

PRINCIPAL COMPONENT ANALYSIS OF FREE THROW SHOT TRAJECTORIES IN RELATION TO SHOOTING PERFORMANCE

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The purpose of this study was to analyze free throw (FT) shot trajectories of individual basketball players using principal component analysis (PCA) and investigate whether variation in trajectories correlated with FT shooting performance. A markerless motion capture system was used to record three-dimensional FT shot trajectories of seven basketball players during practice. Horizontal and vertical positioning of the wrist joint center of the shooting hand were captured in both the sagittal and frontal planes for all trials. These coordinates were used as the input to the PCA. Positive correlations between the amount of variation accounted for by the first principal component and FT shooting performance (in-game FT%) suggest that players with simpler and less variable (i.e., more deterministic) shooting trajectories are better FT shooters.

KEYWORDS: sports, biomechanics, basketball, coordination, variability

INTRODUCTION: The free throw shot is a unique shot in the game of basketball. It is the only shot that takes place during a stoppage in the game clock and is completely uncontested by the opposing players. Also, unlike any other shot, players have enough time to go through a pre-shot ritual that may include dribbling the ball a few times, taking a deep breath, and carefully lining up their shot (Goldschmied, 2021). Whereas other shots are often taken as a split-second decision, free throw shooters have up to 10 seconds to take the shot. Free throw shooting performance (% of made shots) often has a big impact on the outcome of the game, with one study suggesting that around 20% of points in collegiate basketball games come from free throws (Kozar, 1994). That same study found that winning teams scored a significantly higher percentage of points from free throws than did losing teams.

The biomechanics of free throw shooting have been previously studied by several authors. Specifically, studies have looked at variability of shot trajectories through the relationships of different joint angles and/or joint velocities (Button, 2003; Coves, 2020). These studies found that there is no correlation between any kinematic variable and shooting accuracy, and that there is no clear association between variability and skill level. Mullineaux (2010) concluded that traditional magnitudes and timings of kinematic variables are not sufficient and that more complex analysis needs to be done.

One aspect of free throw shooting that is yet to be explored in detail is the shot trajectory, independent of the joints involved with shooting a basketball. Bartlett (2007) found that there was no evidence to suggest that players could generate identical movement patterns from shot-to-shot. However, there is no research that looks at how shot-to-shot variability in shot trajectory correlates to free throw shooting performance. One way to analyze the time-varying dynamics of shot trajectories during free throws would be to use principal component analysis (PCA). PCA is a statistical method used to analyze and extract the underlying structure of two-dimensional time series or spatial data. Previous examples of using PCA include barbell trajectories in weightlifting (Kipp, 2020) and rowing patterns (Warmenhoven, 2019). These previous studies used functional PCA (fPCA) as a statistical approach. In this study, PCA of waveforms is used to quantify the amount of variation in the sagittal plane and frontal plane throughout the entire free throw trajectory. PCA of waveforms and fPCA have been theoretically and experimentally shown to provide very similar results (Warmenhoven, 2021). The purpose of this study was to analyze free throw shot trajectories of individual basketball players using principal component analysis and investigate whether variation in trajectories correlated with free throw shooting performance.

METHODS: Seven collegiate male basketball players (age: 19.6 ± 0.9 ; height: 1.99 ± 0.1 m; mass: 94.6 ± 6.4 kg; dominant shooting hand: 2 left, 5 right) participated in this study. Each

player performed at least 9 free throw shots while 2D video data was recorded (Sony RX0 II). The video data was processed by Theia3D (Theia Markerless, Inc., Kingston, Ontario) software to obtain a 3D pose of skeletal segments. A 6Hz filter was used to smooth the pose from the inverse kinematics and C3D files were exported to Visual3D Professional (C-Motion, Inc., Germantown, MD, USA) for further analysis. Due to technical issues 20 trials were discarded, leaving 55 successful trials for the analysis. Sagittal and frontal plane position of the wrist joint center of the dominant shooting hand was quantified and used as an estimation of the shot path during the free throw shots. The shot was defined from when the vertical (z-coordinate) difference between the center of mass and the wrist joint center was greater than 10 cm, to the last ball contact with the player's fingers (until the frame before the ball release) (Okazaki, 2007). The shot trajectory data was time normalized to 100% with cubic spline interpolation. Data cleaning, processing, and initial visuals were performed in RStudio 2022.12.0 (Posit Software, Boston, MA, USA). The time-normalized horizontal and vertical position data from each shot were concatenated to create a composite function. The composite function for each individual's collection of free throw trials were bound into a single matrix for each player and used as input for the principal component analysis. Principal component analysis (PCA) was performed in MATLAB R2021b (The Mathworks, Inc, Natick, MA, USA). The variance explained by the first principal component (PC1) in each plane was plotted against the player's free throw percentage (FT%) throughout their collegiate career through the 2022 calendar year. Additionally, the first three principal components of the PCA were recorded and graphed for each player in both sagittal and frontal planes.

RESULTS: Three principal components were extracted for each plane and for each player via the PCA. For the sagittal plane, the first principal component accounted for 59.9% to 81.1% of the variation, with all three extracted patterns accounting for 85.4% to 98.2% of the variance (Table 1). For the frontal plane, the first principal component accounted for 47.4% to 87.6% of the variation, with all three extracted patterns accounting for 87.4% to 98.4% of the variance. The shot trajectory patterns extracted by the principal components captured variations in the general forward/backward (sagittal) and side-to-side (frontal) shifts, vertical start and end positions of shots, and crossing of the vertical reference line of wrist joint center trajectory (Figure 1).

Regression analysis indicated that there is a linear relationship between a player's free throw percentage and the amount of variance that is captured within their first principal component (Figure 2). While a significant relationship was present in both the sagittal plane and frontal plane, the sagittal plane correlation ($p = 0.016$; $r^2 = 0.72$) was slightly stronger than that of the frontal plane ($p = 0.026$; $r^2 = 0.66$). Moreover, the strongest correlation came from taking the average of each player's PC1 score in both planes in relation to free throw percentage (p -value = 0.003; $r^2 = 0.85$; Figure 2).

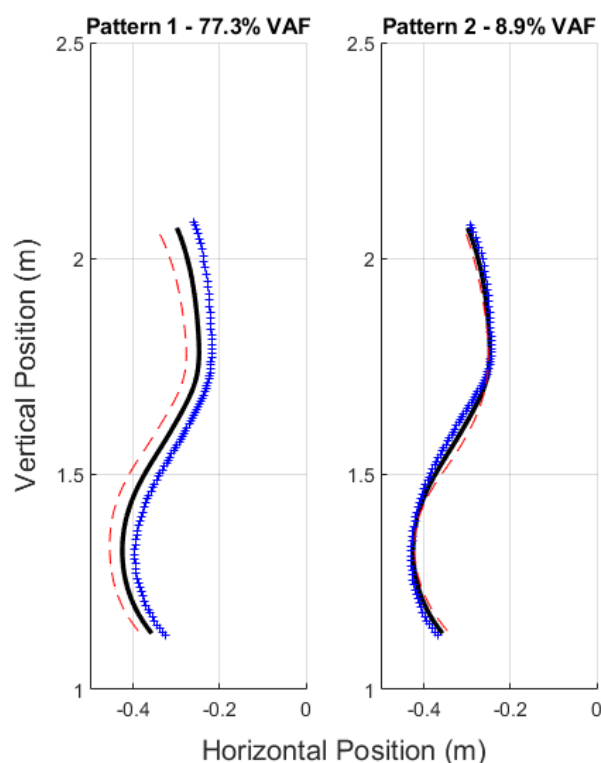


Figure 1: Examples of sagittal plane variation from PC1 and PC2. The ensemble average ball trajectory (black line) was represented with the effects of a one standard deviation change in due the influence of the PC (+/- symbols).

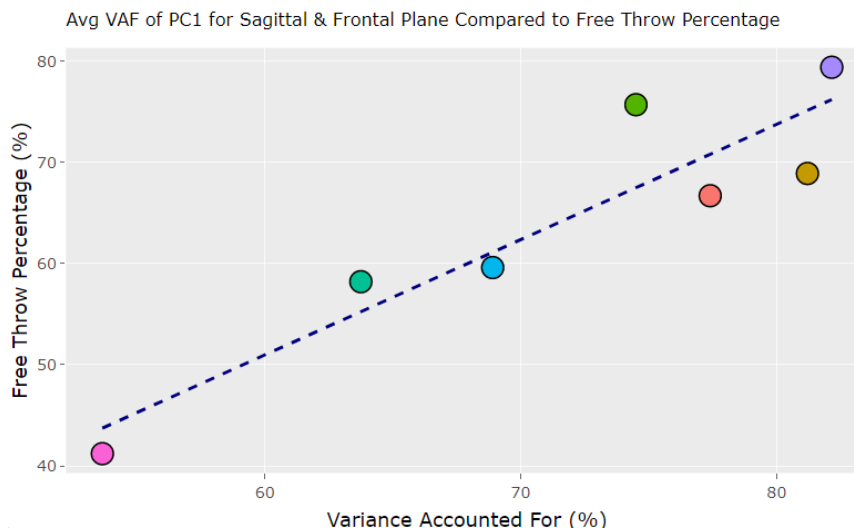


Figure 2: Relationships between the variation accounted for by the first principal component of both planes combined and free throw percentage for all players.

