Development Of A Friction Element For Metal Forming Analysis

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A three-dimensional finite element friction element has been developed. The friction element has been used in the analysis of round-to-square forward extrusion. Comparison with some limited experimental data is presented.

Introduction

Quasi-continuous forming operations—drawing and extruding—are commercially important processes for mass production of metal goods. High-speed production, reduced machining requirements, low scrap losses, improved mechanical properties, one-piece construction, and resultant energy savings promise continuing development for these production techniques. Methods of analysis of the process are required for improving production rate, quality of the product, and longevity of the equipment. An understanding of the flow field, and behavior of the material, induced stress state, and temperature distribution is helpful for selecting and optimizing the process variables such as extrusion rate, reduction in area, die angle, initial billet temperature, die cooling, and lubrication conditions.

Several mathematical formulations have been used in conjunction with finite element techniques to analyze forming processes. Perhaps one of the most promising of these theoretical methods has been the Upper Bound Limit analysis [1]. A three-dimensional finite element computer program utilizing this approach is currently being developed at the University of Missouri-Rolla [2]. The program, entitled FEDA (Finite Element Deformation Analysis), is currently in the refinement stage to make it applicable to industrial process analysis. Incorporating a rapidly converging iterative scheme, the program finds kinematically admissible strain rate fields which successively tend to minimize the power expression prescribed by the Upper Bound theorem.

Davis and Dillard [3] have used "upwinding" to develop a new thermal element for the FEDA program which permits transient, three-dimensional, thermal analysis of continuous forming operations. It permits various complex geometries of the extrusion, general convective boundary conditions on all nonadjoining surfaces, anisotropic thermal conductivities, heat generation in regions of deformation and at frictional surfaces, and it accounts for extrusion rate and losses into surrounding dies.

The next step in the development of the FEDA program is the modeling of friction, especailly for the three-dimensional problems. A three-dimensional finite element has been developed to simulate friction between two curved mating surfaces in metal-forming problems. The results of a simulation of forward extrusion, using the friction or slip element, is compared with extrusions done experimentally by Phu [4].

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Theoretical Development

The friction element of 8 nodes is assumed to be a curved surface with constant thickness, t (Fig. 1). The general equation for a curved surface [5] is

$$X=f(r,s), \tag{1a}$$

$$Y=g(r,s), \tag{1b}$$

and

$$Z=h(r,s). \tag{1c}$$

The strain rates are approximated using the definition,

$$\dot{\epsilon}_n=\frac{d\epsilon_n}{dt_0}=\frac{d}{dt_0}\left(\frac{\partial w_n}{\partial n}\right)\approx\frac{\Delta\dot{w}_n}{\Delta n}=\frac{\Delta\dot{w}_n}{t} \tag{2a}$$

$$\dot{\epsilon}_r=\frac{d\epsilon_r}{dt_0}=\frac{d}{dt_0}\left(\frac{\partial u_r}{\partial n}\right)\approx\frac{\Delta\dot{u}_r}{\Delta n}=\frac{\Delta\dot{u}_r}{t} \tag{2b}$$

$$\dot{\epsilon}_s=\frac{d\epsilon_s}{dt_0}=\frac{d}{dt_0}\left(\frac{\partial v_s}{\partial n}\right)\approx\frac{\Delta\dot{v}_s}{\Delta n}=\frac{\Delta\dot{v}_s}{t} \tag{2c}$$

where $\partial W_n/\partial r$ and $\partial W_n/\partial s$ are assumed to be negligible, and where r and s define the directions of the local coordinate system; n is the direction normal to the surface; and Δw_n, Δu_r, and Δv_s are the changes in velocity in the n, r, and s directions, respectively.

The change in velocity is assumed to be a function of the change in velocity at the corner nodal points of the friction element,

$$\Delta\dot{w}_n=\sum_{k=1}^{4}N_k\Delta\dot{w}_{nk}, \tag{3a}$$

$$\Delta\dot{u}_r=\sum_{k=1}^{4}N_k\Delta\dot{u}_{rk}, \tag{3b}$$

and

$$\Delta\dot{v}_s=\sum_{k=1}^{4}N_k\Delta\dot{v}_{sk}, \tag{3c}$$

where $\Delta\dot{w}_{nk}$, $\Delta\dot{u}_{rk}$, and $\Delta\dot{v}_{sk}$ are the changes in velocity at the nodal points and N_k are the shape functions defined by

$$N_k = \frac{1}{4}(1+r_o)(1+s_o), \tag{4}$$

where

$$r_o = rr_i \quad , \qquad s_o = ss_i \quad ,$$
$$r_i = \pm 1 \quad , \qquad s_i = \pm 1 \quad .$$

