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Abstract: Computer-assisted analysis was used to study the relation among uniform, linear, and nonlinear stress components thermally induced in a composite bridge section for hypothetical parameters of varying span lengths, number of spans, and support conditions, as well as for actual bridges. The results were verified by conventional methods of analysis. The following was concluded for prismatic (constant) sections: (1) For constant proportionality of span lengths, each of the three thermal stress components is independent of span length; (2) variation of the proportionality of span lengths affects only the linear stress component; (3) support reactions and deflections caused by thermal loading are length dependent, but the induced moments and stresses are independent of length; (4) as the number of spans increases, the (thermally induced) moment magnitudes tend to converge; (5) the magnitude of reactions, for constant proportionality of span lengths, varies inversely with span length; and (6) for total end fixity, no exterior or interior vertical support reactions are thermally induced.

Introduction

Thermally induced stresses resulting from support restraint or from inadequate or malfunctioning supporting and expansion devices have been a subject of concern to bridge design engineers for many years. Early attempts to account for thermal stresses and movements in bridges, reviewed in a state of the art by Reynolds and Emanuel (32) in 1974, were hindered by the complexity of analysis and the lack of an accepted rational design criteria.

In recent years it has become accepted practice to eliminate expansion devices by connecting the superstructure to a flexible substructure with either pinned or integral connections at the abutments (8,16). This design has the advantages of eliminating the expenses and maintenance problems associated with expansion devices. However, the structure must be designed to withstand the stresses caused by temperature changes. Recognition of the ubiquitous, nationwide structural distress in highway bridges and the increasing cost of maintenance and repair has reemphasized the need for a rational design procedure to allow for thermally induced stresses in composite streel bridges.

In a study conducted at the University of Missouri-Rolla, Emanuel, et al. (8), concluded that the development of rational design criteria for bridges with semi-integral end bents is feasible. During subsequent studies by Emanuel and Hulsey, a theoretical procedure was developed for determining stresses and strains in composite steel and concrete bridges resulting from thermal loading (9,10,19,20). Also, realistic thermal load-

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ing and temperature distributions were derived from 20 years of recorded weather data (11–13,21). Subsequent experimental studies were conducted on a model composite two-span bridge by Emanuel and coworkers (14,15,18,26,34) to verify the results of the previous theoretical study. Those studies concluded that the developed theoretical procedures are adequate for a reasonable prediction of the behavior of composite-girder bridge structures subjected to thermal loading.

Earlier procedures for determining thermal stresses in composite-girder concrete and steel bridges were presented by Zuk (36), Berwanger (2), and Berwanger and Symko (3,4). Methods of thermal stress analysis have also been developed for other types of bridges, such as solutions for concrete box girder bridges by Priestly (28), and Hunt and Cooke (23). Rahman and George developed a theory to compute temperature-induced stresses and displacements in continuous, skew slab-girder bridges (30) and verified the theory experimentally (31).

The methods now available for calculation of thermal stresses in different types of structures are very diverse, but they all depend on an accurate prediction of the temperaure distribution. The basics of heat transfer are the same for all types of structures, but different methods have been employed to solve the heat transfer equations. Emanuel and Hulsey (11), Berwanger and Symko (3,4), and Lanigan (24) each used a two-dimensional, finite element, heat flow method. Sinusoidal boundary conditions representing actual weather patterns were used by Emanuel and Hulsey, while Berwanger and Symko assumed steady-state boundary conditions. Hunt and Cooke (23) used a finite difference method to solve the one-dimensional heat flow equations for a concrete box-girder bridge. Emerson (17) used a finite difference solution of one-dimensional heat flow supplemented by experimental temperature distributions for different types of bridges. Cundy, et al. (7), have presented a one-dimensional algorithm for prediction of the thermal response of a composite bridge deck. Priestley and Thurston (29) noted very close agreement among finite difference solutions and both two-dimensional finite element and experimental solutions.

A 29-span bridge with an overall length of about 2,700 ft (823.5 m) has been constructed with expansion devices only at the abutments (1,5). However, bridge design engineers generally express concerns regarding the limiting length of composite-girder bridge structures and the relationship of resultant thermal stresses.

