01 Jan 1983

The Application Of Bond Graphs To Electrical Machinery And Power Engineering

K. Sirivadhna

Earl F. Richards
Missouri University of Science and Technology

Max Darwin Anderson
Missouri University of Science and Technology, mda@mst.edu

Recommended Citation

K. Sirivadhna et al., "The Application Of Bond Graphs To Electrical Machinery And Power Engineering," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS thru 102, no. 5, pp. 1176 - 1184, Institute of Electrical and Electronics Engineers, Jan 1983.
The definitive version is available at https://doi.org/10.1109/TPAS.1983.318058

THE APPLICATION OF BOND GRAPHS TO ELECTRICAL MACHINERY AND POWER ENGINEERING

K. SIRIVADHNA E.F. RICHARDS M.D. ANDERSON
University of Missouri-Rolla

ABSTRACT

The bond graph technique, which has been related almost entirely to the field of mechanics, is a modeling procedure where emphasis is placed on the flow of power and energy in a system. Through specific digital simulation programs such as ENPORT IV and V and THTSIM the state space representation, associated output equations and system dynamic response are directly obtainable from the bond graphs. This approach has a great advantage where a complex system is composed of electrical, mechanical, thermal, hydraulic or pneumatic subsystems, such as would exist, for example; in a boiler, turbine, generator exciter system together with its associated controls.

The purpose of this paper is three-fold: (1) to develop interest in the bond graph modeling technique in power engineering (2) to develop bond graph models for typical synchronous and induction machines which are not as well developed in the literature as are the graphs of mechanical components and (3) to complete some of the missing links in the development of bond graphs for electromechanical machines.

Standard well known orthogonal axis transformations are used in the model development. The bond graphs thus developed from accurate mathematical relations can be easily integrated into other electrical or non-electrical systems through the power bonds of the graphs.

INTRODUCTION

The application of bond graphs to electrical power engineering has been very limited. This paper presents basic concepts, identifies contributions which have been made, provides references and presents bond graph structures related to field theory with application to electrical machinery.

The pioneer of the bond graph was Henry M. Paynter [1], who, through his simulation techniques using block diagrams, recognized the need for introducing a concept using power and energy as input-output variables. This differs from block diagram structure where the input-output parameters vary with the components. This concept of using power and energy as input-output parameters easily permits the marriage of mixed components. The general bond graph structure has been further developed in the last decade particularly by Karnopp and Rosenburg [3], [13], [15], [17]. Rosenberg [11] provides a good list of references.

Bond graphs, by definition, are a collection of multiport elements bonded together. At each of the ports two variables are defined whose product defines power. Systems and subsystems are classified according to the number of energy and power ports through which energy or power is exchanged with the environment and also in terms of the internal energy and power transformations involved. A bond graph is analogous to a linear network graphs where the multiport elements correspond to the nodes and the bonds correspond to the branches. The graphs may be of an open or closed loop structure, simple or very complex.

In the past, most applications of bond graphs have been almost entirely applied to mechanical systems. In electromechanical energy conversion devices, such as the rotating electrical machine, mixed energy conversion occurs between electrical circuits and electromagnetic field domains and the use of bond graphs are very applicable.

All systems which involve mixed engineering components of all types (mechanical, electrical, hydraulic, thermal) are conveniently represented by bond graphs. The elements (components) are modeled as energy multiports whose bond to other elements are through power bonds. The overall bond graph structure is easily related to a state-space formulation and the determination of linear system dynamics can be readily acquired using the ENPORT IV or V digital computer simulation programs.

Since the application of bond graphs in electrical engineering has been minimal, the basic concepts of graph formulation have been included in the Appendix.

BOND GRAPH MODELS

In this section bond graph models of an ideal synchronous machine and an ideal induction machine will be formed and developed in two separate sections. The equations describing these machines are summarized from other references and the bond graph elements given in the Appendix will be utilized.

A. Bond Graph Model of an Ideal Synchronous Machine

The following equations are the standard mathematical relationships of the ideal synchronous machine developed for the odq transformed axis obtained from references [5], [18], [19].

82 SM 355-6 A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE PES 1982 Summer Meeting, San Francisco, California, July 18-23, 1982. Manuscript submitted August 31, 1981; made available for printing April 14, 1982.