Table 1: Sagittal and frontal plane PCA results for each player. First three principal components (PC) are listed with the amount of variation accounted for by each one.

Player	Sagittal Plane			Frontal Plane		
	PC1	PC2	PC3	PC1	PC2	PC3
Player1	76.5%	19.4%	2.0%	78.3%	18.7%	1.2%
Player2	74.8%	15.9%	7.2%	87.6%	8.0%	2.8%
Player3	81.1%	11.1%	3.8%	67.9%	22.3%	4.6%
Player4	74.2%	19.8%	4.2%	53.3%	26.4%	13.5%
Player5	78.2%	14.2%	5.3%	59.6%	32.6%	3.3%
Player6	77.3%	8.9%	8.0%	87.0%	6.5%	4.1%
Player7	59.9%	14.2%	11.3%	47.4%	28.4%	11.6%

DISCUSSION: The purpose of this study was to capture the variation in free throw shot trajectories of individual basketball players using principal component analysis and investigate whether variation in trajectories correlated with free throw shooting performance. The results indicated a linear relationship between free throw shooting percentage and the variation accounted for in the first principal component extracted by the PCA. Specifically, players with a high free throw shooting percentage exhibited greater percent variation accounted for by the first PC, which suggests that better free throw shooters exhibit less variation and create a more consistent shot trajectory from shot-to-shot.

The primary findings of the current study were that the variance percentages associated with the extracted principal components correlated with player's free throw shooting ability. In particular, the variances accounted for by the first extracted principal components in the sagittal and frontal planes explained 66-72% of the variation in free throw shooting percentages of all players. Moreover, when the variance accounted for by the first principal component was averaged between the sagittal and frontal plane, the correlation with shooting performance was stronger, as evidenced through a coefficient of determination of 0.85. Therefore, while a strong linear relationship between the PCA results and free throw shooting performance existed, the relationship was even stronger when the results from both planes were combined. This finding likely suggests that three-dimensional variation in free throw shot trajectories is more important than the variation in any single plane.

Another important finding was that the PCA was able to extract player-specific variations (i.e., patterns) in free throw trajectories. These variations included shot-to-shot differences in the general magnitudes of the forward-backward and/or side-to-side movements in the sagittal plane and frontal plane, respectively (e.g., Figure 1). Other variations included shot-

to-shot differences in the “linearity” or “curviness” of the shot trajectories (e.g., Figure 1). It should be noted that the type of variation explained by the principal components differed between individuals, such that for some players the major source of variation came from forward-backward fluctuations whereas for others it came from differences in the shape of the trajectory. These findings illustrate that PCA can be used to effectively quantify and study within- and between-player differences in free throw shooting characteristics. In addition, the extracted principal components, and the patterns that they explain, can be thought of as smaller, independent “building blocks” that make up a player’s overall shooting technique and could therefore provide a basis for further study of shooting mechanics and skill.

There were a few limitations in this study. First, the shot onset was difficult to define because every player had a unique shooting motion. After pilot testing, onset was defined as when the vertical (z-coordinate) difference between the center of mass and the wrist joint center was greater than 10 cm. This may have limited the amount of variation captured at the lower end of the shot path trajectory. Second, the shots were not centered from trial to trial. While the players took all their shots back-to-back (with rebounders passing them the ball so they did not have to move), a player may have slightly moved their feet side-to-side or front-to-back between trials, which may indicate more variation than what was present.

CONCLUSION: This study used principal component analysis to explore the shot path trajectory of free throws by collegiate basketball players. Linear relationships between higher variance proportions for the first extracted principal component and free throw percentage were noted, which indicated that better free throw shooters had more consistent trajectories from shot to shot. The methods presented may also provide coaches and players with a way to break down shots into “building blocks” that are easy to visualize and digest, which may enable them to better understand free throw shooting mechanics and work to improve free throw shooting skill.

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