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Linear State Variable Dynamic Model and Estimator Design for Allison T406 Gas Turbine Engine*

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ABSTRACT

This paper describes a procedure for developing a State Variable Model for the Allison T406 gas turbine engine. This linear model is useful for designing controllers using modern control techniques. The engine and V-22 rotor system is modeled around an operating point by using four state variables and one input variable. For a given power setting, it is observed that two linear models are sufficient to represent the engine dynamics over the entire flight envelope. A relationship between surge margin and the state variables is also developed. It is demonstrated that these linear models are useful in designing an estimator for accommodating hard sensor failures.

NOMENCLATURE

CDP - compressor discharge pressure

CVG - compressor variable geometry

FI - flight idle

FADEC - full authority digital electronic control

I_g - moment of inertia of the gas generator

IRP - intermediate rated power

MGT - measured gas temperature

N_g - gas turbine speed

$\dot{N}_g$ - angular acceleration of the gas turbine

N_p - power turbine speed

$\dot{N}_p$ - angular acceleration of the power turbine

N_R - rotor speed

$\dot{N}_R$ - angular acceleration of the rotor

Q_g - gas generator torque

Q_p - power turbine torque

Q_R - rotor torque

$\dot{Q}_R$ - rate of change of rotor torque

SM - surge margin

TIT - turbine inlet temperature

u - control input vector

W_a - air flow

W_f - fuel flow

x - state vector

$\hat{x}$ - estimated value of state vector

$\tilde{x}$ - state estimation error

δ - pressure correction factor

Δ - perturbation parameter

θ - temperature correction factor

INTRODUCTION

The T406 engine is being developed under U.S. Navy contract for the V-22 Osprey tilt nacelle aircraft. This turboshaft engine is a 6000 SHP class free turbine. Turbine engine performance and dynamics are described by nonlinear component performance maps and equations, which can be modeled using a digital computer. The nonlinear computer model is capable of simulating the engine transient and steady state operation for the entire flight envelope.

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The design of a controller to meet the performance and operational constraints, by using this nonlinear model is very cumbersome. Hence, a linear state variable model is developed using partial derivatives generated by the nonlinear model. This linear model is amenable for designing controllers using modern control techniques.

The T406 engine and a simplified V-22 rotor system is modeled around an operating point by using four state variables, gas turbine speed (N_g), power turbine speed (N_p), rotor torque (Q_R), rotor speed (N_R) and an input variable fuel flow (W_f). The measured output variables are gas turbine speed, power turbine speed and torque. By using the state and input variables, the air flow (W_a), compressor discharge pressure (CDP), turbine inlet temperature (TIT), measured gas temperature (MGT) and surge margin (SM) are computed.

The availability of surge margin information in the model allows controller design which can ensure surge free operation of the engines. A critical comparison is made of the transient and steady state response, between the linear state variable and nonlinear models. For a given power setting, the responses are compared at fourteen operating points covering the entire flight envelope. The transients match very well, with small errors existing in the steady state values. On the basis of linearization error analysis, it is observed that at a given power setting two linear models adequately represent engine response over the entire flight envelope. This analysis is repeated for three power settings from Flight Idle (FI) to intermediate rated power (IRP). The response of the computed variables TIT, MGT, SM, CDP,and Wa, match very well with nonlinear simulations.

An important consideration in the design of high performance control systems for gas turbine engines is the design of fault tolerant control systems. These control systems can enhance the reliability of advanced engines against degraded engine control due to the failure of a sensor. The availability of microcomputers for engine control provides the means to accommodate some failed sensors without requiring redundant hardware sensors. The fault tolerant control system enables the detection of a failed sensor and the estimation of the failed sensor signal which can then be utilized for continuity of safe engine operation. By using the linear state variable model, a feasibility study is completed on the design of an estimator for accommodating hard sensor failures, and a critical comparison is made between the various estimator design techniques.

