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A STOCHASTIC MODEL FOR A SMALL-TIME-INTERVAL-INTERMITTENT HYDROLOGIC PROCESS

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ABSTRACT

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A methodology is developed for the generation of intermittent small-time-interval (15-min.) precipitation. This methodology consists of three components: a probabilistic wet-and-dry sequence component; a Markovian precipitation distribution component; and a regressive spatial-distribution component. The theory of stochastic hydrologic modeling is described and procedures by which this theory can be applied to a hydrologic time series are presented. In particular, the application of the procedures to small-time-interval-intermittent hydrologic processes is given.

The methodology is demonstrated by application to an actual precipitation network, the Boneyard Creek raingage network in Champaign-Urbana, Illinois, U.S.A. The trend in frequency of precipitation amounts from the 89-yr. Morrow plots data are produced by the model using 13-yr. of historical data.

INTRODUCTION

One of the major problems facing hydrologists is the lack of long-term records of precipitation and streamflow. Simulation of hydrologic records by means of stochastic modeling is a commonly accepted statistical procedure to extend the record. In general, there are longer records for precipitation then for streamflow. Hence, it seems logical from a hydrologic point of view to use the longer precipitation data for conversion to longer streamflow data. The stochastically generated data are a statistical projection of the original record, thus making maximum utilization of available information. Secondly, considering the stochastic character of hydrologic processes, a stochastic model is a more statistically complete use of the available record than traditional techniques based only on observed historical data sequences.

A stochastic model is developed to simulate the precipitation of short-time 15-min. increments with temporal and spatial variation. A practical application of this model is to generate precipitation data as the input to a storm-water runoff model. Hence, long-term runoff records can be simulated by using a stochastic model to generate precipitation data as the input to a storm-water runoff model. Classical approaches of frequency analysis can be used to determine the recurrence interval of generated runoff.

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The quantity, rate and quality of urban storm-water runoff as well are affected by spatial and temporal distribution of storm precipitation. Some discrepancies which occur between observed and predicted runoff using existing storm-water runoff models may be due to the input precipitation increments being too long (such as hourly or daily increments) and the lack of consideration of spatial variation. Further, with different degrees of urbanization to be incorporated into the runoff model, various runoff frequencies with respect to changes in the degree and type of urbanization, one may induce conclusions for predicting future and present effects of urbanization on runoff. Urbanization may also effect precipitation. Such effects can be studied by applying a stochastic precipitation model to both urban and rural areas' precipitation and comparing the resulting generated data.

It is the aim of this paper to present methodologies for developing synthetic hydrologic information, specifically precipitation, as applied to the Boneyard Creek network. The methodologies presented can be used at other locations, with minor modification if so warranted by the analyses of historical data. The results obtained with historical and stochastically generated hydrologic information are compared.

MODEL

In order to model the basic underlying components of small-time-interval data, the wet—dry sequences are modeled separately from the precipitation distribution within a storm. Since the precipitation data consist mostly of groups of zero pulses, in most cases of extended length, difficulties arise when one stochastic model is used to model both the wet- and dry-period precipitation processes. Also, a Markov chain is used to model the rainfall distribution with the wet period. A one-step Markov chain is used because the future amount of precipitation for a 15-min. interval is mainly dependent on the present amount of precipitation for a 15-min. interval. An extension and improvement of the classic Markov transition probability matrix is accomplished by fitting an analytical function to the rows of the matrix. The benefit is two-fold: the scheme generates values that have not been observed; and it smoothes the observed data so that distinguishable trends in shape of the probability distribution can be extrapolated to complete extreme values of each of the transition probabilities (rows) of the matrix. Additionally, a mathematical method is developed to fit the extreme values of Weibull distributions by the method of maximum likelihood independently of the remainder of the data, but satisfying the condition that the integration of the probability density function (p.d.f.) for both extreme-value and remainder distributions equals unity.

The intent of this stochastic modeling effort is to reflect the time and spatial distribution in a rain storm as a small-time-interval-intermittent hydrologic process. Such a stochastic model was developed consisting of three components:

(1) A wet-and-dry sequence component which is based on the unique relationship between the probability density function, $f(x)$, and its cumulative distribution function, $F(x)$:

$$F(x_t) = \int_{-\infty}^{x_t} f(x)\mathrm{d}x \tag{1}$$

where $F(x_t)$ is defined over the interval (0,1). The random numbers R^* $(0 \leqslant R^* \leqslant 1)$ distributed uniformly over some interval may be used to represent $P(x)$. For a particular value of R^*, say R_t^*, the corresponding value of $F(x)$ is $F(x_t)$. Then:

$$R_t^* = F(x_t) = \int_{-\infty}^{x_t} f(x)\mathrm{d}x \tag{2}$$

Thus:

$$x_t = F^{-1}(R_t^*) \tag{3}$$

Hence:

$$P[x \leqslant x_t] = F(x_t) = P[x \leqslant F^{-1}(R_t^*)] \tag{4}$$

where $F^{-1}(R_t^*)$ is a variable that has $f(x)$ as its p.d.f. The wet-and-dry sequence component of the model generates internally independent, purely random stochastic series of wet-and-dry sequences.

(2) The precipitation distribution component which uses the Markov chain principle allows sequential dependence in the data. This dependence is taken into account by the transition probabilities, $x_0 + 2, \ldots$; or $x_0 - 1$, $x_0 - 2, \ldots$; which are the units of precipitation following x_0 units of precipitation per given interval of time. This model assumes that the event [$(x_0 + 1)$ units of precipitation] depends only on the previous event (x_0 units of precipitation); hence it is a first-order model. Markov chains reproduce only transitions which have been observed, thus limiting their capabilities to predict more extreme events than have occurred in the historical data used to produce the transition probability matrix. This inherent limitation can be overcome by fitting a p.d.f. to each row, i.e. each state of conditional probabilities, of the matrix or following eq. 4:

$$P[x \leqslant x_{t+1} | x_t] = F(x_{t+1}) = P[x \leqslant F^{-1}(R_{t+1}^*)] \tag{5}$$

which will adequately represent the probability of extreme events.

(3) The spatial distribution component which uses a linear regression equation to measure the association between the observed data of the primary and other stations. The linear association between the data at station x and station y may be predicted by:

$$y = \alpha x + \beta + \epsilon \quad (6)$$

The interstation linear regression coefficient (α) will be used to predict the association between the x (primary station) and y (one of the secondary stations) and β is an intercept value corresponding to an α-value. The ϵ is a normally-distributed random variable $(0,\sigma)$ with a standard deviation (σ) of the residual (ϵ-) values corresponding to the α- and β-values. A primary station is the station which will have precipitation sequences generated by the wet-and-dry sequence component and the precipitation distribution component of the model.

The wet-and-dry sequence of the model

The precipitation data may be considered to consist of groups of nonzero pulses separated by zero-amplitude pulses (periods when no precipitation occurs) of varying lengths. A storm may give rise to two or more groups of precipitation pulses separated by periods with no precipitation. In such cases, it is difficult to say whether the groups of precipitation pulses belong to a single storm or several storms. It is apparent that two groups of precipitation separated by a larger number of zero-amplitude pulses may not belong to the same storm. Therefore, an objective measure is necessary to decide whether two sequences of nonzero pulses separated by a series of zero-amplitude pulses belong to the same storm. Grace and Eagleson (1966) pointed out the existence of significant correlations between successive interval values of precipitation depths. Also, the correlation between successive interval values of precipitation depth appears to increase as the time interval decreases. Thus, one can conceive of a critical time interval separating two sequences of precipitation beyond which the correlation between the two nonzero pulses would be almost zero. This time interval was defined by Grace and Eagleson (1966) as the critical lag, and was taken to be the criterion for separating the precipitation sequences into individual storms. A storm is thus defined as a group of precipitation pulses preceded and followed by zero-amplitude pulses of duration equal to or greater than the critical lag.

Statistics and probability distributions of wet-and-dry periods

Separate probability distributions can be fitted to the wet-and-dry sequences if the cross-correlation between successive wet-and-dry sequences is not significantly different from zero.

