Optimal Design Rules Applied To The Design Of High-speed Mechanisms Under Deflection And Stress Constraints

Frank W. Liou
Missouri University of Science and Technology, liou@mst.edu

Jenny Liu
Missouri University of Science and Technology, jennyliu@mst.edu

OPTIMAL DESIGN RULES APPLIED TO THE DESIGN OF HIGH-SPEED MECHANISMS UNDER DEFLECTION AND STRESS CONSTRAINTS

F. W. Liou and J. D. Liu
Department of Mechanical and Aerospace Engineering and
Engineering Mechanics
University of Missouri, Rolla
Rolla, Missouri

ABSTRACT:

Presented in this paper are two approaches for the design of flexible mechanisms under stress and deflection constraints. Based on the optimality conditions derived in this paper, two procedures are developed to design the minimum weight of mechanisms subject to stress and deflection limitations. The first procedure is the improvement of Thornton's design process [17], and the second procedure is based on the interpolation technique. Several design examples are also presented to demonstrate these methods.

INTRODUCTION:

While most of the studies have been emphasized on the Kineto-Elasto-Dynamic analysis [1-15], only a small number of papers have been given to the synthesis part. Iman and Sandor [4, 7] investigated the synthesis Sandor [4, 7] investigated the synthesis problem under vibrational effects and treated it as a nonlinear under vibrational effects and treated it as a nonlinear programing problem. Khan and his co-workers [16, 17] used optimality criterion technique to develop recursion formula and pointed out that it was much superior to the non-linear programming techniques. Cleghorn et al. [18] presented a new procedure in which the finite difference method was used between cross-sectional diameter and maximum stress in each link to derive Jacobian matrix. Zhang and Gradin [19] improved Khan's and Cleghorn's recursion formulas to develop optimum design procedure. Based on the general design rules [21], Liou and Liu [20] developed recursion formulas for flexible mechanism design and showed that they were very efficient.

Among these research activities, most of the designs involved only single (stress or deflection) constraint. More practical design problems occur when both stress and deflection constraints are included. Thornton et al. [17] solved this problem using stress and displacement recursion formulas derived from optimality criterion techniques to negotiate between the two constraints. Zhang and Gradin [19] changed both cross-sectional parameters and geometrical parameters by using kinematic refinement technique. In each of the methods, the optimal solution is not easy to obtain by iteration due to the stability of the problem. In this paper, the optimality conditions including both stress and deflection constraints are derived. Based on these conditions, two design procedures are developed.

OPTIMALITY CONDITIONS :

A mechanism composed of N members is considered in this derivation. The design goal is to find the cross-sectional sizes of each member, characterized by the design variable Y_i to minimized the total volume V :

$$V = \sum_{i=1}^{N} A_i L_i \tag{1}$$

subject to

$$|\sigma_i| \le \sigma_{ai} \qquad i=1,2,3,\ldots,N \tag{2}$$

$$|X_j| \le X_{aj} \qquad j=1,2,3,\ldots,M \tag{3}$$

where

A_i : cross-sectional area in i^{th} member

L_i : length in i^{th} member

$|\sigma_i|$: maximum absolute stress in the i^{th} member

σ_{ai} : specified allowable stress in the i^{th} member

$|X_j|$: maximum absolute displacement at the j^{th} point

X_{aj} : specified allowable displacement at the j^{th} point

N : number of members

M : number of points with specified deflection

Since variable Y_i can be the cross-sectional area A_i, the diameter of a circular member, or the other similar quantity, the area A_i can be expressed as a function of the design variable Y_i.

$$A_i = CY_i^b \tag{4}$$

where C and b are constants.

Considering the mechanism moves periodically, the allowable stress in the i^{th} member can be written as

$$|\sigma_i| = \frac{|P_i|}{A_i} = \frac{|P_i|}{CY_i^b} \tag{5}$$

where

$|P_i|$ is the maximum absolute force in the i^{th} member during movement

Considering any member i which contains the deflection constraint at point j, from the general design rules, $|X_j|$ is proportional to $1/A_i$ [20]. i.e.,

$$|X_j| = \frac{D_i}{A_i} = \frac{D_i}{CY_i^b} \tag{6}$$

where D_i is a positive constant.

