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Low Order Correction for Subsurface Contributions in Diffusion Experiments

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Penetration plots for the determination of radio tracer diffusion coefficients can be obtained in two ways; either the activity of sections or the residual activity of the diffusion couple can be determined as a function of the sectioning depth. In both cases corrections for absorption of tracer radiation may have to be made. Such corrections are required more often for the residual activity technique and this letter addresses itself to these. A criterion will be given which permits one to decide if the data analysis can be kept simple or if more involved procedures /1/ are required. In developing the criterion it will be assumed that the counting geometry is kept constant.

Consider the residual activity in a diffusion couple sectioned to the penetration depth x,

$$I(x) = \int_x^{\infty} c(y, t) \exp\left[-\mu(y - x)\right] dy \ .$$

Here, c(y, t) gives the tracer distribution in the couple and μ is the linear attenuation coefficient of the tracer radiation. For a thin, planar source the former is given by /2/

$$c(y, t) = q(\pi Dt)^{-1/2} \exp(-y^2/4Dt) \ .$$

After introducing the abbreviation $Q = q(\pi Dt)^{-1/2}$ and the complementary error function, the activity becomes

$$I(x) = Q \exp(\mu^2 Dt + \mu) \operatorname{erfc}\left[x(4Dt)^{-1/2} + \mu (Dt)^{1/2}\right] . \qquad (1)$$

This expression is suitable for an iterative linearized regression analysis /3/ of the data. In some cases it can be simplified, however. If the asymptotic series expansion

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Table 1

sectioning technique	thinnest feasible section (cm)	desirable $\sqrt{Dt}$ (cm)	tracer isotope	$\mu\sqrt{Dt}$ in Fe /5 to 7/	validity of first order approximation (3)
anodic oxidation	$2x10^{-8}$	$1x10^{-6}$	T^1	$1.40x10^{-1}$	no
			Zr^{93}	$4.17x10^{-2}$	no
			C^{14}	$7.68x10^{-3}$	no
			Y^{90}	$1.12x10^{-4}$	no
mechanical lathing	$4x10^{-5}$	$2x10^{-3}$	T^1	$2.80x10^{+2}$	yes
			Zr^{93}	$8.33x10^{+1}$	yes
			C^{14}	$1.53x10^{+1}$	yes
			Y^{90}	$2.25x10^{-1}$	no

of the complementary error function /4/,

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}} \exp(-z^2) \sum_{m=0}^{\infty} \frac{(-1)^m (2m)!}{m!(2z)^{2m+1}} \qquad (2)$$

is used, (1) becomes

$$I(x) = Q \exp(-x^2/4Dt) \sum_{m=0}^{\infty} \frac{(-1)^m (2m)!}{m!(2r)^{2m+1}}, \qquad r = x(4Dt)^{-1/2} + \mu(Dt)^{1/2} .$$

This series approximation is better than 1% for $r > 2$ and for $r > 5$ only its zeroth term need be considered. An accuracy of 1% is assured for the whole penetration plot if $\mu(Dt)^{1/2} > 5$. It thus follows that

$$I(x) = \frac{1}{2} Q \exp(-x^2/Dt) \left[x(4Dt)^{-1/2} + \mu(Dt)^{1/2} \right]^{-1}, \qquad \text{all } x,\ \mu(Dt)^{1/2} > 5 . \qquad (3)$$

This expression reduces to a simple Gaussian in the limit of large attenuations, i.e., in cases where $\mu(Dt)^{1/2} \gg x(4Dt)^{-1/2}$.

A few combinations of penetration depths, x, and linear attenuation coefficients, μ, as they might be encountered in a diffusion experiment are listed in Table 1. The experimental condition $x \sim (4Dt)^{1/2}$ has been incorporated. It can be seen from this Table 1 that (3) is valid in the case of very low energy beta emitters. These isotopes include the important carbon-14 tracer, thus rendering long range carbon interstitial diffusion experiments in transition elements susceptible to a simplified analysis.

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