All of the wet-and-dry sequence data should be used to determine which of the three distributions most frequently used hydrologically (Weibull, exponential, and gamma) should be utilized for modeling the wet-and-dry sequences. The reason for using all of the data is that the more data used, the better one can determine the actual distribution of the data. However, the more data are used the more difficult it is for the fitted distribution to pass the chi-square test. Only the Weibull distribution passed the chi-square test for the data tested, thus, it seems to be the best distribution to use. As the wet-and-dry sequences were uncorrelated, separate univariate probability distributions were fitted to the wet-and-dry sequences. Furthermore, it was also determined that a two-parameter Weibull fitted the test data as well as a three-parameter one. The third parameter is a scale parameter and does not improve the fit of the distribution to the data. Therefore, a two-parameter Weibull distribution was chosen for the wet-and-dry sequences. The histogram and the fitted Weibull distribution for all the wet sequence data used in performing the chi-square test are presented in Table I. The method of maximum likelihood was used in fitting the Weibull distribution to the historical data.

Maximum likelihood estimation of the Weibull distribution parameters with emphasis on extreme values

A major deterrent to wider use of the Weibull distribution has been the difficulty in estimating its parameters, because the calculations involved are not always simple. The Weibull distribution density function:

$$f(x) = ABx^{B-1}\exp(-Ax^B) \tag{7}$$

is defined for A, $B > 0$ and $0 < x < \infty$; when $B = 1$, this distribution becomes the p.d.f. of the well-known one-parameter exponential distribution. The particular form in which eq. 7 is written was chosen for the purpose of simplifying derivation of the maximum likelihood estimating equation. Considering a random sample consisting of n observations fitted by eq. 7, the likelihood function of this sample is:

$$L(x_1, \ldots, x_n; A, B) = \prod_{i=1}^{n} AB\, x_i^{B-1} \exp(-Ax_i^B) \tag{8}$$

By taking logarithms of both sides of eq. 8, differentiating it with respect to A and B in turn, and equating the resulting form to zero, we obtain the following estimating equations:

$$\partial \ln L/\partial A = -An + A^2 \sum_{i=1}^{n} x_i^B = 0 \tag{9}$$

$$\partial \ln L/\partial B = n/B + \sum_{i=1}^{n} \ln x_i - A \sum_{i=1}^{n} x_i^B \ln x_i = 0 \tag{10}$$

Upon eliminating A between these two equations and simplifying, we have:

$$n/B + \sum_{i=1}^{n} \ln x_i - \left(n \Big/ \sum_{i=1}^{n} x_i^B\right) \sum_{i=1}^{n} x_i^B \ln x_i = 0 \tag{11}$$

A large number of authors have considered estimation of parameters of gamma, exponential and Weibull distribution by the method of moments. It has been shown (Harter and Moore, 1965) that except when the distribution closely approximates normality, the method of moments is inefficient because it takes more iterations to determine the values for the parameters. Therefore, it is suggested that the method of maximum likelihood is the more efficient method to fit the distributions. It may be suggested that the Weibull distribution is a more realistic and flexible global model for this data of small interval precipitation. In the following discussion, the maximum likelihood estimates (MLE) of the Weibull parameters will be developed for the extreme values of the data. It is assumed that the cumulative probability density function is:

$$F(x) = 1 - C\exp(-A_{\mathrm{E}}x^{B_{\mathrm{E}}}) \qquad (12)$$

where C is an unknown scalar parameter which maintains the basic property that the total probability of all the data is equal to unity. The two sets of parameters in this derivation are $A_{\mathrm{R}}, B_{\mathrm{R}}$ for the main portion of the distribution and $A_{\mathrm{E}}, B_{\mathrm{E}}$ for the tail (extreme values) of the distribution. The advantage of the above procedure is to assure that the extreme values are used to predict extreme values and not biased by the bulk of the data.

The theoretical procedure for fitting the Weibull distribution development to extreme values is as follows. Consider a random sample of extreme values consisting of n observations when eq. 12 is the applicable density function of the sample. By eq. 8, the likelihood function of said sample is:

$$L_{\mathrm{E}}(x_{k+1}, x_{k+2}, \ldots, x_r, x_{r+1}; A_{\mathrm{E}}, B_{\mathrm{E}})$$

$$= (A_{\mathrm{E}}B_{\mathrm{E}})^{r+1-k} \prod_{i=k+1}^{r+1} x_i^{B_{\mathrm{E}}-1} \exp^{(-A_{\mathrm{E}})} \sum_{i=k+1}^{r} i(x_i^{B_{\mathrm{E}}} - x_{i+1}^{B_{\mathrm{E}}}) \qquad (13)$$

where x_{r+1} is the smallest and x_{k+1} the largest extreme values. By taking logarithms of eq. 13, differentiating it with respect to A and B in turn, and equating the resulting form to zero, we obtain the following estimating equations:

$$\partial \ln L_{\mathrm{E}}/\partial A_{\mathrm{E}} = (r+1-k)/A_{\mathrm{E}} - \sum_{i=k+1}^{r} i(x_i^{B_E} - x_{i+1}^{B_{\mathrm{E}}}) = 0 \qquad (14)$$

$$\partial \ln L_{\mathrm{E}}/\partial B_{\mathrm{E}} = (r+1-k)/B_{\mathrm{E}} + \sum_{i=k+1}^{r} \ln x_i$$

$$-A \sum_{i=k+1}^{r} i[x_i^{B_{\mathrm{E}}} \ln(x_i - x_{i+1}^{B_{\mathrm{E}}}) \ln x_{i+1}] = 0 \qquad (15)$$

Let the extreme values be $(x_{k+1}, x_{k+2}, \ldots, x_r, x_{r+1})$ with $(0 \leqslant k < r < n-1;\ x_{k+1} > x_{k+2} > \ldots > x_r > x_{r+1})$. Then, since practically $A_{\mathrm{E}}, B_{\mathrm{E}}, C > 0$, the maximum likelihood estimates of $(A_{\mathrm{E}}, B_{\mathrm{E}}, C)$ are obtained as follows:

From eq. 12,

$$C = [(r+1)/n] \exp(-A_E)\, x_{r+1}^{B_E} \tag{16}$$

From eq. 14,

$$A_E = (r+1-k) \left[\sum_{i=k+1}^{r} i(x_i^{B_E} - x_{i+1}^{B_E}) \right]^{-1} \tag{17}$$

where $(r+1-k)$ is the number of extreme values. Then eliminating A_E by substituting eq. 16 into eq. 18 and simplifying: B_E is the nonnegative root of:

$$q(B_E) = 0 = 1/B_E + \sum_{i=k+1}^{r} \ln x_i/(r+1-k) - \frac{\sum_{i=k+1}^{r} i(x_i^{B_E} \ln x_i - x_{i+1}^{B_E} \ln x_{i+1})}{\sum_{i=k+1}^{r} i(x_i^{B_E} - x_{i+1}^{B_E})} \tag{18}$$

The above equations for A_E, B_E and C can be solved for any set of extreme values.

Table I gives the results fitting the Weibull distribution to all the data with one set of parameters and Table II gives the results of fitting the Weibull distribution with two sets of parameters by separating extreme values from the data. The difference between the two procedures can be readily seen from the tables. In Table I, $\Sigma\chi^2 = 43.38$ for the one set parameter procedure. In Table II, $\Sigma\chi^2 = 39.22$ for the two set parameter procedure.

The Markov chain transition matrix for the precipitation distribution component of the model

It is assumed, that the first-order Markov model will be adequate to generate the distribution of precipitation amounts within a given storm. In this case, the state i in the transition probability matrix represents $[(i-1)\ 0.001]$ in.* of precipitation. The transition probabilities should be estimated from the historical records by counting the frequency of each particular transition. Table III shows the frequency count in a row (a set of) transition probability matrix. Assuming that the probability distribution of precipitation is different between the first and second halves of the storm, the storm is divided into two parts. In other words, two matrices are developed. One matrix contains the transition probabilities for the first half of the storm and the other contains the transition probabilities for the second half of the storm.