No direct reference was found in the literature relative to the effect of span length on induced thermal stresses. However, in a study of thin-shell concrete cooling towers it was observed that the thermal stresses were dependent only on the temperature gradient and the material properties, and were independent of both the thickness and the radii of curvature of the shell (25). The extrapolation to bimaterial, composite, continuous beams is not immediately evident. Also the question arises as to possible change in reactions (and moments) in continuous structures and coincident variations in transverse movements.

Toward a clarification of bridge length limitations and in an effort to add to the understanding of the behavior of composite-girder bridge structures subjected to thermal loading, this study was initiated. The objective of the study was to determine the effect of different span lengths on the magnitude of thermal stresses. Computer-assisted analysis was used to study different conditions, and the results were verified by using conventional methods of structural analysis.

Explanation of Thermal Stresses

Structures subjected to the natural environment very rarely have a uniform temperature distribution. Nonuniform temperature distributions may produce three different components of the total thermal stress. Churchward and Sokal (6) discussed the decomposition of temperature strains into uniform, linear, and nonlinear components. The components may be explained as follows:

1. The uniform component is the average strain which will produce axial movement without stress if the movement is unrestrained. Total restraint of the axial movement would induce stress without strain, and partial restraint would produce some combination of stress and strain.
2. The linear component is a curvature-inducing strain which will produce vertical deflections and curvature without stress if the vertical movement is completely unrestrained. Total restraint of vertical movement would induce stress without strain, and partial restraint would produce some combination of stress and strain. The curvature is constant for unrestrained prismatic beams (28).
3. The nonlinear component of the total temperature strain is a stress-inducing strain with stresses resulting from the continuity of the cross section and the assumption that plane sections remain plane. Stresses in a simply supported, single-span beam subjected to a nonlinear temperature distribution are produced only by the nonlinear strain component, as the uniform and linear components are unrestrained. An expression for the stresses caused by nonlinear strains is given by Priestley (28) for a general cross section with an arbitrary temperature distribution. The nonlinear component may also be produced by a linear temperature distribution in a nonhomogeneous and anisotropic material or in a composite beam as a result of nonlinear strains caused by different coefficients of thermal expansion.

In summary: for homogeneous isotropic materials, a uniform temperature distribution will produce only the uniform strain component; a linear temperature distribution will produce the linear strain component, and may produce the uniform component if the average temperature of the material changes; and a nonlinear temperature distribution will produce the nonlinear stress-inducing strain component, and may produce the linear and uniform strain components (a nonlinear temperature distribution will usually include a linear component). Any temperature strain component will produce movement without stress if the movement is unrestrained, stress without strain if movement is completely restrained, and some combination of stress and strain if movement is partially restrained.

Thermal stresses in beams will result from a combination of one or more of the three thermal stress components. The magnitude of each component will be dependent on the support conditions, the material

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properties, and the temperature distribution.

The three components of thermal strains and stresses may be demonstrated as in the following examples:

1. A homogeneous simply-supported beam, a cantilever beam, or a continuous beam, pinned at one support and free to expand at the other supports (unrestrained axial movement), subjected to a change in uniform temperature will develop only the uniform component of thermal strain and linear elongation without induced stress.

2. A homogeneous, simply-supported or cantilever beam (unrestrained deflection and rotation) subjected to a linear temperature distribution will develop either the linear component of strain or combined uniform and linear components accompanied by vertical and horizontal displacements without induced stresses.

3. A homogeneous, simply-supported or cantilever beam subjected to a nonlinear temperature gradient will develop a combination of the three thermal strain components and a resultant nonlinear stress gradient (produced by the nonlinear temperature strain component).

4. A change in uniform temperature of the classic homogeneous bar fixed between two walls or of a homogeneous continuous beam pinned or fixed at the abutments (restrained axial movement) initiates the uniform component of the temperature strain. Because the ends are restrained, there is no axial elongation, the length parameter cancels, and axial stresses are induced (which are independent of length).

