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REVISED METHOD OF APPROXIMATE STRUCTURAL ANALYSIS

By R. A. Behr,[1] E. J. Grotton,[2] and C. A. Dwinal[3]

INTRODUCTION

Approximate methods of structural analysis are not obsolete in modern structural engineering practice. Although access to computers has made exact solutions for indeterminate structures far easier to obtain, the ability to obtain a rapid approximate solution without computers is still a relevant skill for the modern structural engineer. Not only do approximate methods offer a means of checking the magnitudes and directions of internal forces and moments obtained by computer analyses, but they also offer insight into the overall structural response and serve as a basis for preliminary design. For these reasons, approximate methods are useful to practicing engineers and form a meaningful part of the undergraduate curriculum for civil engineering students.

The most common set of assumptions underlying many existing textbook treatments of approximate analysis of vertically loaded rectangular frames is:

1. Points of inflection exist at $0.1L$ and $0.9L$ along each girder, where L is the clear span of the girder.
2. The axial force in each girder is zero.

This set of assumptions provides equations equal to three times the number of girders, which, for a rectangular grid frame, makes the system statically determinate (Wang 1983). However, Behr et al. (1989) showed that unrealistic solutions can result from this set of assumptions. This note presents a more accurate, reliable method for the approximate analysis of symmetric rectangular frames under symmetric vertical loads.

REVISED APPROXIMATE METHOD FOR FULLY LOADED FRAMES

After studying inflection-point patterns in many fully loaded rectangular frames, the following set of assumptions produced improved approximate solutions:

1. Points of inflection exist in girders at $0.1L$ and $0.9L$. (In the case of flexible girders and stiff columns, points of inflection can be moved closer to the $0.21L$

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and 0.79L marks; for stiff girders and flexible columns, points of zero moment can be located closer to the ends of the girder.)

2. A point of inflection exists in each column. The location of the inflection point is determined by the position of the column within the structure, i.e.

a. First-story columns with fixed base supports are assumed to have a point of inflection at 0.33H from the column base, where H is the height of the column. A similar suggestion is made by Wang and Salmon (1984). If a hinge exists at the column base, then the point of inflection should be located there.

b. Top-story columns are assumed to have points of inflection at 0.4H, again measured from the base of the column.

c. Intermediate columns are assumed to have points of inflection at mid-height.

Symmetry is an important requirement for the effective use of this method—violation of symmetry has the potential to cause sidesway, which induces a structural response that is incompatible with the underlying assumptions. Factors that can cause sidesway in a vertically loaded rectangular frame include asymmetric loading patterns, asymmetric shape of the structure, asymmetric base-support conditions, and asymmetric distribution of member stiffnesses.

Revised Approximate Method for Checkerboard-Loaded Frames

Checkerboard loading patterns, which involve alternately loaded girders, were also investigated. The following set of assumptions was both simple and appropriate for checkerboard loading cases:

1. Points of inflection exist in girders at 0.1L and 0.9L.
2. A point of inflection exists in each column. The location of the inflection point is determined by the position and function of the column within the structure, i.e.

a. First-story columns with fixed base supports are assumed to have a point of inflection at 0.33H from the column base. If a hinge exists at the column base, then the point of inflection should be located there.

b. External columns that directly support loaded girders have a point of inflection located at 0.2H from the column base.

c. All other columns have a point of inflection located at the base.

Again, total symmetry must be present in order to use this approximate method effectively.