Because $\Delta\dot{w}_n$ was the change in velocity normal to the surface of the friction element, a rotation matrix transformed Cartesian velocities to local velocities. The direction cosines of the normal to a surface are defined as

$$C_x = \frac{j_1}{dA} \quad , \tag{5a}$$

$$C_y = \frac{j_2}{dA} \quad , \tag{5b}$$

and

$$C_z = \frac{j_3}{dA} \quad , \tag{5c}$$

where

$$j_1 = \begin{vmatrix} \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \end{vmatrix} \quad , \tag{6a}$$

$$j_2 = \begin{vmatrix} \frac{\partial z}{\partial r} & \frac{\partial x}{\partial r} \\ \frac{\partial z}{\partial s} & \frac{\partial x}{\partial s} \end{vmatrix} \quad , \tag{6b}$$

$$j_3 = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{vmatrix} \quad , \tag{6c}$$

and

$$dA = (j_1^2 + j_3^2 + j_3^2)^{1/2} \quad . \tag{7}$$

To find the direction cosines of the r- and s-directions, the thickness t is considered. If the direction cosines of the normal to the surface are known, then four new corner nodes can be considered to form a three-dimensional element with constant thickness t where

$$\bar{x} = x + C_{xn} t \quad , \tag{8a}$$

$$\bar{y} = y + C_{yn} t \quad , \tag{8b}$$

and

$$\bar{z} = z + C_{zn} t \quad , \tag{8c}$$

where $\bar{x}$, $\bar{y}$, and $\bar{z}$ and C_{xn}, C_{yn}, and C_{zn} are considered for each node, and x, y, and z are the coordinates of the new nodes.

The direction cosines for the r- and s-directions are calculated using the new coordinates of equation (8) in the equivalent form of equations (5a-c). The set of direction cosines for each node forms a rotation matrix, R.

$$R_i = \begin{bmatrix} C_{xr} & C_{xs} & C_{xn} \\ C_{yr} & C_{ys} & C_{yn} \\ C_{zr} & C_{zs} & C_{zn} \end{bmatrix}_i \qquad i = 1,2,3,4 \tag{9}$$

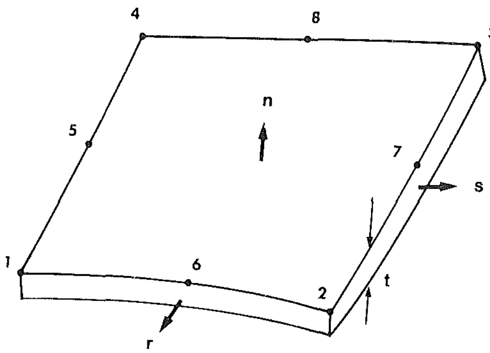

Fig. 1 A three-dimensional friction element

The strain rate tensors are now written as

$$\begin{Bmatrix} \dot{\epsilon}_r \\ \dot{\epsilon}_s \\ \dot{\epsilon}_n \end{Bmatrix} = [B][R] \begin{Bmatrix} \Delta\dot{u} \\ \Delta\dot{v} \\ \Delta\dot{w} \end{Bmatrix} \tag{10}$$

where R is the rotation matrix for each of the four nodes and B is the matrix of the shape functions.

The strains are found by the chain rule

$$\dot{\epsilon} = D_{t_0}\epsilon = \frac{\partial \epsilon}{\partial x}\frac{\partial x}{\partial t_0} + \frac{\partial \epsilon}{\partial y}\frac{\partial y}{\partial t_0} + \frac{\partial \epsilon}{\partial z}\frac{\partial z}{\partial t_0} \quad . \tag{11}$$

The strains are represented by a polynomial function in the local coordinate system,

$$\epsilon = a_1 r + a_2 s + a_3 rs + a_4 r^2 + a_5 s^2 + a_6 \quad . \tag{12}$$

Applying the chain rule to the derivatives of the strain gives

$$\frac{\partial \epsilon}{\partial x} = \frac{\partial \epsilon}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial \epsilon}{\partial s}\frac{\partial s}{\partial x} \quad , \tag{13a}$$

$$\frac{\partial \epsilon}{\partial y} = \frac{\partial \epsilon}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial \epsilon}{\partial s}\frac{\partial s}{\partial y} \quad , \tag{13b}$$

and

$$\frac{\partial \epsilon}{\partial z} = \frac{\partial \epsilon}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial \epsilon}{\partial s}\frac{\partial s}{\partial z} \quad , \tag{13c}$$

where the partial derivatives of the local coordinate system with respect to the Cartesian coordinates are defined by the inverse of the Jacobian matrix.

With the strains at two corner nodes either known from a boundary condition, or solved from the elements between a boundary and the element considered, and the strain rates known at the four corner nodal points, a system of equations is solved for the constants a_1, a_2, a_3, a_4, a_5, a_6, in equation (12). Since stresses are nonzero only when compressive normal strains exist and if stresses are limited to a maximum, σ_{yp}, the stresses can be expressed by

$$\sigma_n = C_{nn}\epsilon_n \quad , \tag{14a}$$

$$\sigma_r = C_{rr}\epsilon_r \quad , \tag{14b}$$

$$\sigma_s = C_{ss}\epsilon_s \quad , \tag{14c}$$

where

$$\begin{aligned} C_{nn} &= E \quad , && \text{if } \frac{\sigma_{yp}}{E} < \epsilon_n < 0 \quad , \\ &= \sigma_{yp}/\epsilon_n && \text{if } \epsilon_n \le \frac{\sigma_{yp}}{E} \quad , \\ &= 0 \quad , && \text{if } \epsilon_n \ge 0 \quad , \\ C_{ss} &= \nu\dot{\epsilon}_s/\epsilon_s && \text{if } \sigma_s < \mu_s\sigma_n \quad , \\ &= 0 \quad , && \text{if } \epsilon_n \ge 0 \quad , \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\epsilon_s}\mu_k\sigma_n \quad , && \text{if } \sigma_s \geq \mu_s\sigma_n \quad , \\
C_{rr} &= \nu\dot{\epsilon}_r/\epsilon_r && \text{if } \sigma_r < \mu_s\sigma_n \quad , \\
&= 0 \quad , && \text{if } \epsilon_n \geq 0 \quad , \\
&= \frac{1}{\epsilon_r}\mu_k\epsilon_n \quad , && \text{if } \sigma_r \geq \mu_s\sigma_n \quad ,
\end{aligned}
$$

where E is the modulus of elasticity, ν is the viscosity for hydrodynamic lubrication, μ_k is the coefficient of friction for Coulomb friction, and μ_s represents the change from Coulomb friction to hydrodynamic lubrication.

Eight nodes are used to define the shape of the friction element and the Jacobian matrix. The shape functions are polynomials to the third degree in two variables.

$$X = \sum_{k=1}^{8} N_k X_k \quad , \tag{15a}$$

$$Y = \sum_{k=1}^{8} N_k Y_k \quad , \tag{15b}$$

and

$$Z = \sum_{k=1}^{8} N_k Z_k \quad , \tag{15c}$$

where

$$N_k = \frac{1}{4}(1+r_o)(1+s_o)(r_o+s_o-1) \tag{16a}$$

at the corner nodes and

$$N_k = \frac{1}{2}(1-r^2)(1+s_o) \tag{16b}$$

or

$$N_k = \frac{1}{2}(1-s^2)(1+r_o) \tag{16c}$$

at the mid-side nodes.

The power dissipated by friction is [6]

$$\dot{w}_f = \int_{s_\Gamma} \tau |\Delta v| ds$$

where s_Γ is the surface on which the friction acts, τ is the shear stress due to friction, and Δv is the velocity discontinuity. Substituting for Δv and integrating over the local coordinate system the power dissipated by friction is

$$\dot{w}_f = \sum_{k=1}^{N} \sum_{m=1}^{N} \{\tau\}[B][R]\alpha_k\alpha_m dA$$

where B_τ is the matrix formed by the first two rows of B,

$$B_\tau = \frac{1}{t}\begin{bmatrix} 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix}$$

τ is the shear stress in the r- and s-directions, α_k and α_m are Gaussian quadrature factors, and dA is defined in equation (7).

Element Performance

Four forward extrusions of 6061–T651 aluminum billets 3.81 cm long with a conical nose of 40 deg were analyzed. The four billets had diameters of 0.991, 1.054, 1.143, and 1.206 cm giving extrusion ratios of 2.25:1, 2.55:1, 2.97:1, and 3.34:1, respectively. The product resulting, which was the same for all four extrusions, had a square area of 0.342 cm^2. The finite element model consisted of 12 hexahedron-shaped elements and 24 friction elements (Fig. 2) modeling a one quarter symmetrical section. Transition zones (the regions where the flow suddenly changes direction) were defined by smaller elements. The coefficient of friction for the lubricated billet and the die was $\mu = 0.08$.

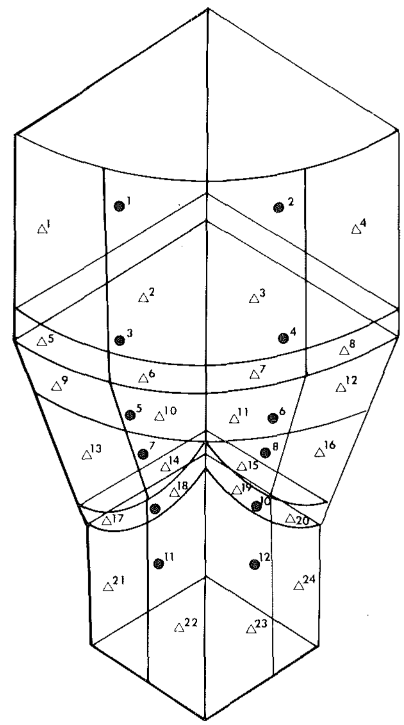

Fig. 2 A one quarter section of a billet consisting of 12 solid elements and 24 friction elements

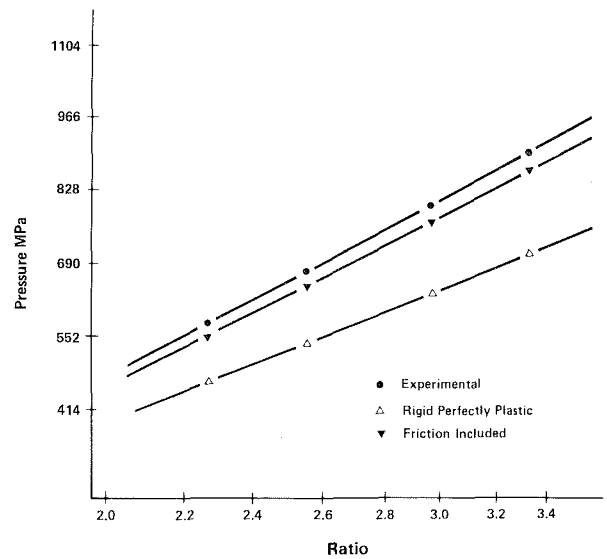

Fig. 3 Comparison of experimental to analytical results for 4 forward extrusions

The power was first minimized with a rigid, perfectly plastic stress-strain curve by setting the coefficient of friction to zero. Using the velocities obtained from the first optimization the effects of friction were included. The results are shown in Fig. 3.

Additional finite element grids for the analysis of round-to-square extrusion with friction elements included are illustrated in Figs. 4 and 5.

Conclusion

The three-dimensional friction element worked well with the Upper Bound Theorem in a Finite Element approach.

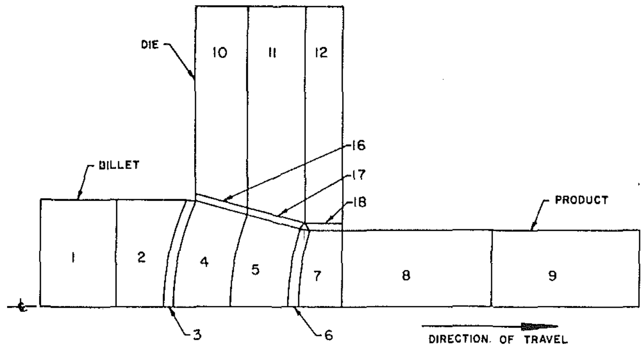

Fig. 4 Cross-section of extrusion process

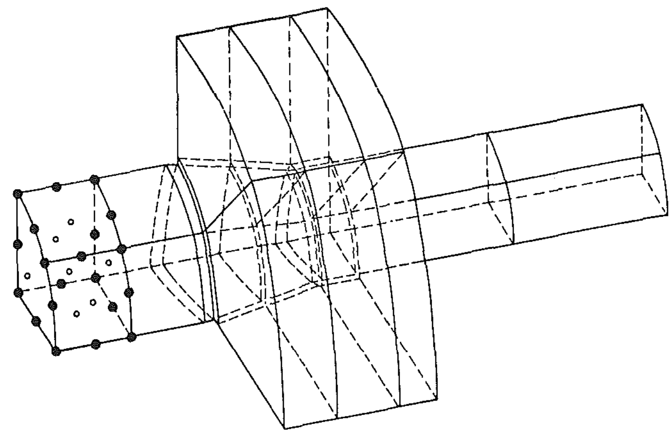

Fig. 5 View of round-to-square extrusion (¼ section)

Since an iterative technique was used to minimize the power, equations (14*a*–*c*) were recalculated at each iterative step. However, when the velocities were not in the region of the optimal solution, the friction elements tended to cause divergence. The friction element lends itself to both Coulomb friction, hydrodynamic lubrication, and combinations of both. In the forward extrusion examples, the increases in power due to friction were developed mainly in the regions where the die changed shape.

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