H(U_b(t) - p_{k_4+1}), \quad k_4 = [\frac{\Phi_4}{2\pi}100] + 1. \tag{17d}$$

Eventually, the total depth of cut is the summation of these depths, as follows:

$$d(t) = d_1 + d_2 + d_3 + d_4. \tag{18}$$

In the following, the aforementioned non-uniform bit is used in the simulations.

Figure 5 compares the behavior of the non-uniform bit modelled by the state-dependent delay model presented in [20] and the proposed model in this paper. In this figure, the nominal velocities and the rock specific energy are considered $\Omega_0 = 6.28\,rad/s$ ($60\,rpm$), $V_0 = 0.0014\,m/s$ ($5\,m/h$), and $\epsilon = 8Mp$, respectively. As can be seen, both the axial and torsional velocities are positive in the whole time domain. Hence, bit-bounce and reverse rotation do not occur during this operation, and there are no multiple-regenerative effects. Consequently, both models predict precisely the same behavior for the drill-string. In this operating condition, the hardness of the formation is not high enough to stubbornly resist the cutting process and impose severe vibrations on the drilling bit. Moreover, the bit is penetrating with a moderate penetration rate in order to avoid severe vibrations. As a result, the vibration amplitude is low, which prevents the multiple-regenerative effects (that invalidate the state-dependent delay model).

To observe the multiple-regenerative effects, a more challenging case is considered: drilling a stiffer formation with a faster rate of penetration. The simulation results for the case with $\Omega_0 = 6.28\,rad/s$ ($60\,rpm$) and with

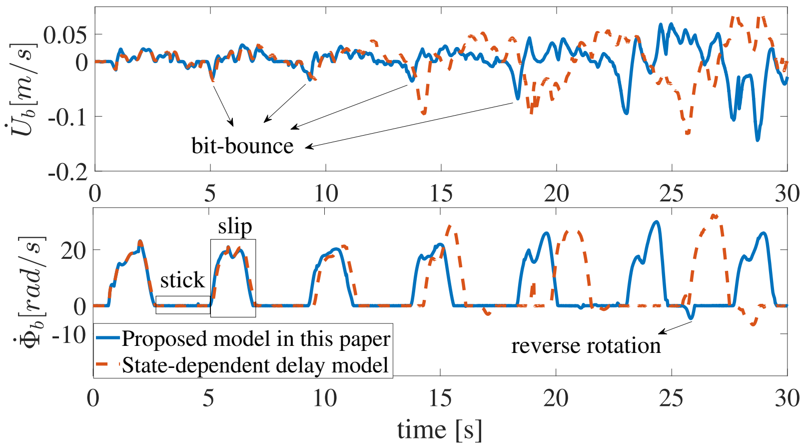



**Fig. 6**: A comparison between the dynamical behavior of a bit with four non-uniformly distributed blades predicted by our model and predicted by the state-dependent delay model presented in [20]. The nominal operating conditions are $\Omega_0 = 6.28\, rad/s$ ($60\, rpm$) and $V_0 = 0.0028\, m/s$ ($10\, m/h$), and the rock intrinsic specific energy is $\epsilon = 120Mpa$.

$V_0 = 0.0028\, m/s$ ($10\, m/h$), and $\epsilon = 120Mpa$ are depicted in Fig. 6. Severe torsional vibrations cause stick-slip phenomena, leading more than half of the operating time to the stick phase. For example, the bit is stuck to the formation between the times $7.5\, s$ and $10\, s$. At this time, on the other side of the

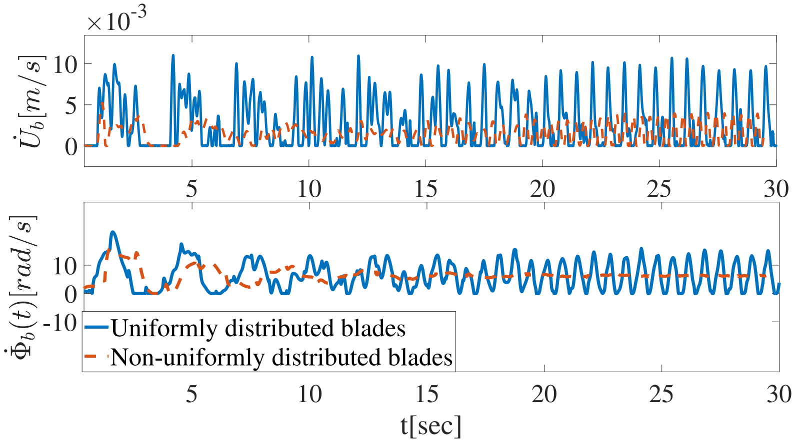



**Fig. 7**: A comparison between the dynamical behavior of a bit with four uniformly distributed blades and a bit with four non-uniformly distributed blades. The nominal operating conditions are $\Omega_0 = 6.28\, rad/s$ ($60\, rpm$) and $V_0 = 0.0014\, m/s$ ($5\, m/h$), and the rock intrinsic specific energy is $\epsilon = 60Mpa$.