Evaluation of Revised Approximate Method

A variety of structural configurations was tested to identify the overall efficacy of the revised method. Frame shape, loading pattern, base fixity condition, and relative flexural stiffness were varied. Shapes of the test frames are shown in Fig. 1. All frames were analyzed with a uniformly distributed load on each girder, called the fully loaded case. Whenever appropriate,

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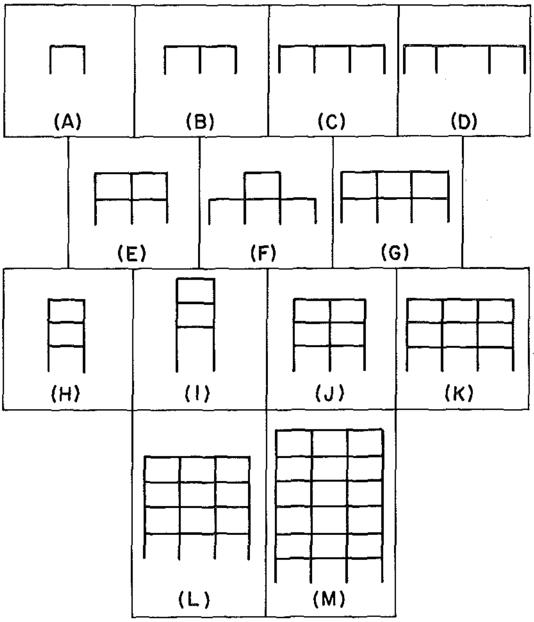

FIG. 1. Frames Used to Evaluate Approximate Methods

frames were also analyzed with a symmetric checkerboard pattern of uniformly distributed loads. Base-support conditions were either all fixed, all pinned, or symmetric variations thereof. Column heights were normally 10 ft (3.1 m) and girder clear spans were normally 15 ft (4.7 m). Each combination of variables, representing one structural configuration, was analyzed using the method prevalent in textbooks, called the old method, the revised method presented herein, and the direct stiffness method, or the exact method, as executed on a microcomputer using an analysis package called *PC-STRAN* (Murphy 1987).

Comparative bending moment diagrams for a typical, fully loaded frame are shown in Fig. 2. The revised approximate method provides a superior match to the exact bending moment diagram. Errors in the base shears and base bending moments for a variety of fully loaded frames are summarized in the upper portion of Table 1. In terms of numerical accuracy, the revised method is clearly superior to the old approximate method in every test frame considered. Also, the revised method always produced correct directions in column moments, whereas the old method often produced sign reversals in key moment values. This is important because proper moment directions are required to visualize the correct deflected shape of a structure.

Comparative bending moment diagrams for a checkerboard-loaded frame are shown in Fig. 3. As before, the revised approximate method is better able to predict the actual moment response of the structure. A summary of base shear and base moment errors for a variety of checkerboard-loaded frames is included in the lower portion of Table 1. The numerical accuracy of the revised method is again superior to that of the old method.

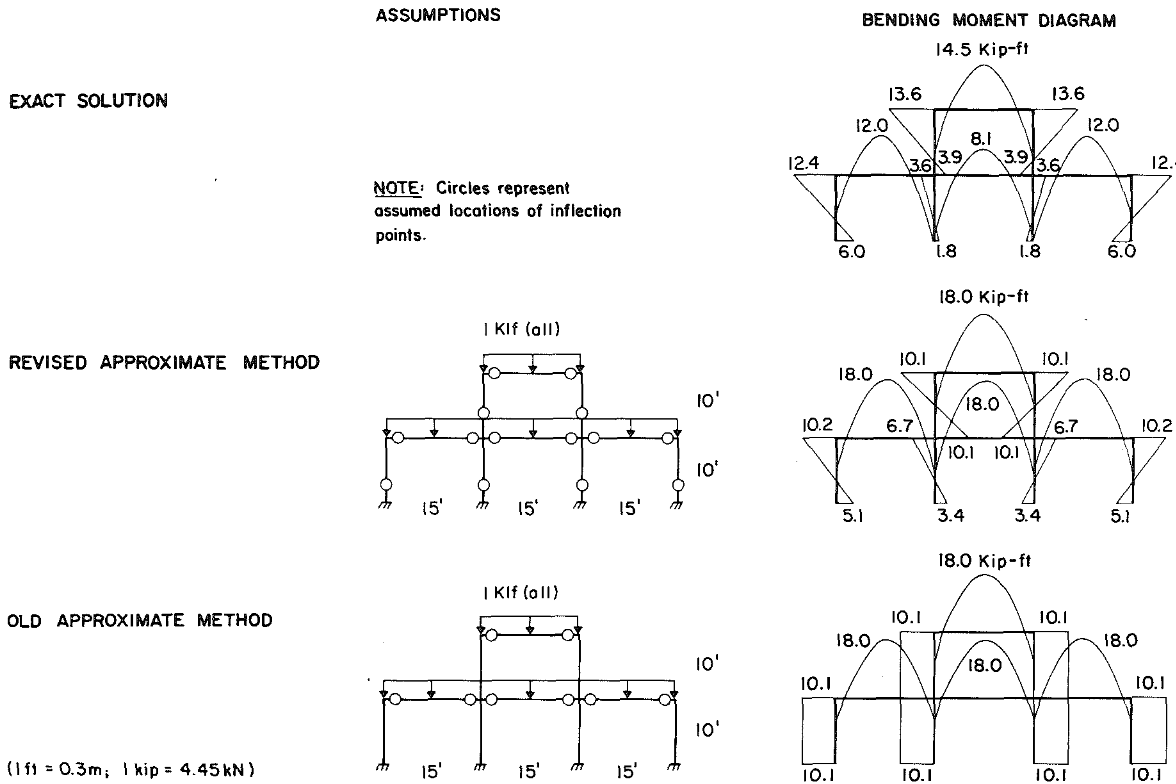

FIG. 2. Comparative Bending Moment Diagrams For Fully Loaded Frame

TABLE 1. Errors in Computer Reactions at First-Story Column Bases

Structure (1)		Moment error (%) (2)	Shear error (%) (3)
(*a*) Fully Loaded Frames			
A	revised	−26.7	−27.3
	old	+46.5	−100.0
B	revised	−9.2	−10.6
	old	+81.7	−100.0
C	revised	−10.1	−12.1
	old	+79.8	−100.0
D	revised	+38.4	+23.0
	old	+177.4	−100.0
E	revised	−49.3	−47.1
	old	+572.8	−100.0
F	revised	−16.2	+17.4
	old	+67.6	−100.0
G	revised	−45.6	−43.4
	old	+551.1	−100.0
H	revised	−7.7	−7.3
	old	+732.4	−100.0
I	revised	+29.4	+30.8
	old	+1,070.6	−100.0
J	revised	−2.0	1.0
	old	+785.6	−100.0
K	revised	−3.8	−1.9
	old	+771.4	−100.0
L	revised	−51.7	−50.8
	old	+1,057.1	−100.0
M	revised	−54.1	−52.1
	old	+1,587.5	−100.0
(*b*) Checkerboard-Loaded Frames			
C	revised	−22.6	−24.0
	old	+55.5	−100.0
D	revised	−23.6	−24.6
	old	+53.2	−100.0
H	revised	−15.8	−24.0
	old	+566.1	−100.0
K	revised	+9.8	+9.3
	old	+335.9	−100.0
L	revised	+17.7	+8.8
	old	+1,501.6	−100.0
M	revised	+22.8	−14.3
	old	+2,686.7	−100.0

Note: Error = $(|A| - |E|)/|E| \times 100\%$, where $|E|$ = the absolute value of the exact solution; and $|A|$ = the absolute value of the approximate solution; revised = revised approximate method used; and old = old approximate method used.