5. A homogeneous, single span, fixed-end (restrained rotation) beam subjected to a linear temperature distribution may be analyzed as the superposition (within the limiting range) of three simple beams: the first, subjected to a linear temperature distribution with resultant constant curvature, vertical displacement, and no induced stress; the second, subjected to fixed end moments to negate the end slopes and produce contrary deflection (under constant moment), producing an undeflected beam with internal flexural stresses; and the third, subjected to the uniform component of temperature strain, developed only if there is a change in the average temperature of the section, producing superimposed axial stresses. The strains of the first beam, resulting from the linear thermal loading, are identical to the strains that would be produced by a constant equivalent thermal moment applied to the entire length of the beam. This equivalent thermal moment will produce deformations without stress, if vertical movement is unrestrained. The fixed end moments of the second beam will be equal and opposite to the thermally induced equivalent moment, and may be calculated by a formula given by Priestly (28)

$$M = -\frac{EI\alpha(T_1 - T_2)}{h} \quad \text{(1)}$$

in which M = the restraining moment; E = the modulus of elasticity; I = the moment of inertia; α = the coefficient of thermal expansion; T_1 = the temperature at the top of the beam; T_2 = the temperature at the bottom of the beam; and h = the depth of the cross section.

6. Interface continuity of a bimaterial simply-supported or cantilever beam will induce a nonlinear thermal strain component and nonlinear

stresses, regardless of the thermal loading (i.e., uniform, linear, or nonlinear).

7. A homogeneous continuous beam pinned at one support and free to expand at the other supports, subjected to a linear temperature distribution will develop flexural stress caused by the induced support reactions and the resultant moments.

8. A homogeneous continuous beam, pinned at one support and free to expand at the other supports, subjected to a nonlinear temperature distribution will experience the nonlinear stress component, flexural stresses and some vertical movement between supports as a result of vertical support restraint, and the uniform strain component if there is a change in the average temperature.

9. A homogeneous continuous beam, pinned at the abutments and free to expand at the other supports, subjected to a linear temperature distribution will experience flexural stresses and some vertical movement between supports as a result of vertical support restraint, and the uniform stress component if there is a change in the average temperature.

10. A homogeneous continuous beam, pinned at the abutments and free to expand at the other supports, subjected to a nonlinear temperature distribution will experience the nonlinear stress component, flexural stresses and some vertical movement between supports as a result of vertical support restraint, and the uniform stress component if there is a change in the average temperature.

11. A homogeneous continuous beam, fixed (rotationally restrained) at the abutments and free to expand at the other supports, subjected to a linear temperature distribution will have flexural stresses and no vertical movement between supports. As the fixed ends negate the equivalent thermal moment and there is no resulting curvature, the vertical reaction at any interior supports is zero, regardless of the number of interior supports or their location.

A fixed end beam subjected to a linear temperature distribution would have only the linear component of temperature strains present if the average temperature were the same as the initial temperature at the time of construction. Since rotation is restrained the linear strain component will produce linear stresses instead of movement. If the average temperature were not the same as the temperature at construction, the uniform component of temperature strains would induce axial stress since axial movement is restrained.

Thermally loaded continuous structures may be analyzed by the method of consistent deformations (8,28). The initial unrestrained curvature of the beam may be calculated as resulting from an equivalent thermal moment, constant along the length of the beam.

Method of Solution

The investigation involved a two-part computer assisted study. In Part I, thermal stresses were determined for hypothetical cases of varying span lengths and support conditions for the bridge section of Hulsey's study (22). The bridge cross-sectional properties and temperature distributions were held constant to determine the relation between span length and thermal stresses for given support conditions. In Part II, thermal stresses were calculated for actual bridge structures of varying span lengths. In Part II, thermal stresses were determined for three theoretical cases: (1) Both the slab and the beam in plane stress; (2) the slab in plane strain and the beam in plane stress; and (3) the slab in some state between plane stress and plane strain (partially restrained) and the beam in plane stress. Thermal stresses for Part I were determined only for theoretical case 3.

Thermal stresses were determined using the Soil-Structure Interaction Program (SSIP) for analysis of thermal stresses in composite bridges developed by Hulsey (22) and used in his and later studies at the University of Missouri-Rolla (9,14,15,18–20,26,34). The subsequent experimental studies confirmed the validity of the program and method of analysis.

The basic assumptions used in the study are the same as those made for Hulsey's derivation (22): (1) The slab and beam of each element act compositely; (2) fatigue stresses are considered to be negligible; (3) Hooke's law applies; (4) plane sections before bending remain plane after bending; (5) the slab and the beam are homogeneous and isotropic; (6) the temperature varies through the cross section; (7) the temperature distribution in each element is constant in the longitudinal, x-direction, i.e., $T(x) \neq f(x)$; (8) the temperature is considered to be constant over the flange width on any given horizontal plane, i.e., the shear stress due to temperature changes is zero, $\tau_{xz} = 0$, $T(z) \neq f(z)$; (9) the temperature distribution in the slab is constant on any given horizontal plane either within the width of the flange or outside the flange, but the temperature for each region may be different; (10) internal member restraints are not imposed in the vertical, y-direction, i.e., $\sigma_y = 0$; (11) longitudinal curvature compatibility ($d\theta_s/dx$), i.e., torsional forces between separated slab sections, is neglected; (12) transverse axial strain compatibility between separated slab sections is neglected; and (13) each interior girder is straight and has a symmetrical cross section.

