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Two-Dimensional Scattering From A Medium Of Finite Thickness

Dwight C. Look
Missouri University of Science and Technology, look@mst.edu

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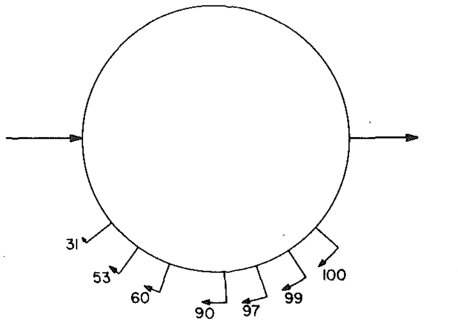

Fig. 3 Time percentages indicating when backflow prevails along the wall of the chamber, Re ~ 40,000 and ϕ = 0 deg

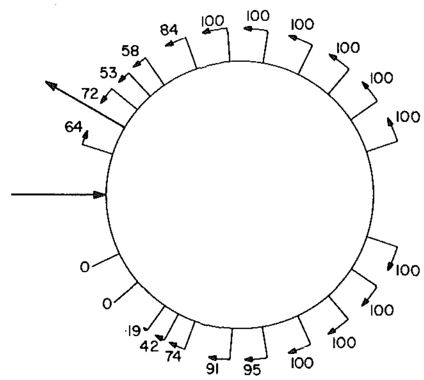

Fig. 4 Time percentages indicating when backflow prevails along the wall of the chamber, Re ~ 40,000 and ϕ = 150 deg

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Two-Dimensional Scattering from a Medium of Finite Thickness[1]

D. C. Look[2]

Nomenclature

C_{sca} = scattering cross section
c = scattering cross section divided by the effective particle volume
I_{exp} = experimental intensity leaving the medium normal to the irradiated surface
I_i = incident flux at $r = 0$
L = depth of distilled water (medium) in tank
N = particle number density
r = radial distance from center of incident (laser) beam
r_0 = effective radius of incident (laser) beam
V_p = volume of paint
V_{sp} = effective volume of a scatterer
V_w = volume of water
ξ = volume of latex paint per unit volume of water plus latex paint
ξ^1 = volume of scattering centers in the paint per unit volume of water plus latex paint
τ_0 = optical depth (= $NC_{sca}L$)
τ_r = radial optical coordinate (= $NC_{sca}r$)
τ_{r_0} = effective optical radius of the incident laser beam (= $NC_{sca}r_0$)

Introduction

The basic assumption of most reported experimental investigations of two-dimensional scattering is that the scattering medium is one of essentially infinite optical depth (e.g., [1, 2]). A few studies have included the finite optical depth as an indirect parameter [3–5].

The objectives of this note are to present (1) the effects of a finite depth on the power escaping normal to the irradiated surface of a scattering volume with a flat black bottom and (2) correct a previously published definition.

Investigation

Experiment. The apparatus used in this experimental investigation has been discussed in detail in [2, 6], and [7]. In the works previously reported, the depth was chosen so that the power received as a result of reflection from the bottom was negligible (an approximate model of a semi-infinite volume). A depth of approximately 21.6 cm was quite adequate when the substrate was black and diffuse [7]. In the case of the work reported here, the depth was decreased. In all instances, the medium was distilled and filtered water with latex paint serving as the scattering centers. The working procedure of this investigation was exactly like that of [2, 6], and [7].

Theory. The optical coordinate, τ, was presented in [2] as

$$\tau = \xi c x.$$

The word definitions of ξ and c are, however, incompatible; thus a slight revision is required. To understand this required revision notice that

$$\begin{aligned}\tau &\equiv \sigma x \\ &\equiv NC_{sca}x \\ &= NV_{sp}\left(\frac{C_{sca}}{V_{sp}}\right)x \\ &= NV_{sp}cx \\ &= \frac{n}{V_p + V_w}V_{sp}cx \\ &= \frac{nV_{sp}}{V_p + V_w}cx \\ &= \xi^1 cx\end{aligned}$$

Thus, in order for c to be consistent with its word definition, ξ^1 must be used instead of ξ to evaluate the optical coordinate. Notice that the c is unaffected by this correction.

This inadvertant confusion of the definitions resulted in an error in the optical coordinates τ_r, τ_0 and τ_{r_0} of [2]. By using this consistent pair, ξ^1C, only a slight correction of the resultants of [2] is required.

In the illustrations that follow, theoretical curves are designated by the solid lines. In Figs. 1 and 2, the theory responsible has been presented in [2]. The theory of Fig. 3 will be published in the open literature later by different authors [8] and is presented here only as an aid for comparison. This upcoming publication will present the back scattered intensity from a finite two-dimensional medium exposed to a Gaussian beam of radiation. The parameters of importance will be the optical thicknesses (used here) and the single scattering

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[2] Professor, Thermal Radiative Transfer Group, Mechanical and Aerospace Engineering, University of Missouri—Rolla, Rolla, MO 65401. Mem. ASME.

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albedo (ω of approximately unity used here). The other assumptions of the theoretical point of view are that the medium is infinite in the radial direction, homogeneous, non-emitting and scatters isotropically while the bottom surface is black.