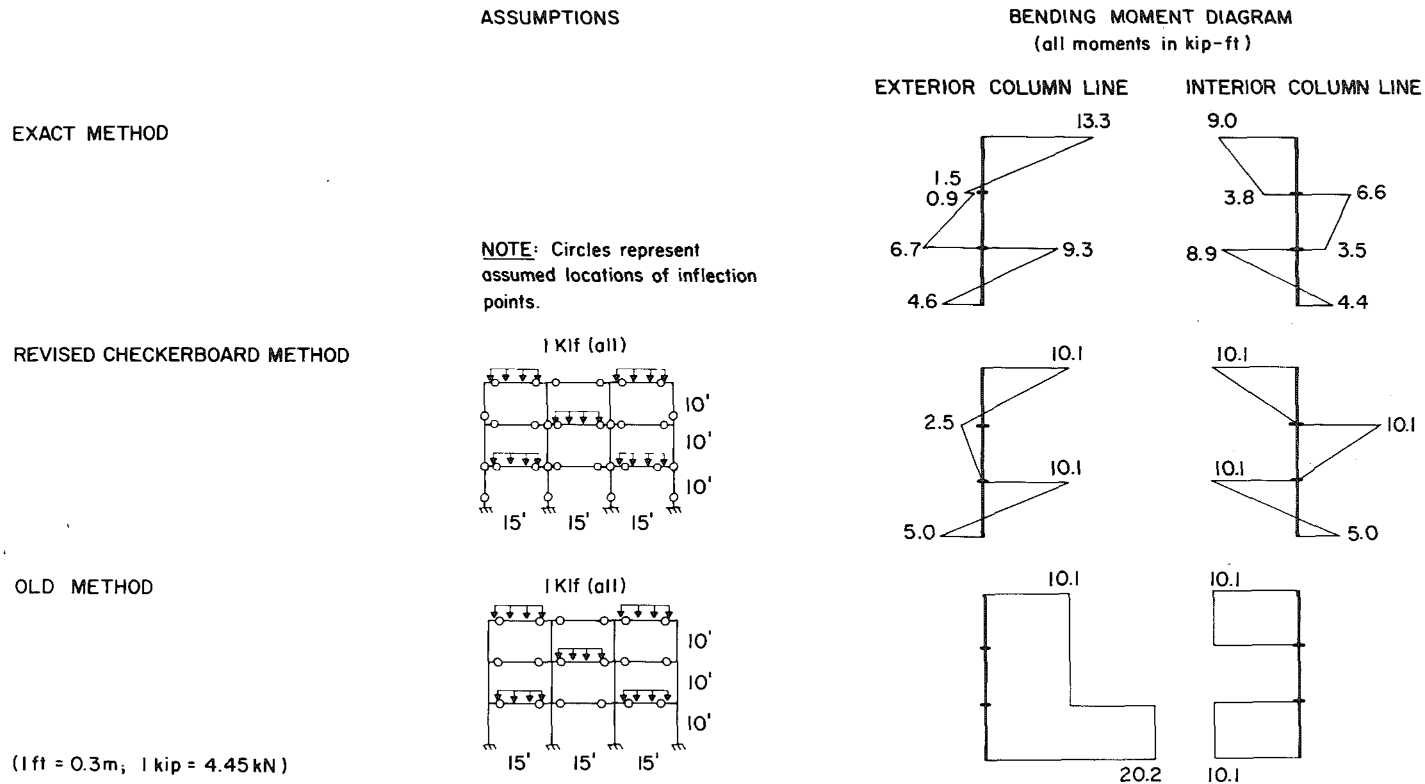

FIG. 3. Comparative Bending Moment Diagrams For Checkerboard-Loaded Frame

Uniqueness of Solutions

For an approximate method of structural analysis to be useful in practice, the solution obtained must be unique, i.e., it must be independent of the order in which free-body diagrams are analyzed in the solution procedure. Concern for nonuniqueness resulted from the realization that the revised approximate method specified more assumptions than the actual degree of indeterminacy of the frame. For example, a two-bay, two-story frame is indeterminate to the 12th degree, yet the revised approximate method incorporates the use of 14 assumptions. To test uniqueness, the structure was analyzed using the 14 assumptions in various combinations of 12, and all solutions were found to be identical. This finding was consistent with Laible (1985), who observed that symmetry of both structure and loading conditions eliminated potential uniqueness problems in vertical load approximate solutions.

Conclusion

A reliable, reasonably accurate approximate method of structural analysis for symmetric, rectangular frames under symmetric vertical loadings has been developed. Using the revised assumption sets for either fully loaded or checkerboard-loaded cases, solutions were superior to those from the slightly different set of assumptions found in many contemporary structural analysis textbooks.

Appendix. References

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