Matrix size needs to be determined for computer analysis. Since actual precipitation rates are limited by meteorological conditions, the number

* 1 in. = 25.4 mm.

TABLE I

Values for the observed and expected histograms of wet sequences and the chi-square test values (χ^2) using one set of Weibull parameters

Hours	Observed	Σ Occurrences	Expected	Σ Occurrences	χ^2	$\Sigma\chi^2$
1.00	183.00	183.00	188.30	188.30	0.15	0.15
2.00	150.00	333.00	146.72	335.03	0.07	0.22
3.00	135.00	468.00	120.52	455.55	1.74	1.96
4.00	89.00	557.00	100.28	555.83	1.27	3.23
5.00	84.00	641.00	84.01	629.83	0.00	3.23
6.00	60.00	701.00	70.68	710.52	1.61	4.85
7.00	61.00	762.00	59.66	770.18	0.03	4.88
8.00	47.00	809.00	50.49	820.67	0.24	5.12
9.00	55.00	864.00	42.80	863.47	3.47	8.59
10.00	29.00	893.00	36.35	899.82	1.49	10.08
11.00	30.00	923.00	30.91	930.74	0.03	10.10
12.00	29.00	952.00	26.32	957.05	0.27	10.38
13.00	22.00	974.00	22.43	979.49	0.01	10.39
14.00	16.00	990.00	19.14	998.62	0.51	10.90
15.00	21.00	1,011.00	16.34	1,014.96	1.33	12.23
16.00	10.00	1,021.00	13.96	1,028.93	1.12	13.35
17.00	13.00	1,034.00	11.94	1,040.87	0.09	13.45
18.00	9.00	1,043.00	10.22	1,051.08	0.14	13.59
19.00	7.00	1,050.00	8.75	1,059.83	0.35	13.94
20.00	9.00	1,059.00	7.49	1,067.32	0.30	14.24
21.00	6.00	1,065.00	6.42	1,073.75	0.03	14.27
22.00	11.00	1,076.00	5.51	1,079.25	5.48	19.75
23.00	3.00	1,079.00	4.72	1,083.98	0.63	20.38
24.00	4.00	1,083.00	4.05	1,088.03	0.00	20.38
25.00	3.00	1,086.00	3.48	1,091.51	0.07	20.45
26.00	7.00	1,093.00	2.99	1,094.51	5.37	25.82
27.00	1.00	1,094.00	2.57	1,097.07	0.96	26.78
28.00	1.00	1,095.00	2.21	1,099.28	0.66	27.44
29.00	2.00	1,097.00	1.90	1,101.18	0.01	27.45

30.00	5.00	1.102.00	1.62	1,102.82	6.94	34.38
31.00	2.00	1,104.00	1.41	1,104.22	0.25	34.64
32.00	2.00	1,106.00	1.21	1,105.43	0.52	35.15
33.00	1.00	1,107.00	1.04	1,106.47	0.00	35.15
34.00	0.00	1,107.00	0.90	1,107.37	0.90	36.05
35.00	0.00	1,107.00	0.77	1,108.14	0.77	36.82
36.00	1.00	1,108.00	0.67	1,108.81	0.17	36.99
37.00	2.00	1,110.00	0.57	1,109.38	3.55	40.54
38.00	0.00	1,110.00	0.49	1,109.88	0.49	41.03
39.00	1.00	1,111.00	0.43	1,110.30	0.77	41.80
40.00	0.00	1,111.00	0.37	1,110.67	0.37	42.17
41.00	0.00	1,111.00	0.32	1,110.99	0.32	42.49
42.00	0.00	1,111.00	0.27	1,111.26	0.27	42.76
43.00	0.00	1,111.00	0.24	1,111.50	0.24	43.00
44.00	0.00	1,111.00	0.20	1,111.70	0.20	43.20
45.00	0.00	1,111.00	0.18	1,111.88	0.18	43.38

Notes:
(1) Weibull parameters: $A = 0.18535; B = 0.95020$.
(2) Total number of observed wet periods: 1113.
(3) The χ^2-values doe not sum to the $\Sigma\chi^2$-values because the values were summed and then rounded to the nearest one-hundredth by the computer program.

TABLE II

Values for the observed and expected histograms of wet sequences and the chi-square test values (χ^2) using two sets of Weibull parameters

Hours	Observed	Σ Occurrences	Expected	Σ Occurrences	χ^2	$\Sigma\chi^2$
(A) *For remaining values:*						
1.00	183.00	183.00	188.30	188.30	0.15	0.15
2.00	150.00	333.00	146.72	335.03	0.07	0.22
3.00	135.00	468.00	120.52	455.55	1.74	1.96
4.00	89.00	557.00	100.28	555.83	1.27	3.23
5.00	84.00	641.00	84.01	639.83	0.00	3.23
6.00	60.00	701.00	70.68	710.52	1.61	4.85
7.00	61.00	762.00	59.66	770.18	0.03	4.88
8.00	47.00	809.00	50.49	820.67	0.24	5.12
9.00	55.00	864.00	42.80	863.47	3.47	8.59
(B) *For extreme values:*						
10.00	29.00	893.00	32.71	896.18	0.42	9.01
11.00	30.00	923.00	29.59	925.76	0.01	9.02
12.00	29.00	952.00	25.59	951.35	0.45	9.47
13.00	22.00	974.00	22.12	973.48	0.00	9.47
14.00	16.00	990.00	19.12	992.60	0.51	9.98
15.00	21.00	1,011.00	16.52	1,009.12	1.21	11.20
16.00	10.00	1,021.00	14.27	1,023.39	1.28	12.47
17.00	13.00	1,034.00	12.32	1,035.72	0.04	12.51
18.00	9.00	1,043.00	10.64	1,046.36	0.25	12.76
19.00	7.00	1,050.00	9.18	1,055.54	0.52	13.28
20.00	9.00	1,059.00	7.93	1,063.46	0.15	13.43
21.00	6.00	1,065.00	6.84	1,070.30	0.10	13.53
22.00	11.00	1,076.00	5.90	1,076.20	4.41	17.94
23.00	3.00	1,079.00	5.09	1,081.29	0.86	18.80
24.00	4.00	1,083.00	4.39	1,085.68	0.03	18.83
25.00	3.00	1,086.00	3.78	1,089.46	0.16	19.00

26.00	7.00	1,093.00	3.26	1,092.72	4.28	23.28
27.00	1.00	1,094.00	2.81	1,095.54	1.17	24.45
28.00	1.00	1,095.00	2.42	1,097.96	0.84	25.28
29.00	2.00	1,097.00	2.09	1,100.05	0.00	25.29
30.00	5.00	1,102.00	1.80	1,101.85	5.69	30.98
31.00	2.00	1,104.00	1.55	1,103.40	0.13	31.11
32.00	2.00	1,106.00	1.34	1,104.73	0.33	31.44
33.00	1.00	1,107.00	1.15	1,105.88	0.02	31.46
34.00	0.00	1,107.00	0.99	1,106.87	0.99	32.45
35.00	0.00	1,107.00	0.85	1,107.73	0.85	33.30
36.00	1.00	1,108.00	0.73	1,108.46	0.10	33.40
37.00	2.00	1,110.00	0.63	1,109.10	2.95	36.35
38.00	0.00	1,110.00	0.54	1,109.64	0.54	36.90
39.00	1.00	1,111.00	0.47	1,110.11	0.60	37.50
40.00	0.00	1,111.00	0.40	1,110.51	0.40	37.90
41.00	0.00	1,111.00	0.35	1,110.86	0.35	38.25
42.00	0.00	1,111.00	0.30	1,111.16	0.30	38.55
43.00	0.00	1,111.00	0.26	1,111.42	0.26	38.81
44.00	0.00	1,111.00	0.22	1,111.64	0.22	39.03
45.00	0.00	1,111.00	0.19	1,111.83	0.19	39.22

Notes:
(A1) Weibull parameters for remaining values: $A = 0.18535; B = 0.95020$.
(A2) Total number of wet periods for remaining values: 871.
(B1) Weibull parameters for extreme values: $A = 0.13777; B = 1.01880; C = 0.82111$.
(B2) Total number of wet periods for extreme values: 242.
(B3) The χ^2-values do not sum to the $\Sigma\chi^2$-values because the values were summed and then rounded to the nearest one-hundredth by the computer program.