0018-9510/83/0500-1176$01.00 © 1983 IEEE

The flux linkage equations are:

$$\begin{bmatrix}\lambda_o\\ \lambda_d\\ \lambda_q\\ \lambda_F\\ \lambda_D\\ \lambda_Q\end{bmatrix} = \begin{bmatrix} L_o & 0 & 0 & 0 & 0 & 0\\ 0 & L_d & 0 & kM_F & kM_D & 0\\ 0 & 0 & L_q & 0 & 0 & kM_Q\\ 0 & kM_F & 0 & L_F & M & 0\\ 0 & kM_D & 0 & M & L_D & 0\\ 0 & 0 & kM_Q & 0 & 0 & L_Q\end{bmatrix}\begin{bmatrix}i_o\\ i_d\\ i_q\\ i_F\\ i_D\\ i_Q\end{bmatrix}$$

The voltage equations are:

$$\begin{bmatrix}v_o\\ v_d\\ v_q\\ v_F\\ v_D\\ v_Q\end{bmatrix} = \begin{bmatrix} r_o & 0 & 0 & 0 & 0 & 0\\ 0 & r_b & 0 & 0 & 0 & 0\\ 0 & 0 & r_c & 0 & 0 & 0\\ 0 & 0 & 0 & r_F & 0 & 0\\ 0 & 0 & 0 & 0 & r_D & 0\\ 0 & 0 & 0 & 0 & 0 & r_Q\end{bmatrix}\begin{bmatrix}i_o\\ i_d\\ i_q\\ i_F\\ i_D\\ i_Q\end{bmatrix} + \begin{bmatrix}\dot\lambda_o\\ \dot\lambda_d\\ \dot\lambda_q\\ \dot\lambda_F\\ \dot\lambda_D\\ \dot\lambda_Q\end{bmatrix} + \begin{bmatrix}0\\ \omega\lambda_q\\ -\omega\lambda_d\\ 0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}3r_s i_o + 3\ell_s \dot i_o\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

The equation for the machine torque is:

$$J\frac{d\omega}{dt} = T_m + T_e$$

$$T_e = (i_d\lambda_q - i_q\lambda_d)$$

Using the power flow approach, each of the voltage equations will be multiplied by the corresponding current flowing in that circuit. The stator o-axis power flow is then:

$$v_o i_o = r_o i_o^2 + \dot\lambda_o i_o + (3r_s i_o + 3\ell_s \dot i_o) i_o$$

$v_o i_o$ = power input to o-axis

$r_o i_o^2$ = power dissipated in r_o (stator resistance)

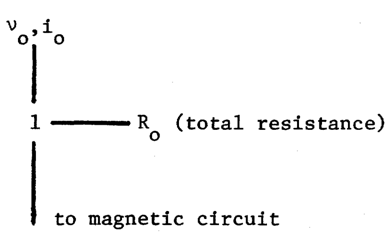
$\dot\lambda_o i_o$ = power transferred into the magnetic circuit

$(3r_s i_o + 3\ell_s \dot i_o) i_o$ = the power lost and stored in the neutral impedance

Under the bond graph modeling technique, two or more of the same types of impedances cannot be presented at a one junction and this equation must be rewritten as:

$$v_o i_o = (r_o + 3r_s) i_o^2 + (\dot\lambda_o + 3\ell_s \dot i_o) i_o$$
$$= R_o i_o^2 + (\dot\lambda_o + 3\ell_s \dot i_o) i_o$$

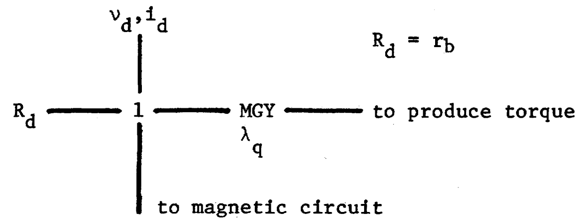
The bond graph of the o-axis power flow is then (see Appendix):



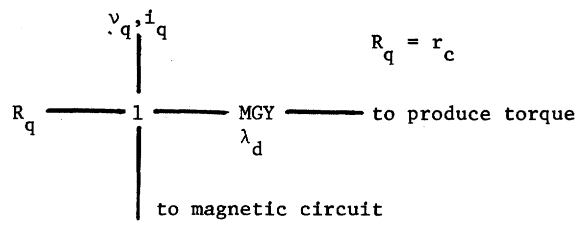
The stator d-axis and q-axis power flow equations are:

$$v_d i_d = r_b i_d^2 + \dot\lambda_d i_d + \omega\lambda_q i_d$$
$$v_q i_q = r_c i_q^2 + \dot\lambda_q i_q - \omega\lambda_d i_q$$

Each term has the same interpretation as in the o-axis equation. The last terms are speed-voltage terms which produce the electrical torque. The bond graph of d-axis power flow is:



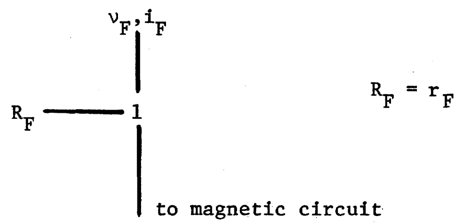
Similarily, the bond graph of q-axis power flow is:



The power flow equation in the rotor field circuit is:

$$v_F i_F = r_F i_F^2 + \dot\lambda_F i_F$$

The bond graph of rotor field circuit power flow is then:



The rotor d-axis and q-axis power flow equations are:

$$v_D i_D = r_D i_D^2 + \dot\lambda_D i_D$$
$$v_Q i_Q = r_Q i_Q^2 + \dot\lambda_Q i_Q$$