drill-string, the rotary table is rotating and increasing the elastic energy of the string by twisting it. At the time $10\,s$, the stored elastic energy becomes sufficient to overcome the formation, the bit accelerates (torsionally), and the slip phase starts. The result of this sudden energy release is an overshoot in the angular velocity of the bit. It is seen that after one second, at the time $11\ s$, the angular velocity becomes $20\,rad/s$, which is more than two times greater than the nominal angular velocity ($6.28 rad/s$). On the other hand, in the axial direction, bit-bounce occurs frequently. In the beginning, before $t = 15\ s$, at $t = 5.8\ s$, $t = 10\ s$, and $t = 14.2\ s$ the bit-bounce occurs in very short time intervals. But, after $t = 15s$, the bit-bounce occurs more severely with longer time intervals. Subsequently, multiple-regenerative effects frequently take place making the difference between the predictions of the two models significant. More severely, even bit reverse rotation occurs at $t = 26.4s$. Note that the simulation results of the model presented in [14] diverge since the well-trajectory becomes discontinuous when the reverse rotation occurs. In this case, differentiating the well-trajectory for the governing PDE give infinite values. These simulation results, which are presented for practical field condition, illustrates the necessity of using the proposed model in this paper that captures the reverse rotation phenomenon as well as the bit bouncing.

To show the effectiveness of non-uniformly arranging the cutting blades, a comparison between the behavior of the drill-string equipped with the non-uniform bit and a drill-string equipped with a bit with four uniformly distributed blades is presented in Fig. 7. In this case, the non-uniform arrangement of the cutting blades reduces the amplitude of unwanted vibrations significantly. In order to make a quantitative comparison between the results, the deviation of the axial and angular velocities from the nominal values is computed, and the pick values in the time interval between $t = 20s$ and $t = 30s$ are compared for the two models. For the bit with uniformly distributed blades, the maximum deviation of the axial velocity from the nominal value is $0.0079\,m/s$, which occurs at $t = 25.53\,s$. On the other hand, for the bit with non-uniformly distributed blades, the maximum deviation occurs when the bit axially sticks (this takes place in several time intervals between $t = 20\ s$ and $t = 30\ s$), and its value is equal to the nominal axial velocity which is $0.0028\,m/s$. For the torsional dynamics, the maximum deviation of the angular velocity from the nominal value is $9.64\,rad/s$ (occurring at $t = 24.13\ s$), for the bit with uniformly distributed blades, and $0.82\,rad/s$ (occurring at $t = 20.53\,s$) for the bit with non-uniformly distributed blades. Accordingly, the axial vibration amplitude is reduced by more than 64 percent and the torsional one is reduced by more than 90 percent (in the sense of the above quantitative criterion).

# 4 Conclusions

In this paper, coupled axial-torsional dynamics of a distributed drill-string with a bit equipped with non-uniformly distributed blades has been studied. A modified model has been proposed for the calculation of the depth of cut for a bit with non-uniformly distributed cutting blades. The cut-surface profile has been determined by providing an implicit function of the bit torsional and axial trajectories. This modification enables the model to capture the multiple-regenerative effects caused by the bit reverse rotation in addition to the bit bouncing. The proposed model is illustratively compared with other models in the literature by employing a distributed drill-string model in terms of neutral-type delay differential equations. The simulation results illustrate the existence of the bit-bounce in a wide range of operating conditions, which indicates the necessity of considering multiple-regenerative effects. Moreover, it is illustrated that non-uniformly arranging the cutting blades can mitigate the unwanted vibrations. Future work under consideration is using the proposed model in this paper for dynamical analysis and parametric design of the drill-string with the aim of reducing undesirable vibrations.

# References

[1] E. Detournay, P. Defourny, A phenomenological model for the drilling action of drag bits, in: International journal of rock mechanics and mining sciences & geomechanics abstracts, volume 29, Elsevier, 1992, pp. 13–23.

[2] M. Zhao, B. Balachandran, Dynamics and stability of milling process, International journal of solids and structures 38 (2001) 2233–2248.

[3] T. Richard, C. Germay, E. Detournay, A simplified model to explore the root cause of stick–slip vibrations in drilling systems with drag bits, Journal of sound and vibration 305 (2007) 432–456.

[4] X. Liu, N. Vlajic, X. Long, G. Meng, B. Balachandran, Coupled axial-torsional dynamics in rotary drilling with state-dependent delay: stability and control, Nonlinear Dynamics 78 (2014) 1891–1906.

[5] X. Zheng, B. Balachandran, State-dependent delay and drill-string dynamics, Procedia IUTAM 22 (2017) 31–38.

[6] X. Liu, N. Vlajic, X. Long, G. Meng, B. Balachandran, Nonlinear motions of a flexible rotor with a drill bit: stick-slip and delay effects, Nonlinear Dynamics 72 (2013) 61–77.

[7] X. Liu, N. Vlajic, X. Long, G. Meng, B. Balachandran, State-dependent delay influenced drill-string oscillations and stability analysis, Journal of Vibration and Acoustics 136 (2014).