TABLE III
Frequency counts of transition from the state $i = 1$ or zero precipitation

Amount	Counts	Amount	Counts	Amount	Counts
0.000	11	0.022	7	0.043	0
0.001	170	0.023	3	0.044	8
0.002	187	0.024	6	0.045	0
0.003	109	0.025	4	0.046	6
0.004	72	0.026	4	0.047	0
0.005	61	0.027	5	0.048	1
0.006	48	0.028	5	0.049	0
0.007	28	0.029	5	0.050	2
0.008	33	0.030	5	0.051	0
0.009	21	0.031	2	0.052	4
0.010	18	0.032	3	0.053	0
0.011	28	0.033	0	0.054	4
0.012	12	0.034	7	0.055	0
0.013	16	0.035	0	0.056	1
0.014	17	0.036	8	0.057	0
0.015	10	0.037	0	0.058	0
0.016	17	0.038	6	0.059	0
0.017	12	0.039	0	0.060	1
0.018	13	0.040	5	0.061	0
0.019	8	0.041	0	0.062	5
0.020	10	0.042	6	0.063	0
0.021	7				

Note: 15-min. precipitation amounts.

of columns in the matrix is limited by the rare occurrence of very high values of precipitation. A value of 3 in. of precipitation in 15 min. was selected as the highest limit. The number of rows also has to be limited more severely because there are very few observations at the highest limit. As a result, a nonsquare matrix of 3000 columns × 14 rows was adopted. As mentioned before, one state represents a range of 0.001 in. of precipitation. Then each of the first nine rows contain one state; each of the next four rows contains a combined range of nine states; and the fourteenth row contains a combined range of all remaining states. This process was utilized in order to insure that there is enough data to fit a Weibull distribution.

Linear regression analysis of the spatial-distribution component of the model

An examination of observed precipitation data indicates that the calculated critical lag closely represents the time of separation of two storm cells moving over a gage. In other words, an average storm appears to move across the watershed in a time interval approximately equal to the calculated critical lag. Thus, the critical lag may be used to define the duration of a storm. Therefore, if precipitation occurs at one gage, zero precipitations should be filled at the other gage within the critical lag. Once the precipitations at the two gages have been matched, a linear regression analysis involving the least-squares method was performed. Also the residuals of the least squares was used to determine the standard deviation of ϵ (eq. 6).

Model generation of precipitation data

The classical approach in generating synthetic sequences of independent random variables for a given probability distribution (derived from a set of given data) is to use a graphical method to transform independent random numbers of uniform probability distribution (interval 0 to 1), to those of the given probability distribution. However, with the advent of computer technology the graphical method can now be replaced by the use of computers to generate synthetic sequences. The computerized method employs a conversion table to convert the random numbers of a uniform probability distribution to those of the given probability distribution. Let t be the random number of the given probability distribution with subscript i denoting a particular random number then, the conversion table relates t_i to x_i for the same cumulative probabilities $P(t \leqslant t_i) = P(x \leqslant x_i)$. Thus, a series of N-values of t_i can be transformed to a series of N-values of x_i by means of this conversion table. A better way to generate random variables is to use mathematical probabilities function instead of conversion tables. The cumulative probability function of a continuous random variable y with probability density $f(y)$ is:

$$F(y) = \int_{-\infty}^{y} f(y)\,\mathrm{d}y \tag{19}$$

Assume that $F(y)$ is a random variable uniformly distributed over the interval (0,1), its probability density function is $f\{F(y)\}$ so that its cumulative distribution function is:

$$g[F(y)] = \int_{0}^{F(y)} f[F(y)]\,\mathrm{d}[F(y)] \tag{20}$$

or $f[F(y)] = 1$ with $0 \leqslant F(y) \leqslant 1$. Thus, a random value y from a (Weibull) p.d.f. $f(y)$ can be obtained as follows:

(1) Generate a random value $F(h)$ from a uniform distribution over (0,1).

(2) Solve the following equation for a Weibull distribution with given A and B:

$$F(y) = 1 - \exp(-Ay^B), \quad \text{for } y \tag{21}$$

then one obtains:

$$y = [-\ln\{1 - F(y)\}/A]^{B^{-1}} \tag{22}$$

which is the random variable of a Weibull distribution.

By the above procedure, the wet-period length in hours taken as a random variable is generated.

By the same procedure, the 15-min. precipitation amounts in the generated wet period are to be generated as follows. Generate $F(y)$ for eq. 22 with values A and B, obtained for the first half of a storm at a given present state. The corresponding 15-min. precipitation amount y is then solved by eq. 22. This procedure begins with state one and proceeds until the 15-min. precipitation amounts of the first half of the storm are all so generated. Similarly, the 15-min. precipitation amounts of the second half of the storm are generated except the last time interval of the storm is taken as the first state and then the generation procedure steps backward in time. By the procedure described above a dry-period length in days is generated following the preceding wet period. The above process is repeated until the desired number of years of data has been generated.

After a 15-min. precipitation amount at the primary gage is generated, a corresponding 15-min. precipitation amount can be generated at the secondary gage by eq. 6. The values of α, β and σ for ϵ in eq. 6 are used to calculate simultaneous precipitation amounts for all secondary gages. In eq. 6 the random component ϵ is normally distributed with zero mean and standard deviation σ.

MODEL APPLICATION

Boneyard Creek network

The Boneyard Creek network shown in Fig. 1 and described in Table IV consists of six recording gages in 13-km^2 area in Champaign-Urbana, Illinois, U.S.A.

This network was originally installed in 1949 by the Civil Engineering Department of the University of Illinois in cooperation with the Illinois State Water Survey and the U.S. Geological Survey for the primary purpose of investigating the urban precipitation—runoff relationship (Chow, 1952).

The six raingages of interest in this study are Nos. *1*–*5* and *7* as shown in Fig. 2. Gage *2* was selected as the primary gage for a reason to be given later and gage *3* as an example of a secondary gage in this study. Thirteen years, 1949–1961, of continuous precipitation data were used in this investigation.

15-min. Precipitation data

The recorded rain chart taken from a raingage was converted to 15-min. amounts of precipitation in thousandths of an inch with the use of a digitizer and a PL-*1* computer program. Also, a FORTRAN computer program was written to compute some basic statistics of the data. This program's output is used to check for missing and/or faulty data. Approximately 200,000 individual nonzero 15-min. precipitation pulses were analyzed by this program.

From this analysis, gage *2* was found to have the least amount of missing and/or faulty data. Also, by using the Thiessen method to compute areal

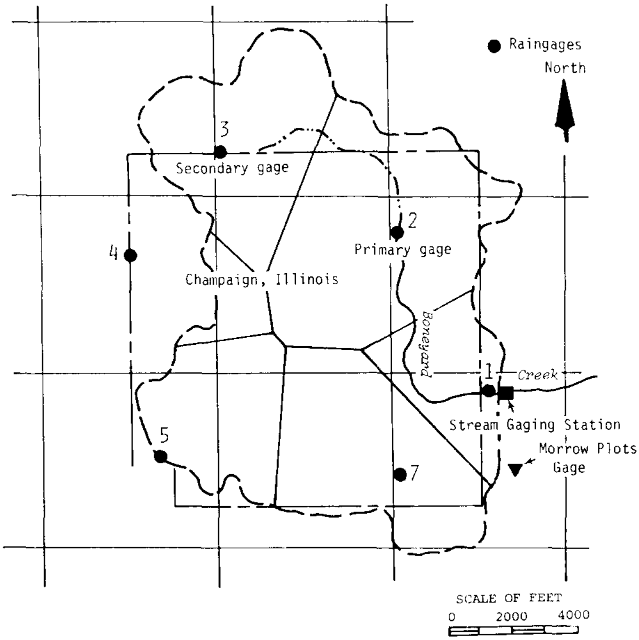

Fig. 1. Boneyard basin and raingage network (1 ft. = 0.3048 m).

precipitation, the Thiessen polygon for gage *2* covers the largest portion of the watershed. Therefore, gage *2* was selected as the primary gage for the development of the proposed stochastic model. The data from gage *2* consist of ~ 27,000 individual nonzero 15-min. precipitation pulses. The missing and/or faulty data at gage *2* were filled and/or replaced with the data from gage *1* for the corresponding storms. Since only a small percentage of data (< 1%) is possibly in error and because of the structure of the model, smoothing the observed data by determining distinguishable trends in the shape of the p.d.f., these data error will not have a significant effect upon the generated synthetic data. The complete data set for gage *2* was edited and sorted on the computer to obtain a composite temporal record of the precipitation and was then recorded on magnetic tape.

