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Reduced Order Modeling Methods For Turret-gun System

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REDUCED ORDER MODELING METHODS FOR TURRET - GUN SYSTEM

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Abstract

Reduced order modelling techniques are used to design lower order robust controllers for a Turret-Gun System. The balance-truncation, Routh approximation, Litz's modal techniques are used to derive reduced order models for a Turret-Gun System. A critical comparison of time and frequency response characteristics between original and reduced order models is made. The spillover problem associated with a reduced order model is also investigated.

1. INTRODUCTION

One of the important problems in the control of a Turret-Gun system is to approximate a high-order, complex mathematical model of the system with a low-order, simpler model. The resulting reduced order models are useful for designing and implanting robust controllers for a Turret-Gun System. This methodology will provide simplicity of implementation and reduction in hardware requirements.

The Turret-Gun System contains nonlinear elements, such as gear trains, servo valves, hydraulic motors etc. In this system, the firing disturbances excite the structural modes. Integrated Systems Inc. (ISI) has developed a detailed nonlinear model and identified various parameters of the model [1]. A linear quadratic Gaussian with loop transfer recovery (LQG/LTR) controller is designed for the gun system using linearized models. We encountered the convergence/numerical integration problems in the simulation of this controller along with the non-linear plant. It is also noticed that there is a large spread of eigenvalues of the linear model [2]. For the convergence of the control algorithms and their easy implementation, controllers are designed using reduced order models.

A large number of techniques are available in the literature for deriving reduced order models. In this paper reduced order models for Turret-Gun Systems are designed using balanced realization [3], Litz's modal technique [4] and Routh Approximation [5,6] methods. A critical comparison of time and frequency response characteristics between an original and reduced order models is made. The choice of a reduced order model is very significant for the validity of reduced order robust controllers. The linear quadratic Gaussian with loop transfer recovery methodology is employed to design reduced order robust controllers. The discarded modes in the original system representation can be excited by a reduced order controller and may destabilize the control system. This problem is known as spillover problem and is eliminated in the proposed design methodology.

2. DESCRIPTION AND STATE VARIABLE MODEL OF THE TURRET-GUN SYSTEM

The Turret-Gun System consists of a 30 mm chain gun driven by an electrical motor and is capable of firing 600 rounds per minute. Weapon pointing commands are generated by an integrated helmet and display sight system using a fire control computer.

The gun is mounted within a cradle using a brass slide mechanism which allows for recoil movement. Recoil adapters are mounted between the recoiling mass of the gun and the cradle to absorb some of the recoil force. The cradle and gun assemblies are attached to a fork using two trunnion pins. One trunnion pin has a resolver built into it. This resolver provides the elevation pointing error to the turret control box. The elevation axis positioning is accomplished through the use of a servo valve controlled, double acting hydraulic cylinder. The piston has unequal cross-sectional areas to account for gravitational effects. A delta hydraulic pressure transducer provides rate feedback information to the turret control box.

The fork assembly is held in place by the Azimuth housing. The housing holds a rotary hydraulic motor and a gearbox. The housing also holds a train rate sensor which measures the angular velocity of the Gun/Cradle/Fork unit, and a resolver for measuring the angular position.The Azimuth housing, fork, cradle and gun are attached to the vehicle's hull.

A detailed nonlinear model of the Turret-Gun System [1] was developed by Integrated Systems Inc. (ISI). Preliminary testing has shown that very little coupling exists between the elevation and azimuth axes. Hence independent models were developed for azimuth and elevation axes. The azimuth axis system (Fig 1) consists of three physically identifiable sections: (1) servovalve (2) hydraulic motor/gearbox and (3) gun plant. A block diagram representation of this system is shown in Fig. 2. A linear state variable model of the Turret-Gun System is obtained by using system build and analyzing system features of matrix x.