Temperature distributions were determined for each section of the bridge by using the finite element heat transfer program called THERM. The program was developed by Wilson and Nickell (35) and modified by Hulsey (22) to produce temperature distribution data in the form suitable for use in SSIP.

Thermal Loading.—From a computerized reduction of 20 years of weather data recorded by the National Weather Service at a station in Columbia, Missouri, Emanuel and Hulsey (11,12,21) developed expressions for approximation of thermal loadings for bridges. From these, the maximum ambient air temperature may be determined as

$$T_{a_{\max}} = 15 \sin \frac{2\pi(h-9)}{24} + 17 \sin \frac{2\pi\left(d - 110 + \dfrac{h}{24}\right)}{365} + 74 \qquad (2)$$

in which h = the hour of the day; d = the day of the year; and $T_{a_{\max}}$ = the maximum ambient air temperature in °F. Similarly, the total solar energy incident on a horizontal surface during the maximum air temperature day, $Q_{a_{\max}}$ may be determined as

$$Q_{a_{max}} = 945 \sin \frac{2\pi(d - 82)}{365} + 1{,}636 \qquad (3)$$

in which d = the day of the year.

TABLE 1.—Element Properties for Typical Interior Girder, Part I

Section (1)	Property (2)	Value: Slab (3)	Value: Beam (4)	Units (5)
1, 2, 3	Coefficient of thermal expansion	4.0×10^{-6} (26.2×10^{-6})	6.5×10^{-6} (11.7×10^{-6})	in inches per inch per degrees Fahrenheit (meters per meter per degrees Celsius)
	Conductivity	1.0 (0.16)	31.0 (4.98)	in British thermal units per (hour-foot-degrees Fahrenheit) [watts per (meter·degrees Celsius)]
	Emissivity	0.9	—	
	Heat transfer coefficient	1.0 (0.027)	1.0 (0.027)	in British thermal units per (hour-square foot) (watts per square meter)
	Modulus of elasticity	3.8×10^3 (26.2)	29×10^3 (200.0)	in kips per square inch (gigapascals)
	Poisson's ratio	0.2	0.3	
1	Effective slab width	78 (1,981.2)	—	inches (millimeters)[a]
	Slab thickness	7.5 (190.5)	—	
	Top and bottom flange width		11 (279.4)	
	Thickness		0.875 (21.8)	
	Web thickness		0.4375 (11.1)	
	Web depth		42 (1,066.8)	
2	Effective slab width	156 (3,962.4)	—	
	Slab thickness	7.5 (190.5)	—	
	Top and bottom flange width		22 (558.8)	
	Thickness		0.875 (21.8)	
	Web thickness		0.875 (22.2)	
	Web depth		42 (1,066.8)	
3	Effective slab width	78 (1,981.2)	—	
	Slab thickness	15 (381.0)	—	
	Top and bottom flange width		11 (279.4)	
	Thickness		1.75 (44.5)	
	Web thickness		0.4375 (11.1)	
	Web depth		84 (2,133.6)	

[a]All of the following properties are in inches (millimeters).

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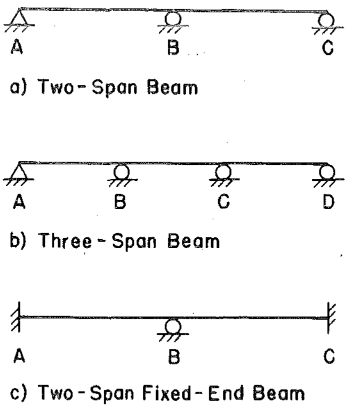