Autocorrelation analysis of data and determination of critical lag

A computer program was written to generate the two-dimensional matrix of two nonzero pulses of 15-min. precipitation given a specified lag time in the autocorrelation analysis. This output is then executed on the IBM® *360/75* SOUPAC statistical program to generate the autocorrelation coefficient for various lags. The plot of the autocorrelation coefficient as a function of lag time is shown in Fig. 2. It shows that the precipitation values have high positive correlation at smaller lags. In particular, the lag-one (successive precipitation values separated by one 15-min. interval) correlation is 0.67799 for a sample size of 12,439 (15-min. nonzero

TABLE IV
Boneyard Creek raingage network

Gage No.	Location					Gage type	Period of record at each location
	legal description				general description		
	corner	section	township	range			
1	NW 1/4 NW 1/4	18	19N	9E	24.5 m north of EE bldg.	Stevens recorder	March 1949 to present
2	NW 1/4 NW 1/4	7	19N	9E	68.5 m south of Cap & Gown bldg. (several minor moves in immediate area)	Stevens recorder	Nov. 1948 to present
3	NW 1/4 SW 1/4	1	19N	8E	southwest of Pioneer plant	Stevens recorder	Nov. 1948 to present
4	SW 1/4 NE 1/4	11	19N	8E	southwest of Nelsen concrete plant, formerly State Garage	Stevens recorder	Nov. 1948 to present
5	SE 1/4 NE 1/4	14	19N	8E	yard of J.J. Doland home near Country Club	Stevens recorder	Nov. 3, 1948 to Oct. 20, 1951
Moved	SE 1/4 NE 1/4	22	19N	8E	south of Dean's dairy on S. Mattis	Stevens recorder	Oct. 20, 1951 to May 7, 1953
Moved	SE 1/4 SE 1/4	11	19N	8E	Shell Station prospect and sprg. aves.	Stevens recorder	May 7, 1953 to present
7	cent. NE 1/4	13	19N	8E	west of Prairie Farms creamery	Stevens recorder	Nov. 2, 1948 to Oct. 21, 1950
Moved	NE 1/4 SE 1/4	13	19N	8E	12 m west of Warm Air Research bldg. (now gone)	Stevens recorder	March 24, 1951 to Oct. 5, 1957

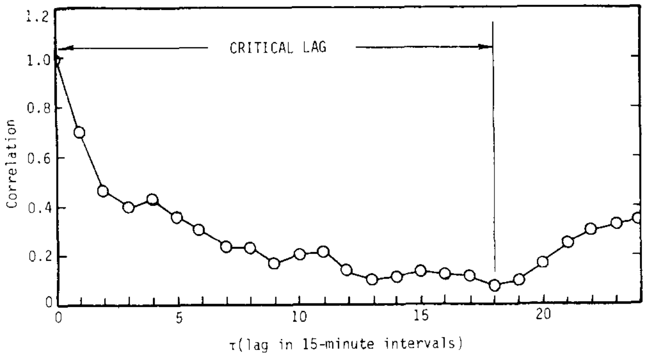

Fig. 2. Correlogram (cor $x(t)$, $x(t + \tau)$) for 13 years, 1949—1961, of recorded historical Boneyard data.

precipitation amounts) which indicates definite dependence between pairs of adjoining precipitation values in the series. The values of autocorrelation coefficients decrease with increasing lags indicating weaker correlation between precipitation pulses separated by larger lags. Using the t-statistic to test the hypothesis that the correlation in the populations of two variables is zero, all autocorrelation coefficients (Fig. 2) calculated including the critical lag autocorrelation were found to be significantly different from zero at the 1% level. The critical lag (minimum correlation amounts) for the 15-min. data was 270 min., or minimum dependence between precipitation.

Parameter estimation

The computer program which constitutes the stochastic model can be divided into four parts as follows:

(1) Evaluation of the wet-and-dry sequence Weibull parameters (A and B).

(2) Evaluation of Weibull parameters for precipitation amounts distributed in a wet period.

(3) Evaluation of spatial parameters (α, β, σ).

(4) Generation of a wet-period and its following dry-period length, and also the precipitation amount in the wet period.

The parameter for the Boneyard Creek network data to be described later, are given in Table V. All four parts of the computer programs can be combined into a monitoring program with each part becoming a subprogram of the monitoring program. For the sake of understanding and clarity, Table VI shows how these computer programs are utilized for computation of the proposed stochastic model. In addition, an operational flow-chart given in Fig. 3 illustrates the use of this model for generation of precipitation amounts in the intermittent precipitation process.

Fig. 4 gives a sample output of the proposed stochastic precipitation model. Fig. 5 shows the frequency of the simulated data at gage *2* produced by the proposed model and the historical frequency data for 13 yr. of

TABLE V

Weibull parameters for the three model components

(1) *Wet-and-dry sequence component:*

$A = 0.10261$	$B = 1.04995$	winter wet periods	$N = 251$
$A = 0.21426$	$B = 0.96223$	spring wet periods	$N = 333$
$A = 0.25321$	$B = 1.02526$	summer wet periods	$N = 247$
$A = 0.12616$	$B = 1.02290$	fall wet periods	$N = 243$
$A = 0.25871$	$B = 0.95041$	winter dry periods	$N = 251$
$A = 0.41761$	$B = 0.80215$	spring dry periods	$N = 332$
$A = 0.25455$	$B = 0.90868$	summer dry periods	$N = 247$
$A = 0.27220$	$B = 0.90601$	fall dry periods	$N = 243$

(2) *Precipitation-distribution component*:

				*	
$A = 0.36311$	$B = 0.69910$	1st half of storm	previous state	1	$N = 595$
$A = 0.35422$	$B = 0.70441$	2nd half of storm	previous state	1	$N = 553$
$A = 0.21024$	$B = 1.62109$	1st half of storm	previous state	2	$N = 407$
$A = 0.21923$	$B = 1.50940$	2nd half of storm	previous state	2	$N = 643$
$A = 0.14468$	$B = 1.37012$	1st half of storm	previous state	3	$N = 548$
$A = 0.11356$	$B = 1.58252$	2nd half of storm	previous state	3	$N = 728$
$A = 0.02374$	$B = 2.38330$	1st half of storm	previous state	4	$N = 449$
$A = 0.04830$	$B = 1.93115$	2nd half of storm	previous state	4	$N = 533$
$A = 0.98698$	$B = 1.32129$	1st half of storm	previous state	5	$N = 271$
$A = 0.02577$	$B = 2.02051$	2nd half of storm	previous state	5	$N = 360$
$A = 0.01479$	$B = 2.09375$	1st half of storm	previous state	6	$N = 261$
$A = 0.01414$	$B = 2.14648$	2nd half of storm	previous state	6	$N = 263$
$A = 0.00502$	$B = 2.55078$	1st half of storm	previous state	7	$N = 218$
$A = 0.02782$	$B = 1.66797$	2nd half of storm	previous state	7	$N = 179$
$A = 0.00799$	$B = 2.17188$	1st half of storm	previous state	8	$N = 168$
$A = 0.01045$	$B = 2.01563$	2nd half of storm	previous state	8	$N = 180$
$A = 0.00736$	$B = 2.12109$	1st half of storm	previous state	9	$N = 103$
$A = 0.00609$	$B = 2.20117$	2nd half of storm	previous state	9	$N = 107$
$A = 0.00376$	$B = 2.27734$	1st half of storm	previous state	10	$N = 167$
$A = 0.01562$	$B = 1.66016$	2nd half of storm	previous state	10	$N = 163$
$A = 0.00236$	$B = 2.16895$	1st half of storm	previous state	20	$N = 743$
$A = 0.00872$	$B = 1.71045$	2nd half of storm	previous state	20	$N = 478$
$A = 0.00068$	$B = 2.17285$	1st half of storm	previous state	30	$N = 247$
$A = 0.00233$	$B = 1.82422$	2nd half of storm	previous state	30	$N = 171$
$A = 0.00055$	$B = 2.04492$	1st half of storm	previous state	40	$N = 130$
$A = 0.00002$	$B = 3.03125$	2nd half of storm	previous state	40	$N = 76$
$A = 0.00025$	$B = 1.92578$	1st half of storm	previous state.GE.	50	$N = 104$
$A = 0.00784$	$B = 1.19922$	2nd half of storm	previous state.GE.	50	$N = 87$
$A = 0.28680$	$B = 0.61304$	1st half of storm	previous state	1	$N = 597$
$A = 0.31038$	$B = 0.61279$	2nd half of storm	previous state	1	$N = 619$
$A = 0.35500$	$B = 0.79590$	1st half of storm	previous state	2	$N = 254$
$A = 0.23420$	$B = 1.24219$	2nd half of storm	previous state	2	$N = 432$
$A = 0.25586$	$B = 0.84680$	1st half of storm	previous state	3	$N = 405$
$A = 0.17202$	$B = 1.12292$	2nd half of storm	previous state	3	$N = 469$
$A = 0.10749$	$B = 1.28955$	1st half of storm	previous state	4	$N = 325$
$A = 0.05445$	$B = 1.73486$	2nd half of storm	previous state	4	$N = 362$
$A = 0.11904$	$B = 1.04590$	1st half of storm	previous state	5	$N = 165$
$A = 0.04639$	$B = 1.58789$	2nd half of storm	previous state	5	$N = 295$
$A = 0.04605$	$B = 1.44141$	1st half of storm	previous state	6	$N = 107$

TABLE V (*continued*)

$A = 0.02116$	$B = 1.92969$	2nd half of storm	previous state	6 $N = 148$
$A = 0.01223$	$B = 2.04297$	1st half of storm	previous state	7 $N = 123$
$A = 0.01872$	$B = 1.84766$	2nd half of storm	previous state	7 $N = 125$
$A = 0.06654$	$B = 1.08887$	1st half of storm	previous state	8 $N = 129$
$A = 0.01557$	$B = 1.76953$	2nd half of storm	previous state	8 $N = 132$
$A = 0.00830$	$B = 1.89258$	1st half of storm	previous state	9 $N = 95$
$A = 0.03988$	$B = 1.30469$	2nd half of storm	previous state	9 $N = 105$
$A = 0.02509$	$B = 1.37891$	1st half of storm	previous state	10 $N = 108$
$A = 0.04696$	$B = 1.17383$	2nd half of storm	previous state	10 $N = 95$
$A = 0.01590$	$B = 1.39551$	1st half of storm	previous state	20 $N = 507$
$A = 0.02352$	$B = 1.31592$	2nd half of storm	previous state	20 $N = 443$
$A = 0.01167$	$B = 1.28760$	1st half of storm	previous state	30 $N = 254$
$A = 0.01408$	$B = 1.27148$	2nd half of storm	previous state	30 $N = 188$
$A = 0.01267$	$B = 1.17773$	1st half of storm	previous state	40 $N = 152$
$A = 0.02991$	$B = 0.95313$	2nd half of storm	previous state	40 $N = 72$
$A = 0.00378$	$B = 1.23145$	1st half of storm	previous state.GE.50	$N = 253$
$A = 0.04062$	$B = 0.76367$	2nd half of storm	previous state.GE.50	$N = 127$
$A = 0.33018$	$B = 0.47342$	1st half of storm	previous state	1 $N = 486$
$A = 0.34516$	$B = 0.47730$	2nd half of storm	previous state	1 $N = 476$
$A = 0.28921$	$B = 0.90186$	1st half of storm	previous state	2 $N = 112$
$A = 0.25801$	$B = 1.02197$	2nd half of storm	previous state	2 $N = 226$
$A = 0.11411$	$B = 1.46387$	1st half of storm	previous state	3 $N = 193$
$A = 0.13600$	$B = 1.28418$	2nd half of storm	previous state	3 $N = 250$
$A = 0.02293$	$B = 2.38672$	1st half of storm	previous state	4 $N = 120$
$A = 0.03033$	$B = 2.08203$	2nd half of storm	previous state	4 $N = 169$
$A = 0.18062$	$B = 0.74121$	1st half of storm	previous state	5 $N = 106$
$A = 0.09729$	$B = 1.10547$	2nd half of storm	previous state	5 $N = 128$
$A = 0.06336$	$B = 1.23242$	1st half of storm	previous state	6 $N = 77$
$A = 0.03208$	$B = 1.62305$	2nd half of storm	previous state	6 $N = 92$
$A = 0.10537$	$B = 0.92090$	1st half of storm	previous state	7 $N = 65$
$A = 0.02668$	$B = 1.56250$	2nd half of storm	previous state	7 $N = 86$
$A = 0.07289$	$B = 0.99609$	1st half of storm	previous state	8 $N = 74$
$A = 0.01412$	$B = 1.83594$	2nd half of storm	previous state	8 $N = 73$
$A = 0.01208$	$B = 1.79688$	1st half of storm	previous state	9 $N = 68$
$A = 0.01262$	$B = 1.73438$	2nd half of storm	previous state	9 $N = 65$
$A = 0.01040$	$B = 1.73438$	1st half of storm	previous state	10 $N = 32$
$A = 0.01672$	$B = 1.58594$	2nd half of storm	previous state	10 $N = 43$
$A = 0.01576$	$B = 1.40186$	1st half of storm	previous state	20 $N = 244$
$A = 0.04098$	$B = 1.05225$	2nd half of storm	previous state	20 $N = 179$
$A = 0.00880$	$B = 1.41504$	1st half of storm	previous state	30 $N = 115$
$A = 0.02102$	$B = 1.11914$	2nd half of storm	previous state	30 $N = 101$
$A = 0.01317$	$B = 1.12891$	1st half of storm	previous state	40 $N = 59$
$A = 0.02106$	$B = 1.07813$	2nd half of storm	previous state	40 $N = 54$
$A = 0.01071$	$B = 0.97070$	1st half of storm	previous state.GE.50	$N = 249$
$A = 0.07584$	$B = 0.65576$	2nd half of storm	previous state.GE.50	$N = 127$
$A = 0.26908$	$B = 0.61145$	1st half of storm	previous state	1 $N = 493$
$A = 0.36287$	$B = 0.62622$	2nd half of storm	previous state	1 $N = 456$
$A = 0.27185$	$B = 1.19629$	1st half of storm	previous state	2 $N = 345$
$A = 0.19240$	$B = 1.54321$	2nd half of storm	previous state	2 $N = 661$
$A = 0.12961$	$B = 1.47510$	1st half of storm	previous state	3 $N = 640$
$A = 0.13347$	$B = 1.43140$	2nd half of storm	previous state	3 $N = 657$
$A = 0.12885$	$B = 1.20337$	1st half of storm	previous state	4 $N = 315$