There is no direct power input to the D and Q damper windings, so

$$v_D i_D = 0\ ,\ v_Q i_Q = 0$$

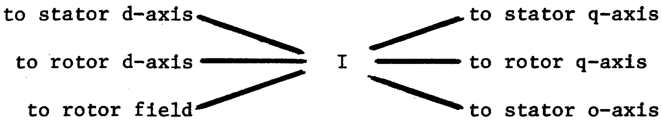
The bond graph of rotor d-axis power flow is:

R_D —— 1 —— to magnetic circuit $\qquad R_D = r_D$

The rotor q-axis power flow bond graph is:

R_Q —— 1 —— to magnetic circuit $\qquad R_Q = r_Q$

The magnetic circuit equations are the flux linkage equations which can be represented by



The swing equation power flow is:

$$\omega J \frac{d\omega}{dt} = \omega T_m + \omega T_e$$
$$= \omega T_m + \omega(\lambda_q i_d - \lambda_d i_q)$$
$$= \omega T_m - \omega\lambda_d i_q + \omega\lambda_q i_d$$

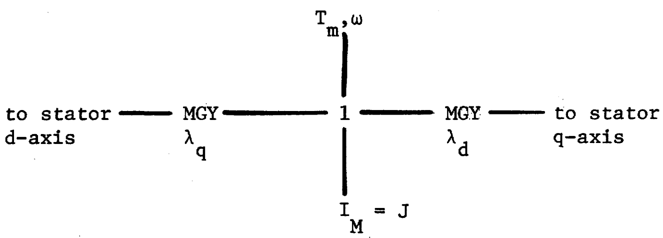
$\omega J \frac{d\omega}{dt}$ = machine accelerating power

ωT_m = mechanical input of the machine

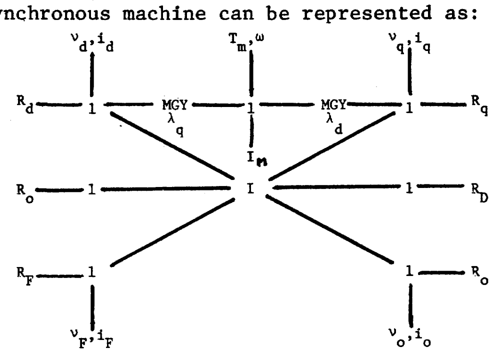
$\omega\lambda_q i_d$ = electromechanical power in stator d-axis

$\omega\lambda_d i_q$ = electromechanical power in stator q-axis.

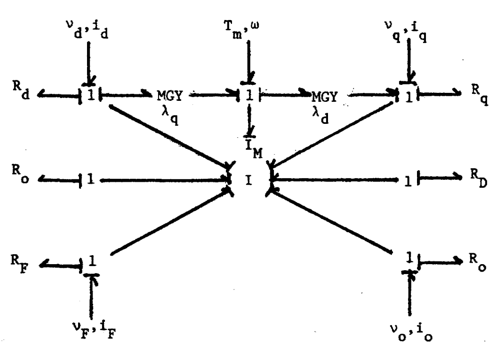
The swing equation bond graph is:



Now by combining the bond graphs developed, the entire ideal synchronous machine can be represented as:



By using the ENPORT IV program, the power flow and causality are identified as:



B. Bond Graph of the Ideal Induction Machine

Using the same approach as in the last section, the mathematical equations of the ideal induction machine are listed below [5], [19].

The flux linkage matrix in the odq reference frame is:

$$\begin{bmatrix} \lambda_o \\ \lambda_d \\ \lambda_q \\ \lambda_O \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ss} & 0 & 0 & L_{MM} & 0 \\ 0 & 0 & L_{ss} & 0 & 0 & L_{MM} \\ 0 & 0 & 0 & L_{RR} & 0 & 0 \\ 0 & L_{MM} & 0 & 0 & L_{RR} & 0 \\ 0 & 0 & L_{MM} & 0 & 0 & L_{RR} \end{bmatrix} \begin{bmatrix} i_o \\ i_d \\ i_q \\ i_O \\ i_D \\ i_Q \end{bmatrix}$$

The voltage equations are:

$$\begin{bmatrix} v_o \\ v_d \\ v_q \\ v_O \\ v_D \\ v_Q \end{bmatrix} = \begin{bmatrix} r_a & 0 & 0 & 0 & 0 & 0 \\ 0 & r_b & 0 & 0 & 0 & 0 \\ 0 & 0 & r_c & 0 & 0 & 0 \\ 0 & 0 & 0 & r_A & 0 & 0 \\ 0 & 0 & 0 & 0 & r_B & 0 \\ 0 & 0 & 0 & 0 & 0 & r_C \end{bmatrix} \begin{bmatrix} i_o \\ i_d \\ i_q \\ i_O \\ i_D \\ i_Q \end{bmatrix} + \begin{bmatrix} \dot\lambda_o \\ \dot\lambda_d \\ \dot\lambda_q \\ \dot\lambda_O \\ \dot\lambda_D \\ \dot\lambda_Q \end{bmatrix} - \begin{bmatrix} 0 \\ \omega\lambda_q \\ -\omega\lambda_d \\ 0 \\ s\omega\lambda_Q \\ -s\omega\lambda_D \end{bmatrix} + \begin{bmatrix} 3r_s i_o + 3\ell_s \dot i_o \\ 0 \\ 0 \\ 3r_r i_O + 3\ell_r \dot i_O \\ 0 \\ 0 \end{bmatrix}$$