After solving equations (1) to (6), the following equations can be obtained.

$$V = \sum_{i=1}^{N} C\, Y_i^b\, L_i \tag{7}$$

$$g_i = \frac{|P_i|}{CY_i^b} - \sigma_{ai} \tag{8}$$

$$g_j = \frac{D_i}{CY_i^b} - X_{aj} \tag{9}$$

where V is the objective function, g_i and g_j are the constraints. From Kuhn-Tucker conditions, the following equations can be defined :

$$\phi = \sum_{i=1}^{N} CY_i^b L_i + \lambda_i\left(\frac{|P_i|}{CY_i^b} - \sigma_{ai} \right) + \lambda_j\left(\frac{D_i}{CY_i^b} - X_{aj} \right) \tag{10}$$

$$\frac{\partial\phi}{\partial Y_i} : CY_i^{b-1}L_i - \frac{\lambda_i|P_i|}{CY_i^{b+1}} - \frac{\lambda_j D_i}{CY_i^{b+1}} = 0 \tag{11}$$

$$\frac{\partial\phi}{\partial\lambda_i} : \frac{|P_i|}{CY_i^b} - \sigma_{ai} = 0 \tag{12}$$

$$\frac{\partial\phi}{\partial\lambda_j} : \frac{D_i}{CY_i^b} - X_{aj} = 0 \tag{13}$$

Because the stress constraint can be applied to every member of the mechanism under the strength consideration, and the deflection constraint can be applied to certain point(s), two optimality conditions can be discussed by solving equations (11) to (13).

CONDITION 1 :

If any member i does not include the deflection constraint (i.e., only stress constraint g_i is active in this member), then the deflection constraint can be ignored in equations (11) to (13). After solving equations (11) to (13), it is found that only when $|\sigma_i| = \sigma_{ai}$ is the optimality condition.

CONDITION 2 :

If any member i contains the deflection constraint at point j, then three cases can be discussed :

<a>
If both g_i and g_j are active, the optimality condition is when both $|\sigma_i| = \sigma_{ai}$ and $|X_j| = X_{aj}$.

<b>
If g_i is active but g_j is inactive, then only when $|\sigma_i| = \sigma_{ai}$ is the optimality condition.

<c>
If g_i is inactive but g_j is active, then only when $|X_j| = X_{aj}$ is the optimality condition.

These conditions can be used to determine whether the current design has reached the optimum or not. For example, if a design satisfies the stress constraint, (i.e. the maximum stress equals the allowable stress in each member of the mechanism), and if all of the deflections in the specified points are below the allowable deflections, then this case satisfies the optimality condition 2-b. The solution obtained under the stress constraint is the optimum solution.

Based on the above discussion, the following design procedures can be developed for designing high-speed mechanisms.

DESIGN PROCEDURE I :

Thornton et al. [16, 17] developed a design procedure to include stress constraint with the recursion formula :

$$(A_i)_{v+1} = \left(\frac{|\sigma_i|_v}{\sigma_{ai}} \right)^{\eta} (A_i)_v \tag{14}$$

where η is the relaxation factor which is equals to unity [16, 17].

The same formula can also be obtained from design rule approach [20]. This formula can be used to calculate the cross-sectional area A_i which satisfies the stress constraint. Zhang and Gradin [19] reported that η can be replaced by a variable η' to reduce the iteration time.

$$\eta' = \frac{\log(A_i)_{v+1} - \log(A_i)_v}{\log|\sigma_i|_v - \log|\sigma_i|_{v+1}} \tag{15}$$

From the design rule approach [20], the relationship between areas and deflections can be expressed as :

$$(A_i)_{v+1} = \left(\frac{|X_i|_v}{X_{ai}} \right)^{\varsigma} (A_i)_v \tag{16}$$

where ς is also a relaxation factor.

Formula (16) is easier to resize the cross-sectional area corresponding to the deflection constraint. The design procedure is suggested as follows :

1. Use formula (14) to determine a set of areas A_i which satisfy the stress constraint.

2. Input the above A_i to analysis software to obtain the maximum absolute deflections $|X_i|$ in those specified points. Locate those points whose $|X_i|$ are greater than the allowable deflections X_{ai}. For convenience, these points (members) are called "group D". Those members in group D satisfy the optimality condition 2-c; i.e., only the deflection constraint is active.

