Modeling and Analysis of Dual Three-Phase Self-Excited Induction Generator

M. Ayala, O. Gonzalez, J. Rodas, R. Gregor, L. Comparatore and D. Caballero
Laboratory of Power and Control Systems, Facultad de Ingeniería, Universidad Nacional de Asunción
E-mail: {mayala, ogonzalez, jrodas, rgregor, lcomparatore & dcaballero}@ing.una.py

***Abstract*—Multiphase self-excited induction generators have recently become more interesting due to its advantages compared to its equivalent three-phase generator. However, as an induction generator, it has some disadvantages regarding poor voltage and frequency regulation under varying load and speed regimes. A steady state analysis is fundamental to comprehend the machine's behavior. This paper presents a detailed mathematical model and analysis of the dual three-phase self-excited induction generator. Simulation results are provided to show the behavior of the machine is build up voltage through different conditions, such as, variation of the excitation capacitance and variation of the mechanical rotor speed. For a more precise modeling, it is considered a non-linear variation of the magnetizing inductance.**

***Index Terms*—Magnetizing inductance, minimum capacitance, self-excited induction generator.**

I. Introduction

Recently, the interest in multiphase machines has risen due to intrinsic features such as lower torque ripple, power splitting or better fault tolerance than three-phase machines. Current research works and developments support the prospect of future more widespread applications of multiphase machines [1]. In industrial applications, any multiphase machine would be operated under variable-speed conditions, meaning that a multiphase power electronic converter is required [2]. At the same time, multiphase generation systems appear in renewable applications in very recent times, particularly in grid-connected wind energy conversion and stand-alone systems [3].

Among the most common generations systems, wind power generation is increasingly becoming more cost competitive comparing to all the environmentally safe and clean renewable energy sources. Fixed Speed wind power generating systems have gained much popularity among the manufacturers and developers in this field mainly due to the fact that fixed speed wind turbines choice of generator is squirrel cage induction generator [4].

The design of larger wind turbines and reliability requirements match the features of multiphase machines due to their capability to split power and provide fault tolerance. The most investigated option has been the use of dual three-phase self-excited induction generators (SEIG) supplied by two-level voltage source converters (VSCs) [5].

SEIG are a good option for wind powered generation system especially in remote areas, because they do not need any external source of power supply to produce the required excitation magnetic field. Permanent magnet generators may also be used for wind energy applications, however, the generated voltage increases linearly with the wind turbine speed. SEIG can cope with a small increase in speed from their rated value because, due to saturation, the increasing rate of the generated voltage is not linear with the mechanical speed [6].

Understanding the terminal voltage build up process of SEIG and its performance under steady state and dynamic condition, it becomes a fundamental step toward the development of more efficient and competitive SEIG technology [4]. For a SEIG, the external elements that can modify the voltage profile are the mechanical speed, the terminal capacitance and the load impedance [7], [8].

This paper presents a mathematical model and analysis of a dual three-phase SEIG and lists some of the critical parameters of the machine. The voltage build up process is analyzed by using the MATLAB/Simulink program. The modeling is tested under different conditions.

This paper is organized as follows: Section II analyzes the mathematical model of the dual three-phase SEIG. Section III details the magnetizing inductance behavior and the self-excitation process. Simulation results are provided in Section IV, showing the output voltage and load current under different conditions. The conclusions are finally summarized in the last section.

II. Self excited induction generator model

The mathematical model of the dual three-phase SEIG is very similar to the dual three-phase induction motor. However, the SEIG has a capacitor bank connected to its terminals, which has to be considered in this model. Besides, in this case, the rotor speed, provided by the turbine, is considered as an input, unlike the motor case [5], [9].

In order to comprehend the mathematical model of the dual three-phase SEIG, first it is necessary to convert the six-phase (a-b-c-d-e-f) to a two-phase (α - β) model using Clark's transformation proposed in [10].

In this section, it is presented the mathematical model in the (α - β) stationary frame to study both transient and steady state behaviors during the voltage build up process, the modeling of excitation system with a fixed load. The equivalent electric circuit of a dual three-phase SEIG is shown in Fig 1.

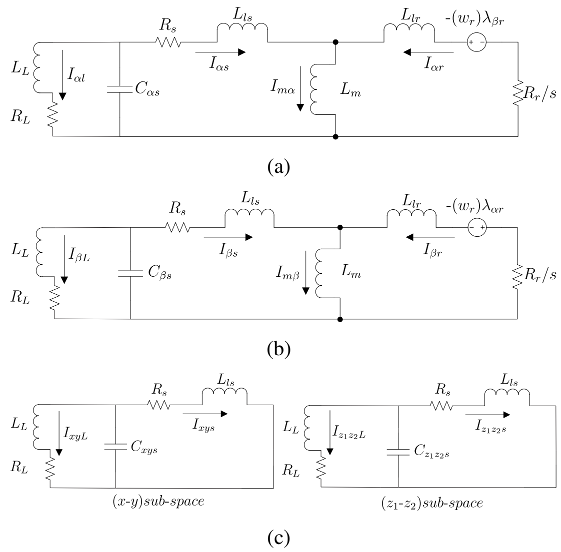

Fig. 1. Equivalent circuit of the dual three-phase SEIG in: (a) α sub-space; (b) β sub-space; (c) x-y and z_1-z_2 sub-space.

where $C_{\alpha s}$, $C_{\beta s}$ and C_{xys}, $C_{z_1 z_2 s}$ are capacitances from the (α-β), (x-y) and (z_1-z_2) sub-space respectively. R_s and R_r are resistances from the stator and rotor. L_{ls} and L_{lr} are dispersion inductances from the stator and rotor. L_m is the magnetizing inductance. ω_r is the electrical speed of the motor. $\lambda_{\alpha r}$ and $\lambda_{\beta r}$ are rotor flux linkages from (α-β) sub-space. $I_{\alpha s}$ and $I_{\alpha r}$ are stator and rotor currents from α sub-space. $I_{\beta s}$ and $I_{\beta r}$ are stator and rotor currents from β sub-space. $I_{m\alpha}$ and $I_{m\beta}$ are magnetizing currents from (α-β) sub-space. I_{xys} and $I_{z_1 z_2 s}$ are stator currents from (x-y) and (z_1-z_2) sub-spaces.

For two isolated neutrals configuration, both $(z_1 - z_2)$ currents cannot flow, so the $(z_1 - z_2)$ components can be ignored [11].

The equations which describe the circuits representing the dual three-phase SEIG are:

$$\begin{aligned}
V_{c\alpha s} + R_s I_{\alpha s} + p\lambda_{\alpha s} &= 0\\
R_r I_{\alpha r} - \omega_r \lambda_{\beta r} + p\lambda_{\alpha r} &= 0\\
V_{c\beta s} + R_s I_{\beta s} + p\lambda_{\beta s} &= 0\\
R_r I_{\beta r} + \omega_r \lambda_{\alpha r} + p\lambda_{\beta r} &= 0
\end{aligned} \tag{1}$$

where p is the differential operator d/dt and $V_{c\alpha s}$ and $V_{c\beta s}$ are the capacitors voltages, which are equivalent to:

$$\begin{aligned}
V_{c\alpha s} &= \tfrac{1}{C_{\alpha s}} \int I_{\alpha s}\, dt + V_{c\alpha s0}\\
V_{c\beta s} &= \tfrac{1}{C_{\beta s}} \int I_{\beta s}\, dt + V_{c\beta s0}
\end{aligned} \tag{2}$$

where $V_{c\alpha s0}$=$V_{c\alpha s}|_{t=0}$ and $V_{c\beta s0}$=$V_{c\beta s}|_{t=0}$, are the initial capacitor voltages.

Links stator windings flow are given by:

$$\begin{aligned}
\lambda_{\alpha s} &= L_s I_{\alpha s} + L_m I_{\alpha r}\\
\lambda_{\beta s} &= L_s I_{\beta s} + L_m I_{\beta r}
\end{aligned} \tag{3}$$

and the links rotor windings flow are given by:

$$\begin{aligned}
\lambda_{\alpha r} &= L_r I_{\alpha r} + L_m I_{\alpha s} + \lambda_{\alpha r0}\\
\lambda_{\beta r} &= L_r I_{\beta r} + L_m I_{\beta s} + \lambda_{\beta r0}
\end{aligned} \tag{4}$$

where $\lambda_{\alpha r0}$ and $\lambda_{\beta r0}$ are the links residual flow in (α-β) sub-space and are considered as a constant. L_s and L_r are the inductances of the stator and rotor windings, respectively, and are given by:

$$\begin{aligned}
L_s &= L_{ls} + L_m\\
L_r &= L_{lr}^{'} + L_m
\end{aligned} \tag{5}$$

The electrical speed of the rotor and the flow links produce a rotational voltage, so, this is equivalent to:

$$\begin{aligned}
\omega_r \lambda_{\alpha r} &= \omega_r (L_r I_{\alpha r} + L_m I_{\alpha s} + \lambda_{\alpha r0})\\
\omega_r \lambda_{\alpha r} &= \omega_r (L_r I_{\alpha r} + L_m I_{\alpha s} + K_{\beta r})\\
\omega_r \lambda_{\beta r} &= \omega_r (L_r I_{\beta r} + L_m I_{\beta s} + \lambda_{\beta r0})\\
\omega_r \lambda_{\beta r} &= \omega_r (L_r I_{\beta r} + L_m I_{\beta s} + K_{\alpha r})
\end{aligned} \tag{6}$$

where $K_{\alpha r}$ and $K_{\beta r}$ are induced initial voltages caused by the residual magnetic flux in the (α-β) sub-space and are constant values.

By replacing (3) and (6) in (1), it is obtained the equations which describe the dual three-phase SEIG in the stationary reference frame. Thus we have:

$$\begin{aligned}
& V_{c\alpha s} + R_s I_{\alpha s} + L_s p I_{\alpha s} + L_m p I_{\alpha r} && = 0\\
& V_{c\beta s} + R_s I_{\beta s} + L_s p I_{\beta s} + L_m p I_{\beta r} && = 0\\
& - K_{\alpha r} - \omega_r L_m I_{\beta s} + R_r I_{\alpha r} - \omega_r L_r I_{\beta r} && \\
& + L_m p I_{\alpha s} + L_r p I_{\alpha r} && = 0\\
& K_{\beta r} + \omega_r L_m I_{\alpha s} + R_r I_{\beta r} + \omega_r L_r I_{\alpha r} && \\
& + L_m p I_{\beta s} + L_r p I_{\beta r} && = 0
\end{aligned} \tag{7}$$

Eq. (7) can be written with a matrix form:

$$\begin{aligned}
\begin{bmatrix} 0\\0\\0\\0 \end{bmatrix} &= \begin{bmatrix} r_s & 0 & 0 & 0\\ 0 & r_s & 0 & 0\\ 0 & -\omega_r L_m & R_r & -\omega_r L_r\\ \omega_r L_m & 0 & \omega_r L_r & R_r \end{bmatrix} \begin{bmatrix} I_{\alpha s}\\ I_{\beta s}\\ I_{\alpha r}\\ I_{\beta r} \end{bmatrix} +\\
& \begin{bmatrix} L_s & 0 & L_m & 0\\ 0 & L_s & 0 & L_m\\ L_m & 0 & L_r & 0\\ 0 & L_m & 0 & L_r \end{bmatrix} p \begin{bmatrix} I_{\alpha s}\\ I_{\beta s}\\ I_{\alpha r}\\ I_{\beta r} \end{bmatrix} + \begin{bmatrix} V_{c\alpha s}\\ V_{c\beta s}\\ -K_{\alpha r}\\ K_{\beta r} \end{bmatrix}
\end{aligned} \tag{8}$$

From (8), clearing the derivatives of currents we obtain the simulation model for the dual three-phase SEIG.In this way we get:

$$
\begin{bmatrix} \dot{I}_{\alpha s} \\ \dot{I}_{\beta s} \\ \dot{I}_{\alpha r} \\ \dot{I}_{\beta r} \end{bmatrix} = \frac{1}{LL} \begin{bmatrix} L_s & 0 & -L_m & 0 \\ 0 & L_s & 0 & -L_m \\ -L_m & 0 & L_r & 0 \\ 0 & -L_m & 0 & L_r \end{bmatrix} \Bigg\{
$$

$$
- \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & -\omega_r L_m & R_r & -\omega_r L_r \\ \omega_r L_m & 0 & \omega_r L_r & R_r \end{bmatrix} \begin{bmatrix} I_{\alpha s} \\ I_{\beta s} \\ I_{\alpha r} \\ I_{\beta r} \end{bmatrix} + \begin{bmatrix} -V_{c\alpha s} \\ -V_{c\beta s} \\ K_{\alpha r} \\ -K_{\beta r} \end{bmatrix} \Bigg\} \quad (9)
$$

where:

$$
LL = L_s L_r - L_m^2 \quad (10)
$$

For an inductive load (RL) the components of differential load voltage equations can be written as:

$$
\begin{aligned} \frac{dV_{\alpha L}}{dt} &= \frac{1}{C_{\alpha s}} I_{\alpha C_{\alpha s}} \\ \frac{dV_{\beta L}}{dt} &= \frac{1}{C_{\beta s}} I_{\beta C_{\beta s}} \end{aligned} \quad (11)
$$

where $I_{\alpha C_{\alpha s}} = I_{\alpha s} - I_{\alpha L}$ and $I_{\beta C_{\beta s}} = I_{\beta s} - I_{\beta L}$. The components of differential load current equations can be written as:

$$
\begin{aligned} \frac{dI_{\alpha L}}{dt} &= \frac{1}{L_L}(V_{\alpha L} - R_{L I_{\alpha L}}) \\ \frac{dI_{\beta L}}{dt} &= \frac{1}{L_L}(V_{\beta L} - R_{L I_{\beta L}}) \end{aligned} \quad (12)
$$

The mechanical part of the electrical drive is given by the following equations:

$$
T_g = 3P\left(\psi_{\alpha s} i_{\beta s} - \psi_{\beta s} i_{\alpha s}\right) \quad (13)
$$

$$
J\frac{d}{dt}\omega_r + B\omega_r = P\left(T_i - T_g\right) \quad (14)
$$

where T_i denotes the input torque, T_g is the generated torque, J the inertia coefficient, P the number of pairs of poles, $\psi_{\alpha s}$ and $\psi_{\beta s}$ the stator flux and B the friction coefficient.

The equations in (x-y) sub-space do not link to the rotor side and consequently do not contribute to the air-gap flux, nonetheless, they are an important source of energy losses. These equations can be written as:

$$
\begin{bmatrix} V_{cxs} \\ V_{cys} \end{bmatrix} = \begin{bmatrix} L_{ls} & 0 \\ 0 & L_{ls} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_{cxs} \\ I_{cys} \end{bmatrix} + \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \begin{bmatrix} I_{cxs} \\ I_{cys} \end{bmatrix} \quad (15)
$$

As mentioned before, (9)-(15) represent the simulation model for the dual three-phase SEIG.

TABLE I
PARAMETERS OF THE DUAL THREE-PHASE SEIG

PARAMETER	SYMBOL	VALUE	UNIT
Stator resistance	R_s	0.62	Ω
Rotor resistance	R_r	0.63	Ω
Stator inductance	L_s	206.2	mH
Rotor inductance	L_r	203.3	mH
Magnetizing inductance	M	199.8	mH
System inertia	J	0.27	kg.m^2
Pairs of poles	P	3	–
Friction coefficient	B	0.012	kg.m^2/s
Nominal frequency	f_a	50	Hz
Electrical power	P_w	15	kW

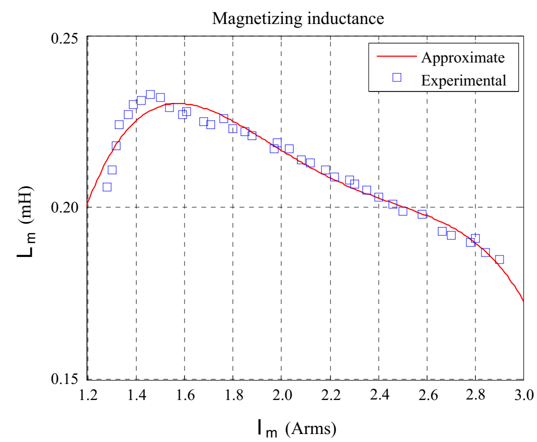

Fig. 2. Magnetizing inductance curve related to the magnetizing current.

III. Magnetizing Inductance and Self-Excitation Process Analysis

The variation of the magnetizing inductance is the main factor in the dynamics of the voltage build up and stabilization in dual three-phase SEIGs. The relationship between the magnetizing inductance L_m and magnetizing current I_m is obtained experimentally from open-circuit test at synchronous speed with the induction motor parameters listed in Table I.

The magnetizing inductance curve as function of the magnetizing current is given in Fig. 2 and it is a nonlinear function, which can be represented by a four order polynominal curve fit, obtained through the MATLAB/Simulink software:

$$
L_m = -0.0667 I_m^4 + 0.5901 I_m^3 - 1.93 I_m^2 + 2.7304 I_m - 1.1774 \quad (16)
$$

where the measured magnetizing current is:

$$
I_m = \frac{\sqrt{I_{m\alpha}^2 + I_{m\beta}^2}}{\sqrt{2}} \quad (17)
$$

where $I_{m\alpha} = I_{\alpha s} + I_{\alpha r}$ and $I_{m\beta} = I_{\beta s} + I_{\beta r}$

The dual three-phase SEIG requires a minimal value for the mechanical rotor speed and the capacitance connected to the

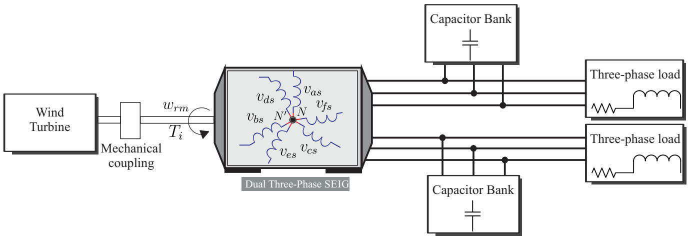

Fig. 3. Block diagram of the dual three-phase SEIG.

stator terminals of the machine. An effective way to obtain it for the self-exciting process is through the next equation:

$$C_{min} = \frac{1}{P^2 \omega_{rm}^2 M} \approx 51\mu F \tag{18}$$

where ω_{rm} is the mechanical rotor speed in rad/s, P is the number of pairs of poles and M is the magnetizing inductance to an average voltage in mH.

IV. Simulation Results

MATLAB/Simulink simulation environment has been designed for the dual three-phase SEIG. Simulation tests have been performed to show the behavior of the mathematical model of the SEIG. Numerical integration using first order Eulers method algorithm with an integration time of $1\mu s$ has been applied to compute the build up process in the time domain. A block diagram of the mathematical model of the dual three-phase SEIG is provided in Fig. 3.

Fig. 4 and 5 show the inducted voltage and the load current for a dual three-phase RL load. It is considered a capacitor bank of $67\mu F$, a constant mechanical rotor speed of $\omega_{rm} =$ 1000 rpm and a load of $R = 19.45\Omega$, $L = 356.1$ mH.

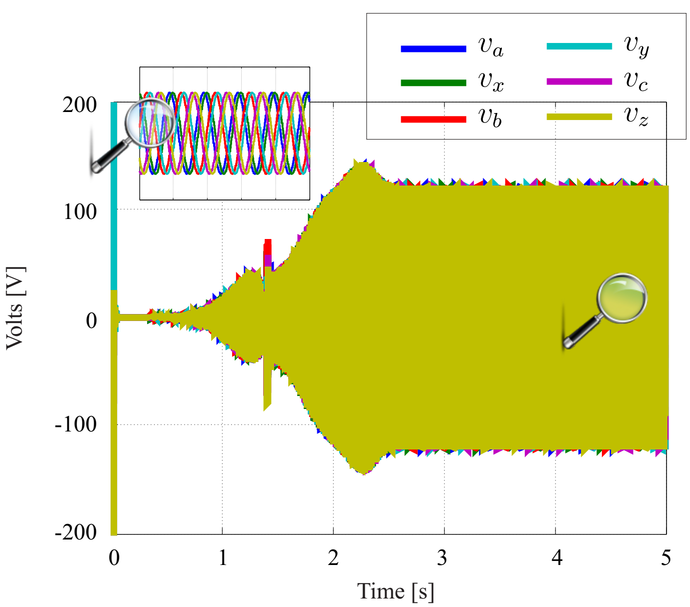

Fig. 4. Output voltage of the dual three-phase SEIG.

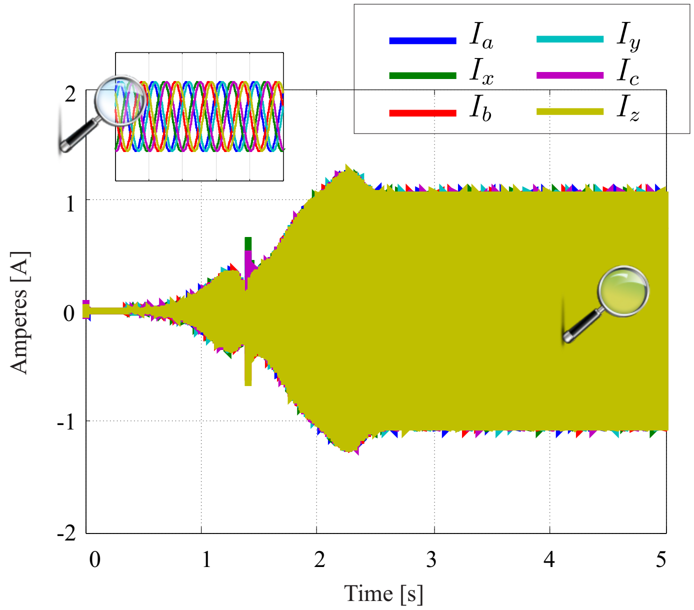

Fig. 5. Load current generated by the dual three-phase SEIG.

The build up process of the generated voltage is successful, as the amount of reactive power produced from the excitation capacitor is sufficient to the dual three-phase SEIG excitation process. In both figures, it can be seen the phase shift, being equivalent to 120° between phases of the same three-phase system and 30° between the two three-phase systems.

It is demonstrated in Fig. 6 the negative effect of using a capacitor bank of capacitance lower than the minimal calculated on (18), the amount of the reactive power produced from the excitation capacitor bank is insufficient to the excitation process. For this case, it was used a constant mechanical rotor speed of 1000 rpm. It could be obtained a successful excitation process by increasing the mechanical rotor speed to compensate the decreasing of the capacitance.

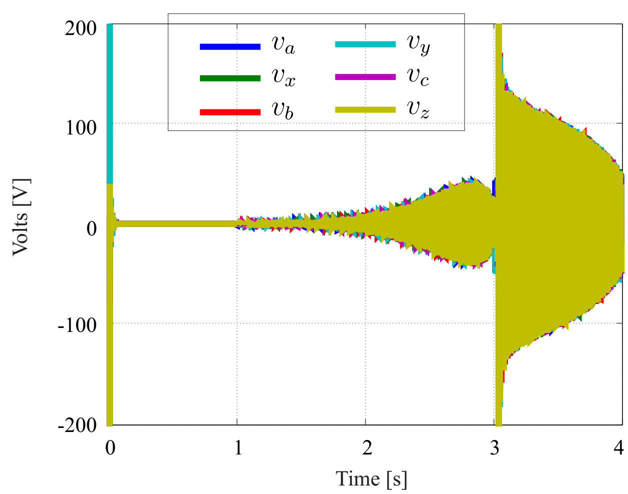

Fig. 6. Output voltage waveform of the dual three-phase SEIG when the capacitor bank value is $40\mu F$.

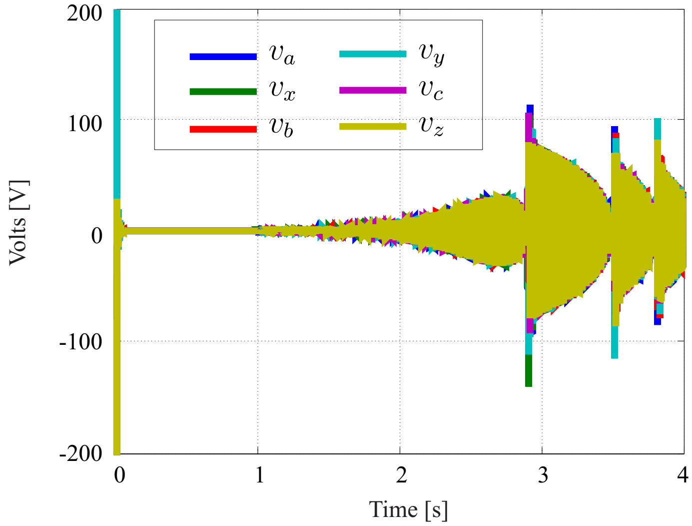

Fig. 7. Output voltage waveform of the dual three-phase SEIG when the rotor mechanical speed value is 772.5 rpm.

Fig. 7 shows the output of the dual three-phase SEIG by using a lower mechanical rotor speed, obtaining an unsuccessful build up process. This is because the mean value of the mechanical rotor speed is insufficient to excite the dual three-phase SEIG, it was used a excitation capacitance of $67\mu F$. Just as the last test, it could be obtained a successful excitation process, this time, by increasing the capacitance in order to support the reducing of the mechanical rotor speed.

V. Conclusion

In this paper, the model of the dual three-phase self-excited induction generator in the fundamental (α-β) sub-space is presented. Since the output voltage and frequency depend on severed variables such as: the mechanical rotor speed, the excitation capacitor and the connected load, it is important to perform the steady-state analysis in order to observe its behavior under different conditions. Therefore, a study and analysis of the dual three-phase SEIG performance is also shown in this paper. The study demonstrates how the excitation capacitance and the mechanical rotor speed can affect the steady state performance of the dual three-phase SEIG under resistive and inductive loads. The results have shown that the capacitance of the excitation capacitor must be precisely calculated in order to obtain a successful build up process of the dual three-phase SEIG. Furthermore, it was analyzed that the dual three-phase SEIG has a critical excitation capacitor value at constant mechanical rotor speed. The output voltage magnitude depends of the variation of the mechanical rotor speed, where decreasing the rotor speed will lead to decrease the output voltage. This demonstrates the voltage regulation problem of the dual three-phase SEIG when it is used for wind application (rotor speed varying with wind speed varying). The output build up voltage process also follows the variation of the magnetizing inductance. The relationship between the magnetizing inductance and the magnetizing current needs to be obtained experimentally to obtain a very precise model of the dual three-phase SEIG.

Acknowledgment

The authors would like to thank to the Paraguayan Government for the economical support provided by means of a CONACYT grant project 14-INV-101.

References

[1] F. Barrero and M. J. Duran, "Recent advances in the design, modeling, and control of multiphase machines: Part I," *IEEE Trans. Ind. Electron.*, vol. 63, no. 1, pp. 449–458, 2016.

[2] E. Levi, "Advances in converter control and innovative exploitation of additional degrees of freedom for multiphase machines," *IEEE Trans. Ind. Electron.*, vol. 63, no. 1, pp. 433–448, 2016.

[3] M. J. Duran and F. Barrero, "Recent advances in the design, modeling, and control of multiphase machines part: II," *Industrial Electronics, IEEE Transactions on*, vol. 63, no. 1, pp. 459–468, 2016.

[4] T. M. Masaud and P. Sen, "Modeling and analysis of self-excited induction generator for wind energy conversion," in *Proc. ISGT*. IEEE, 2015, pp. 1–5.

[5] J. Rodas, R. Gregor, Y. Takase, D. Gregor, and D. Franco, "Multimodular matrix converter topology applied to the six-phase wind energy generator," in *Proc. UPEC*. IEEE, 2015, pp. 1–6.

[6] L. Kalamen, P. Rafajdus, P. Sekerak, and V. Hrabovcova, "A novel method of magnetizing inductance investigation of self-excited induction generators," *IEEE Transactions on Magnetics*, vol. 48, no. 4, pp. 1657–1660, 2012.

[7] R. Gunawan, F. Yusivar, and B. Yan, "The magnetizing characteristic in the air gap of the SEIG using Runge Kutta method and correction factor," *Journal of Theoretical & Applied Information Technology*, vol. 72, no. 3, 2015.

[8] J. Dalei and K. B. Mohanty, "A novel method to determine minimum capacitance of the self-excited induction generator," in *Students Technology Symposium (TechSym), 2014 IEEE*. IEEE, 2014, pp. 408–413.

[9] S. Toledo, M. Rivera, R. Gregor, J. Rodas, and L. Comparatore, "Predictive current control with reactive power minimization in six-phase wind energy generator using multi-modular direct matrix converter," 2016.

[10] Y. Zhao and T. A. Lipo, "Space vector PWM control of dual three-phase induction machine using vector space decomposition," *Industry Applications, IEEE Transactions on*, vol. 31, no. 5, pp. 1100–1109, 1995.

[11] H. S. Che and W. P. Hew, "Dual three-phase operation of single neutral symmetrical six-phase machine for improved performance," in *Proc. IECON*, 2015, pp. 001 176–001 181.