FIG. 1.—Structures Investigated in Part I

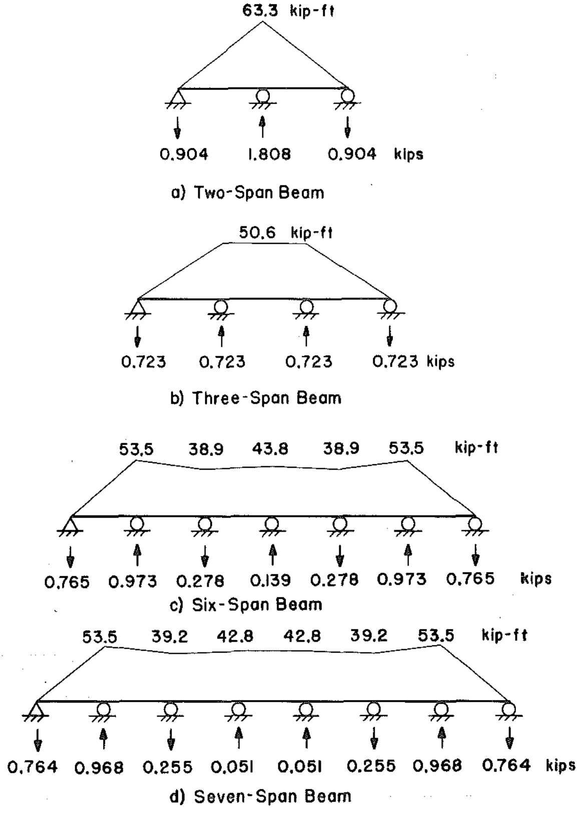

FIG. 2.—Moment Diagrams (Plotted on Tension Side) and Support Reactions for Continuous Beams With Varying Number of Spans (1 ft-kip = 1.357 kN·m, 1 kip = 4.45 kN)

For convenience, the temperature distribution for the highest bridge temperature on the day of maximum ambient air temperature was used in this study. This distribution occurred at hour 14 of day 201. The 122 element finite element mesh of Hulsey (22) was also used for this study. Thermal properties of the materials are shown in Table 1.

TABLE 2.—Reactions, Moments, and Stresses, Part I, Two-Span Beam

	Span Lengths, in Feet (Meters)			
Item (1)	100 (30.5) (2)	200 (61.0) (3)	300 (91.5) (4)	Unit (5)
Reaction at support A	−633 (−2,817)	−316 (−1,406)	−211 (−939)	in pounds-force (newtons)
Reaction at support B	1,265 (−5,629)	633 (−2,817)	422 (1,878)	in pounds-force (newtons)
Reaction at support C	−633 (−2,817)	633 (−1,406)	211 (−939)	in pounds-force (newtons)
Maximum moment	−63.3 (−85.9)	−63.3 (−85.9)	−63.3 (−85.9)	in kip-feet (kilonewton · meters)
Stress at top of slab	−280 (−1,929)	−280 (−1,929)	−280 (−1,929)	in pounds per square inch (kilopascals)[a]
Stress at bottom of slab	414 (2,852)	414 (2,852)	414 (2,852)	
Stress at top of beam	−2,326 (−16,026)	−2,326 (−16,026)	−2,326 (−16,026)	
Stress at bottom of beam	−718 (−4,947)	−718 (−4,947)	−718 (−4,947)	

[a]All of the following items are in pounds per square inch (kilopascals).

TABLE 3.—Reactions, Moments, and Stresses, Part I, Three-Span Beam

	Span Lengths, in Feet (Meters)			
Item (1)	100 (30.5) (2)	200 (61.0) (3)	300 (91.5) (4)	Unit (5)
Reaction at support A	−506 (−2,252)	−253 (−1,126)	−169 (−752)	in pounds-force (newtons)
Reaction at support B	506 (2,252)	253 (1,126)	169 (752)	in pounds-force (newtons)
Reaction at support C	506 (2,252)	253 (1,126)	169 (752)	in pounds-force (newtons)
Reaction at support D	−506 (2,252)	−253 (1,126)	−169 (−752)	in pounds-force (newtons)
Maximum moment	−50.6 (−68.7)	−50.6 (−68.7)	−50.6 (−68.7)	in kip-feet (kilonewtons · meters)
Stress at top of slab	−288 (1,984)	−288 (−1,984)	−288 (−1,984)	in pounds per square inch (kilopascals)[a]
Stress at bottom of slab	410 (2,825)	410 (2,825)	410 (2,825)	
Stress at top of beam	−2,359 (−16,254)	−2,359 (−16,254)	−2,359 (−16,254)	
Stress at bottom of beam	−507 (−3,493)	−507 (−3,493)	−507 (−3,493)	

[a]All of the following items are in pounds per square inch (kilopascals).

