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# Disk And Strip Forging With Side Surface Foldower: Part 2: Evaluation Of The Upper-bound Solutions

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# Disk and Strip Forging with Side Surface Foldover

## Part 2: Evaluation of the Upper-Bound Solutions

*The upper-bound solutions developed in Part 1 are evaluated with regard to their ability to produce a lower value for required power (load, pressure, or work). Comparisons made with existing solutions such as the triangular field solution and one-zone bulge solution show that for strip, each solution has a domain of geometry and friction in which it is superior. The new solution produces a lower upper-bound for conditions of high interface friction and relatively thin specimen, the area where foldover is the observed mode of flow. For solid cylindrical disks, the solution fails to improve upon existing analyses, but comes sufficiently close to warrant additional study.*

*After evaluation, these solutions were then used in an incremental technique to model the geometry and flow as a function of reduction in height. Results appear most encouraging, and the relative simplicity of the technique when compared with present alternatives is quite attractive.*

### The Limit Analysis Approach

The limit analysis approach has proved to be a powerful tool for the study of metal forming problems. Two approximate solutions are developed: one, an upper-bound and the other, a lower-bound. The exact forming pressure is never greater than the pressure predicted by the upper bound, which requires flow to be described by a kinematically admissible velocity field, and never less than that predicted by the lower-bound, which requires a statically admissible stress field. Thus, the exact pressure is located between these limiting values, which converge toward the exact solution as the velocity field and the stress field respectively become more realistic.

Since the stress-based lower-bound solution is usually considerably more difficult to develop, the upper-bound technique has become the more practical approach for theoretical analysis of metal flow. The method considers an admissible velocity field that satisfies the incompressibility, continuity, and velocity boundary conditions. From it the internal deformation, shear, and friction powers are computed to give the total power required for forming and the forming load or pressure. If properly developed, these computed powers and loads will be greater than or equal to the actual requirements.

A spectrum of solutions is often available, with varying degree of complexity dependent upon proposed modes of flow, etc. Sometimes the velocity field contains one or more parameters which are determined by minimizing the total forming power with respect to those parameters. Thus, a general-type solution can be utilized to give the best upper-bound solution for any given set of conditions.

Several observations can usually be made:

1 With increasing number of parameters or increased velocity field complexity, the solution tends to improve at the expense of more involved mathematical computations.

2 Upper-bound estimates tend to become lower as the proposed velocity field becomes a closer model of reality.

3 No one solution will emerge as best for all conditions of a given process. Solutions should compliment each other with the solution that provides the lowest prediction of power being regarded as the best for that given case of geometry and friction.

For a more complete presentation of the state of the art in the application of limit analysis to metal forming processes see reference [1].[1]

### Evaluation of Upper-Bound Solutions for Strip Forging

Part 1 of this publication presents a solution for strip forging which incorporates the phenomena of side-surface bulging and foldover of side-surface into contact with the platens. In order to assess the merits of this solution, it should be evaluated with respect to other analyses

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[1] Numbers in brackets designate References at end of paper.

regarding its ability to produce a lower prediction of required power for given conditions of the process.

**The Parallel Field.** The simplest upper-bound analysis for strip forging is probably the parallel velocity field, which can be found in Section 13.2.1 of reference [1]. As the platens force a decrease in height, the specimen compensates with an increase in width with no bulging or barreling of the free surface. This solution, however, is incorporated as a limiting case of the bulge velocity field and as such, will not be treated as a separate identity.

**The Bulge Field.** Section 13.2.2 of reference [2] presents a velocity field which incorporates an exponential cusp-shaped bulging of variable magnitude. Mathematically, the velocity components can be represented as a function of position by:

$$\dot{U}_x = \frac{b}{1-e^{-b}}\dot{U}\frac{x}{t}e^{-by/t}$$
$$\dot{U}_y = \frac{-1}{1-e^{-b}}\dot{U}(1-e^{-by/t}) \quad (1)$$
$$\dot{U}_z = 0$$

The parameter, $b$, is a variable parameter which may assume any positive number. That value of b which provides the lowest required power is the optimum value of the bulge parameter and serves to estimate the expected degree of bulge. If $b = 0$, the bulge velocity field reduces to the parallel field solution.