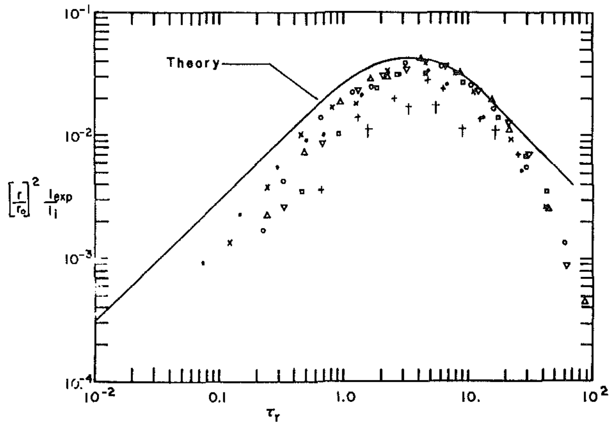

Fig. 1 The weighted ratio of the normal emergent intensity to the flux incident normally upon the scattering volume $(r/r_0)^2(I_{exp}/I_i)$ versus τ_r for a 10.8 cm deep medium with a flat black, diffuse substrate: ● $r/r_0 = 15$, x $r/r_0 = 25$, ○ $r/r_0 = 35$, △ $r/r_0 = 50$, ▽ $r/r_0 = 70$, □ $r/r_0 = 100$, + $r/r_0 = 140$, and † $r/r_0 = 180$

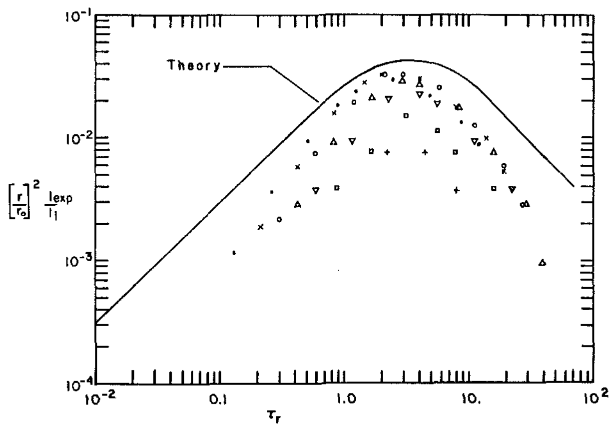

Fig. 2 The weighted ratio of the normal emergent intensity received to the flux incident normally upon the scattering volume $(r/r_0)^2(I_{exp}/I_i)$ versus τ_r for a 5.4 cm deep medium with a flat black diffuse substrate (Same legend as Fig. 1 except no $r/r_0 = 180$)

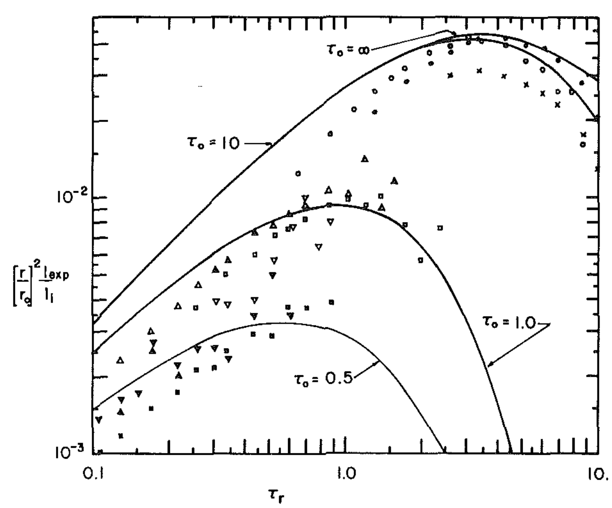

Fig. 3 The weighted ratio of normal emergent intensity received to the flux incident normally upon the scattering volume $(r/r)^2(I_{exp}/I_i)$ versus τ_r for a flat black, diffuse substrate: $\tau_0 = 9.4$ and depths of ● (21.6 cm), ○ (10.8 cm), and x (5.4 cm); $\tau_0 = 0.94$ and depths of ▽ (21.6 cm), △ (10.8 cm), and □ (5.4 cm); and $\tau_0 = 0.47$ and depths of ▼ (21.6 cm), ▲ (10.8 cm) and ■ (5.4 cm)

Results

The results of this investigation are presented in Figs. 1–3. Figs 1 and 2 present $(r/r_0)^2$ (I_{exp}/I_i) versus τ_r with the radial coordinate of the detector as the parameter. The solid curves (theory) come directly from Fig. 8 of [2]. These are typical of the data presented in [2, 6], and [7], with the slight exceptions that the finite depth effects are noticeable. The theory curves of these two figures are for an infinite depth (i.e., $\tau_0 \rightarrow \infty$) and are presented here to emphasize the effect of a finite depth. Thus, as for Fig. 1, the water depth of 10.8 cm provides a medium whose scattering characteristics are noticeably different than that of a semi-infinite medium. The peak and general shape of the experimental points are similar to those of the theoretical curve. As the depth of the scattering medium decreases (Fig. 2) the spread of the experimental data points depart significantly from the semi-infinite case (theory). Neither shape or peak position may be pin-pointed. Thus, the received energy is uncharacteristic of the semi-infinite theory.

Fig. 3 illustrates the experimental data of Figs. 1 and 2, but it is regrouped according to the parameter $\tau_0(=NC_{sca}L)$. The theoretical curves are developed for a model more characteristic of this experimental situation [9]. The theoretical curves and the experimental data are in general agreement. That is, though dispersed, the data points follow the general increasing-to-a-peak-and-decreasing characteristic of the theory curves.

Conclusion

An investigation of the effects of finite depth on the two-dimensional scattering from a planar medium with a highly absorbing (flat black) bottom has been carried out. Data representative of the power emerging normally from the top surface of the scattering medium are included. Ordinary flat white latex paint was used as the source of scattering centers in a medium of distilled water. The agreement between theory and experiment using only one empirically determined coefficient (c) is at least fair. That is, the results of the experimental data and the values from the theory are in fair agreement in magnitude, shape, and position. Careful measurement of the volume of scattering centers has been used to correct the optical dimension in the presentation of data.

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