The swing equation is:

$$J\frac{d\omega}{dt} = T_m + T_e$$
$$T_e = (i_D\lambda_Q - i_Q\lambda_D)$$

Equations of power transfer are:

$$P_s = (i_o\dot\lambda_o + i_d\dot\lambda_d + i_q\dot\lambda_q) + (i_o^2 r_a + i_d^2 r_b + i_a^2 r_c)$$
$$+\omega(i_d\lambda_q - i_q\lambda_d) + (3r_s i_o + 3\ell_s \dot i_o) i_o$$
$$P_r = (i_O\dot\lambda_O + i_D\dot\lambda_D + i_Q\dot\lambda_Q) + (i_O^2 r_A + i_D^2 r_B + i_Q^2 r_d)$$
$$+s\omega(i_D\lambda_Q - i_Q\lambda_D) + (3r_r i_O + 3\ell_r \dot i_O) i_O$$

The stator o-axis power flow is:

$$v_o i_o = r_a i_o^2 + \dot\lambda_o i_o + (3r_s i_o + 3\ell_s \dot i_o) i_o$$

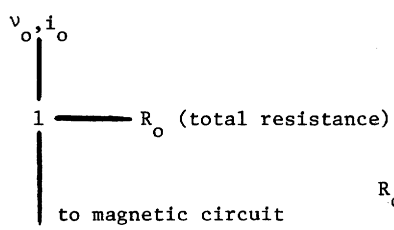
$v_o i_o$ = power input to o-axis

$r_a i_o^2$ = power dissipated is r_a

$\dot\lambda_o i_o$ = power transferred in to magnetic circuit

$(3r_s i_o + 3\ell_s \dot i_o) i_o$ = the power lost and stored in the neutral impedance

The bond graph of the o-axis power flow then becomes:



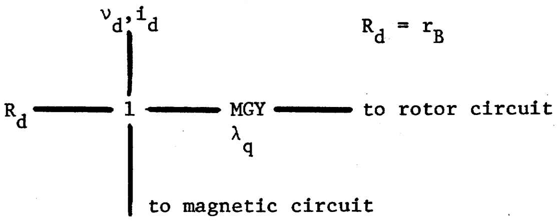
$$R_o = r_a + 3r_s$$

The stator d-axis and q-axis power flow equations are:

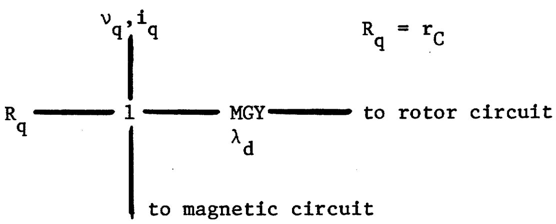
$$\nu_d i_d = r_b i_d^2 + \dot{\lambda}_d i_d + \omega\lambda_q i_d$$
$$\nu_q i_q = r_c i_q^2 + \dot{\lambda}_q i_q - \omega\lambda_d i_q$$

Each term has the same meaning as the o-axis relationships. The last terms again are speed-voltage terms which represent power transferred across the air-gap to the rotor circuit.

The d-axis power flow bond graph is:



and the bond graph of q-axis power flow is:



The rotor o-axis power flow equation is:

$$\nu_0 i_0 = r_A i_0^2 + \dot{\lambda}_0 i_0 + (3r_r i_0 + 3\ell_r \dot{i}_0) i_0$$

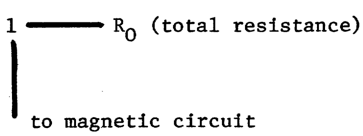
$\nu_0 i_0$ = power input to rotor o-axis = 0

$r_A i_0^2$ = power dissipated in r_A

$\dot{\lambda}_0 i_0$ = power transferred into the magnetic circuit

$(3r_r i_0 + 3\ell_r \dot{i}_0) i_0$ = the power lost and stored by the neutral impedance

The bond graph of the rotor o-axis power flow becomes:

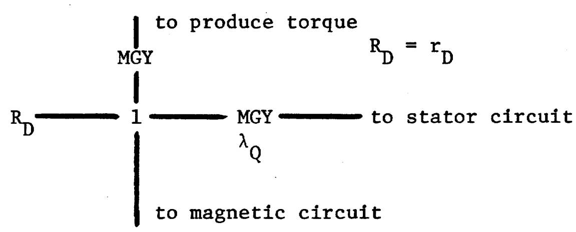


The rotor d-axis and q-axis power flow equations are:

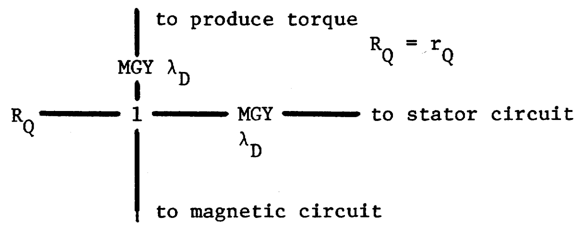
$$\nu_D i_D = r_B i_D^2 + \dot{\lambda}_D i_D + \omega\lambda_Q i_D - (1-s)\omega\lambda_Q i_D$$
$$\nu_Q i_Q = r_C i_Q^2 + \dot{\lambda}_Q i_Q - \omega\lambda_D i_Q + (1-s)\omega\lambda_D i_Q$$
$$\nu_D i_D = 0 \quad , \quad \nu_Q i_Q = 0$$

The third terms on the right hand side represent the power transferred from stator circuit across the air-gap by transformer action and the fourth terms on the right hand side are the electromechanical power output.

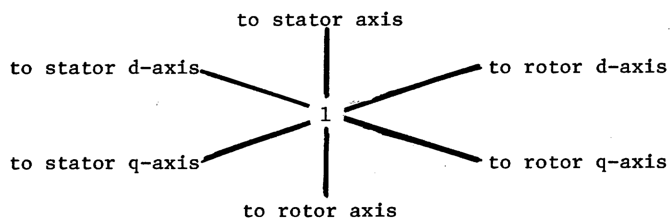
Finally, the rotor d-axis power flow bond graph is:



The bond graph of the rotor q-axis power flow is:



The magnetic circuit equations are the flux linkage equations which can be represented by



The swing equation power flow is:

$$\omega_r J\frac{d\omega_r}{dt} = \omega_r T_m + \omega_r T_e$$
$$= \omega_r T_m + \omega_r(\lambda_Q i_D - \lambda_D i_Q)$$
$$= \omega_r T_m + \omega_r \lambda_D i_Q - \lambda_r \lambda_Q i_D$$

ω_r = rotor angular velocity

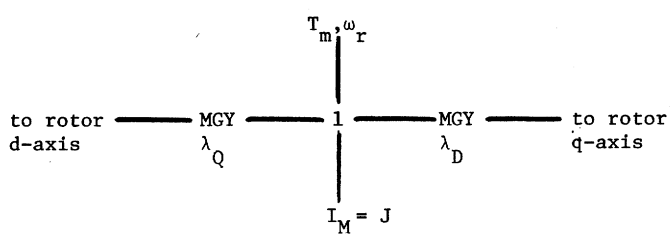
$\omega_r J\frac{d\omega_r}{dt}$ = power stored in inertia of the machine

$\omega_r T_m$ = mechanical input of the machine

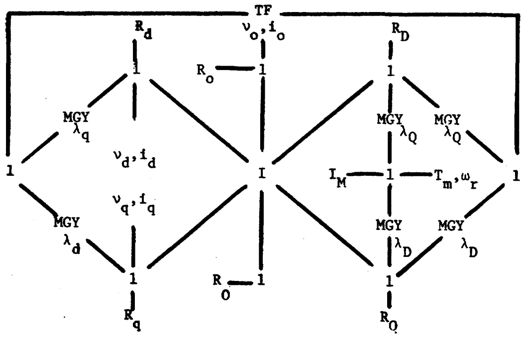
$\omega_r \lambda_Q i_D$ = electromechanical power in rotor d-axis

$\omega_r \lambda_D i_Q$ = electromechanical power in rotor q-axis

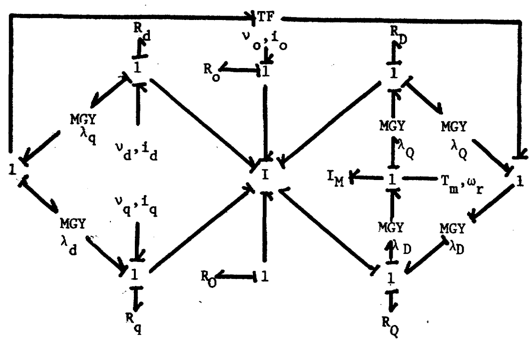
The bond graph of the swing equation is:



The bond graph of the ideal induction machine thus becomes:



By using the ENPORT IV program, the power flow and causalities are identified as:

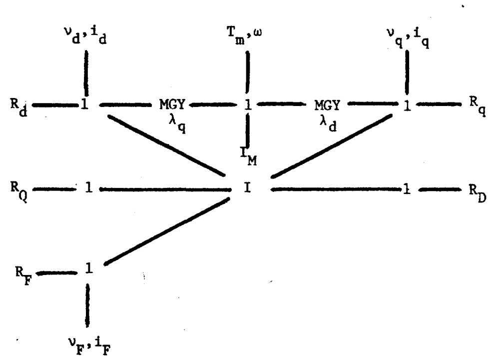


C. Remarks Concerning ENPORT IV and V Digital Programs

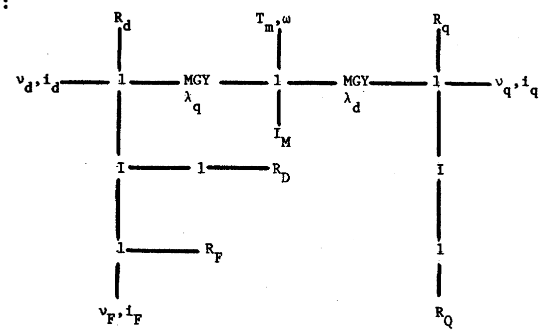
The ENPORT IV or V programs do not have the capability of solving the non-linear bond graph. Because of the MGY elements, the bond graph model of the synchronous and induction machine are nonlinear. Even though this program cannot compute the dynamic solutions of these models, it can however, be used to find power flow and causalities of the models as has been previously indicated. Since these models are formed from accurate mathematical equations representing the ideal synchronous and induction machines, there is a very high degree of confidence that these models will generate representative characteristics of the machines if these models were run on a non-linear digital computer program.

The bond graph model of the synchronous machine can be simplified by assuming balanced conditions, and equal phase resistance in each stator circuit. If the three phase circuit is balanced, there is no current or voltage in the zero sequence and the o-axis circuit can then be deleted from the model.

This leads to a simplified bond graph of the synchronous machine:

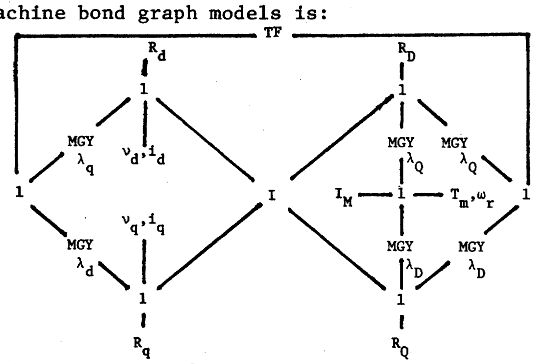


This model can be compared to the bond graph model of the synchronous machine by D. Sahn [7] which is shown below:

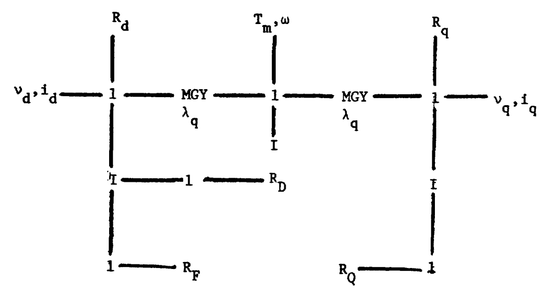


If the magnetic circuit is separated in the simplified model, both models are exactly the same.

The induction machine model can also be simplified by assuming balanced conditions and the modified induction machine bond graph models is:



The bond graph model of the induction machine given by D. Sahn [7] is



They are not alike. The model from [7] does not include the relationship to rotor speed and also does not include transformer action. Both of these characteristics are usually presented in the conventional model of the induction machine, otherwise they would be equivalent.

Unfortunately, these models developed cannot be used to simulate the machine dynamics at this time because the bond graph digital computer programs available are unable to solve the non-linear bond graph models. The authors were recently informed by Prof. R. C. Rosenberg of Michigan State University that the program named THTSIM is a bond graph digital program which has been recently formulated to solve the non-linear bond graph models. This program has been written by Prof. J. J. van Dixhoorn of the Department of Electrical Engineering, Enschede, Netherlands for a PDP system (LSI-11 under RSX 11M or Rt-11) which must be translated to run on an IBM system. With this program, a system utilizing the non-linear characteristics of these machines can be studied easily by using the bond graph technique of modeling.

CONCLUSIONS

The basic bond graph concepts have been introduced considering the concept of power flow. The real advantage in using this technique is (1) mixed energy systems can be bonded together without regard to the form of the energy (2) the procedure provides a powerful tool in developing a systematic approach to obtaining the state equations for dynamic systems (3) the technique predicts problems in developing the system dynamics before equations are written through causality considerations and (4) it provides a very pictoral view of the interactions of the various components.

The odq power invariant transformations have been used to obtain the bond graphs for the synchronous and induction machine. Through the use of the ENPORT program causality and power flow directions have been obtained.

The usefulness of the bond graph formulation for linear and non-linear systems and the applications of the ENPORT digital program to linear graphs for obtaining causality and dynamic response is very appealing. Future applications using the THTSIM program or equivalent programs, which are being developed with increasing interest, will remove the non-linear constraint from the bond graph approach and open up many new applications of the procedure.