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TABLE 4.—Reactions, Moments, and Stresses, Part I, Two-Span Fixed-End Beam

Item (1)	Span Lengths, in Feet (Meters): 100 (30.5) (2)	200 (61.0) (3)	300 (91.5) (4)	Unit (5)
Reaction at support A	0 (0)	0 (0)	0 (0)	in pounds-force (kilonewtons)
Reaction at support B	0 (0)	0 (0)	0 (0)	in pounds-force (kilonewtons)
Reaction at support C	0 (0)	0 (0)	0 (0)	in pounds-force (kilonewtons)
Maximum moment	−42.2 (−57.3)	−42.2 (−57.3)	−42.2 (−57.3)	in kip-feet (kilonewton · meters)
Axial thrust	1,260 (5,607)	1,260 (5,607)	1,260 (5,607)	in kips (kilonewtons)
Stress at top of slab	−1,641 (−11,306)	−1,641 (−11,306)	−1,641 (−11,306)	in pounds per square inch (kilopascals)[a]
Stress at bottom of slab	−940 (−6,477)	−940 (−6,477)	−940 (−6,477)	
Stress at top of beam	−14,910 (−102,730)	−14,910 (−102,730)	−14,910 (−102,730)	
Stress at bottom of beam	−12,890 (−88,812)	−12,890 (−88,812)	−12,890 (−88,812)	

[a]All of the following items are in pounds per square inch (kilopascals).

The initial temperature of the bridge was assumed to be a steady-state condition equal to the average air temperature for the day of the year when calculations were started. Hulsey (22) recommended that the effect of the assumed initial temperature on the final temperature distribution obtained by the finite element method should be studied. In limited observations during the course of this study the assumed initial temperature distribution had very little effect. For calculations beginning at hour 0 of day 200 and continued through hour 14 of day 201, a reasonable assumption for the inital temperature of 75° F (23.9° C) and an unrealistic assumption of 0° F (−17.8° C) produced the same results. The temperature distributions for the two assumptions converged after iterations for approximately 24 hr of time step periods.

Part I—Hypothetical Cases.—A two-span continuous prismatic beam similar to that of Fig. 1(*a*) was analyzed to determine thermal stresses for span lengths of 100, 200, and 300 ft (30.5, 61.0, and 91.5 m). A three-span continuous prismatic beam similar to that of Fig. 1(*b*) was also studied for span lengths of 100, 200, and 300 ft (30.5, 61.0, and 91.5 m). Other support conditions were investigated, such as a fixed-end beam similar to that of Fig. 1(*c*) with span lengths of 100, 200, and 300 ft (30.5, 61.0, and 91.5 m). Continuous prismatic beams with varying numbers of spans such as those of Fig. 2, were also investigated. The cross-sectional properties of Sections 1 of Table 1 were used for each case. The two-span continuous beam of Fig. 1(*a*) was also studied for the cross-sectional properties of Sections 2 and 3 of Table 1. For Section 2 with doubled width and Section 3 with doubled depth the results were the same as those for Section 1. The results of Part I are shown in Tables 2, 3, and 4.

Part II—Actual Structures.—In Part II, thermal stresses were calculated for four actual bridge structures of varying span lengths as follows: Bridge 1, 48-62-48 ft (14.6-18.9-14.6 m); Bridge 2, 99-99 ft (30.2-30.2 m); Bridge 3, 120-160-120 ft (36.6-48.8-36.6 m); and Bridge 4, 140-190-190-140 ft (42.7-58.0-58.0-42.7 m). Bridge layouts, cross-sectional properties, and calculated thermal stresses for theoretical cases 1, 2, and 3 at points of support and change of section are shown by Taylor (33). The maximum calculated stresses for case 3 are summarized in Table 5. As designed, the bridges were pinned at one support only, and free of the uniform thermal stress component.

The calculated thermal stresses are greatly dependent on the material properties and other values chosen for the heat transfer solution. Thus, actual environmental stresses will vary somewhat from the calculated values, but the calculated stresses do demonstrate the parametric effects of span length, support conditions, and cross-section geometry.