With the assumption of a constant interfacial friction, represented by a constant shear friction factor, m, the bulge velocity field predicts a relative average forging pressure of:

$$\frac{p_{\text{ave}}}{\sigma_o} = \frac{1}{\sqrt{3}}\left\{\sqrt{1+\frac{b^2}{4D^2}} + \frac{2D}{b} \times \ell n\left[\frac{b}{2D} + \sqrt{1+\frac{b^2}{4D^2}}\right] + \frac{mbe^{-b}}{2(1-e^{-b})D}\right\} \quad (2)$$

The relative average forging pressure is a function of three variables: the strip dimension ratio ($D = t/w$), the platen-strip friction factor ($m$), and the degree of bulging ($b$). Of these, the first two are usually preset to describe the condition of the forging operation. The third variable, $b$, assumes that value which minimizes the pressure predicted by equation (2).

Expressions have been developed to compute $b_{\text{opt}}$ as a function of the other process variables, such as:

$$b_{\text{opt}} \approx \frac{3}{1+\left(\frac{2}{m}\right)\left(\frac{1}{D}\right)} \quad (3)$$

The above expression is approximate in nature, is convenient in reducing the mathematical difficulty in using the results, and retains the upper bound property of equation (2). However, approximating the value of $b_{\text{opt}}$ by use of equation (3) increases the value of $P_{\text{ave}/\sigma_o}$ as computed by equation (2). When used for the purposes of comparison with other solutions, equation (2) is used directly with the value of $b$ being obtained by optimization and not through an approximation technique.

**The Triangular Field.** The basic triangular velocity field is a simple approach wherein the strip is divided into four triangular rigid bodies which shear over one another along surfaces of velocity discontinuity (Fig. 1). No plastic deformation occurs, so no internal power of deformation is computed. Since the triangles which contact the platens undergo no relative motion with respect to the platen surfaces, friction losses are zero. All required power is associated with shear along the diagonal surfaces of velocity discontinuity, with the field being applied to describe the conditions at a given instant of deformation. The relative average forging pressure for such a field becomes:

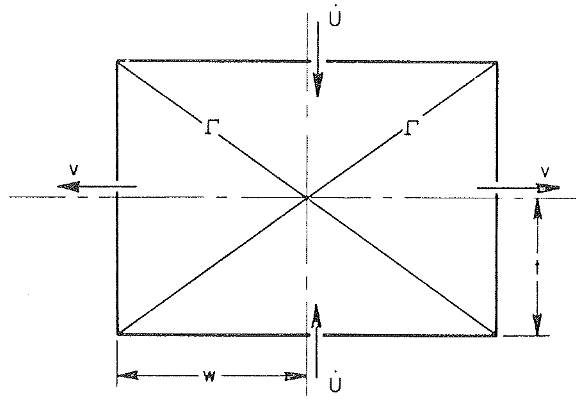


**Fig. 1 The triangular field**

$$\frac{p_{\text{ave}}}{\sigma_0} = \frac{1}{\sqrt{3}}\left(D + \frac{1}{D}\right) \quad (4)$$

As deformation produces a wider strip, where width is significantly greater than height, improved solutions can be obtained by including additional triangular elements on either side of the original as in Fig. 2. Additional elements are always added in pairs such that the total number of units is $n + 1$ where $n$ is an even positive integer. The width of the central region is arbitrary, with all additional units being uniform in size. Minimization of the resulting solution establishes the "best" values for element size. Section 13.2.3 of reference [2] develops this solution to produce a relative average forging pressure expression of:

$$\frac{p_{\text{ave}}}{\sigma_0} = \frac{D}{\sqrt{3}}\left\langle 1 + n + \frac{1}{D^2}\left[\left(\frac{w_s}{w}\right)^2 + \frac{1}{n}\left(1-\frac{w_s}{w}\right)^2\right]\right\rangle + \frac{m}{2\sqrt{3}D}\left[1-\left(\frac{w_s}{w}\right)^2\right] \quad (5)$$

where:

$w_s$ = half width of central region

$$\frac{w_s}{w} = \frac{1}{1+n-mn/2}$$

## Nomenclature

$b$ = bulge parameter
$b_{\text{opt}}$ = value of $b$ which produces minimum required power
$D$ = dimension ratio ($t/w$ or $T/R_0$)
$m$ = constant shear stress friction factor
$n$ = number of additional triangular units
$p_{\text{ave}}$ = average forging pressure
$p_{\text{ave}}/\sigma_o$ = relative average forging pressure
$R, \theta, y$ = cylindrical coordinate system
$S$ = zone size parameter ($\xi/w$ or $\xi/Ro$)
$S_{\text{opt}}$ = value of $S$ which produces minimum required power
$t$ = half thickness of the strip
$\dot{U}$ = velocity vector
$\dot{U}_i$ = component of a vector
$w$ = half width of the strip
$w_s$ = half width of central region of multiple triangle field
$x, y, z$ = Cartesian coordinate system
$\Delta y$ = increment of deformation
$\xi$ = depth of penetration of Zone II along the strip or disk centerline
$\sigma_o$ = flow stress in a uniaxial tensile test of a round rod
$\Gamma$ = surface of velocity discontinuity

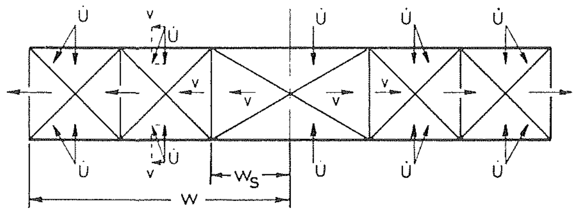


**Fig. 2 Multiregion triangular field**

Since the additional elements now undergo relative sliding with respect to the forging surfaces, friction becomes an integral part of the above expression.

**Comparison of Strip Solutions.** One way[2] to evaluate the merits of the two-zone solution developed in Part 1 of this presentation is to compare the results to those of the alternative solutions. Providing all solutions being compared are rigorous upper-bounds and use the same friction representation, that solution which produces the lowest value for total required power or relative average forging pressure should be regarded as being superior.

To effect such a comparison, tables such as Table 1 were prepared to present the optimized single zone bulge solution of equation (2), the simple triangular field solution of equation (4), the optimized multiple triangle solution of equation (5), and the new two-zone solution of Part 1. With friction held constant, the thickness-to-width ratio, $D$, was varied from 1.0 to 0.1 in increments of 0.1 and from 0.10 to 0.01 in increments of 0.01. Friction was then changed and additional tables prepared for the different values. Fig. 3 graphically depicts the regions of superiority of the various solutions.

The single triangle solution dominates the region of thick strip, namely from thickness equal to width to thickness of roughly ½ the width of the strip. Since the solution assumes complete sticking on the platens, other solutions are more able to compete at lower friction conditions and the range of domination of this solution will diminish with a decrease in friction.

[2] Simplicity of the derivation, convenience for the user and other factors may also serve as criteria. Also, verification of the results as a true prediction of the phenomena studied (fold over in present study) should be a major criteria.

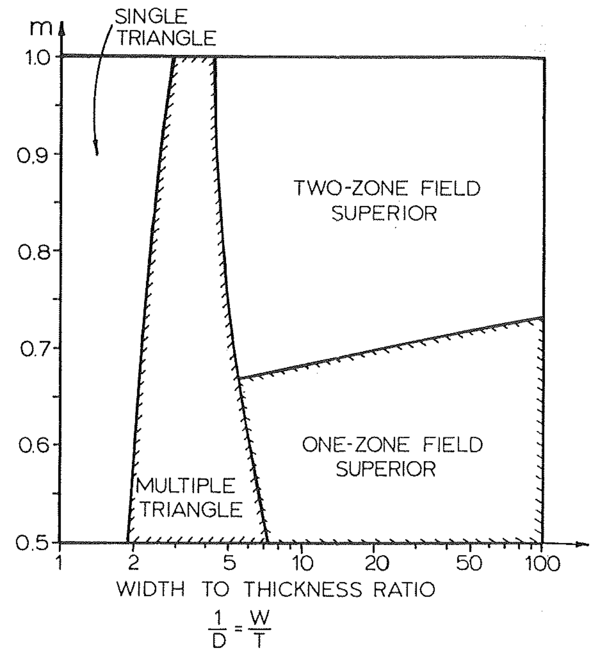


**Fig. 3 Comparison at strip solutions**

The multiple triangle solution becomes superior as the strip becomes thinner and occupies a region adjacent to that of the single triangle. Superior solutions usually occur for low values of $n$, either $n = 2$ or $n = 4$. For thinner strip, where higher values of n produce the best solution for the multiple triangle approach, other non-triangular solutions offer an advantage.