[8] B. Besselink, T. Vromen, N. Kremers, N. Van De Wouw, Analysis and control of stick-slip oscillations in drilling systems, IEEE transactions on control systems technology 24 (2015) 1582–1593.

[9] U. J. F. Aarsnes, N. van de Wouw, Dynamics of a distributed drill string system: Characteristic parameters and stability maps, Journal of Sound and Vibration 417 (2018) 376–412.

[10] S. K. Gupta, P. Wahi, Criticality of bifurcation in the tuned axial–torsional rotary drilling model, Nonlinear Dynamics 91 (2018) 113–130.

[11] S. Tashakori, G. Vossoughi, H. Zohoor, N. van de Wouw, Suppression of axial-torsional vibrations in drilling system described by neutral-type delay differential equations, in: NODYCON 2019: First International Nonlinear Dynamics Conference, 2019.

[12] M. A. Faghihi, S. Tashakori, E. A. Yazdi, M. Eghtesad, N. van de Wouw, Suppression of axial-torsional vibrations of a distributed drilling system by the eigenvector contradiction method, IFAC-PapersOnLine 55 (2022) 109–114.

[13] X. Liu, N. Vlajic, X. Long, G. Meng, B. Balachandran, Multiple regenerative effects in cutting process and nonlinear oscillations, International Journal of Dynamics and Control 2 (2014) 86–101.

[14] S. K. Gupta, P. Wahi, Global axial–torsional dynamics during rotary drilling, Journal of Sound and Vibration 375 (2016) 332–352.

[15] X. Liu, X. Long, X. Zheng, G. Meng, B. Balachandran, Spatial-temporal dynamics of a drill string with complex time-delay effects: bit bounce and stick-slip oscillations, International Journal of Mechanical Sciences 170 (2020) 105338.

[16] M. Wiercigroch, K. Nandakumar, L. Pei, M. Kapitaniak, V. Vaziri, State dependent delayed drill-string vibration: theory, experiments and new model, Procedia IUTAM 22 (2017) 39–50.

[17] Y. Yan, M. Wiercigroch, Dynamics of rotary drilling with non-uniformly distributed blades, International Journal of Mechanical Sciences 160 (2019) 270–281.

[18] J. Niu, Y. Ding, L. Zhu, H. Ding, Mechanics and multi-regenerative stability of variable pitch and variable helix milling tools considering runout, International Journal of Machine Tools and Manufacture 123 (2017) 129–145.

[19] B. Balachandran, M. Zhao, A mechanics based model for study of dynamics of milling operations, Meccanica 35 (2000) 89–109.

[20] K. Tian, R. Ganesh, E. Detournay, Influence of bit design on the stability of a rotary drilling system, Nonlinear Dynamics 100 (2020) 51–75.

[21] K. Tian, E. Detournay, Influence of pdc bit cutter layout on stick–slip vibrations of deep drilling systems, Journal of Petroleum Science and Engineering 206 (2021) 109005.

[22] K. Tian, E. Detournay, H. Zhang, An alternative formulation for modeling self-excited vibrations of drillstring with pdc bits, in: International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, volume 85468, American Society of Mechanical Engineers, 2021, p. V009T09A047.

[23] K. Tian, E. Detournay, H. Zhang, An alternative formulation for modeling self-excited vibrations of drillstring with polycrystalline diamond compact bits, Journal of Computational and Nonlinear Dynamics 17 (2022) 051002.

[24] M. A. Faghihi, S. Tashakori, E. A. Yazdi, M. Eghtesad, Multiple regenerative effects of the bit-rock interaction in a distributed drill-string system, in: International Conference on Wave Mechanics and Vibrations, Springer, 2023, pp. 491–500.

[25] R. Wildemans, A. Aribowo, E. Detournay, N. van de Wouw, Modelling and dynamic analysis of an anti-stall tool in a drilling system including spatial friction, Nonlinear Dynamics 98 (2019) 2631–2650.

[26] J. Bailey, I. Finnie, An analytical study of drill-string vibration (1960).

[27] I. Finnie, J. Bailey, An experimental study of drill-string vibration (1960).

[28] B. Saldivar, S. Mondié, Drilling vibration reduction via attractive ellipsoid method, Journal of the Franklin Institute 350 (2013) 485–502.

[29] B. Saldivar, S. Mondié, J. J. Loiseau, V. Rasvan, Suppressing axial-torsional coupled vibrations in drillstrings, Journal of Control Engineering and Applied Informatics 15 (2013) 3–10.

[30] S. Tashakori, G. Vossoughi, H. Zohoor, N. van de Wouw, Prediction-based control for mitigation of axial–torsional vibrations in a distributed drill-string system, IEEE Transactions on Control Systems Technology 30 (2021) 277–293.

[31] S. Tashakori, G. Vossoughi, H. Zohoor, E. A. Yazdi, Modification of the infinite-dimensional neutral-type time-delay dynamic model for the coupled axial–torsional vibrations in drill strings with a drag bit, Journal of Computational and Nonlinear Dynamics 15 (2020).