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Non Deterministic Zero Divisor Graph

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Abstract

A non-deterministic zero divisor graph refers to an element in a ring or algebraic structure that can multiply with another element to give zero, but the specific outcome of the multiplication is not uniquely determined. In other words, there may be multiple elements that can multiply with the given element to produce zero. This concept is typically encountered in non-commutative rings or algebras where the order of multiplication matters. In such structures, the existence of non-deterministic zero divisors can complicate calculations and lead to different results depending on the order of operations. In this paper, we examine that all the zerodivisor graphs are nondeterministic graph but the converse need not. We manifest the nondeterministic zero divisor graph is possible only with nonprimes. Hereby, we study about weighted graphs, Weiner index and golden ratio rule. Also we provide an algorithm for zero divisor graph of n parameters and thereby, explore the given graph is either deterministic or nondeterministic graph using python.

Keywords: Zero divisor graphs, Non deterministic graph, Weiner index, Weighted graph, Golden ratio rule.
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1 Introduction

The concept of zero divisor graph was first introduced by Beck in 1988 [2]. Anderson and Livingston [1] were the first who simplify Beck's zero divisor graph. In this paper, we find the zero divisors Z_p and Z_q, where p and q are either prime or composite. In 2002, Redmond [11] extended the definition of zero divisor graph to the ring. In 2005, Costa and Faridi [3] introduced Zero-divisor graphs of noncommutative rings can have various structures and properties. They may be either connected or disconnected or have specific coloring properties. In 2018, Fan and Liu [4] suggested the zero-divisor graphs from a commutative ring theory perspective.

Deterministic and nondeterministic graphs are mathematical structures used in computer science to model systems that have a finite number of states and transitions between those states. These graphs are used in a wide range of applications, including natural language processing, machine learning, and computer vision. The concept of deterministic graphs was introduced by Stephen Kleene in the 1950s, as part of his work on regular expressions and finite automata. A deterministic graph is a graph in which each state has exactly one transition for each input symbol. This means that, given a current state and an input symbol, there is only one possible next state. In the 1960 s, researchers such as Michael O. Rabin and Dana S. Scott [10] developed the concept of nondeterministic finite automata (NFA), which are more powerful than deterministic finite automata (DFA) because they allow for multiple possible transitions from one state to another. This led to the development of algorithms for converting NFA into DFA, which are more efficient to work with.

A nondeterministic graph is a graph in which each state may have multiple transitions for the same input symbol. This means that, given a current state and an input symbol, there may be multiple possible next states. Nondeterministic graphs are more expressive than deterministic graphs, but they are also more difficult to work with computationally.

Stuart A. Kurtz [9] wrote a paper in 1973 titled "Deterministic and Nondeterministic Graphs," which presented a unified framework for deterministic and nondeterministic graphs. This paper provided a foundation for further research on the topic and helped to establish the importance of deterministic and nondeterministic graphs in computer science.

Today, deterministic and nondeterministic graphs are important concepts in computer science and are used in a wide range of applications. Researchers continue to explore new algorithms and applications for these graphs, making them an active area of research in computer science.

Throughout this paper, we consider the zerodivisor graph z_n.

In section 2, the basic definition and preliminary results of the contents are provided. In section 3, we examined that the nondeterministic zero divisor graph with suitable illustrations. In section 4, we provide an algorithm for zero divisor graph at n parameters and verify the given graph is either deterministic or non deterministic graph using python.

2 Preliminaries

We will step over some fundamental definitions in this section that are related to our main concept.

Definition 2.1. [1] Let z_n be graph with a vertex set v. The two distinct vertices v_1 and v_2 are adjacent if and only if $v_1v_2 = 0$. It is said to be a *zero divisor graph.*

Definition 2.2. [1] *Incidence matrix* is a two-dimensional Boolean matrix, in which the rows represent the vertices and columns represent the edges. The entries indicate whether the vertex at a row is incident to the edge at a column. Incidence matrix is one of the ways to represent a graph.

Definition 2.3. [9] A *non-deterministic graph* is a mathematical representation of a system in which the outcome of certain actions or events is not uniquely determined. In a non-deterministic graph, multiple valid paths or results may exist.

Definition 2.4. [11] The *Wiener index* (W) is a graph invariant in mathematics that is defined as the sum of the distances between all pairs of vertices in a graph.

Definition 2.5. [6] A *weighted graph* is a type of graph where a numerical value, called a weight, is assigned to each edge in the graph.