3. Use formula (16) to resize those members in group D and formula (14) to determine the cross-sectional areas for all the members (including those in group D). Choose the one with larger area between those from formula (16) and those from formula (14) for the members in group D. η' value in formula (14) is set between the range of 0.01 and 1.0. ς value in formula (16) is set between 0.001 to 0.4. At the begin of the iteration, a large value of ς (say 0.4) can be chosen for fast convergence. If the maximum stress in any member consequently increases twice in the successive iterations (this means the iteration may diverge), then reduce ς by one half.

4. If all of the members satisfy the optimality conditions then exit; Otherwise back to step 3 to continue.

This design procedure can be used to design most of the flexible mechanism problems quite efficiently. However, in some cases, such as when the specified deflection are too small or when too many points belong to group D (say more than two points), this procedure can perform badly. Therefore, design procedure II is suggested to solve these problems.

DESIGN PROCEDURE II :

If considering the normal case that the lengths of the mechanism are much greater than its thicknesses, the axial stress can be neglected. Also assume that the mechanism is composed of symmetric cross-sectional areas and moves with constant velocity. From the general rules [21], both maximum stress $|\sigma|$ and maximum deflection $|X|$ in the same member are proportional to $1/A^2$ (A is the cross-sectional area of the member). Thus $|\sigma|$ are proportional to $|X|$. For those members in group D (same as described in design procedure I), $|\sigma|$ should be reduced to allow $|X|$ equals X_a to satisfy the optimality condition 2-C. The degree of reducing $|\sigma|$ can be done by the interpolation technique. Referring to Figure 1, Data No.1 and Data No.2 are used to obtain Data No.3, then a cubic interpolation function is used to approximate the actual function, and the final point F can be obtained. The design procedure is described below :

1. Same as step 1 in design procedure I.

2. Same as step 2 in design procedure I to obtain group D. Each member in this group has an active deflection constraint. Also gather those members with active stress constraint as group S. From the optimality condition 2-b, the maximum stress $|\sigma_i|$ in group S should equal the allowable stress σ_{ai}, and $|\sigma_i|$ should be reduced to allow $|X_j|$ equal X_{aj} in group D.

3. Compute the value of $|X_j|/X_{aj}$ for each point of group D, and call this relative value as ψ for convenience. Find the most critical member; i.e., the member with maximum ψ. Record $|\sigma_i|$ and $|X_j|$ in the critical member. This will be the first set of data for numerical interpolation later, and then be expressed as $|\sigma_i|_1$ and $|X_j|_1$.

4. Reduce the allowable stress σ_{ai} in the critical member to some value $(\sigma_{ai})_1$ and keep σ_{ai} in other members the same as before. Repeat the process based on the new allowable stresses $(\sigma_{ai})_1$ to obtain the second set of data $|\sigma_i|_2$ and $|X_j|_2$. The choice of $(\sigma_{ai})_1$ depends on the difference between the allowable deflection X_{aj} and $|X_j|_1$. If X_{aj} is quite close to $|X_j|_1$, then $(\sigma_{ai})_1$ can be chosen near σ_{ai}. Normally $(\sigma_{ai})_1$ can be chosen as half the value of σ_{ai}.

5. Interpolate the first and second sets of data $|\sigma_i|_1$, $|X_j|_1$, $|\sigma_i|_2$ and $|X_j|_2$ to calculate the approximate value of $(\sigma_{ai})_2$ for the desired X_{aj}. Use the same process as step 4 to obtain $|\sigma_i|_3$ and $|X_j|_3$. If $|X_j|_3$ is quite close to X_{aj}, then set σ_{ai} equal $(\sigma_{ai})_2$ and back to step 2 to continue the calculation; otherwise use three sets of data $|\sigma_i|_1$, $|X_j|_1$, $|\sigma_i|_2$, $|X_j|_2$, $|\sigma_i|_3$ and $|X_j|_3$ for interpolation to obtain $(\sigma_{ai})_3$. This will be the appropriate stress value.

6. After all of the $|X_j|$ in group D are within the acceptable range (say the relative error of both stress and deflection between the maximum value and the allowable value is smaller than 5 %), then exit; Otherwise, update σ_{ai} to equal $(\sigma_{ai})_3$ and back to step 2 to iterate again.

In both of the design procedures, if only deflection constraint is active in all of the members, then it can easily be solved by equation (16). This type of problem has been studied by Liou and Liu [20], where the value of ς is a variable defined as

$$\varsigma = \frac{\log(A_i)_{v+1} - \log(A_i)_v}{\log|X_i|_v - \log|X_i|_{v+1}} \tag{17}$$

From experience, ς is within the range of 0.01 to 1.0 and will ensure the convergence.

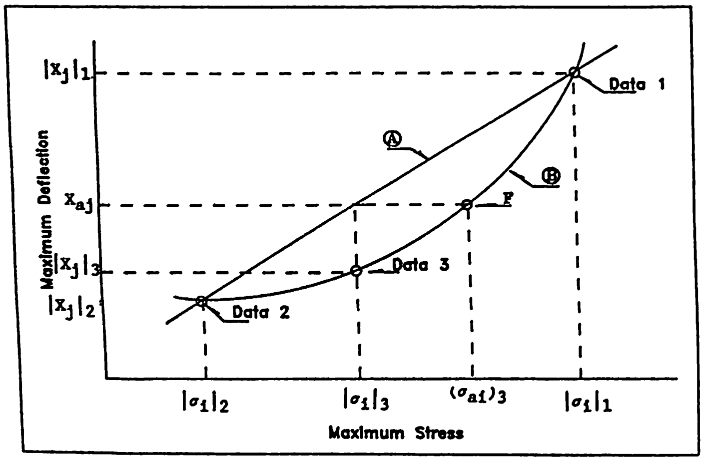

Figure 1 : Numerical interpretation in the critical member for Design Procedure II (Line A -linear interpolation function, Line B -actual function)

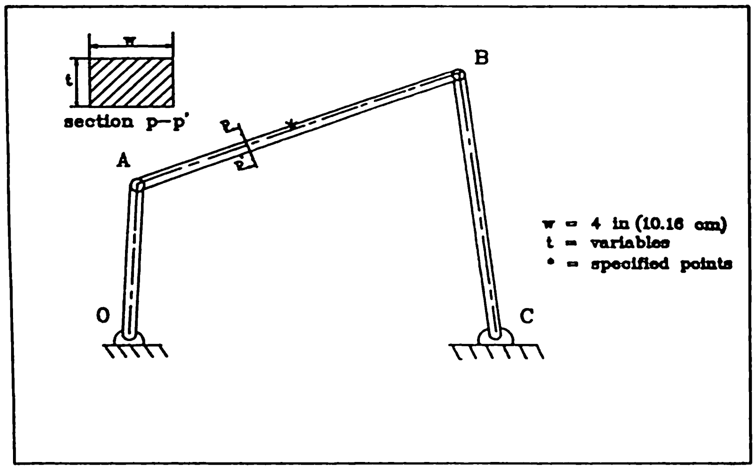

Figure 2 : Four-bar linkage and the specified point for deflection for Example 1

EXAMPLES :

(A) A four-bar mechanism is shown in Figure 2 with specifications as follows :

Length of OA = 12.0 in (30.48 cm)
Length of AB = 38.0 in (96.52 cm)
Length of BC = 38.0 in (96.52 cm)
Length of OC = 48.0 in (121.92 cm)
Crank speed = 31.4 rad/sec (counter-clockwise)
Modulus of elasticity = 1.03×10^7 psi (7.1×10^{10} N/m^2)
Mass density = 0.1 lbf/in^3 (2678 Kg/m^3)
Cross-sectional shape = rectangular
[constant width equals 4 in (10.16 cm), and design variable is the thickness of the individual link]
The allowable stress for each link = 5000 psi (3.447×10^7 N/m^2)

The allowable deflection at the mid-point of the coupler link = 0.08 in (0.2032 cm)

Both quasi-static and full dynamic methods are used for the design with design procedure I. The design results are shown in Table 1. Only six iterations are needed for the full dynamic analysis and five iterations are needed for the quasi-static analysis. It is found that both the stress and deflection constraints can be satisfied.

(B) Another four-bar mechanism is shown in Figure 3 with specifications as follows :

Length of OA = 12.0 in (30.48 cm)
Length of AB = 36.0 in (91.44 cm)
Length of BC = 36.0 in (91.44 cm)
Length of OC = 48.0 in (121.92 cm)
Crank speed = 31.4 rad/sec (counter-clockwise)
Modulus of elasticity = 1.03×10^7 psi (7.1×10^{10} N/m^2)
Mass density = 0.1 lbf/in^3 (2678 Kg/m^3)
Cross-sectional shape = rectangular
[constant width equals 4 in (10.16 cm), and design variable is the thickness of the individual link]

Method	Link	Designed Thickness (in.)	Max. Deflection in Specified Point (in.)	Maximum Stress (psi)
Full Vibration	Crank	1.718	——	4987.17
	Coupler	1.053	0.081	2683.87
	Follower	0.652	——	5001.56
Quasi-Static	Crank	1.010	——	4993.11
	Coupler	0.800	0.080	2014.37
	Follower	0.414	——	4999.97

Table 1 : Designed results for Example 1

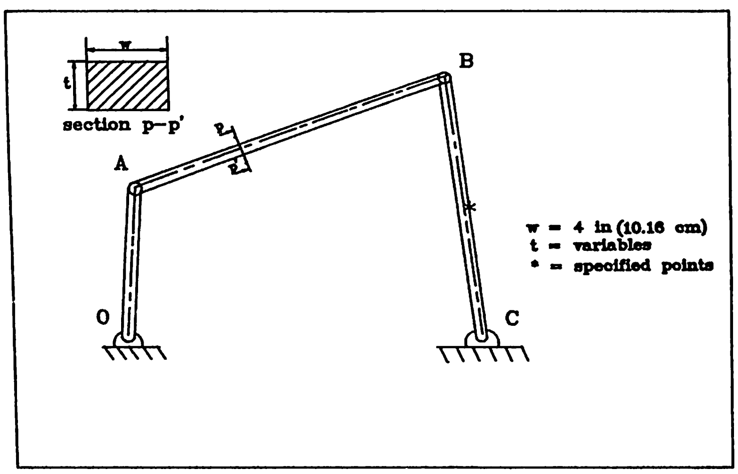

Figure 3 : Four-bar linkage and the specified point for deflection for Example 2

The allowable stress for each link = 5000 psi (3.447×10^{7} N/m^2)
The allowable deflection at the mid-point of the follower link = 0.02 in (0.0508 cm)

This problem can not easily be solved by using design procedure I, although ζ value has been reduced quite close to zero. Design procedure II can be used to solve this problem. Both the full dynamic and the quasi-static methods are used for the design. The design history for full dynamic analysis is shown in Table 2 and quasi-static analysis is shown in Table 3. Twenty-six iterations are needed by using full dynamic method and sixteen iterations are needed for the quasi-static method. It can be found that both the stress and deflection constraints are satisfied.

Step	Link	Specified Stress (psi)	Designed Thickness (in)	Max. Stress (psi)	Max. Disp. in Specified Point (in)	Iteration Number
1	2	5000	1.115	4983.94	——	6
	3	5000	0.554	4945.08	——	
	4	5000	0.579	4993.54	0.241	
2	2	5000	3.184	4965.26	——	6
	3	5000	0.700	4838.09	——	
	4	415.0	9.215	403.29	0.009	
3	2	5000	2.588	4996.61	——	6
	3	5000	0.676	4751.10	——	
	4	621.24	5.626	614.04	0.011	
4	2	5000	2.000	4917.52	——	8
	3	5000	0.702	4821.06	——	
	4	1157.52	2.810	1168.35	0.019	

Table 2 : Design history for Example 2 by using full dynamic method

Step	Link	Specified Stress (psi)	Designed Thickness (in)	Max. Stress (psi)	Max. Disp. in Specified Point (in)	Iteration Number
1	2	5000	0.650	5150.75	——	4
	3	5000	0.294	5000.00	——	
	4	5000	0.359	5000.00	0.366	
2	2	5000	1.893	5002.15	——	5
	3	5000	0.302	5000.04	——	
	4	300.0	6.495	300.00	0.013	
3	2	5000	1.629	5043.07	——	4
	3	5000	0.299	4999.69	——	
	4	401.86	4.736	401.86	0.015	
4	2	5000	1.311	5210.67	——	3
	3	5000	0.297	5000.75	——	
	4	616.20	3.024	615.46	0.020	

Table 3 : Design history for Example 2 by using quasi-static method

(C) The third mechanism is shown in Figure 4 with specifications as follows :

Length of OA = 12.0 in (30.48 cm)
Length of AB = 36.0 in (91.44 cm)
Length of BC = 36.0 in (91.44 cm)
Length of OC = 48.0 in (121.92 cm)
Crank speed = 31.4 rad/sec (counter-clockwise)
Modulus of elasticity = 1.03×10^{7} psi (7.1×10^{10} N/m^2)

Mass density = 0.1 lbf/in^3 (2678 Kg/m^3)
Cross-sectional shape = rectangular
[constant width equals 4 in (10.16 cm), and design variable is the thickness of the individual link]
The allowable stress for each link = 5000 psi (3.447×10^{7} N/m^2)

The allowable deflections :

crank link (at mid-point) : 0.1 in (0.254 cm)
coupler link (at mid-point) : 0.1 in (0.254 cm)
follower link (at mid-point) : 0.1 in (0.254 cm)

Again, since this problem can not easily be solved by using design procedure I, design procedure II is suitable for this problem. Quasi-static method is used for the this example. The design history is shown in Table 4. Twenty iterations are needed to complete the design. The optimal design results are shown in Table 5.

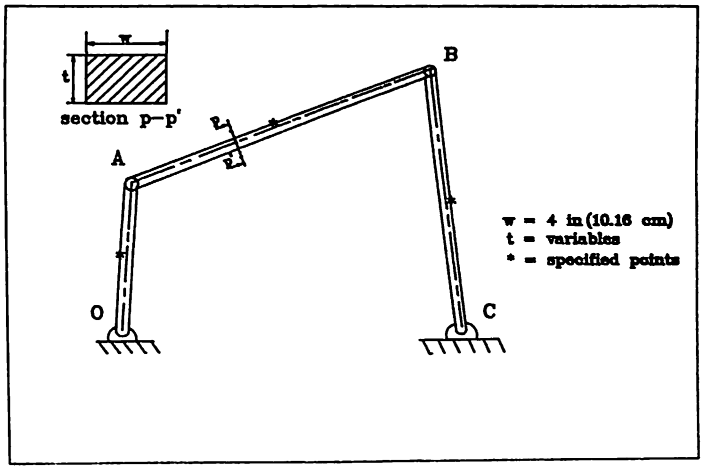

Figure 4 : Four-bar linkage and the specified points for deflection for Example 3

Step	Link	Specified Stress (psi)	Designed Thickness (in)	Max. Disp. in Specified Point (in)	Iteration No.
1	2	5000	0.650	0.023	4
	3	5000	0.294	0.488	
	4	5000	0.359	0.366	
2	2	5000	0.831	0.017	4
	3	2500	0.590	0.122	
	4	5000	0.361	0.361	
3	2	5000	0.852	0.017	4
	3	2349.73	0.628	0.107	
	4	5000	0.361	0.361	
4	2	5000	0.954	0.015	4
	3	2349.73	0.628	0.107	
	4	2500	0.726	0.094	
5	2	5000	0.950	0.015	4
	3	2349.73	0.628	0.107	
	4	2556.18	0.709	0.098	

Table 4 : Design history for Example 3

Link	Designed Thickness (in)	Maximum Stress (psi)	Max. Displacement in Specified Point (in)	Total Volume (in^3)
Crank	0.950	5024.80	0.015	238.13
Coupler	0.628	2349.73	0.107	
Follower	0.709	2556.18	0.098	

Table 5 : Designed results for Example 3

(D) Another four-bar design problem is shown in Figure 5 with specifications as :

Length of OA = 12.0 in (30.48 cm)
Length of AB = 30.0 in (76.20 cm)
Length of BC = 36.0 in (91.44 cm)
Length of OC = 36.0 in (91.44 cm)
Crank speed = 31.4 rad/sec (counter-clockwise)
Modulus of elasticity = 1.03×10^7 psi (7.1×10^{10} N/m^2)
Mass density = 0.1 lbf/in^3 (2678 Kg/m^3)
Cross-sectional shape = rectangular

[constant width equals 4 in (10.16 cm), and design variable is the thickness of the individual link]
The allowable stress for each link = 5000 psi ($3.447\times10^7 N/m^2$)
The allowable deflections :
crank link (at end-point) : 0.01 in (0.0254 cm)
coupler link (at mid-point) : 0.1 in (0.254 cm)
follower link (at mid-point) : 0.1 in (0.254 cm)

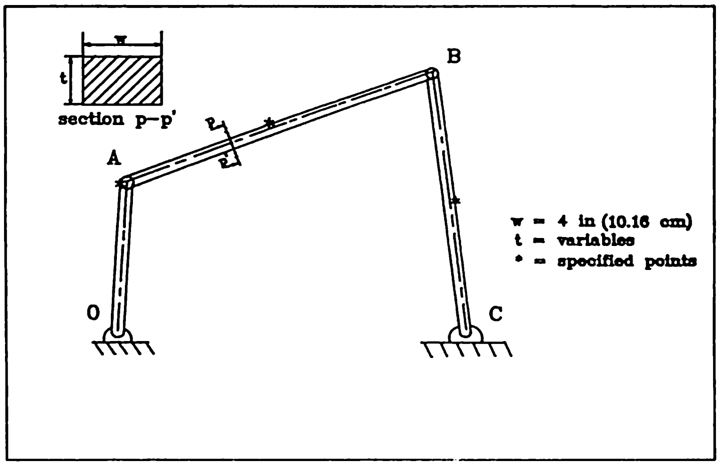

Figure 5 : Four-bar linkage and the specified points for deflection for Example 4

It can be found that only deflection constraint is active in this example. When equations (16) and (17) are used, it takes only six iterations for either full dynamic or quasi-static model to complete the design. The design results are shown in Table 6. Both stress and deflection constraints can be satisfied.

In each of the examples, the initial values of the cross-sectional areas A_i for iteration are equal to unity.

Method	Link	Designed Thickness (in.)	Max. Deflection in Specified Point (in.)	Maximum Stress (psi)
Full Vibration	Crank	2.129	0.010	2386.71
	Coupler	0.616	0.099	3546.59
	Follower	0.878	0.099	3408.61
Quasi–Static	Crank	1.893	0.010	2076.13
	Coupler	0.520	0.100	2808.85
	Follower	0.805	0.100	3081.75

Table 6 : Designed results for Example 4

CONCLUSION :

The optimality conditions are derived in this paper when both stress and deflection constraints are considered. Two design procedures are also suggested. Procedure I is the improved Thornton's design procedure where the iterating equation (16) is much simplified and the relaxation factor can be automatically updated. Equation (16) is very simple and is only valid for the mechanisms modeled with beam element, due to the basic assumption of these design rules.

In design procedure II, without using complicated procedure to compromise stress and deflection constraints, interpolation technique can easily be used to obtain the optimal design. In most of the cases, design procedure I can be used to solve the problem quite efficiently. Design procedure II consumes more CPU time yet more stable. Therefore, it can be used to solve those problems that can not be solved with design procedure I.

An obvious extension of this work will be the development of the optimality conditions when stress, displacement, natural frequency,... etc. constraints are all included. Some exquisite methods can be developed to involve these optimality conditions.

ACKNOWLEDGEMENT :

The authors would like to express their appreciation for the financial support of the Department of Mechanical and Aerospace Engineering and Engineering Mechanics, University of Missouri-Rolla.

REFERENCES :

1. Dubowsky, S., and F. Freudenstein, "Dynamic Analysis of Mechanical Systems with Clearances, Part I: Formulation of Dynamic Model; Part II: Dynamic Response," Journal of Engineering for Industry, 93, No. 1, pp. 305-316, (February 1971).

2. Bagci, C. Kalagcioglu, S., "Elastodynamics of Planar Mechanisms Using Actual Finite Line Elements, Lumped Mass Systems, Matrix-Exponential Method, and the Method of CGKES," Journal of Mechanical Design, Vol 101, No. 3, 1979, pp.417-427.

3. Sadler, J. P. and G. N. Sandor, "Nonlinear Vibration Analysis of Elastic Four-bar Linkage," Journal of Engineering for Industry, 96, No. 2, pp. 411-419, 1974.

4. Imam, I., G. N. Sandor, and S. N. Kramer, "Deflection and Strain Analysis in High-Speed Planar Mechanisms with Elastic Links," Journal of Engineering for Industry , 95B, pp. 541-548, (1973).

5. Bahgat, B. M., K. D. Willmert, "Finite Element Vibrational Analysis of Planar Mechanisms," Mechanism and Machine Theory, 11 (1), pp. 47-71, (1976).

6. Winfrey, R. C., "Elastic Link Mechanisms Dynamics," ASME Journal of Engineering in Industry, Vol. 93, pp. 268-272, (1971).

7. Imam, I., and G. N. Sandor, "High-Speed Mechanism Design --A General Analytical Approach," ASME Journal of Engineering for Industry, Vol. 97, pp. 609-628, (May 1975).

8. Midha, A., A.G. Erdman, D.A. Frohrib, "A Computational Efficient Numerical Algorithm for the Transient Response of High-Speed Elastic Linkages," ASME Journal of Mechanical Design, Vol.101, 138-148 (1979).

9. Turcic, D. A., A. Midha, and J. R. Bosnik, " Dynamic Analysis of Elastic Mechanism Systems, Part II: Applications and Experimental Results," ASME, Journal of Dynamic Systems, Measurement, and Control, Vol. 106, 255-260, 1984.

10. Song J.O. and E.J. Haug, "Dynamic Analysis of Planar Flexible Mechanisms," Journal of Computer Methods in Applied Mechanics and Engineering, Vol. 24, 1980, pp. 359-381.

11. Dado, M. and A. H. Soni,"Complete Dynamic Analysis of Elastic Linkages," ASME Paper 86-DET-52. 13. Shabana, A. and Wehage, R. A., "Spatial Transient Analysis of Inertia-Variant Flexible Systems," Journal of Mechanisms, Transmissions, and Automation in Design. Vol. 106, No. 2, March 1984, pp.126-132.

12. Shabana, A. and Wehage, R. A., "Spatial Transient Analysis of Inertia-Variant Flexible Systems," Journal of Mechanisms, Transmissions, and Automation in Design. Vol. 106, No. 2, March 1984, pp.126-132.

13. Liou, F.W. and A.G. Erdman, "Analysis of A High-Speed Flexible Four-bar Linkage, Part I: Formulation and Solution", Journal of Vibration, Acoustics, Stress, and Reliability in Design, Vol. 111, No. 1, pp.35-41, 1989.

14. Liou, F.W. and A.G. Erdman, "Analysis of A High-Speed Flexible Four-bar Linkage, Part II: Analytical and Experimental Results on the Apollo", Journal of Vibration, Acoustics, Stress, and Reliability in Design, Vol. 111, No. 1, pp.42-47, 1989.

15. Liou, F.W. "Dynamic Analysis of High-Speed Mechanisms with Elastic Members," Doctoral dissertation, University of Minnesota, 1987.

16. Khan, M. R., W. A. Thornton, and K. D. Willmert, "Optimality Criterion Techniques Applied to Mechanism Design," ASME Journal of Mechanical Design, Vol. 100, pp. 319-327, (1978).

17. Thornton, W. A., K. D. Willmert, and M. R. Khan, "Mechanism Optimization via Optimality Criterion Techniques," ASME Journal of Mechanical Design, Vol. 101, pp. 392-397, (1979).

18. Cleghorn, W. L., R. G. Fenton, and B. Tabarrok, "Optimum Design of High-Speed Flexible Mechanisms," Mechanism and Machine Theory, Vol. 16, No. 4, pp. 399-406, (1981).

19. Zhang, Ce. and H. T. Grandin, "Optimum Design of Flexible Mechanisms," ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 105, pp. 267-272.

20. Liou F. W. and Liu J. D., "Optimal Design of Flexible Mechanisms Using General Design Rules," to be presented in First International Applied Mechanical Systems Design Conference, 1988, Cookeville, Tennessee.

21. Liou F. W., A. G. Erdman and K. Stelson, "General Design Rules for High-Speed, Flexible Mechanisms" 1988 20th Biennial Mechanisms Conference, 1988, Kissimmee, Florida, Sep. 25-28, 1988, also being reviewed for publication in Journal of Mechanisms, Transmissions, and Automation in Design.