DESCRIPTION OF T406 ENGINE AND ROTOR SYSTEM

The T406-AD-400 gas turbine engine, currently under development at Allison, is designed to be installed on the V-22 Osprey tilt rotor aircraft. The V-22 is a vertical lift aircraft, being developed by Bell/Boeing, as a joint service program to fulfill a variety of mission needs of the Marines, Army, Air Force and Navy. An illustration of the aircraft is shown in Figure 1. This aircraft combines the vertical takeoff, landing and hover capabilities of a helicopter with the high speed and long range of a turboprop aircraft.

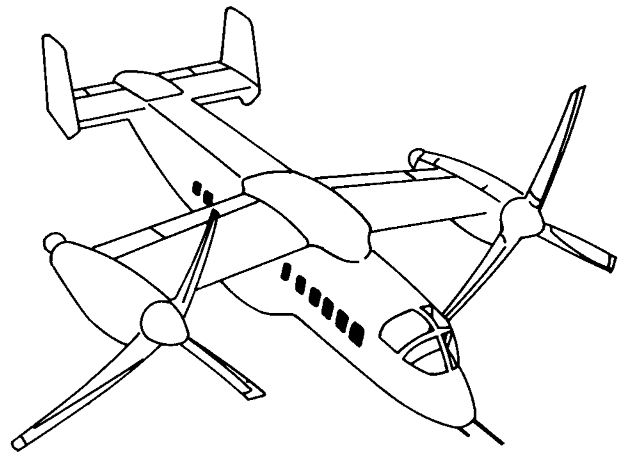


FIG. 1 V-22 OSPREY TILT ROTOR AIRCRAFT

A cross sectional view of the Allison T406 turboshaft engine is shown in Figure 2. This free turbine engine consists of a fourteen stage axial flow compressor directly coupled to a two stage gas generator turbine. The compressor includes six variable geometry stator stages. A two stage power turbine is directly coupled to the power output shaft to provide power for driving the aircraft rotors. Fuel is burned in an annular combustor to provide the energy that runs the engine. An accessory gear box is attached to the engine for driving the fuel pump, starter, alternator, and oil pumps.

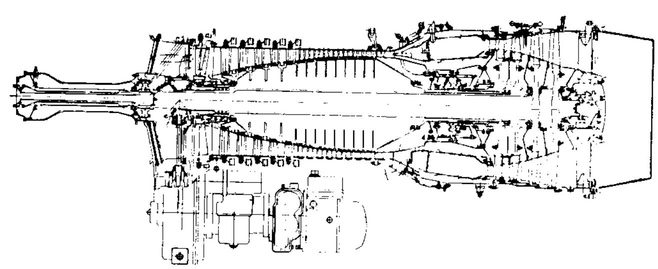
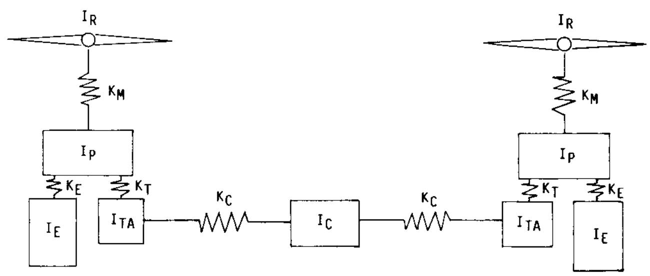


FIG. 2 ENGINE CROSS SECTION

Basic engine control consists of positioning the compressor variable geometry and metering the fuel flow to the engine. The control system must provide prompt response to the pilot's power commands yet maintain sufficient surge margin, limit engine temperatures and speeds to safe levels, and maximize operating efficiency. These tasks are performed using a full authority digital electronic control (FADEC). A power demand signal from the flight control computer is sent to the FADEC where it is converted to a gas turbine speed reference. Proportional plus integral control of the gas turbine speed error determines the engine fuel flow demand. Compressor variable geometry position is determined as a function of corrected gas turbine speed. For redundancy, the engine has two single channel FADECs plus an independent analog backup electronic control.

The V-22 Osprey propulsion system consists of two Allison T406 turboshaft engines. One engine is mounted on the end of each wing. Aircraft transition between helicopter and airplane modes is accomplished by rotating the engine nacelle between vertical and horizontal positions. The output shaft of each engine drives a proprotor gear box which in turn directly drives one rotor and shares torque with the other rotor through the tilt gear boxes and the mid wing gear box. A simple schematic of this rotor system using inertias and torsional spring rates is shown in Figure 3.



I_R Rotor Inertia
I_P Proprotor Gear Box Inertia
I_E Engine Inertia
I_{TA} Tilt Axle Gear Box Inertia
I_C Mid Wing Gear Box Inertia
K_M Mast and Backup Structure Torsional Stiffness
K_E Engine Output Shaft Torsional Stiffness
K_T Tilt Axle Gear Box Shaft Torsional Stiffness
K_C Cross Axle Shaft Torsional Stiffness

FIG. 3 SIMPLIFIED V-22 ROTOR SYSTEM

STATE VARIABLE MODEL FORMATULATION

Nonlinear Computer Models

Development of the linear state variable model required use of a nonlinear digital engine model to generate steady state data for calculating partial derivatives at various operating points, and to run transient response simulations for comparison with the state variable model response. This detailed nonlinear computer model is a thermodynamic representation of the gas path through the engine. Thermodynamic properties of the gas path are calculated at various stations from engine inlet through the exhaust nozzle. The performance of individual components, and the overall engine performance are also calculated in this program. The program follows an iterative matching procedure to maintain continuity in the gas path and to match other performance parameters from component to component for the conditions at which the engine is running.

The model considers such factors as ambient pressure, ambient temperature, and inlet conditions due to the effect of aircraft velocity. The program will calculate steady state engine parameters when running to a specified power turbine speed and either gasifier speed, temperature, horsepower, or fuel flow. The program also allows for the inclusion of horsepower extraction. It is in this mode that data for calculating partial derivatives is obtained.

Linear State Variable Model

The T406 engine and a simplified V-22 rotor system is modeled around an operating point by using four state variables, gas turbine speed (N_g), power turbine speed (N_p), rotor torque (Q_R) and rotor speed (N_R). This engine has two input variables, fuel flow (ω_f) and compressor variable geometry (CVG), however in this analysis only fuel flow is considered as an input. In order to generate a linear model, the system was linearized about an operating point, thus all state, input and output variables represent perturbations around that point.

Gas generator torque (Q_g) and power turbine torque (Q_p) were linearized as functions of N_g, N_p and ω_f.

$$\Delta Q_g = (\frac{\delta Q_g}{\delta N_g})\ \Delta N_g + (\frac{\delta Q_g}{\delta N_p})\ \Delta N_p + (\frac{\delta Q_g}{\delta \omega_f})\ \Delta \omega_f \qquad (1)$$

$$\Delta Q_p = (\frac{\delta Q_p}{\delta N_g})\ \Delta N_g + (\frac{\delta Q_p}{\delta N_p})\ \Delta N_p + (\frac{\delta Q_p}{\delta \omega_f})\ \Delta \omega_f \qquad (2)$$

Using the fact that torque is the product of the moment of inertia and angular acceleration, we have

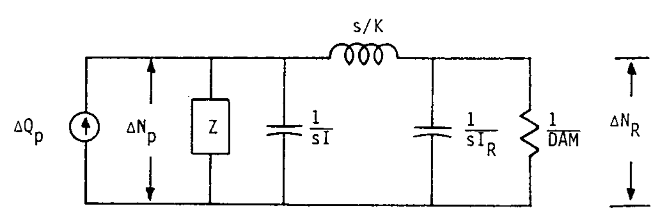
$$\Delta \dot{N}_g = \frac{1}{I_g}\ \Delta Q_g$$

$$= \frac{1}{I_g}\ (\frac{\delta Q_g}{\delta N_g})\ \Delta N_g + \frac{1}{I_g}\ (\frac{\delta Q_g}{\delta N_p})\ \Delta N_p$$

$$+ \frac{1}{I_g}\ (\frac{\delta Q_g}{\delta \omega_f})\ \Delta \omega_f \qquad (3)$$

where I_g and $\Delta \dot{N}_g$ are the moment of inertia and angular acceleration of the gas generator.

For analysis purposes, only one engine and the rotor system model is considered, as both engines are similar. A simplified equivalent analogous electrical circuit of this system is given below.



$I = I_p + I_E$
Z = Cross Shaft Loading
K = Equivalent Torsional Stiffness
DAM = Aerodamping Load

FIG. 4 EQUIVALENT CIRCUIT FOR SINGLE ENGINE AND ROTOR SYSTEM

Let the torque absorbed (delivered) by the cross shaft be $\Delta Q_{ps} = (\pm\sigma)Q_p$, where $0 \leq \sigma \leq 1$. From the equivalent circuit (Figure 4), the following relationships are developed.

$$\Delta\dot{N}_p = \left(\frac{1\mp\sigma}{I}\right) \Delta Q_p - \frac{1}{I} \Delta Q_R \quad (4)$$

$$\Delta\dot{Q}_R = K \Delta N_p - K \Delta N_R \quad (5)$$

$$\Delta\dot{N}_R = \frac{1}{I_R} \Delta Q_R - \frac{DAM}{I_R} \Delta N_R \quad (6)$$

Equations (2) through (6) yields a linear state variable model.

$$\begin{bmatrix} \Delta\dot{N}_g \\ \Delta\dot{N}_p \\ \Delta\dot{Q}_R \\ \Delta\dot{N}_R \end{bmatrix} = \begin{bmatrix} \frac{1}{I_g}\left(\frac{\delta Q_g}{\delta N_g}\right) & \frac{1}{I_g}\left(\frac{\delta Q_g}{\delta N_p}\right) & 0 & 0 \\ \frac{1}{I}(1\mp\sigma)\left(\frac{\delta Q_p}{\delta N_g}\right) & \frac{1}{I}(1\mp\sigma)\frac{\delta Q_p}{\delta N_p} & -\frac{1}{I} & 0 \\ 0 & K & 0 & -K \\ 0 & 0 & \frac{1}{I_R} & -\frac{DAM}{I_R} \end{bmatrix} \begin{bmatrix} \Delta N_g \\ \Delta N_p \\ \Delta Q_R \\ \Delta N_R \end{bmatrix} + \begin{bmatrix} \frac{1}{I_g}\left(\frac{\delta Q_g}{\delta \omega_f}\right) \\ (1\mp\sigma)\left(\frac{\delta Q_p}{\delta \omega_f}\right) \\ 0 \\ 0 \end{bmatrix} \Delta\omega_f \quad (7)$$

Output Equation:

$$\begin{bmatrix} \Delta N_g \\ \Delta N_p \\ \Delta Q_p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \left(\frac{\delta Q_p}{\delta N_g}\right) & \left(\frac{\delta Q_p}{\delta N_p}\right) & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta N_g \\ \Delta N_p \\ \Delta Q_R \\ \Delta N_R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \left(\frac{\delta Q_p}{\delta \omega_f}\right) \end{bmatrix} \Delta\omega_f \quad (8)$$

In order to effectively control or monitor engine operation, knowledge of air flow (ω_a), compressor discharge pressure (CDP), turbine inlet temperature (TIT), measured gas temperature (MGT) and surge margin (SM) is required. A relationship between these desired variables and state variables is developed and given below:

For small variations around the operating point, the perturbed variables ($\Delta\omega_a$, ΔCDP, ΔTIT, ΔMGT) are dependent primarily on ΔN_g and $\Delta\omega_f$ and can be expressed as

$$\Delta\omega_a = \left(\frac{\delta\omega_a}{\delta N_g}\right) \Delta N_g + \left(\frac{\delta\omega_a}{\delta\omega_f}\right) \Delta\omega_f \quad (9)$$

$$\Delta CDP = \left(\frac{\delta CDP}{\delta N_g}\right) \Delta N_g + \left(\frac{\delta CDP}{\delta\omega_f}\right) \Delta\omega_f \quad (10)$$

$$\Delta TIT = \left(\frac{\delta TIT}{\delta N_g}\right) \Delta N_g + \left(\frac{\delta TIT}{\delta\omega_f}\right) \Delta\omega_f \quad (11)$$

$$\Delta MGT = \left(\frac{\delta MGT}{\delta N_g}\right) \Delta N_g + \left(\frac{\delta MGT}{\delta\omega_f}\right) \Delta\omega_f \quad (12)$$

The partial terms $\frac{\delta\omega_a}{\delta N_g}$, $\frac{\delta\omega_a}{\delta\omega_f}$, ---- contained in equations (9) through (12)are calculated at a given operating point from the nonlinear computer simulation.

Computation of Surge Margin:

Surge is a phenomena common to gas turbine compressors under certain operating conditions. As the flow through an axial compressor is throttled from the design steady state point to the limit, the steady axisymmetric flow pattern becomes unstable. The phenomenon resulting from this instability is known as surge. When operating the gas turbine engine, it is essential to avoid surge under all operating conditions. A linear equation for calculating surge margin in terms of state variables and the input variable is developed.

The compressor surge lines for airflow and pressure ratio as a function of corrected N_g were linearized for normal operating conditions as,

$$R_C \text{ Surge} = m_1 \frac{N_g}{\sqrt{\theta}} + C_1 \quad (13)$$

$$\omega_{ac} \text{ Surge} = m_2 \frac{N_g}{\sqrt{\theta}} + C_2 \quad (14)$$

where m_1, m_2, C_1 and C_2 are constants.

The surge margin is computed by using the relationship

$$SM = \frac{R_C \text{ Surge} \times \omega_{ac}}{R_C \times \omega_{ac} \text{ Surge}} - 1 \quad \times 100\ [\%] \quad (15)$$

where θ and δ are correction factors ,

$$R_C = \frac{CDP}{14.7\delta}, \quad \omega_{ac} = \frac{\omega_a\sqrt{\theta}}{\delta}$$

Substitution of Eqns. (13 and 14) in Eq. (15) yields

$$SM = \left[\frac{14.7\ \omega_a(m_1 N_g\sqrt{\theta} + C_1\theta)}{CDP\ (m_2 N_g + C_2\sqrt{\theta})} - 1\right] \times 100 \quad (16)$$

Linearization of Eq. (16) and then substitution of Eqn. (9 and 10) yields,

$$\Delta SM = \left[\left(\frac{\delta SM}{\delta N_g}\right) + \left(\frac{\delta SM}{\delta CDP}\right) \left(\frac{\delta CDP}{\delta N_g}\right) + \left(\frac{\delta SM}{\delta \omega_a}\right) \left(\frac{\delta \omega_a}{\delta N_g}\right) \right] \Delta N_g$$

$$+ \left[\left(\frac{\delta SM}{\delta \omega_a}\right) \left(\frac{\delta \omega_a}{\delta \omega_f}\right) + \left(\frac{\delta SM}{\delta CDP}\right) \left(\frac{\delta CDP}{\delta \omega_f}\right) \right] \Delta \omega_f \quad (17)$$

where

$$\left(\frac{\delta SM}{\delta N_g}\right) = \frac{1470\ \omega_{ao}}{CDP_o} \quad \frac{(m_2 N_{go} + C_2\sqrt{\theta})(m_1\sqrt{\theta}) - (m_1\frac{N_{go}}{\sqrt{\theta}} + C_1\theta)m_2}{(m_2\ N_{go} + C_2\ \sqrt{\theta})^2} \quad (18)$$

$$\left(\frac{\delta SM}{\delta CDP}\right) = -\ 1470\ \omega_{ao} \frac{(m_1\ N_{go}\sqrt{\theta} + C_1\ \theta)}{(m_2\ N_{go} + C_2\sqrt{\theta})(CDP_o)^2} \quad (19)$$

$$\left(\frac{\delta SM}{\delta \omega_a}\right) = \frac{1470\ (m_1\ N_{go}\sqrt{\theta} + C_1\theta)}{CDP_o\ (m_2\ N_{go} + C_2\sqrt{\theta})} \quad (20)$$

N_{go}, ω_{ao}, CDP_0 are the steady state values at the operating point.