TABLE V *(continued)*

$A = 0.04522$	$B = 1.91260$	2nd half of storm	previous state	4 $N = 392$
$A = 0.02758$	$B = 1.98438$	1st half of storm	previous state	5 $N = 264$
$A = 0.06393$	$B = 1.44824$	2nd half of storm	previous state	5 $N = 292$
$A = 0.05798$	$B = 1.36133$	1st half of storm	previous state	6 $N = 170$
$A = 0.00801$	$B = 2.41211$	2nd half of storm	previous state	6 $N = 215$
$A = 0.01123$	$B = 2.03320$	1st half of storm	previous state	7 $N = 147$
$A = 0.01733$	$B = 1.86230$	2nd half of storm	previous state	7 $N = 164$
$A = 0.02361$	$B = 1.64453$	1st half of storm	previous state	8 $N = 110$
$A = 0.00500$	$B = 2.35938$	2nd half of storm	previous state	8 $N = 141$
$A = 0.00591$	$B = 2.14844$	1st half of storm	previous state	9 $N = 115$
$A = 0.01257$	$B = 1.78906$	2nd half of storm	previous state	9 $N = 81$
$A = 0.01316$	$B = 1.73828$	1st half of storm	previous state	10 $N = 81$
$A = 0.00337$	$B = 2.28906$	2nd half of storm	previous state	10 $N = 64$
$A = 0.00765$	$B = 1.69482$	1st half of storm	previous state	20 $N = 530$
$A = 0.01386$	$B = 1.47266$	2nd half of storm	previous state	20 $N = 397$
$A = 0.00141$	$B = 1.94922$	1st half of storm	previous state	30 $N = 185$
$A = 0.01307$	$B = 1.33789$	2nd half of storm	previous state	30 $N = 128$
$A = 0.00024$	$B = 2.29297$	1st half of storm	previous state	40 $N = 115$
$A = 0.00014$	$B = 2.49219$	2nd half of storm	previous state	40 $N = 93$
$A = 0.00570$	$B = 1.11621$	1st half of storm	previous state.GE.50	$N = 200$
$A = 0.00211$	$B = 1.48047$	2nd half of storm	previous state.GE.50	$N = 94$

(3) *Spatial-distribution component:*

0.56889	0.00338	0.01553
0.40873	0.00862	0.03255
0.31623	0.01344	0.04960
0.33880	0.00653	0.02287

*Refers to previous state plus one.

record at gage *2* and for the regional historical data derived by Illinois State Water Survey. These data were derived from the partial-duration series of 15-min. precipitation amounts.

Fig. 6 shows the simulated frequency data at gages *2* and *3* produced by the proposed model and the historical frequency data for 89 yr. of record at Morrow plots and for the Illinois State Water Survey regional data. These data were derived from the partial-duration series of daily precipitation amounts.

Fig. 7 shows the correlogram for the simulated 100 yr. of 15-min. precipitation. As shown in Figs. 4—7, the proposed model appears to perform as intended.

MAJOR ACCOMPLISHMENTS OF THE PROPOSED MODEL

(1) The proposed model can produce as many extreme values without an upper bound as one wishes. Therefore, it can produce such values greater than the historical values. This is clearly shown in Fig. 5.

TABLE VI

Utilization of computer programs for the proposed stochastic model

Inputs	Program	Outputs
Primary gage data (precipitation amounts and time of occurrence) Critical lag time	program "SEQ": (calculates length of wet period in hours and length of dry period in days); appendix A*	lengths of wet-and-dry sequences
Lengths of wet-and-dry sequences	program "WEIB": (calculates by method of maximum likelihood the parameters for the Weibull distribution for wet-and-dry sequences); appendix B* or appendix C*	parameters for Weibull distribution for each season of wet-and-dry sequences; table 5-5
Primary gage data (precipitation amounts and time of occurrence) Critical lag time	program "DIS": (calculates by method of maximum likelihood the parameters for the Weibull distribution for the transition probability matrix for precipitation distribution); appendix D*	parameters for Weibull distribution for each season of the transition probability matrix of precipitation distribution; table 5-6
Primary and secondary gage(s) "data" (precipitation amounts and time of occurrence)	program "STORM": (time match of precipitation amounts between primary gage and and secondary gage, then use standard package program for regression analysis with residuals normally distributed); appendix E*	parameters for each season of the spatial regression component; table 5-7
Parameters for each season of the three model components; table 5-1	program "RAIN": (generates precipitation amounts and time of occurrence for any number of years specified); appendix F*	"*N*" years synthesized precipitation

* Morris (1978).

(2) The proposed model seems to produce frequency data of similar trend. This is shown in Fig. 6 that curves for the simulated data of gages *2* and *3* and for the 89-yr. Morrow plots data appear to be parallel. The relationship by the Illinois State Water Survey does not follow the trend of the data probably due to spatial variation of the precipitation. This spatial variation may also account for the differences among the other curves.

(3) The proposed stochastic model can generate 15-min. precipitation data for a network of stations for use as input to urban watershed modeling. In such application, the following characteristics of the precipitation data can be reproduced adequately:

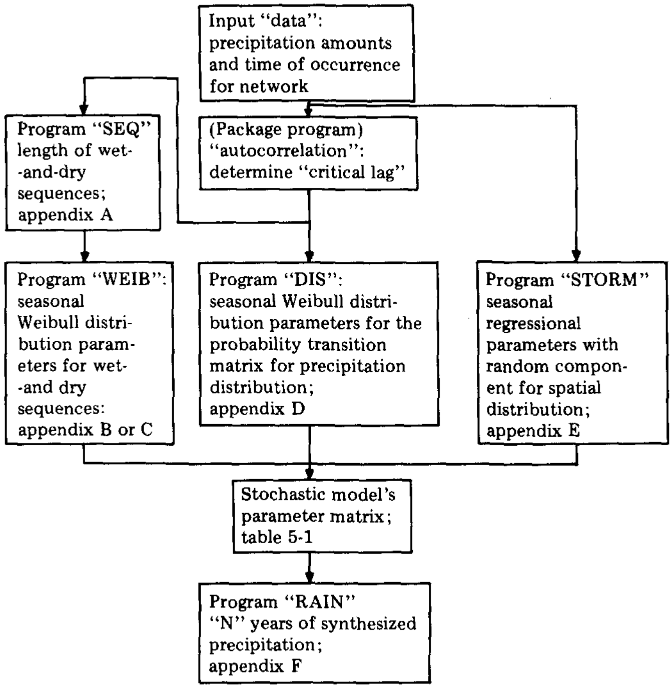

Fig. 3. Operational flow-chart for the stochastic model.

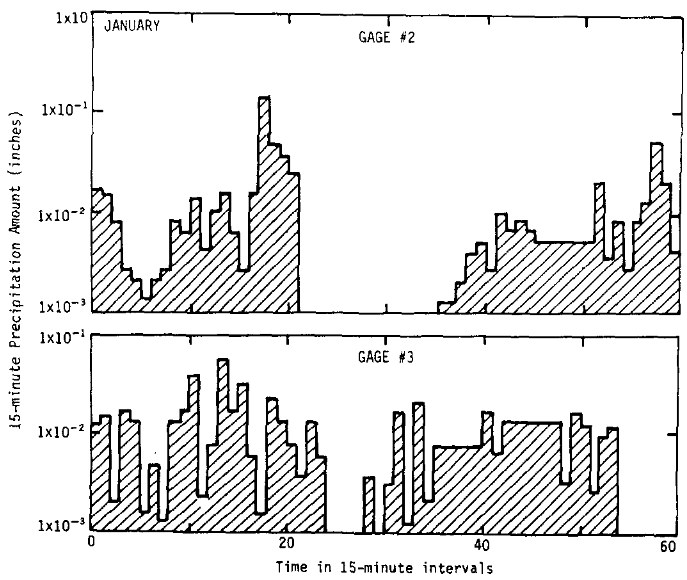

Fig. 4. Simulated precipitation by the proposed model.