LIST OF SYMBOLS

o = stator o-axis winding
d = stator d-axis winding
q = stator q-axis winding
O = rotor o-axis winding
D = rotor d-axis winding
Q = rotor q-axis winding
F = rotor field winding
i = instantaneous current
ν = instantaneous voltage
λ = instantaneous flux linkage
L = inductance
r = resistance
R = total resistance
ω = synchronous speed
J = angular momentum
T_m = mechanical torque
T_e = electrical torque
r_s = stator neutral resistance
ℓ_s = stator neutral inductance
r_r = rotor neutral resistance
ℓ_r = rotor neutral inductance
ω_r = instantaneous rotor angular velocity
s = slip

REFERENCES

[1] Paynter, H. M., "Analysis and Design of Engineering Systems", MIT Press, 1961.

[2] Takahashi, Y., Rabins, M. J. and Auslander, D.M., "Control and Dynamic Systems", Addison-Wesley Publishing Company, second edition, 1972.

[3] Rosenberg, R. C., "A User's Guide to ENPORT IV", A Wiley-Interscience Publication, 1972.

[4] Fitzgerald, A. E. and Kingsley, C., "Electric Machinery", McGraw-Hill Book Company, Inc., second edition, 1961.

[5] Anderson, P. M., "Power System Control and Stability", The Iowa State University Press, Ames, Iowa, 1977.

[6] Ridzuan, M. and Laughton, M.A., "Dynamic Analysis of Field Energy Systems Using Bond Graphs", Int. J. Control, Vol. 21, No. 1, pp. 21 - 38, Jan., 1975.

[7] Sahn, D., "A Two-Axis, Bond Graph Model of the Dynamics of Synchronous Electrical Machines", J. Franklin Inst., Vol. 308, No. 3, pp. 205 - 218, Sept. 1979.

[8] Karnopp, D.C., "Power-Conserving Transformations: Physical Interpretations and Applications Using Bond Graphs", J. Franklin Inst., Vol. 288, No. 3, pp. 175 - 201, Sept., 1969.

[9] Brown, F. T., "Direct Application of the Loop Rule to Bond Graphs", Journal of Dynamic Systems, Measurement, and Control, Sept., 1972.

[10] Martens, H. R., "Simulation of Nonlinear Multiport Systems Using Bond Graphs", Journal of Dynamic Systems, Measurement, and Control, Mar., 1973.

[11] Karnopp, D. C. and Rosenberg, R. C., "Guest Editorial", Journal of Dynamic Systems, Measurement, and Control, Sept., 1972.

[12] Martens, H. R. and Bell, A. C., "A Logical Procedure for the Construction of Bond Graphs in Systems Modeling", Journal of Dynamic Systems, Measurement, and Control, Sept., 1972.

[13] Rosenberg, R. C., "State-Space Formulation for Bond Graph Models of Multiport Systems", Journal of Dynamic Systems, Measurement, and Control, Mar., 1971.

[14] Oster, G. F., and Auslander, D. M., "The Memristor: A New Bond Graph Element", Journal of Dynamic Systems, Measurement, and Control, Sept. 1972.

[15] Rosenberg, R. C., "Multiport Models in Mechanics", Transactions of the ASME, Sept., 1972.

[16] Brown, F. T., "Lagrangian Bond Graphs", Journal of Dynamic Systems, Measurement, and Control, Sept., 1972.

[17] Rosenberg, R. C. and Karnopp, D. C., "A Definition of the Bond Graph Language", Journal of Dynamic Systems, Measurement, and Control, Sept., 1972.

[18] R. H. Park, "Two-Reduction Theory of Synchronous Machines, Generalized Method of Analysis", AIEE Trans., Vol. 48, pp. 716 - 727, July, 1929.

[19] A.E. Fitzgerald, C. Kingsley, Electric Machinery, McGraw-Hill, 1961.

APPENDIX

BASIC BOND GRAPH CONCEPTS

A bond graph is a type of linear graph which gives a clear display of the relationship between the variables of a dynamic system. In bond graph notation [2], [3], there are nine basic multiport elements and four power-energy variables which are necessary in describing them. The multiport elements are considered to be nodes of the graph and the bonds the branches. These generalized four variables are:

a. Effort e(t)

Examples : voltage, force, torque, pressure, temperature

b. Flow f(t)

Examples : current, velocity, angular velocity, volume flow, heat flow

c. Momentum p(t) (time integral of effort)

Examples : flux linkage, linear momentum, angular momentum, pressure momentum

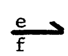
d. Displacement q(t) (time integral of flow)

Examples : charge, displacement, rotation, volume, heat energy

The bond representing a pair of effort and flow variables gives a product which is scalar power.

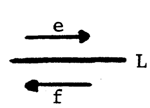
$$P(t) = e(t)f(t) \quad (1)$$

This bond may be represented as follows:



The half-arrow describes the direction for assumed positive power flow. Each bond may be marked by a short single vertical stroke which establishes a sense of input-output to each of the elements in the system, and defines the concept of causality.