Equation (17) gives information about surge margin in terms of the state variable (ΔN_g) and input variable ($\Delta\omega_f$).

The partial derivatives are approximated by $\frac{\delta x}{\delta y} = \frac{\Delta x}{\Delta y}$ and these derivatives are calculated numerically using the nonlinear computer model.

Comparison of Step Response at Various Operating Points

In order to find appropriate linearization points, partial derivatives were evaluated at six different operating points at the 3000 hp level, thus yielding six linear models. The step response of each of these linear models was compared to that of the nonlinear simulations at fourteen operating points covering the entire flight envelope. Based on these comparisons, it was determined that two linear models can adequately represent engine performance over the entire flight envelope for a given power setting. For high altitude operating conditions, a model derived at 20,000 ft., 0.2 MN provides satisfactory approximations. In the case of low altitude operating points, a model derived either at 10,000 ft., 0.4 MN or at sea level, static provides good approximations. The transient responses between linear and nonlinear models matched very well, however small steady state errors were present. This procedure was repeated for 2000 hp and 4000 hp level operating points and similar results were obtained. Figure 5 shows typical transient response comparisons at one operating point.

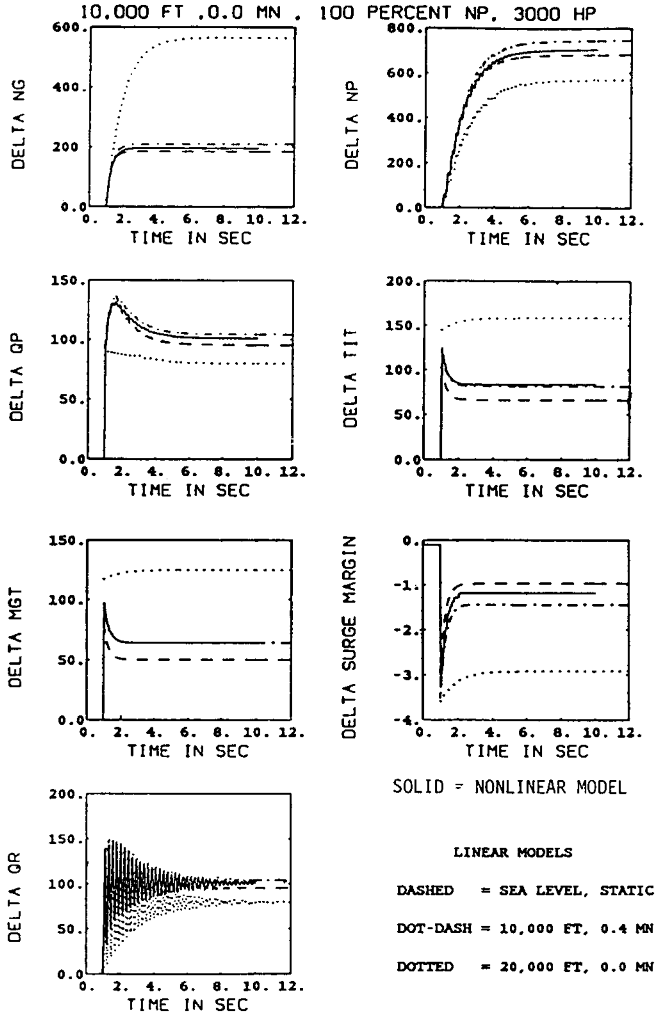


FIG. 5. TRANSIENT RESPONSE COMPARISONS.

In order to quantify these steady state errors, a percent error was defined as

$$\%\ Error = \frac{\left(\begin{array}{c}\text{steady state value}\\ \text{of linear model}\end{array}\right) - \left(\begin{array}{c}\text{steady state value}\\ \text{of nonlinear model}\end{array}\right)}{(\text{steady state value of nonlinear model})} \times 100$$

The percent error in ΔN_g versus operating point number is plotted in Fig. 6. Similar plots are generated for ΔN_p, ΔQ_R, ΔQ_p, ΔTIT, ΔMGT and ΔSM, however, they are not included in this paper in order to limit the number of pages.

On the basis of these steady state error plots, two appropriate models for each power setting have been selected. One for the low altitude region and another for high altitude conditions. Using this scheme, it is found that the average magnitude of the steady state errors is less than 12%.

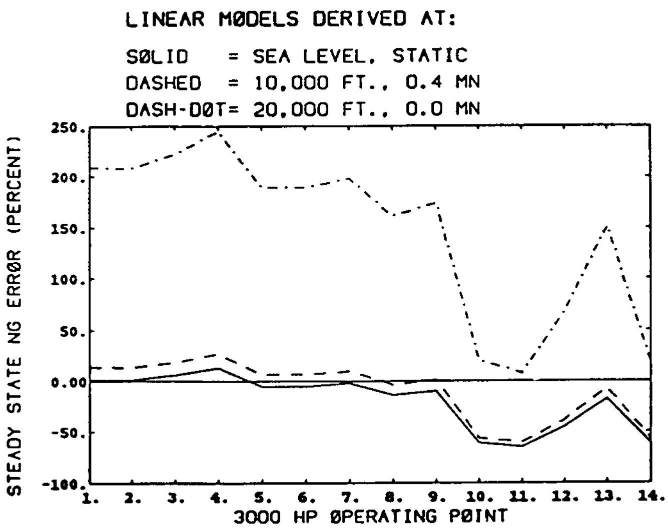


FIG. 6 PERCENT ERROR V/S OPERATING POINT

DESIGN OF ESTIMATORS FOR ACCOMMODATING HARD SENSOR FAILURES

Various methods for detecting sensor failures in engine control systems using estimator techniques have been introduced in recent years [2]. These methods are based on the principle of analytical redundancy and offer advantages in cost, weight and reliability over hardware redundancy techniques. The availability of microcomputers for engine control provides the means to accommodate some failed sensors by generating an estimate of the failed sensor signal which can then be utilized for continuity of safe engine operation. By using the linear state variable model developed in the previous section, an estimator is designed for accommodating hard sensor failures.

The estimator is a dynamic system in which the output of the estimator approaches asymptotically the sensor value that is to be reconstructed. The state estimator utilizes the available input and output to reconstruct the failed sensor value of the system. Amongst the various methods available for designing an estimator, the Eigenstructure assignment procedure [3] has many advantages. The eigenstructure assignment procedure is used for designing an estimator for the engine control system.

The dynamical equation of the state estimator is given by

$$\dot{\hat{x}} = (A - LC)\,\hat{x} + Ly + Bu \qquad (21)$$

The gain matrix L must be designed so that the error ($\tilde{x} = x - \hat{x}$) decays to zero in a reasonable amount of time and has a satisfactory time response.

By using the eigenstructure assignment procedure [3] we can determine L as

$$L^T = -Z\,w^{-1} \qquad (22)$$

where $w = [\omega_1\ \omega_2\ - - -\ \omega_n]$

$\omega_1^T,\ \omega_2^T,\ - - -\ \omega_n^T$ = Reciprocal eigenvectors.

$Z = [Z_1\ Z_2\ - - -\ Z_n]$

The vector $[\omega_i^T\ Z_i^T]^T$ must lie in the kernel or null space of the matrix

$$S(\lambda_i) = [A^T - \lambda_i I C^T] \qquad \text{for } i = 1, 2, - - - n \qquad (23)$$

where λ_i = desired eigenvalues.

The eigenstructure assignment procedure is applied to the linear model of the engine-rotor system for one operating condition. The transient response of the original and estimated state variables is given in Fig.7. It can be seen from these graphs, that the estimated value approaches the true value within 0.2 seconds. We have also applied reduced order estimation design procedures and obtained similar results.