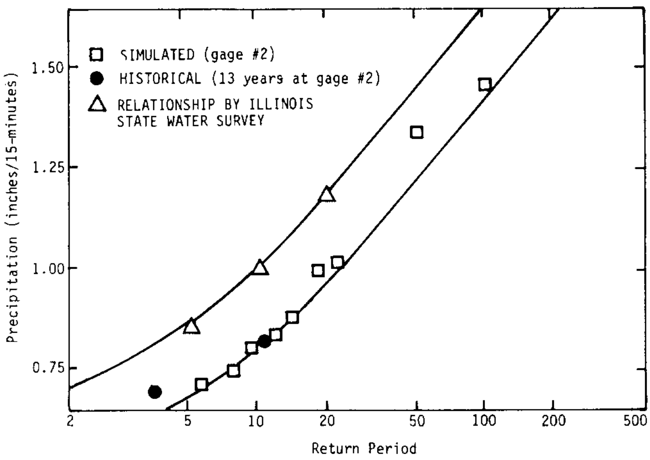

Fig. 5. Frequency data of historical and simulated 15-min. precipitations.

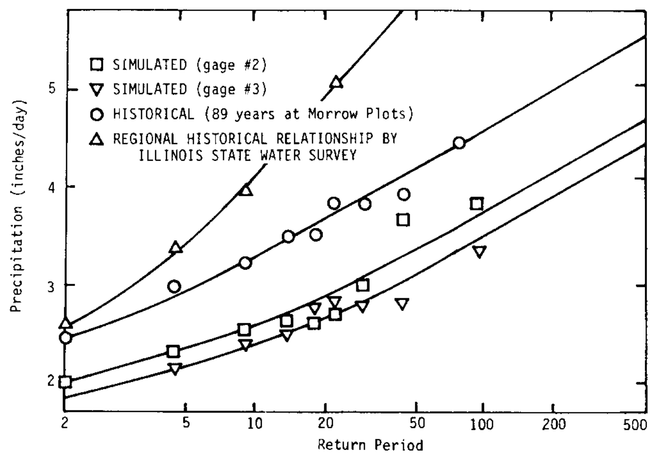

Fig. 6. Frequency data of historical and simulated daily precipitations.

(a) the mean and variance of the storm precipitation (Tables VII and VIII);

(b) the distribution of the storm lengths, storm precipitation, and lengths of dry periods (Table I);

(c) the autocorrelation structure of the precipitation (Figs. 2 and 7).

CONCLUSIONS

The wet-and-dry sequences of data need to be modeled separately from the precipitation distribution within a storm. Since the precipitation data consist mostly of groups of zero pulses, in most cases of extended length,

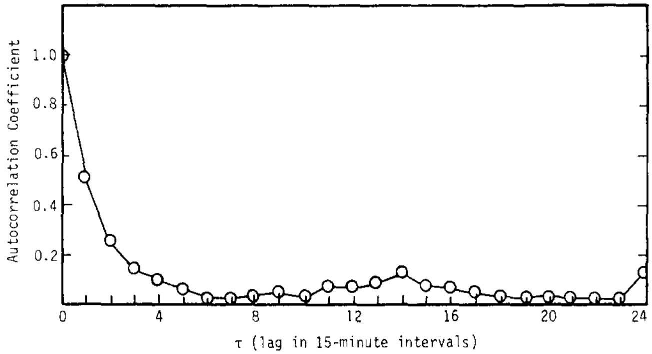

Fig. 7. Correlogram (cor $x(t), x(t + \tau)$) for simulated data by proposed stochastic model.

difficulties arise when a stochastic model is used to model both the wet- and dry-period precipitation processes. Also, a Markov chain is used to model the rainfall distribution within the wet-period. A one-step Markov chain is

TABLE VII

Frequency counts for 13 years of historical data

Amounts	Absolute frequency		Relative frequency		Accumulated frequency	
Gage	*2*	*3*	*2*	*3*	*2*	*3*
0.001	3,228	2,019	12.6	10.9	12.6	10.9
0.002	4,121	2,637	16.0	14.2	28.6	25.0
0.005*	1,463*	1,142*	5.7*	6.0*	53.5*	47.6*
0.007*	1,111*	639*	4.3*	3.4*	62.6*	56.0*
0.010*	627*	443*	2.4*	2.4*	71.5*	65.0*
0.020*	224*	163*	0.9*	0.9*	84.9*	79.2*
0.050*	43*	31*	0.2*	0.2*	95.1*	93.5*
0.070*	15*	15*	0.1*	0.1*	96.9*	96.0*
0.100*	1*	2*	0.0*	0.0*	98.2*	97.6*
0.200*	2*	2*	0.0*	0.0*	99.3*	99.2*
0.508*	1*	1*	0.0*	0.0*	99.9*	99.9*
0.730*	1*	1*	0.0*	0.0*	100.0*	100.0*
0.811*	1*	1*	0.0*	0.0*	100.0*	100.0*
Statistics for 13 years of historical data:						
Mean	0.014	0.017				
Mode	0.002	0.002				
St. dev.	0.034	0.040				
Median	0.005	0.006				
Variance	0.001	0.002				

*Intermediate values not shown in table.

Total number of data points = 25,712 (gage *2*); and 18,597 (gage *3*).

TABLE VIII
Frequency counts for 100 years of generated data

Amounts	Absolute frequency		Relative frequency		Accumulated frequency	
Gage	*2*	*3*	*2*	*3*	*2*	*3*
0.001	16,712	3,382	9.4	2.4	9.4	3.6
0.002	16,154	3,264	9.0	2.4	18.4	6.0
0.005*	12,369*	3,482*	6.9*	2.5*	41.2*	13.4*
0.007*	9,667*	3,434*	5.4*	2.5*	52.8*	18.4*
0.010*	6,641*	3,504*	3.7*	2.6*	65.4*	26.1*
0.020*	1,903*	2,950*	1.1*	2.2*	84.3*	49.3*
0.050*	222*	803*	0.1*	0.6*	95.1*	85.9*
0.070*	96*	343*	0.1*	0.3*	96.8*	93.8*
0.100*	62*	104*	0.0*	0.1*	98.0*	98.1*
0.200*	20*	1*	0.0*		99.4*	99.9*
0.495*	3*	1*	0.0*		100.0*	100.0*
0.660*	3*		0.0*		100.0*	
1.000*	1*		0.0*		100.0*	
1.326*	1*		0.0*		100.0*	
1.451*	1*		0.0*		100.0*	
Statistics for 100 years of generated data:						
Mean	0.015	0.027				
Mode	0.001	0.010				
St. dev.	0.032	0.025				
Median	0.007	0.021				
Variance	0.001	0.001				

*Intermediate values not shown in table.
Total number of data points = 178,664 (gage *2*); and 136,555 (gage *3*).

used because the future amount of precipitation for a 15-min interval is mainly dependent on the present amount of precipitation for a 15-min. interval. Weekly stationary precipitation sequences are obtained by reducing the seasonal periodicity by employing stochastic models for each season of winter, spring, summer and fall.

An extension and improvement of the classic Markov chain transition probability matrix is developed and applied. This is done by fitting an analytical function to describe the matrix. The benefit is two-fold:

(1) the scheme generates values that have not been observed.

(2) it smoothes the observed data so that distinguishable trends in the shape of the probability distribution can be extrapolated to complete the tail end of each row of the matrix.

The computer time requirement by the stochastic hydrologic model algorithm is not excessive; it is estimated to be ~ 1 s of CPU time on a CDC®-*175* computer time per year of data generated. The parameter estimation component for the precipitation distribution requires ~ 100 s of CPU time per year of observed input data.

The principal advantage of this model is that it can statistically predict or project extreme events and preserve the statistical moments of the data base. This predictive capability is intrinsically incorporated into each component of the model.

SUMMARY

The major contributions are as follows:

(1) An intermittent small-interval precipitation stochastic model in time and space.

(2) A procedure to predict the probability of occurrences of the extreme events.

(3) A tool for studying urban runoff by generating precipitation from past data, which can then be converted to runoff using a runoff model.

Through the application of the proposed stochastic model to the Boneyard Creek network data, the following specific conclusions can be made:

(1) The model can reproduce the trend of the frequency of precipitation amounts from the recorded 89-yr. Morrow plots data, with small spatial variations.

(2) The model reproduces the mean and variance of storm precipitations, the storm lengths, the dry-period lengths, the transition probability distributions of the precipitations, and the autocorrelation structure of the Boneyard Creek network data.

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