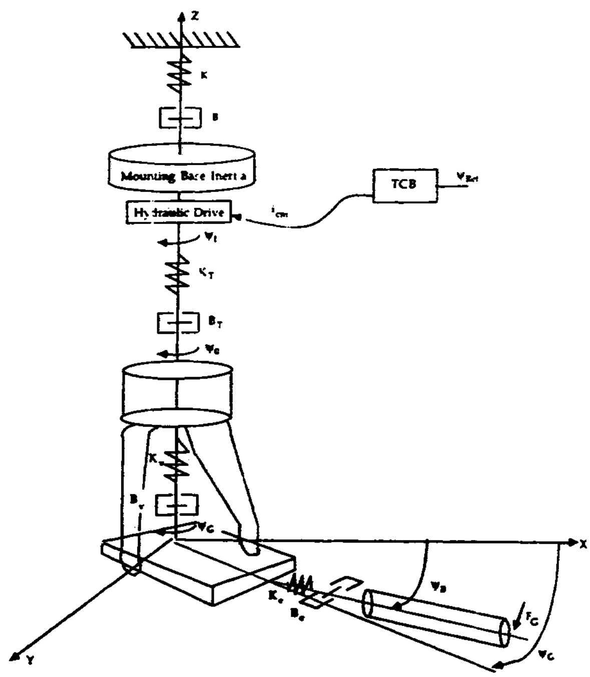

Fig 1. Schematic Diagram of Azimuth Axis System

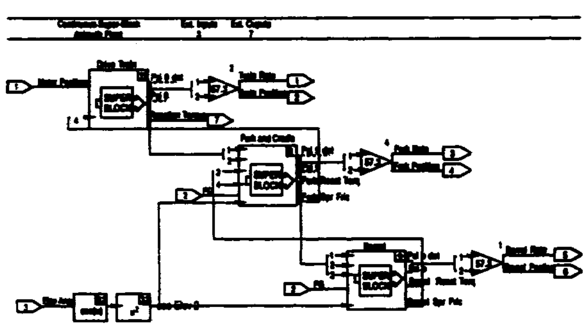

Fig 2. Block Diagram

3. REDUCED - ORDER MODELING TECHNIQUES

In this Section a brief review of various reduced order modeling techniques is given.

(A) Balanced Realization Method:

Among the various model reduction techniques, reduced order models derived using the balancing-truncation [3] technique have many advantages. This technique is based on controllability and observability (location of actuators and sensors) considerations of the plant. The subsystem corresponding to dominant (non-dominant) singular values of the balanced Grammians is termed the strong (weak) subsystem. By approximating the weak subsystem at $\omega = 0$, a reduced order model is derived. Let the state variable representation of a Turret-Gun be given by

$$\dot{x} = Ax + Bu \quad \ldots(1)$$

$$y = Cx + Du \quad \ldots(2)$$

Let $$x_b = Tx \quad \ldots(3)$$

where T is a linear transformation matrix which transforms the system representation into balance realization form.

$$\dot{x}_b = A_{bal}x_b + B_{bal}u \quad \ldots(4)$$

$$y = C_{bal}x_b + D_{bal}u \quad \ldots(5)$$

The Hankel singular values (σ_i) are determined by

$$\sigma_i = \sqrt{\lambda_i(PQ)} \qquad i = 1, 2, \ldots n \quad \ldots(6)$$

where P and Q are controllability and observability grammians.

The balanced realization Eqns (4 and 5) are partitioned as

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \quad \ldots(7)$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u \quad \ldots(8)$$

$$y = C_1x_1 + C_2x_2 + Du \quad \ldots(9)$$

The approximation employed is

$$X_2(s) \approx -A_{22}^{-1}A_{21}X_1(s) - A_{22}^{-1}B_2U(s) \quad \ldots(10)$$

and the reduced order model is given by

$$\dot{x}_r = A_rx_r + B_ru \quad \ldots(11)$$

$$y_r = C_rx_r + D_ru \quad \ldots(12)$$

where

$$A_r = A_{11} - A_{12}A_{22}^{-1}A_{21} \quad \ldots(13)$$

$$B_r = B_1 - A_{12}A_{22}^{-1}B_2 \quad \ldots(14)$$

$$C_r = C_1 - C_2A_{22}^{-1}A_{21} \quad \ldots(15)$$

$$D_r = D_1 - C_2A_{22}^{-1}B_2 \quad \ldots(16)$$

The reduced order models possess the following properties [7] which are very useful in the design of reduced order LQG/LTR controllers.

(i) The reduced order models preserve stability, controllability, observability and minimality.
(ii) The steady state values of the original system and reduced order model are identical.
(iii) Error bound of the reduced order model

$$\bar{\sigma}[G(j\omega) - G_r(j\omega)] \quad < \quad 2\sum_{i=r+1}^{n}\sigma_i \quad \forall\ \omega \quad \ldots(17)$$

(iv) Reduced order models have good frequency response match at low frequencies.

(B) Routh Approximation Method

The Routh Approximation method proposed by Hutton and Friedland [5]. is based on the Routh Stability criterion. This reduced order methodology has the following properties.

(i) The reduced order model is guaranteed stable, if the original system is stable.