Definition 2.6. [6] The *golden ratio rule* is a design principle that uses the mathematical ratio of approximately 1.618 to create aesthetically pleasing and balanced compositions, by dividing a line or shape into two unequal parts in a specific way.

3 Non deterministic zero divisor graph

An element in a ring or other mathematical structure that can multiply with another component to yield zero is also referred to as a non-deterministic zero divisor graph because the exact outcome of the multiplication cannot be determined in advance. The order of multiplication matters in non-commutative rings or algebras, wherever this topic is frequently discussed. The existence of non-deterministic zero divisor graphs in such structures can render calculations more difficult and generate different outcomes depending on the order in which operations are performed.

Definition 3.1. Let z_n be a zero divisor graph, represented by a directed graph with vertices V and edges E. z_n is said to be *non deterministic zero divisor graph* if, for every vertex set $S \subseteq V$, there exists a specific set of choices C such that the outcome of certain actions or events is not uniquely determined.

Lemma 3.1. *If n is nonprime, then z_n is non deterministic zero divisor graph.*

Proof. Let $n = p_1, p_2, \ldots\ldots, p_n$ are distinct non primes. Then the zerodivisor z_n can be partitioned into disjoint sets. Then, $|A_1|$ = (P_2 - 1)(P_3 - 1), $|A_2|$ = (P_1 - 1)(P_3 - 1), $|A_3|$ = (P_3 - 1)(P_2 - 1),$|A_4|$ = P_3 - 2,$|A_5|$ = (P_5 - 1)(P_4 - 2). Hence, $|V|$ $\neq \sum_{i=1}^{n}|A_i|$. The nonprime numbers of zero divisor in which every edges are not uniquely determined. □

Example 3.1 Let n=15 be non prime. The zero divisor graph of z_{15} is as follows:

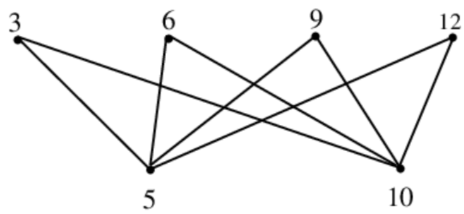

Fig 3.1 : z_{15}

The graph is nondeterministic. Therefore, z_{15} is nondeterministic zerodivisor graph.

Lemma 3.2. *If z_n is nondeterministic zero divisor graph, then n is nonprime.*

Proof. Let z_n be a non deteministic zero divisor graph. The incidence matrix M is defined as:

$$M = [m_{ij}] = \begin{cases} 1, & \text{if the } i^{th} \text{ vertex is incident to the } j^{th} \text{ edge,} \\ 0, & \text{otherwise.} \end{cases}$$

By using weiner index of incidence matrix, we concluded that n is non prime. □

Example:3.2 The incidence matrix for z_{15} is,

$$M=\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Here, the Weiner index(W) is 14. Hence,it is nonprime.

Theorem 3.1. *If z_n is nondeterministic zero divisor graph iff n is nonprime.*

Proof. Clearly,from the above lemmas. □

Theorem 3.2. *If n is nonprime iff z_n is nondeterministic zerodivisor graph.*

Proof. Let n be nonprimes. The fibonacci sequence is the sequence formed by $z(n)^2 = 2z(n-1)^2 + 2z(n-2)^2 - z(n-3)^2$. By using, golden ratio rule, z_n=$\emptyset^n(1-\emptyset)^n/\sqrt{5}$ (where z_n denotes the the sequence) n denotes the number of the sequence. $\emptyset\rightarrow$ Golden ratio. The value of $\emptyset = 1.618$. Hence, z_n is nondeterministic zerodivisor graph. Conversely, Let z_n be a nondeterministic zerodivisor graph. Let us take a multiple of two vertices which are adjacent to be the weighted graph. Clearly, the sum of weighted graph is nonprime. □

Example:3.2
To find the 9^{th} Fibonacci number using the golden ratio formula, we can substitute n = 9 into the formula: F_n = ($\emptyset^n$ - (1-$\emptyset$)^n) / sqrt(5).
where $\emptyset$ (phi) is the golden ratio, which is approximately equal to 1.61803398875.
Thus, we have: F_9 = ($\emptyset^9$ - (1-$\emptyset$)^9) / sqrt(5) = (34.091 - 0.188) / sqrt(5) = 21.000
Therefore, the 9^{th} Fibonacci number is 21, which is the same as the value obtained by the recursive or iterative methods.

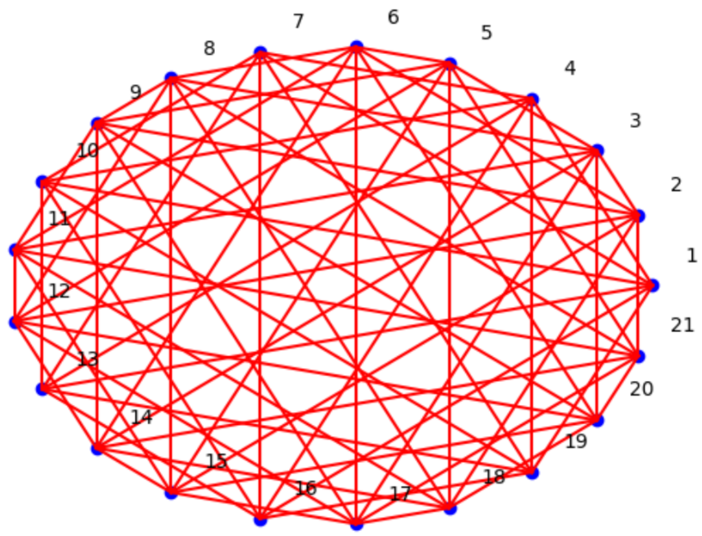

Fig 3.2 : z_{21}

Additionally, we have used the nondeterministic zerodivisor to demonstrate that n is not prime. The result of the figure is depicted above.
In fig 3.3, the multiple of two vertices and noted as weighted graph. The sum of

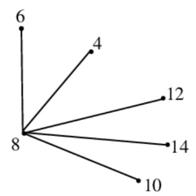

Fig 3.3 : z_{16}

weighted graph is nonprime.

4 Algorithm

4.1 The algorithm for zero divisor graph for a given n parameters

In this section, we provide an algorithm for zero divisor graph using the matplotlib library in Python(pseudocodes)
1. Initializing num-values, as any non prime number
2. Define the nodes and their positions
nodes = list(range(1, num-nodes+1))
theta = evenly spaced values between(0, 2 * Pi, num-nodes, excluding endpoint)
3.calculate positions using polar coordinates(theta)
4.Define the edges and their weights
edges = []
weights = []
for i = 0 to num-nodes - 1 do:
for j = i + 1 to num-nodes - 1 do:
if (i + j) add edge(edges, i + 1, j + 1)
add weight(weights, 0)
5.Draw the nodes
figure, axis = create figure and axis()
for each node, position in zip(nodes, positions) do:
plot node(axis, position)
6. Draw the edges representing zero divisors in red
for each edge, weight in zip(edges, weights) do:
if weight == 0 then:
node1 = positions[find index of node(nodes, edge[0])]
node2 = positions[find index of node(nodes, edge[1])]
plot edge(axis, node1, node2, 'red')
7. Show the graph
turn off axis(axis)
show figure(figure)

4.2 Manifest the graph is either deterministic or nondeterministic graph

In this section, we provide an algorithm to explore the given graph is either deterministic or nondeterministic graph using python(pseudocodes).
1. Initializing the states, alphabets and transitions.

2. Assigning the set of initial states and accepting states
3. Define a function to stimulate the automaton
function simulate(inputString):
current states $<$ set(initial states)
next states $<$ set()
for each character in input string:
for each state in current states:
if (state, character) is a key in transitions:
next states $<$ next states union transitions[(state, character)]
current states $<$ next states
next states $<$ set()
4. Test the automaton for each state in current States
current states $<$ set(initial states)
next states $<$ set()
print(simulate('ab')) :"true"
print(simulate('bab')) :"true"
print(simulate('aabb')) :"false"
print(simulate('bbab')) :"false"

5 Conclusions

In this paper, we introduced deterministic and nondeterministic zero divisor graph over z_n, where n is nonprime. Throughout this paper we considered the graph is zerodivisor graph z_n, where n is nonprime. We examined that the zerodivisor graph is either deterministic zerodivisor graph or nondeterministic zero divisor graph. Finally, we determined that the zero divisor graph z_n is nondeterministic onaccount of nonprime n but, the converse need not be true. We proved the above result with suitable illustrations. we provided an algorithm for constructing the zero divisor graph with n parameters and also we furnished an algorithm to manifest the given graph is either deterministic or nondeterministic graph. The implementation of the algorithm overdone in the Python programming language. The Applications of nondeterministic zero divisor graphs in Neural Network and coding theory are under our consideration.

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