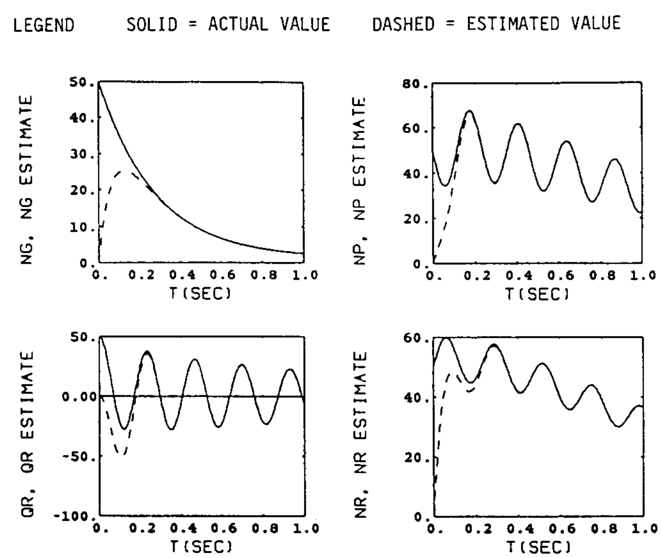


FIG. (7a) COMPARISON OF TRUE AND ESTIMATED VALUES WITH FAILED N_g SENSOR.

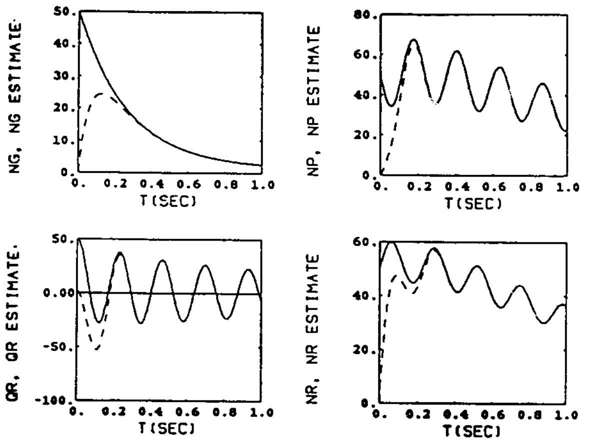


FIG. (7b) COMPARISON OF TRUE AND ESTIMATED VALUES WITH FAILED N_p SENSOR.

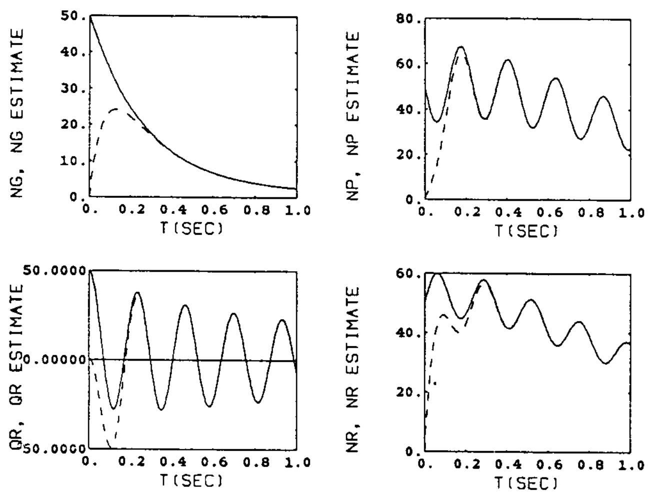


FIG. (7c) COMPARISON OF TRUE AND ESTIMATED VALUES WITH FAILED Q_p SENSOR.

CONCLUSIONS

It is demonstrated that a satisfactory linear state variable model can be derived for the engine-rotor system by using partial derivative terms. At a given power setting, two linear models adequately represent the transient response over the entire flight envelope. Obviously, increasing the number of models to cover the flight envelope will decrease the errors, but the complexity of analysis will be increased.

The linear model contains information about ω_a, CDP, TIT, MGT and surge margin. This data enables the design of surge free controllers using the linear state variable models.

By using the linear state variable model, an estimator is designed for accommodating hard sensor failures. The preliminary results obtained are very encouraging in the design of a fault tolerant control system.

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