Discussion of Results

Part I—Hypothetical Cases.—As previously discussed, both a two-span and a three-span structure, similar to Figs. 1(*a*) and 1(*b*), respectively, were analyzed for varying span lengths and support conditions. The two-span structure, with pinned-roller-roller supports was analyzed for span lengths of 100, 200, and 300 ft (30.5, 61.0, and 91.5 m). As shown in Table 2, in each case the maximum moment (resulting from vertical support restraint) was −63.3 kip-ft (−85.9 N · m) and the maximum stresses in the concrete were 280 psi (1,929 kPa) in compression and 414 psi (2,852 kPa) in tension. The maximum stress in the steel was 2,326 psi (16,026 kPa) in compression.

The three-span structure, with pinned-roller-roller-roller supports was analyzed for span lengths of 100, 200, and 300 ft (30.5, 61.0, and 91.5 m). As shown in Table 3, in each case the maximum moment was −50.6 kip-ft (−68.7 N · m), and the maximum stresses in the concrete were 288 psi (1,984 kPa) in compression and 410 psi (2,825 kPa) in tension. The maximum stress in the steel was 2,359 psi (16,254 kPa) in compression.

Thus, for the conditions studied, the thermal stresses were independent of span length. The thermal stresses would, of course, vary for different temperature distributions. However, independence of length would not be affected. Also, it should be noted that the reactions varied inversely with span length.

For a comparison of support conditions, a two-span beam with fixed-roller-fixed supports and the cross-sectional properties of Section 1, Table 1, was analyzed for span lengths of 100, 200, and 300 ft (30.5, 61.0, and 91.5 m). Again, as shown in Table 4, the moments, stresses, and axial thrusts were identical for each span length. Also as a result of the end fixity, no vertical support reactions were induced. It should be noted that although the thermal stresses are independent of span length, the stress magnitudes, as a result of the uniform stress component, are much greater than those of the two and three-span axially unrestrained structures.

Independence of span length may be readily verified by conventional methods of structural analysis. Axial stresses caused by restraint of the uniform component of the temperature strains (previously discussed) are independent of length as shown in the equation

$$\sigma = \alpha \Delta T E \quad (4)$$

in which σ = the stress; α = the thermal coefficient of expansion of the member; ΔT = the change in the temperature of the member; and E = the modulus of elasticity of the material.

Similarly, although it is not as obvious, the bending stresses caused by restraint of the linear component of the temperature strains in a composite beam are also independent of the span length as may be shown by conventional methods of structural analysis such as numerical integration (27). For the two-span beam of Fig. 1(*a*), the actual induced moment, M, at the center support is

$$M = 1.5 M_T \quad (5)$$

in which M_T = the equivalent thermal moment. The reaction at the center support, R, is dependent on span length, L, and

$$R = \frac{0.75 M_T}{L} \quad (6)$$

Thus, the support reactions and deflections are length dependent, but the moments are independent of length. Also, the induced thermal stresses will be the same for varying span lengths as long as the support conditions remain the same and the ratio of span lengths is held constant. For example, the thermal stresses in the structure of Fig. 1(*a*) would be the same for span lengths of either 50 or 100 ft (30.5 and 61.0 m). Similarly, the thermal stresses would be identical for span lengths of either 50 and 100 ft (15.3 and 30.5 m) or 100 and 200 ft (30.5 and 61.0 m) for Span 1 (A-B) and Span 2 (B-C), respectively.

The influence of the number of spans of constant length and section is illustrated in Fig. 2. As may be seen, the maximum moment is produced in a two-span structure [Fig. 2(*a*)], and the minimum moment in a three-span structure [Fig. 2(*b*)]. As the number of spans increases, the moment magnitudes tend to converge. However, the support reactions show a variation both in magnitude and sign, especially with regard to an even or odd number of spans.

None of the three component types of thermal stresses varied directly with span length. The thermal stresses may vary indirectly with length due to other factors such as stiffer cross-sections required to support gravity loads over longer span lengths, resulting from the dependency of thermal stresses on the layout of the structure and support conditions and on the temperature distribution (which in turn is dependent upon material properties and cross-section geometry).

Also, it should be remembered that an important factor affecting thermal stresses in composite sections is the difference in thermal coefficients of the component members, i.e., the deck and the beam. In this study, the coefficient of expansion of concrete was less than that of the steel (for both Parts I and II). Sections with different coefficients for the concrete would experience different magnitudes of stresses and reac-

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TABLE 5.—Maximum Theoretical Stresses, Part II

Location (1)	Unit Stress, in Pounds per Square Inch (Kilopascals)			
	Bridge 1 (2)	Bridge 2 (3)	Bridge 3 (4)	Bridge 4 (5)
Top of slab	−309 (−2,129)	−390 (−2,687)	−381 (−2,625)	−309 (−2,129)
Bottom of slab	379 (2,611)	412 (2,839)	415 (2,859)	428 (2,948)
Top of beam	−3,178 (−21,586)	−2,670 (−18,396)	−2,607 (−17,962)	−2,681 (18,472)
Bottom of beam	−503 (3,543)	−1,022 (−7,042)	−1,344 (−9,260)	−616 (−4,244)
	530 (3,651)	325 (2,239)	463 (3,190)	371 (2,556)

tions, and might, for coefficients greater than that of steel, have a reversal of signs. However, the independence of span length would remain unchanged.

Part II—Actual Structures.—The maximum calculated stresses for the four bridges of varying sections, span lengths, and number of spans are summarized in Table 5. Detailed results at the various stations for theoretical cases 1–3 are shown by Taylor (33). These maximums show a close correlation with the exception of the compressive stress in the bottom of the beam, which is a result of the nonlinear stress component at the ends of the structure (zero moment and zero flexural stress). Although unsubstantiated, it is believed that the variation in tensile stress at the bottom of the beam may be the result of varying support arrangements and the splicing of nonprismatic sections, rather than of varying span lengths.

Conclusions

Based on the results of varying the parameters of span lengths, number of spans and support conditions, the following conclusions were reached:

1. For prismatic (constant) sections and constant proportionality of span lengths, each of the three thermal stress components is independent of span length.
2. For prismatic (constant) sections, the uniform and nonlinear thermal stress components are completely independent of span lengths. Variation of the proportionality of span lengths affects only the linear stress component.
3. Thermal stresses are not directly dependent on the size of the cross section, but may be indirectly dependent on the cross section. The thermal stresses are dependent on the temperature distribution which in turn is dependent on the cross-sectional properties.
4. Support reactions and deflections caused by thermal loading are

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length dependent, but the induced moments and stresses are independent of length.

5. As the number of spans increases, the (thermally induced) moment magnitudes tend to converge. Also, the magnitude of thermally induced reactions may vary with an odd or even number of spans, and the magnitude of reactions, for constant proportionality of span length, varies inversely with span lengh.

6. For total end fixity, no exterior or interior vertical support reactions are thermally induced.

The results provide an insight into better understanding of thermal response, but should be extrapolated with caution, especially with respect to possible positioning of expansion devices. It should be noted that expansion devices, if operable, would relieve the uniform component, but they would not relieve the linear and nonlinear thermal stress components.

The wide variations in individual bridge layouts preclude a general conclusion concerning the indirect effect of span length on thermal stresses. However, recently developed methods of thermal analysis and interactive computer analysis and design allow comparison of alternate support and span arrangements and sectional properties at the planning stage.

Recommendations for Further Study

During the course of any investigation, questions arise as a result of the research. Many of the questions are usually beyond the scope of the study and remain unanswered.

The following topics would be of practical value toward development of rational design procedures and better understanding of thermal behavior of bridge structures and should be explored:

1. Further study of the effects on thermal response of partial axial restraint produced by piers and abutments.
2. A study of the interaction between superstructures and integral abutments or partial rotational restraint.
3. A study of the possible effect of proportionality of span lengths on the point of zero movement for superstructures tied to flexible substructures.

Other studies of interest to bridge engineers and those in related fields were suggested in the previous studies (14–16,18,22,26,34).

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

d = day of the year;
$d\theta_s/dx$ = longitudinal curvature;
E = modulus of elasticity;
$f(\ )$ = function of the variable in parentheses;
h = depth of cross section, hour of the day;
I = moment of inertia of cross section;
L = span length;
M = stress-inducing moment resulting from thermal loading;
M_T = nonstress-inducing equivalent thermal moment;
Q = solar energy flux;
R = vertical support reaction;
T = temperature;
α = thermal coefficient of expansion;
Δ = change in a quantity, e.g., ΔT;
σ = normal stress; and
τ = shear stress.

Subscripts

a = ambient;
max = maximum;
1 = top;
2 = bottom;
x = longitudinal direction;
y = vertical direction; and
z = transverse direction.

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