A Comparative Study of Sliding-Mode-Based Control Strategies of a Quad-Rotor UAV

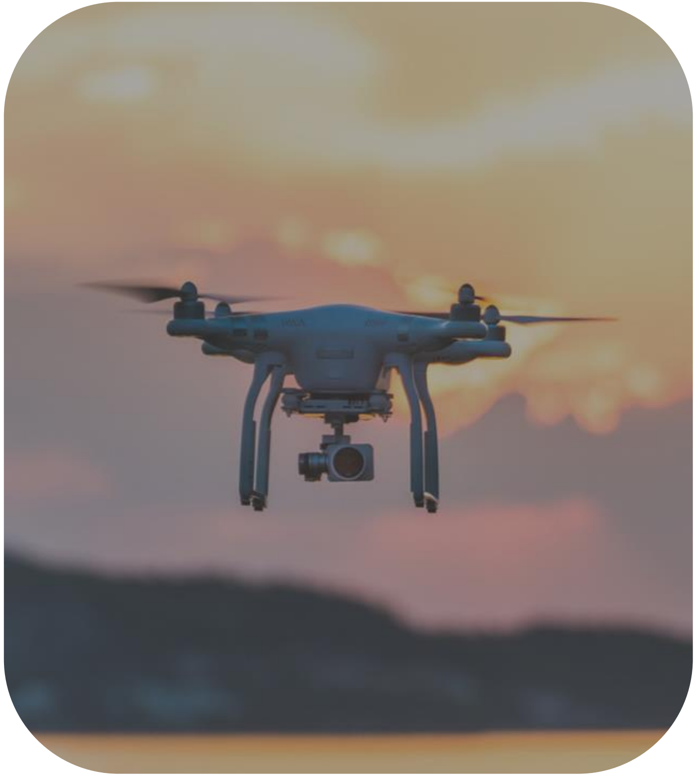
E. Paiva , H. Fretes , J. Rodas, M. Saad , Y. Kali, J.L. Lesme , F. Lesme

IECON 2021
47th Annual Conference of the
IEEE Industrial Electronics Society
OCTOBER 13 - 16, 2021 | VIRTUAL CONFERENCE

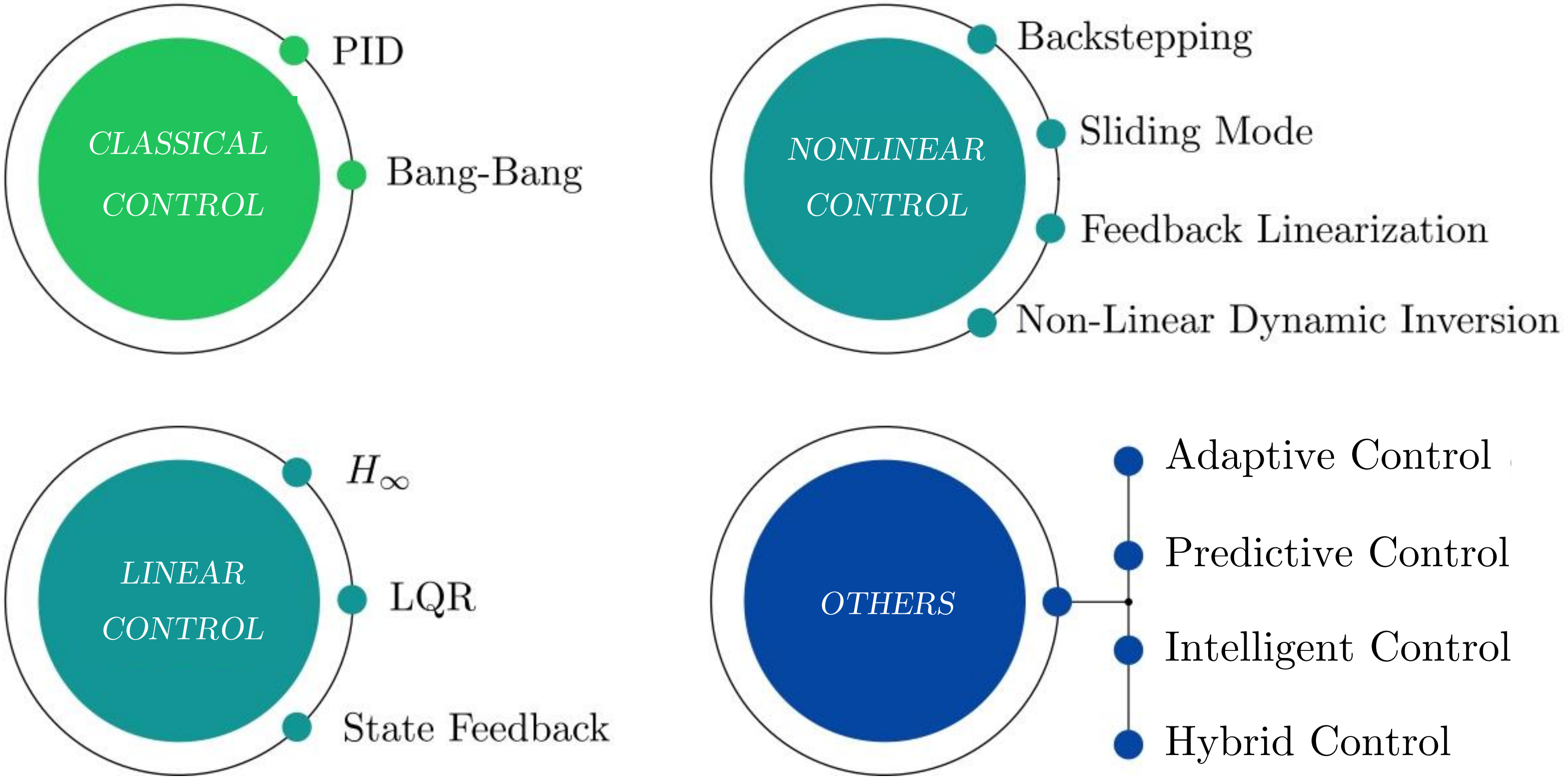
Introduction

An unmanned aerial vehicle (UAV) is an unmanned, reusable aircraft capable of autonomously maintaining a controlled and sustained level of flight.

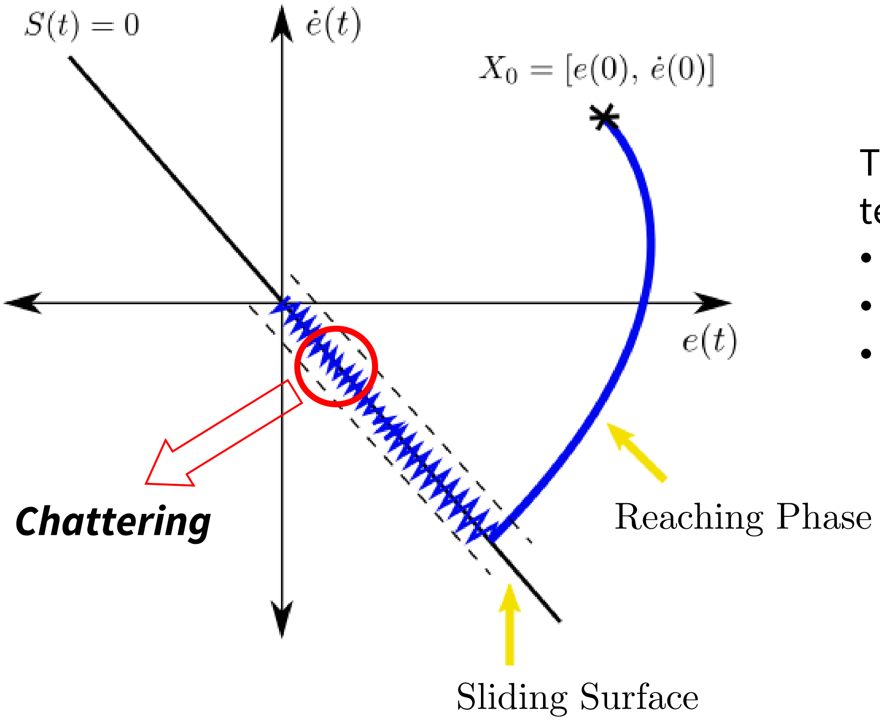
Today, there are multiple types of UAVs, which can be differentiated according to their weight, size, number of propellers, application.



Introduction



Introduction

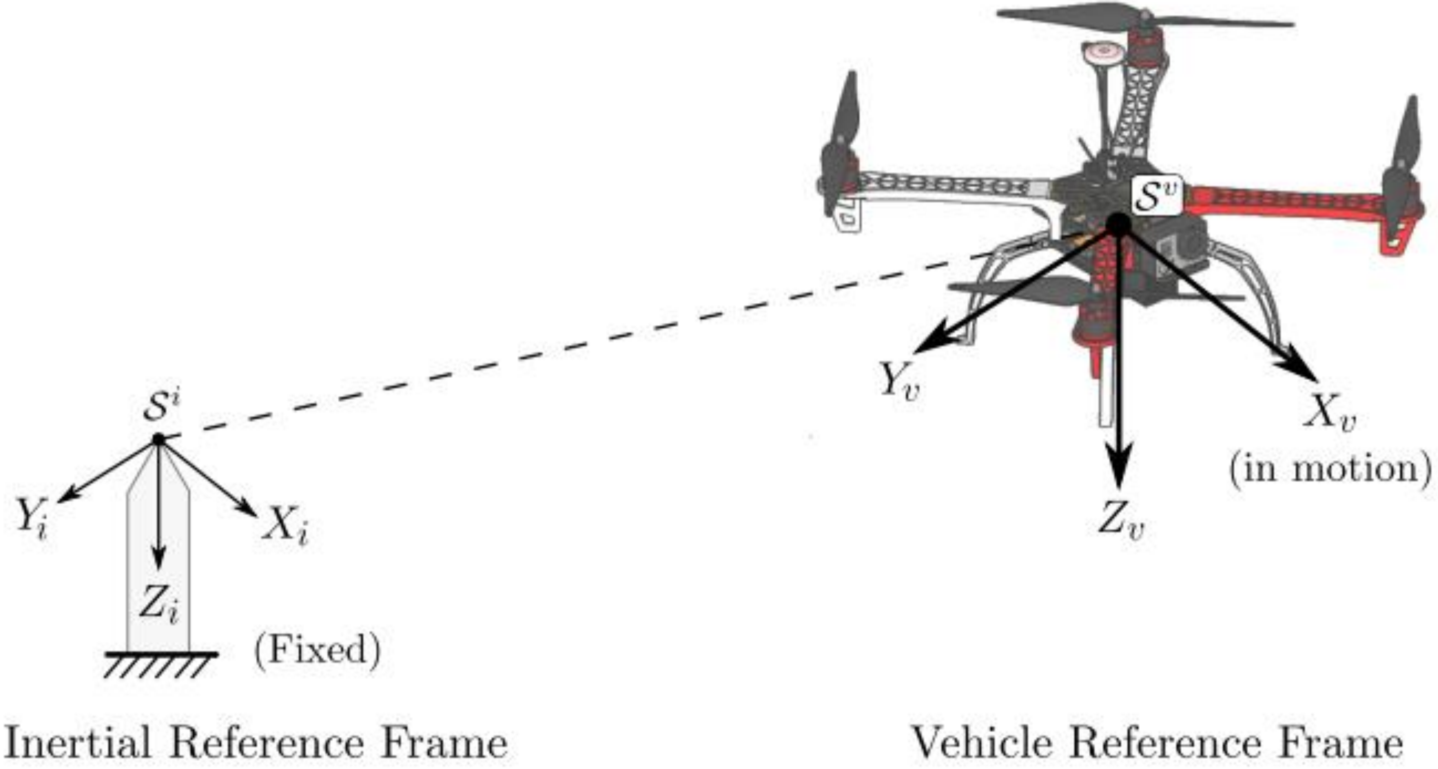


Sliding Mode Control (SM)