(ii) The sequence of Routh approximants converge monotonically to the original system in terms of the impulse response energy.
(iii) The method does not require any information about the system eigenvalues and eigenvectors.

Rao, et al [6] have developed a Routh canonical realization in time domain for single input - single output systems. This procedure eliminates the reciprocal transformations used in the Hutton and Friedland [5] procedure.

Consider a linear system represented in the phase canonical forms

$$\dot{x} = Ax + Bu \qquad ...(18)$$
$$y = Cx \qquad ...(19)$$

where x is an n-vector, u and y are scalar.

The Routh canoncial form is obtained by considering a linear transformation

$$z = Px \qquad ...(20)$$

and is given by

$$\dot{z} = Fz + Gu \qquad ...(21)$$
$$y = Ez \qquad ...(22)$$
where $F = PAP^{-1}$, $G = PB$, $E = CP^{-1}$ ...(23)

The system matrices F and G can be written as

$$F = \begin{bmatrix} -\gamma_1 & 0 & -\gamma_3 & 0 & -\gamma_5 & \cdots & -\gamma_n \\ 0 & 0 & \gamma_3 & 0 & \gamma_5 & \cdots & \gamma_n \\ -\gamma_1 & -\gamma_2 & -\gamma_3 & 0 & -\gamma_5 & \cdots & -\gamma_n \\ \cdot \\ \cdot \\ -\gamma_1 & -\gamma_2 & -\gamma_3 & -\gamma_4 & -\gamma_5 & \cdots & -\gamma_n \end{bmatrix}$$
$$G = [1\ 0\ 1\ 0\ 1\ \ldots\ 1]^T \quad \text{for n odd} \qquad ...(24)$$

$$F = \begin{bmatrix} 0 & \gamma_2 & 0 & \gamma_4 & \cdots & \gamma_n \\ -\gamma_1 & -\gamma_2 & 0 & -\gamma_4 & \cdots & -\gamma_n \\ 0 & 0 & 0 & \gamma_4 & \cdots & \gamma_n \\ \cdot \\ \cdot \\ -\gamma_1 & -\gamma_2 & -\gamma_3 & -\gamma_4 & \cdots & -\gamma_n \end{bmatrix}$$
$$G = [0\ 1\ 0\ 1\ \ldots\ 1]^T \quad \text{for n even} \qquad (25)$$

where the values of γ_i are obtained from a Routh table using the characteristic equation of the transfer function $C(sI-A)^{-1}B$.

Let the characteristic equation be

$$f(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots a_n = 0 \qquad ...(26)$$

Then the Routh table can be constructed as

$$\begin{matrix} a_n & a_{n-2} & a_{n-4} & \cdots & a_2 & 1 \\ a_{n-1} & a_{n-3} & a_{n-5} & \cdots & a_1 & 0 \\ a_0^2 & a_2^2 & a_4^2 & \cdots \\ a_0^3 & a_2^3 & \cdots \\ \cdot \\ \cdot \\ a_0^n \end{matrix} \qquad ...(27)$$

The γ_i are given by

$$\gamma_1 = \frac{a_n}{a_{n-1}}, \quad \gamma_2 = \frac{a_{n-1}}{a_0^2}, \quad \gamma_2 = \frac{a_0^2}{a_0^3} \cdots \gamma_n = \frac{a_0^{n-1}}{a_0^n}$$

The transformation matrix P can be extracted from the Routh table as

$$P = \begin{bmatrix} a_{n-1} & 0 & a_{n-3} & 0 & \cdots & 1 \\ 0 & a_0^2 & 0 & a_2^2 & \cdots & 0 \\ 0 & 0 & a_0^3 & 0 & & 1 \\ \cdot \\ \cdot \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \quad \text{for n odd} \qquad ...(28)$$

For an even order n, the last column of P is replaced by

$$[0\ 1\ 0\ 1\ \ldots\ 0\ 1]^T$$

The output matrix E can be evaluated using the relationship

$$E = CP^{-1} \qquad ...(29)$$

A reduced order model is derived by discarding $z_{r+1}, \ldots z_n$ and is given by

$$\dot{x}_r = A_r x_r + B_r u \qquad ...(30)$$
$$y_r = C_r x_r \qquad ...(31)$$

where

$$A_r = DFD^T, \quad B_r = DG, \quad C_r = ED^T$$
and $D = [I_r \ \vdots\ 0]_{r \times n}$

(C) Litz's Modal Method

The reduced order models derived on the basis of modal techniques play a very important role in the design of controllers for large scale systems. A large number of modal techniques are available in the literature [8]. Litz [4] has developed a reduced order model based on a dominance measure and permits larger reductions in size for the same accuracy. A brief review of this method is given below:

Consider a linear system represented by

$$\dot{x} = Ax + Bu \qquad ...(32)$$
Let $$x = Pz \qquad ...(33)$$

where P is a transformation matrix which transforms the given system into Jordan canonical form

$$\dot{z} = \Lambda z + Gu \qquad ...(34)$$

A dominance measure is introduced by Litz, to determine dominant modes to be retained, most sensitive state variables and an appropriate order for the reduced model. Equations (33) and (34) are partitioned as shown below:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad ...(35)$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} u \qquad ...(36)$$

where z_1 and z_2 represent dominant and nondominant states respectively.

To derive a reduced order model, the following approximation is introduced:

$$\tilde{z}_2 \approx E z_1 \qquad ...(37)$$

From Equations (35,36 and 37)

$$x_1 = (P_{11} + P_{12}E) z_1 = F Z_1 \qquad ...(38)$$

$$\dot{z}_1 = \Lambda_1 z_1 + G_1 u$$

Let $x_r = F z_1$ ...(39)

where $F = (P_{11} + P_{12}E)$ is a linear transformation matrix. The reduced order model is given by

$$\dot{x}_r = A_r x_r + B_r u \qquad ...(40)$$

$$\text{where} A_r = F \Lambda_1 F^{-1} \text{ and } B_r = F G_1 \qquad ...(41)$$

The matrix E is determined optimally through the use of Lagrange multipliers to minimize a weighted integral of the square of the error between $\tilde{z}_2$ and z_2. The matrix E is given by

$$E = \Lambda_2^{-1}\left[S + (G_2 - S T^{-1} G_1)(G_1^* T^{-1} G_1)^{-1} G_1^*\right] T^{-1} \Lambda_1 \qquad ...(42)$$

where

$$(S)_{i,j} = -\frac{1}{\lambda_{m+i} + \lambda_j} (G_2 Q G_1^*)_{ij}$$

$$(T)_{i,j} = -\frac{1}{\lambda_{m+i} + \lambda_j} (G_2 Q G_1^*)_{ij}$$

4. DERIVATION OF REDUCED ORDER MODELS FOR TURRET-GUN SYSTEM

A 12th order linear state variable model of the Turret-Gun System is derived from a nonlinear system by using analyzing systems features of matrix x. The input and output variables are azimuth current and train rate respectively. It is noticed that there is a large spread in the eigenvalues of this model. A 7th order reduced model is selected on the basis of eigenvalues and singular values. The reduced order models are derived by using balance-truncation, Litz's modal technique and Routh approximation methods. The eigenvalues of the original system and reduced order models are given in Table I. The frequency response plots of original and reduced order models are given in Fig (3). There is an excellent low frequency match between original and reduced order models. Step response comparisons were made between original and reduced order models and are given in Fig (4). It is very hard to see the difference between original and reduced order models.

Table I
Comparison of Eigenvalues

Original System	Reduced Order Models (7th Order)		
	Balanced - Truncation	Litz's Modal	Routh Method
-1.713×10^{-3}	-1.713×10^{-3}	-1.713×10^{-3}	-1.713×10^{-3}
$-5.375 \pm j52.045$	$-5.375 \pm j52.045$	$-5.375 \pm j52.045$	$-5.375 \pm j52.045$
$-20.265 \pm j137.94$	$-20.31 \pm j138.01$	$-20.265 \pm j137.94$	$-20.912 \pm j138.21$
$-19.819 \pm j294.8$	$-19.909 \pm j294.8$	$-19.819 \pm j294.8$	$-311.8 \pm j541.3$
-945.26			
$-1573.2 \pm j4550.7$			
$-122.23 \pm j6397.9$			

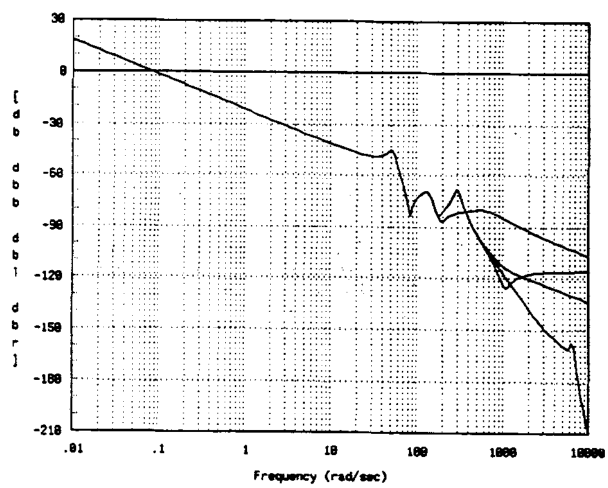

Fig 3. Frequency Response Plots

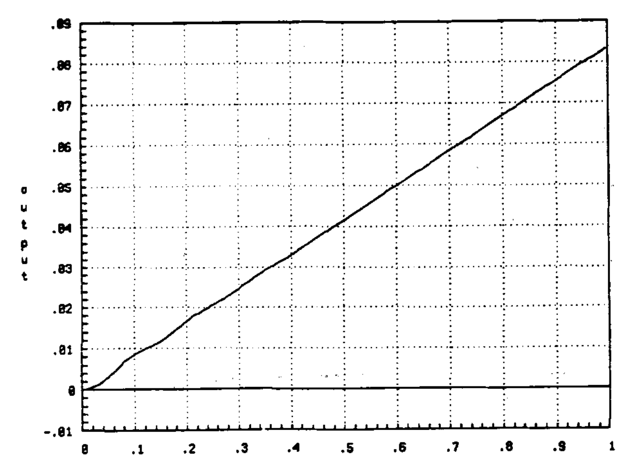

Fig 4. Step Response Plots

5. DESIGN OF REDUCED ORDER CONTROLLERS

In order to minimize the computational and implementation of LQG/LTR methodology, the reduced order models are used for design of controllers. The reduced order models derived by using balance-truncation, Litz's modal and Routh Approximation methods gave identical frequency responses in the range of 0.01 to 1000 rad/sec. The reduced order model derived by using the balance-truncation method is used to design reduced order LQG/LTR controllers.

Let K(s) and $K_r(s)$ represent the transfer functions of LQG/LTR controllers designed using the original 12th order system and 7th order reduced model respectively. The singular value plots of the target feedback loop and open loop transfer functions G(s) K(s) and $G(s)K_r(s)$ are shown in Fig (5). The eigenvalues of the closed loop systems full and reduced order controllers are given in Table II. From the eigenvalues of closed loop system, it is evident that the spillover problem is not present in this system.

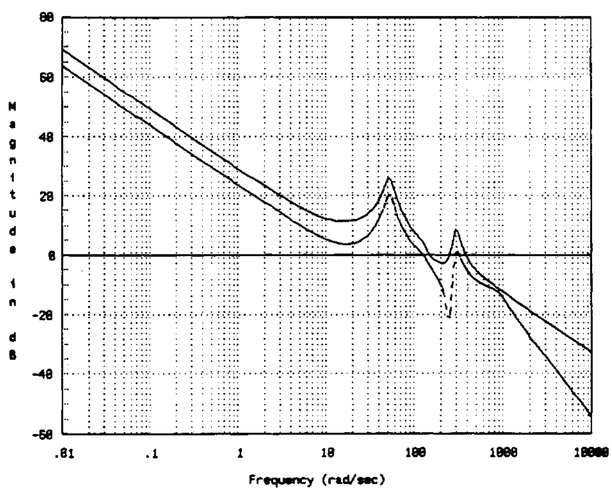

Fig. 5. Singular Value Plots

Table II
Eigenvalue Comparisons of Closed Loop System

12th Order System 12th Order Controller	12th Order System 7th Order Controller
-54.056	-7.6669
-13.177 ± j63.377	-3.4081 ± j82.891
-21.011 ± j138.87	-75.8 ± j57.122
-21.487 ± j295.08	-16.448 ± j140.42
-945.26	-15.721 ± j173.18
-1573.2 ± j4550.7	-62.465 ± j307.78
-122.23 ± j6397.9	-550.88
-0.9278	-320.9 ± j759.85
-2.999 ± j169.96	-1522.2
-217.35 ± j172.77	-1572.9 ± j4550.6
-2258 ± j4380	-122.24 ± j6397.8
-8098.5	
-5984.6 ± j6310.4	
-963.16 ± j9854.0	

6.CONCLUSIONS

Reduced order models are derived for a Turret-Gun System by using balance-truncation, Routh approximation and modal techniques. The eigenvalues of reduced order models compared with original system. The frequency responses of a reduced order model and an original system are identical in the low frequency range. The reduced order LQG/LTR controller is designed using simplified models. The comparison of eigenvalues and simulation results indicate that the reduced order design gave satisfactory results. The spillover problems are not present in this closed loop system.

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