When thickness becomes substantially smaller than width, the one-zone and two-zone bulge solutions compete for superiority, with the two-zone solution dominating for high friction values. This is expected, for the foldover phenomenon incorporated in this solution is mainly observed under conditions of thin specimen and high friction.

One benefit of the upper-bound approach can be clearly seen from the above description. The occurrence of phenomena in metal deformation processing can be "predicted" by simply determining the conditions under which a solution incorporating the given phenomenon produces the lowest predicted power or pressure. Cracking or rupture, for example, could be incorporated in a solution for strip forging and might possibly produce a solution that would occupy a region in Fig. 3. In cases of complex deformation or where numerous variables are present, the predictive features of the approach are a considerable asset. When coupled with a companion lower-bound solution as given in Chapter 13 of reference [2] an estimate of possible error can also be obtained. For a wide range of parameters the present upper bound solution is very close to the lower bound solution. Any improvement is measured in only a few percentage points. Regions of superiority may well be reversed by changes in secondary factors such as strain and strain rate effects, temperature gradients, and others, as suggested in reference [1].

**Table 1**
**Comparison of Solutions—Strip**
**Relative Average Forging Pressure ($P_{ave}/\sigma_o$)**
**$m = 1.0$**
**$D$ = variable**

| | Relative Average Forging Pressure Predicted | | | |
|---|---|---|---|---|
| $D = t/w$ | Single Zone Bulge | Simple Triangular | Multiple Triangle | Two-Zone Bulge & Fold |
| 1.0 | — | 1.1547 | 2.1651 $n = 2$ | — |
| .9 | 1.3982 | 1.1611 | 2.0400 $n = 2$ | 1.4050 |
| .8 | 1.4354 | 1.1836 | 1.9269 $n = 2$ | 1.4217 |
| .7 | 1.4838 | 1.2289 | 1.8310 $n = 2$ | 1.4536 |
| .6 | 1.5490 | 1.3087 | 1.7609 $n = 2$ | 1.5067 |
| .5 | 1.6413 | 1.4434 | 1.7321 $n = 2$ | 1.5922 |
| .4 | 1.7814 | 1.6743 | 1.7754 $n = 2$ | 1.7312 |
| .3 | 2.0171 | 2.0977 | 1.9629 $n = 2$ | 1.9722 |
| .2 | 2.4928 | 3.0022 | 2.5018 $n = 4$ | 2.4589 |
| .1 | 3.9299 | 5.8312 | 3.9837 $n = 8$ | 3.9119 |
| .09 | 4.2499 | 6.4670 | 4.3136 $n = 10$ | 4.2337 |
| .08 | 4.6502 | 7.2630 | 4.7179 $n = 10$ | 4.6358 |
| .07 | 5.1650 | 8.2883 | 5.2384 $n = 12$ | 5.1523 |
| .06 | 5.8516 | 9.6571 | 5.9322 $n = 14$ | 5.8408 |
| .05 | 6.8131 | 11.5759 | 6.8993 $n = 18$ | 6.8041 |
| .04 | 8.2557 | — | — | 8.2486 |
| .03 | 10.6606 | — | — | 10.6552 |
| .02 | 15.4711 | — | — | 15.4675 |
| .01 | 29.9041 | — | — | 29.9023 |

## Evaluation of the Disk Forging Solution

The two-zone disk forging solution developed in Part 1 was also evaluated with regard to its ability to provide a lower prediction for required power. As a first stage of evaluation, the new solution was compared to the single-zone bulge solution upon which it was based. This solution is developed and presented in Section 7.5 of reference [2] and predicts a relative average forging pressure of:

**Table 2**
**Comparison of Solutions—Disk**
**Relative Average Forging Pressure ($P_{ave}/\sigma_o$)**
**$m$ = 1.0**
**$D$ = variable**

| $D = t/w$ | Relative Average Forging Pressure Predicted: Single Zone Bulge Solution | Relative Average Forging Pressure Predicted: Two-Zone Bulge & Fold |
|---|---|---|
| .9 | 1.3423 | 1.4608 |
| .8 | 1.3930 | 1.5020 |
| .7 | 1.4589 | 1.5542 |
| .6 | 1.5474 | 1.6324 |
| .5 | 1.6722 | 1.7464 |
| .4 | 1.8609 | 1.9234 |
| .3 | 2.1776 | 2.2264 |
| .2 | 2.8146 | 2.8473 |
| .1 | 4.7340 | 4.7502 |
| .09 | 5.1612 | 5.1757 |
| .08 | 5.6952 | 5.7081 |
| .07 | 6.3820 | 6.3933 |
| .06 | 7.2979 | 7.3075 |
| .05 | 8.5803 | 8.5883 |
| .04 | 10.5042 | 10.5107 |
| .03 | 13.7112 | 13.7160 |
| .02 | 20.1256 | 20.1288 |
| .01 | 39.3700 | 39.3716 |

$$\frac{p_{ave}}{\sigma_o} = \frac{8b}{D}\left\{\left[\frac{1}{12} + \frac{D^2}{b^2}\right]^{3/2} - \frac{D^3}{b^3} - \frac{m}{24\sqrt{3}}\frac{1}{1 - e^{-b/2}}\right\} \quad (6)$$

As with the strip solution, approximations have been developed to determine the value of $b$ which optimizes the solution, $b_{opt}$. Once again, these were not utilized so that a true, uncompromised, evaluation of the solutions could be obtained. Integral terms in the two-zone solution were solved by numerical integration techniques rather than use the approximated expressions to again achieve a better comparison of the solutions.

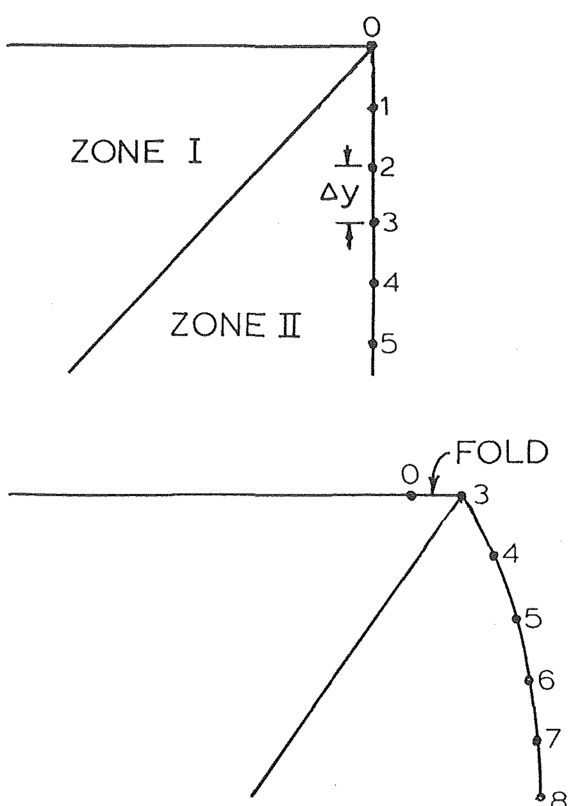


**Fig. 4 Deformation model**

Results obtained by evaluating both solutions for a range of geometries and friction values revealed that the two-zone solution was nowhere lower than its one-zone counterpart. Table 2 shows the comparison of solutions for $m$ = 1.0.

Understanding the success of the two-zone strip solution but apparent failure of the parallel disk solution requires an analysis of component terms. Comparison of like power terms reveals that the internal power of deformation increases for the two-zone solution due largely to the added shear over the surface of velocity discontinuity separating the zones. Friction losses decrease, however, as foldover reduces the extent of shearing over the contact surfaces. For those conditions where the decrease dominates the increase, the two-zone approach will produce a superior solution.

For the strip geometry, the ratio of platen-workpiece surface area experiencing a drop in friction losses to the added interzone shear surface is such as to substantially favor the two-zone solution for high values of interface friction. For the disk geometry, however, the interzone surface of velocity discontinuity now wraps around the entire circumference of the specimen and the additional power required dominates the reduced friction losses. The upper-bound technique requires all interior surfaces to be characterized by maximum shear resistance, and this may be a substantial overestimate of reality. Should a more accurate representation of the resistance to shear become acceptable to upper-bound analysis, the result may well be a superior two-zone solution. The proximity of the two current solutions for thin disk geometries is sufficient to suggest that the two-zone approach may well merit additional study. As an isolated solution, the present two-zone velocity field does offer a means to predict the occurrence of the foldover phenomenon and estimate its severity with ease and a high degree of reliability as sill be seen in the next section.

## Application of Solutions to Modeling Deformation

The successful development of the two-zone solutions and subsequent optimization of relative average forging pressure has provided the optimum values of both $b$ (the bulge parameter) and $S$ (the zone size parameter) for a variety of friction conditions and process geometries. Since these values are all that is required to completely characterize the velocity of all points within the deforming body at a given instant, it now becomes possible to: increment the downward motion of the platen or press surface, apply the instantaneous velocity field for the given increment, and model the entire deformation sequence. Moreover, since the analysis has been designed to permit foldover, the development of this phenomenon with deformation can be modeled.

As a preliminary investigation of modeling capabilities, the strip solution was selected and the two-zone solution was assumed to be appropriate for all geometries encountered during the deformation. Data for the optimum values of $b$ and $S$ (the minimization parameters in the solution) were curve fit by polynomial regression techniques for rapid evaluation in the incremental modelling. As a further simplification, only the paths taken by a select family of points were computed, namely, the original corner of the strip and the points on the free surface. The height of the original free surface was divided into 1000 increments such that ten units of incrementation would be required for each percent reduction in height.

Fig. 4 shows the initial distribution of reference points, with Point 0 being associated with Zone I. After three deformation increments, the platen has begun to overtake an outwardly bulged surface and produces a foldover equivalent to the separation of points 0 and 3.

Fig. 5 presents a typical deformation sequence for a strip of original thickness equal to ½ the width and maximum interfacial friction ($m$ = 1.0). The small arrow on the top surface indicates the location of the original corner and serves to determine the relative amounts of top surface spread and side surface foldover. The plot presents the profile for the upper-right quadrant of the strip and appears to predict a double-bulging of the side-surface. Although observed in some actual forgings, little significance has been attributed to its occurrence in the model, being most likely an outcome of the selected velocity field.

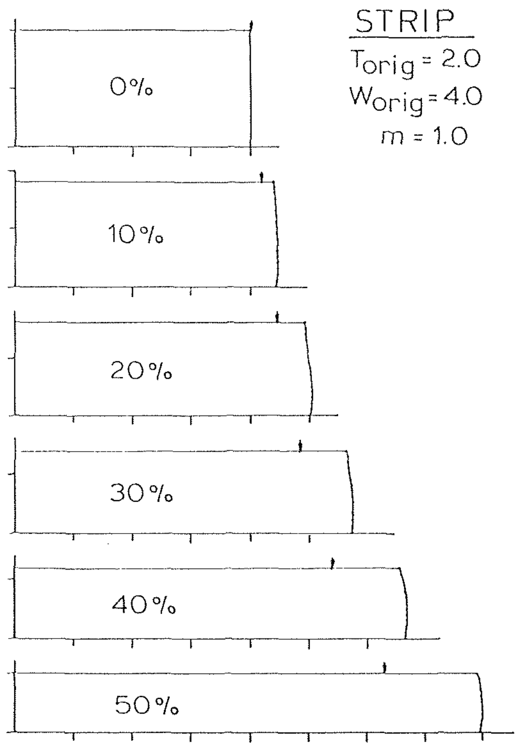


Fig. 5 Deformation sequence for strip

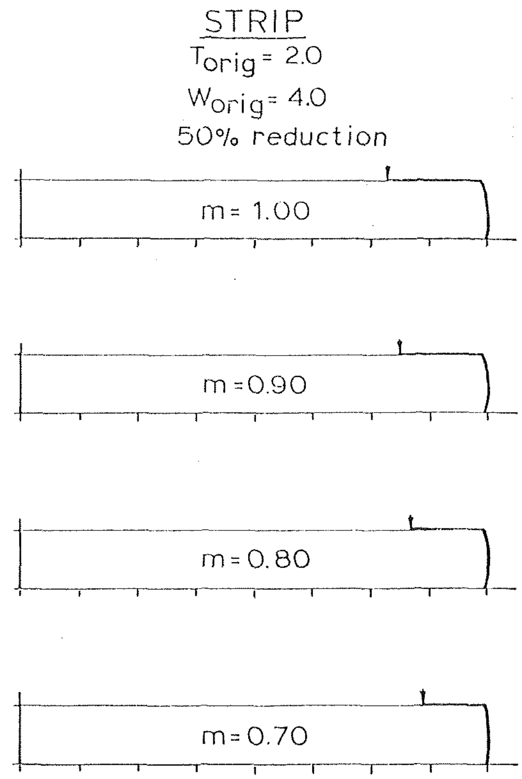


Fig. 6 Effect of the friction factor

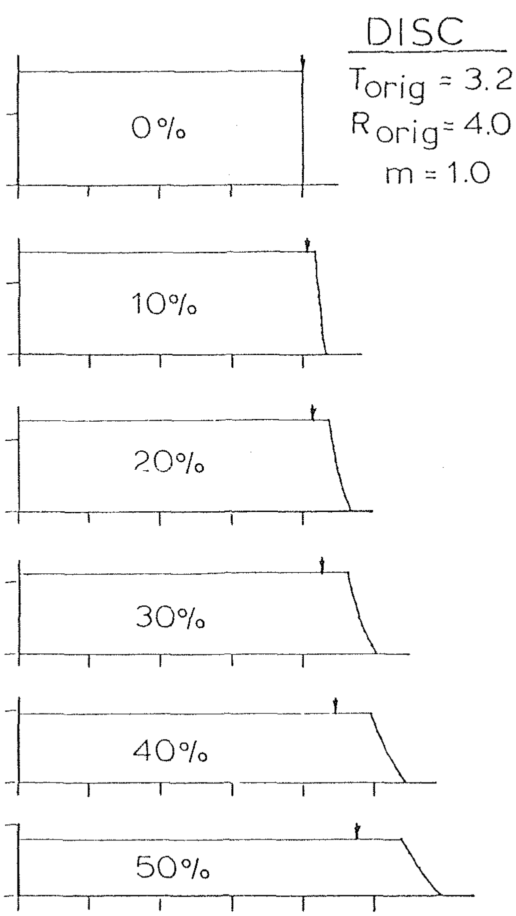


Fig. 7 Deformation sequence for disk

Fig. 6 illustrates the effect of a variation in friction by comparing identical 50 percent reductions for friction factors from $m = 1.0$ to $m = 0.7$. While the overall geometry shows little variation, the relative contributions of the original top surface and folded side surface differ noticeably. Increased foldover is produced by the higher interface friction. Although still in a rather primitive state, development has already begun on the application of this phenomenon to the measure of high interfacial frictions during forging (references [3] and [4]).

Previous effort to analytically model bulge and foldover in plane strain strip forging (references [5] and [6]) were based on the slip line analytical technique. Extensive computer utilization was required, however, because the slip line net must be totally recomputed for each increment of deformation. The upper-bound approach provides a single set of optimized equations which describe the motion of all points within the body and requires little time to compute an increment. Both expense and complexity can be reduced.

A similar modeling technique was applied to the disk solution to demonstrate its feasibility. Fig. 7 illustrates a typical result, with the arrow again indicating the position of the original corner. Alternative techniques for modeling foldover in the disk geometry (references [7–9]) rely primarily on the finite element technique which again suffers from a high degree of mathematical complexity and extensive utilization of computer techniques. The current upper-bound approach to modeling disk deformation is only an initial step toward a workable solution, but if improved, has the promise of providing an attractive alternative to current options.

## Conclusions

A new capability has been added to the upper-bound technique of analysis. The two-zone velocity field for strip forging appears to be an adequate upper-bound solution for describing the combined phenomena of bulge and fold and provides a superior solution for the expected conditions of geometry and friction. Application of the solution to a modeling technique using incremental deformations provides an acceptable model of the forging process. Such modeling has previously been accomplished only by far more complex and costly techniques.

The parallel analysis for the disk geometry, while not producing a lower solution, does provide a means to predict foldover and a starting point for additional refinements and improvements.


## Acknowledgments

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