The Chattering effect is reduced by using different techniques:

- Non-Linear Gains
- Dynamic Extensions
- High order sliding mode controllers.

Mathematical model of the UAV



Controllers design

Mathematical structure of SM control

$$\dot{\chi}_1(t) = \chi_2(t)$$

$$\dot{\chi}_2(t) = \mathrm{F}(\chi(t)) + \mathrm{G}(\chi(t))\, u(t) + h(t)$$

where:

- $\chi(t) = [\chi_1(t), \chi_2(t)]^T$.
- $\chi_1(t)$ are the state variables.
- $\chi_2(t)$ are the derivatives of the state variables.
- $u(t)$ are the system inputs.
- $h(t)$ is the uncertainty function.
- $\mathrm{F}(\chi(t))$ is the vector of nonlinear dynamics.
- $\mathrm{G}(\chi(t))$ is the nonlinear control matrix.

Controllers design

Mathematical structure of the position.

$$u_x = \frac{\tau_1}{\mathrm{m}} \left(\sin(\phi)\sin(\psi) + \cos(\phi)\sin(\theta)\cos(\psi) \right)$$

$$u_y = \frac{\tau_1}{\mathrm{m}} \left(\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \right)$$

$$u_z = \mathrm{g} - \frac{\tau_1}{\mathrm{m}} \left(\cos(\phi)\cos(\theta) \right)$$

$$\ddot{X} = u_x + d_x$$

$$\ddot{Y} = u_y + d_y$$

$$\ddot{Z} = u_z + d_z$$

$$\mathrm{F}_P(\chi) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathrm{G}_P(\chi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

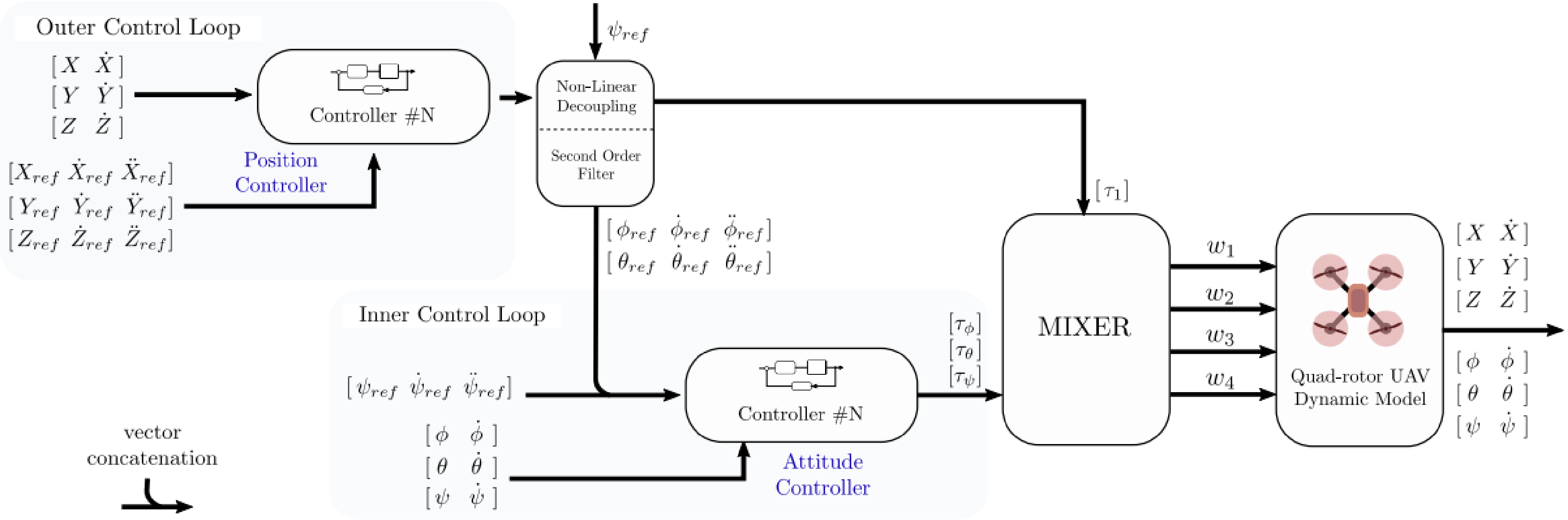
Controllers design

Mathematical structure of attitude.

$$\ddot{\phi} = \frac{\tau_\phi}{J_x} + \frac{\tau_\theta}{J_y}\sin(\phi)\tan(\theta) + \frac{\tau_\psi}{J_z}\cos(\phi)\tan(\theta) + d_\phi$$
$$\ddot{\theta} = \frac{\tau_\theta}{J_y}\cos(\phi) - \frac{\tau_\psi}{J_z}\sin(\phi) + d_\theta$$
$$\ddot{\psi} = \frac{\tau_\theta}{J_y}\sin(\phi)\sec(\theta) + \frac{\tau_\psi}{J_z}\cos(\phi)\sec(\theta) + d_\psi$$

$$\mathrm{F}_\Phi(\chi) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathrm{G}_\Phi(\chi) = \begin{bmatrix} \frac{1}{J_x} & \frac{1}{J_y}\sin(\phi)\tan(\theta) & \frac{1}{J_z}\cos(\phi)\tan(\theta) \\ 0 & \frac{1}{J_y}\cos(\phi) & -\frac{1}{J_z}\sin(\phi) \\ 0 & \frac{1}{J_y}\sin(\phi)\sec(\theta) & \frac{1}{J_z}\cos(\phi)\sec(\theta) \end{bmatrix}$$

Controllers design



Controllers design

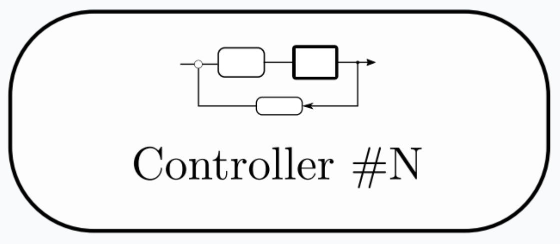
Step 1: Error calculation

$$e(t) = \chi_1(t) - \chi_{1d}(t)$$

Step 2: Sliding Surface

$$S(t) = \dot{e}(t) + \lambda_P \, e(t) + \lambda_I \int_0^t e(t)\mathrm{dt}$$

Step 3: Proposed Controllers



Controllers design

- Controller #1: Conventional SM (SMC)

$$\dot{S}_i(t) = -K_{Ci}\,\text{sign}(S_i(t)) \qquad \textit{for} \quad i = 1, 2, 3$$

- Controller #2: SM Exponential Reaching Law (SMC-ERL)

$$\dot{S}_i(t) = -\frac{K_{Ei}}{N(S_i(t))}\,\text{sign}(S_i(t)) \qquad \textit{for} \quad i = 1, 2, 3$$

$$N(S_i) = \delta_{0i} + (1 - \delta_{0i})\,\exp(-a_i\,|S_i(t)|^{p_i})$$

- Controller #3: SM Modified Super Twisting Algorithm (SMC-MST)

$$\begin{aligned} \dot{S}_i(t) &= -K_{1i}|S_i(t)|^{0.5}\,\text{sign}(S_i(t)) - K_2\,S_i(t) + \varpi_i(t) \\ \dot{\varpi}_i(t) &= -K_{3i}\,\text{sign}(S_i(t)) - K_{4i}\,\varpi_i(t) \end{aligned} \qquad \textit{for} \quad i = 1, 2, 3$$

Controllers design

Step 4: Control Effort

By applying the first derivative to the sliding surface:

$$\begin{aligned}\dot{S}(t) &= \ddot{e}(t) + \lambda_P\,\dot{e}(t) + \lambda_I\,e(t)\\ &= \dot{\chi}_2(t) - \dot{\chi}_{2d}(t) + \lambda_P\,\dot{e}(t) + \lambda_I\,e(t)\\ &= \mathrm{F}(\chi) + \mathrm{G}(\chi)\,u(t) + h(t) - \dot{\chi}_{2d}(t) + \lambda_P\,\dot{e}(t) + \lambda_I\,e(t)\end{aligned}$$

the control effort is:

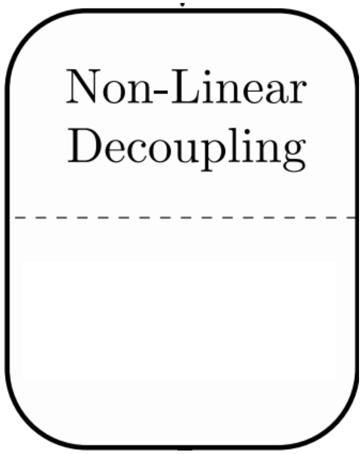
$$u(t) = \mathrm{G}(\chi)^{-1}\left[\dot{S}(t) + \dot{\chi}_{2d}(t) - \lambda_P\,\dot{e}(t) - \lambda_I\,e(t) - \mathrm{F}(\chi)\right]$$

Controllers design

$$\begin{bmatrix} \tau_1 \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \underbrace{\begin{bmatrix} C_t & C_t & C_t & C_t \\ -C_t\, l_d \frac{\sqrt{2}}{2} & C_t\, l_d \frac{\sqrt{2}}{2} & C_t\, l_d \frac{\sqrt{2}}{2} & -C_t\, l_d \frac{\sqrt{2}}{2} \\ -C_t\, l_d \frac{\sqrt{2}}{2} & -C_t\, l_d \frac{\sqrt{2}}{2} & C_t\, l_d \frac{\sqrt{2}}{2} & C_t\, l_d \frac{\sqrt{2}}{2} \\ C_d & -C_d & C_d & -C_d \end{bmatrix}}_{T} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix}$$

$$\begin{bmatrix} w_1^2 & w_2^2 & w_3^2 & w_4^2 \end{bmatrix}^T = T^{-1} \begin{bmatrix} \tau_1 & \tau_\phi & \tau_\theta & \tau_\psi \end{bmatrix}^T$$

Controllers design



$$u_x = \frac{\tau_1}{\mathrm{m}} \left(\sin(\phi)\sin(\psi) + \cos(\phi)\sin(\theta)\cos(\psi) \right)$$

$$u_y = \frac{\tau_1}{\mathrm{m}} \left(\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \right)$$

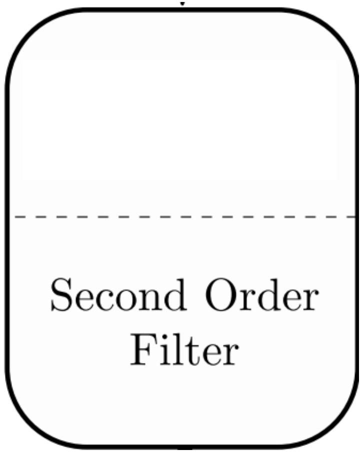
$$u_z = \mathrm{g} - \frac{\tau_1}{\mathrm{m}} \left(\cos(\phi)\cos(\theta) \right)$$

$$\tau_1 = \mathrm{m}\sqrt{u_x^2 + u_y^2 + (u_z - \mathrm{g})^2}$$

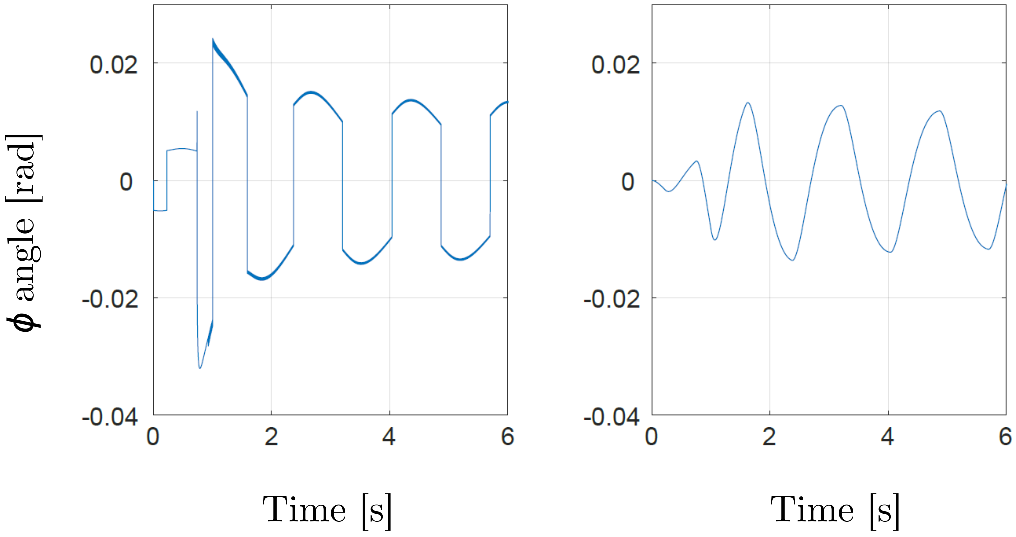
$$\phi^* = \arcsin\left(\frac{\mathrm{m}}{\tau_1}(u_x \sin(\psi^*) - u_y \cos(\psi^*))\right)$$

$$\theta^* = \arctan\left(\frac{1}{u_z - \mathrm{g}}(u_x \cos(\psi^*) + u_y \sin(\psi^*))\right)$$

Controllers design



$$H(s) = \frac{w_n^2}{s^2 + 2\,\zeta\,w_n\,s + w_n^2}$$



Results

Simulation #1: stationary flight

Simulation Conditions

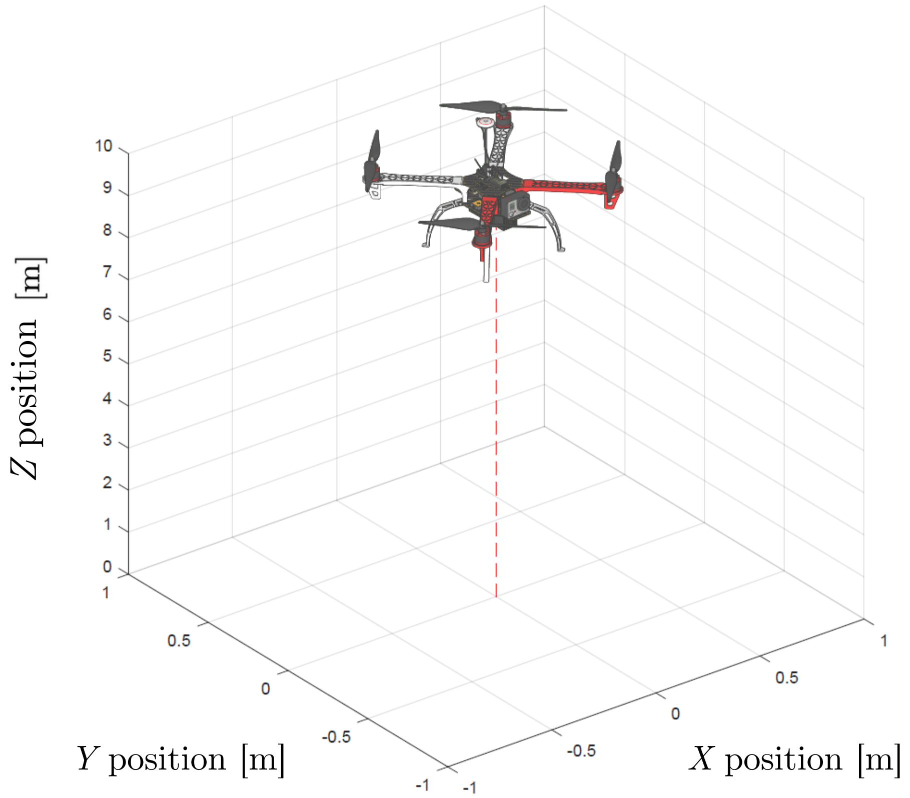
$$X_{ref} = 0\,[\mathrm{m}] \quad Y_{ref} = 0\,[\mathrm{m}]$$

$$Z_{ref} = 10\,[\mathrm{m}] \quad \psi_{ref} = 0\,[\mathrm{rad}]$$

$$\dot{X}_{ref} = \dot{Y}_{ref} = \dot{Z}_{ref} = 0\,[\mathrm{m/s}]$$

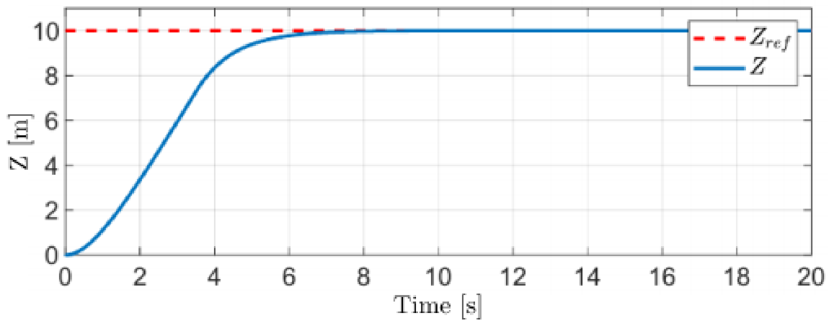
$$\dot{\psi}_{ref} = 0\,[\mathrm{rad/s}]$$

Uncertainty due to the difference in the mathematical model between the system and the controller.

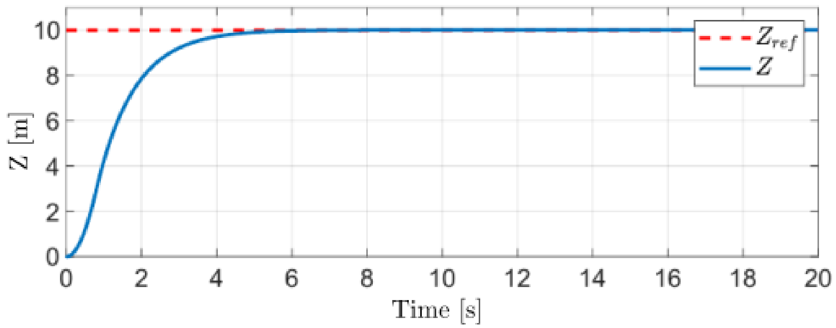


Results

SMC



SMC-ERL



SMC-MST

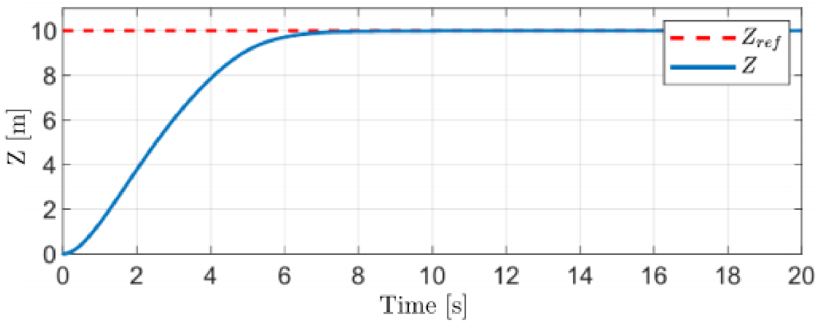
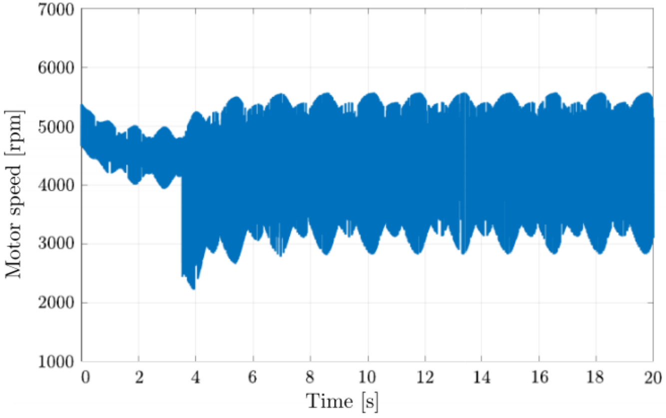


TABLE II
SIMULATION #1 RESULTS SUMMARY

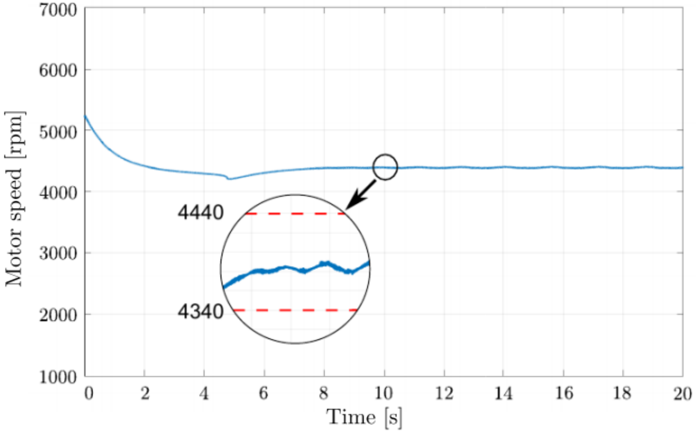
Performance metrics	SMC	SMC-ERL	SMC-MST
Rise time [s]	7.5	5.9	7.8
Chattering range [rpm]	2700	100	40
Precision $X-Y$ [cm]	15	1.6	2.5

Results

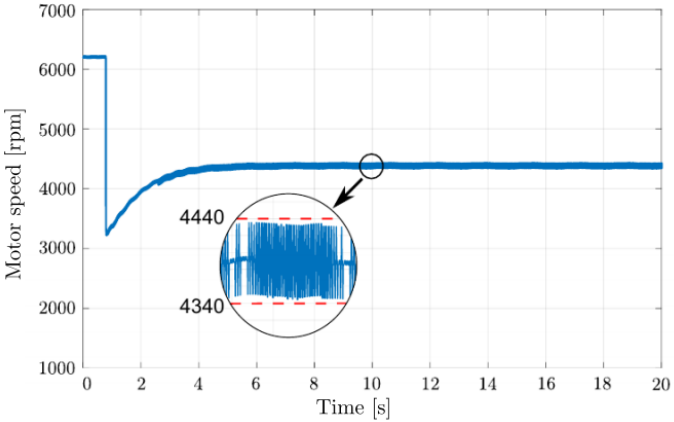
SMC



SM-STM



SM-ERL



Chattering Range [rpm]

- SMC : **2700**
- SM-ERL : **100**
- SM-STM : **40**

Results

Simulation #2: waypoints tracking
Simulation Conditions

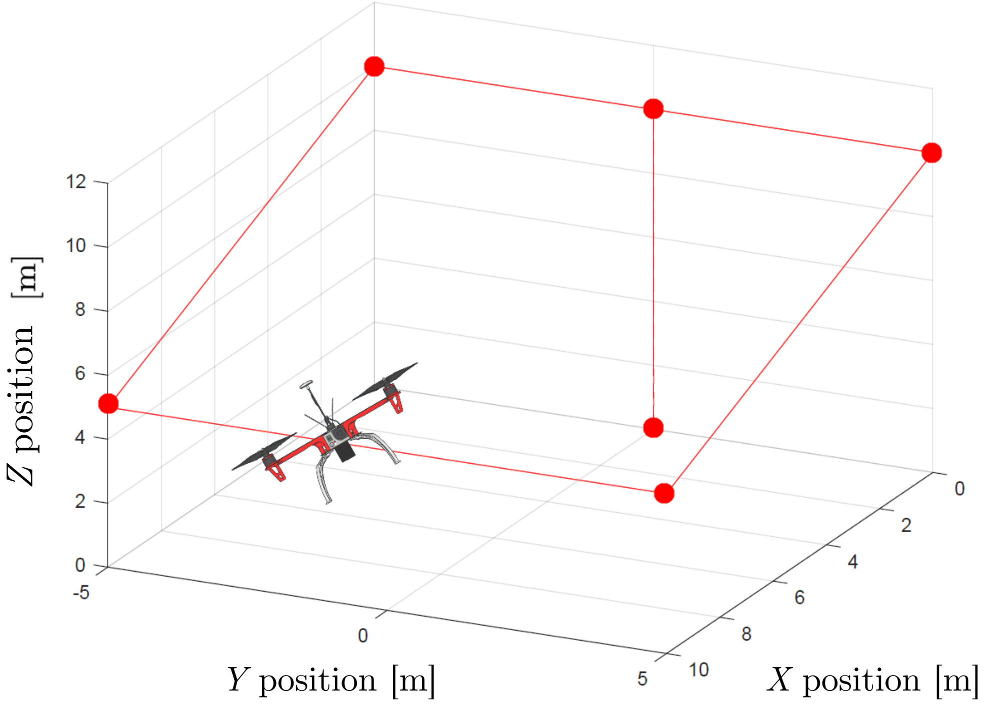
Desired trajectory
Fifth-order smooth polynomial curve

Disturbance

$$h_{XYZ} = 0.5\,\mathrm{g}\,[\,1\;\;1\;\;-1\,]^{T}\;\;[\mathrm{m/s^2}]$$

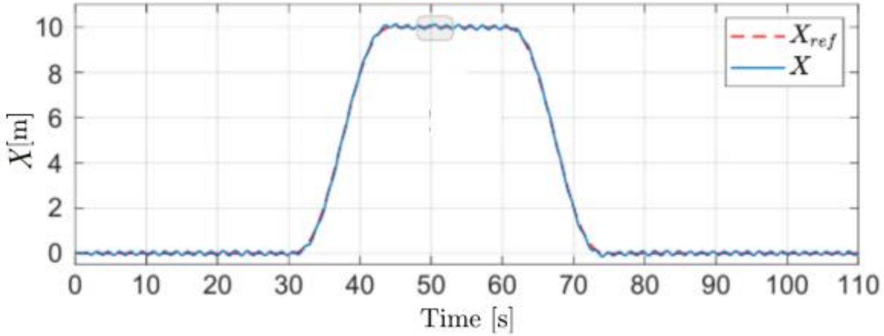
$$\mathrm{t}\;\in\;[50, 50.2]$$

- Uncertainty due to the difference in the mathematical model between the system and the controller.
- Uncertainty due to white Gaussian noise. $\Upsilon \sim \mathcal{N}(0,\ 1\mathrm{e}-6)$

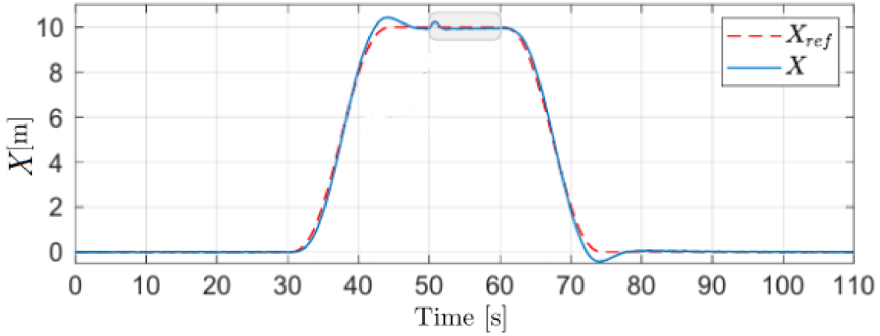


Results

SMC



SM-ERL



SM-STM

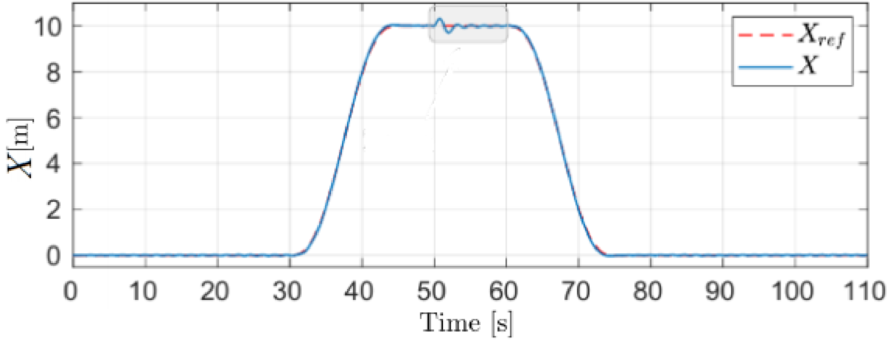
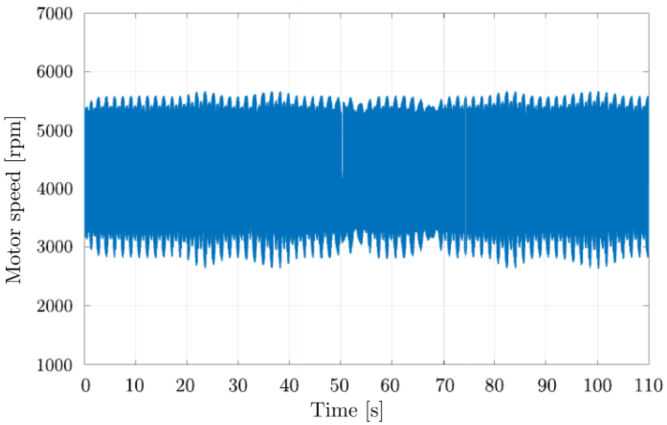


TABLE IV
SIMULATION #2 RESULTS SUMMARY

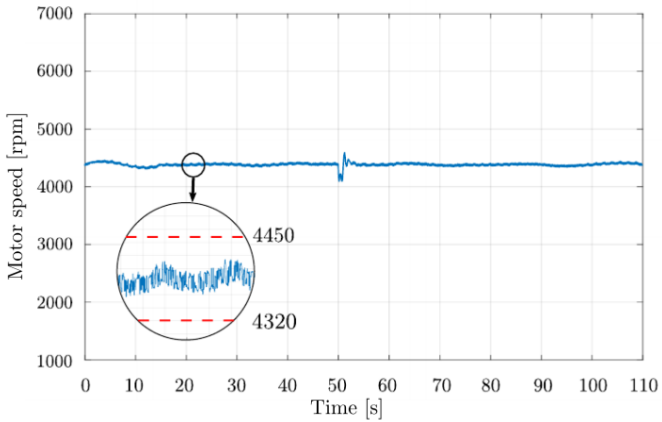
Performance metrics	SMC	SMC-ERL	SMC-MST
Oscillations $\phi - \theta$ [rad]	0.2	< 0.035	< 0.035
Chattering range [rpm]	3000	130	65
Overshoot $X - Y$ [%]	6	< 1	< 1

Results

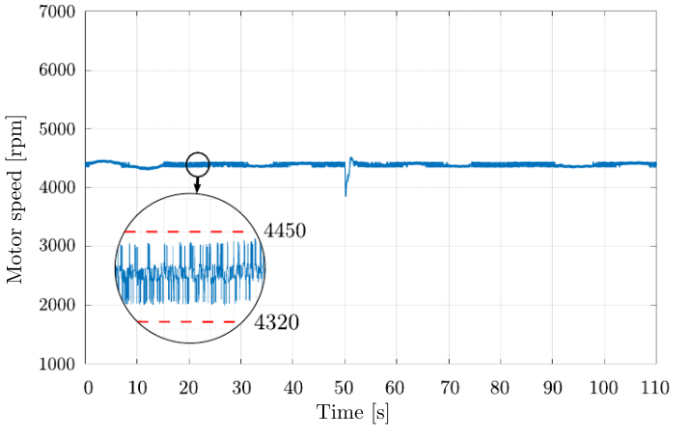
SMC



SM-STM



SM-ERL



Chattering Range [rpm]

- SMC : **3000**
- SM-ERL : **130**
- SM-STM : **65**

Conclusions

Three various SMC-based control schemes were modelled and simulated under two different test conditions to compare their tracking performance and the resulting chattering in the motors' speed. Although all three controllers showed similar performance in terms of the tracking, the conventional SMC controller showed excessive chattering in the motors' speed, both in the stationary flight and the waypoints tracking tests, leading to practical issues. Both the SMC-ERL and SMC-MST showed a significant reduction of the chattering effect, with the latter having a lower amplitude by a small margin. The SMC-ERL had the lower rising time in the stationary flight test. Consequently, the motors work at maximum speed for a short time, which can also wear out the engines. All three controllers showed robustness against disturbances, having a slight overshoot in the response for a simulated impact against an object with half the mass of the UAV but achieving the tracking objective. These results show that both techniques, SMC-ERL and SMC-MST, effectively reduce the chattering while maintaining robustness against uncertainties in the model and disturbances without adding excessive complexity in the controller design.