This concept is important. For example: In the case of an inductance (element), if we attempt to introduce a step change in current (flow) an impulse in voltage (effort) appears, which is unnatural. Therefore a more appropriate input to the inductance is a voltage which preserves energy continuity. This is indicated in causality notation on a bond graph for element L as:



or using the shortstroke notation

The nine interdisciplinary basic multiport elements without causality indicated are as follows:

1. The inertance (storage element) represented as:

$$\overset{e\ \rightarrow}{\underset{\leftarrow\ f}{\rightharpoonup}}\quad L$$

is defined by:

$$e(t) = \frac{1}{L}\,Df(t) \tag{2}$$

2. The capacitance (storage element) represented as:

$$\overset{\leftarrow\ e}{\underset{f\ \rightarrow}{\rightharpoonup}}\quad C$$

is defined by:

$$f(t) = CDe(t) \tag{3}$$

3. The resistance (dissipation element) represented as

$$\overset{e\ \rightarrow}{\underset{\leftarrow\ f}{\rightharpoonup}}\quad R \qquad \text{or} \qquad \overset{\leftarrow\ e}{\underset{f\ \rightarrow}{\rightharpoonup}}\quad R$$

is defined by:

$$e(t) = Rf(t) \tag{4}$$

4. The effort source, represented as $SE \overset{e}{\rightharpoonup}$ is defined by:

$$e(t) = SE(t) \tag{5}$$

5. The flow source, represented as $SF \overset{f}{\rightharpoonup}$, is defined by:

$$f(t) = SF(t) \tag{6}$$

6. The transformer (2-port) represented as:

$$\overset{e_1}{\underset{f_1}{\rightharpoonup}}\ TF\ \overset{e_2}{\underset{f_2}{\rightharpoonup}}$$

is defined by:

$$\begin{bmatrix} e_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} e_2 \\ f_2 \end{bmatrix} \tag{7}$$

7. The gyrator (2-port) represented as:

$$\overset{e_1}{\underset{f_1}{\rightharpoonup}}\ GY\ \overset{e_2}{\underset{f_2}{\rightharpoonup}}$$

is defined by:

$$\begin{bmatrix} e_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 0 & m \\ \frac{1}{m} & 0 \end{bmatrix} \begin{bmatrix} e_2 \\ f_2 \end{bmatrix} \tag{8}$$

8. The common flow junction (a 3-port indicated) is sometimes called a one junction and is represented as:

$$\begin{tikzcd} {} \arrow[r, harpoon, "e_1", "f_1"'] & 1 \arrow[r, harpoon, "e_3", "f_3"'] & {} \\ & {} \arrow[u, harpoon, "e_2", "f_2"'] & \end{tikzcd} \tag{9}$$

where:

$$e_1 + e_2 = e_3$$

$$f_1 = f_2 = f_3$$

NOTE: In an electrical circuit this would appear as a series connection.

9. The common effort junction (a 3-port shown) is sometimes called a zero junction and is represented as:

$$\begin{tikzcd} {} \arrow[r, harpoon, "e_1", "f_1"'] & 0 \arrow[r, harpoon, "e_3", "f_3"'] & {} \\ & {} \arrow[u, harpoon, "e_2", "f_2"'] & \end{tikzcd}$$

where by definition:

$$e_1 = e_2 = e_3$$

$$f_1 + f_2 = f_3 \tag{10}$$

NOTE: In an electrical circuit this would be a parallel connection.

Both the zero and one junction are power conservative, i.e., the net power at the junction is zero and is taken into account by directionality of the power half arrows.

Special elements useful to the development of the synchronous and induction machine are:

A. Modulated Gyrator (MGY)

A gyrator is a 2 port element. Mathematical equations of the gyrator are:

$$\begin{bmatrix} e_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 0 & r \\ \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} e_2 \\ f_2 \end{bmatrix}$$

where

e = effort

f = flow

r = gyrator modulus

The bond graph notation of the gyrator is:

$$\overset{e_1}{\underset{f_1}{\leftharpoondown}}\ \underset{r}{GY}\ \overset{e_2}{\underset{f_2}{\leftharpoondown}}$$

The modulated gyrator is a gyrator where its modulus is a function of a variable in the model, $g(x)$, instead of a constant.

The bond graph notation of the modulated gyrator is given as:

$$\overset{e_1}{\underset{f_1}{\rightharpoonup}}\ \underset{g(x)}{MGY}\ \overset{e_2}{\underset{f_2}{\rightharpoonup}}$$

B. Multiport Inertance

One port inertance (I_1) can be represented by:

$$p_1(t) = p_1(t_0) + \int_{t_0}^{t} e_1(s)ds \quad , \quad s = \text{a dummy variable}$$

$$f_1 = g(p_1)$$

The bond graph notation of one port inertance is:

$$\xrightarrow[f_1]{e_1} I$$

Multiport inertance has mathematical representation as:

$$f_i = g_i(p_1, p_2, p_3 \ldots p_n), \; i = 1,\ldots,n$$
$$p_i(t) = p_i(t_0) + \int_{t_0}^{t} e_i(s)ds, \; i = 1,\ldots,n$$

The bond graph notation is: