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Finite-Element Analysis of Pressure Vessels

The finite-element technique has been applied in the analysis of a variety of pressure vessel problems. The example problems described in this paper suggest that the finite-element method is perhaps the most suitable means currently available for obtaining quick and accurate solutions for real-life pressure vessel problems. Finite-element programs can be used by the practicing engineer. Companion programs are available that can be used to check the input data and graphically display both the input and output data.

Introduction

The "finite-element" method has recently evolved into a most powerful tool for performing both stress and heat conduction analyses of complex engineering structures. The ability of the finite-element method to include, in the study of real engineering problems, the effects of (*a*) nonisotropic and non-homogeneous material properties; (*b*) plastic deformation; (*c*) nonisotropic subsequent yield surfaces; (*d*) arbitrary thermal and mechanical loads; and (*e*) large deformations, allows the analyst to draw closer to the actual solution. However, even though the analyst may have a confident knowledge of the complete analytical stress state within a structure, it is not completely known if these stresses correspond to a safe design or not. The inability to relate a general 3-dimensional stress state to a safe-design criteria is largely due to the lack of sufficient material test data. That is, most material testing is performed under uniaxial, or, at best, biaxial stress states, and the results are extrapolated to more general stress states.

There are numerous reports [1–6][1] that illustrate the effect of the stress state on material behavior. Thus it is imperative that material response be known for any combination of stresses, and this information must be acquired from proper experimentation and not from an extension of a criteria that is based on uniaxial or biaxial test data.

Unfortunately, experimental tests in which a triaxial (all three normal stress components can vary independently) stress state can be developed usually do not have geometrical and loading configurations such that a direct analytical method can be used for determining the existing stresses. With the finite-element technique, specimens of variable geometry (for example, specimens containing grooves, notches [7], or other discontinuities), and subjected to combined loads and boundary conditions, can be analyzed and used in conjunction with the experimental observations to determine actual material response under a controllable stress state.

The overall project at the University of Missouri–Rolla has concerned itself with: (*a*) the collection of yield and fracture test data acquired under combined stress states (stress state determined with finite-element program); (*b*) the construction of stress dependent yield and fracture models [6] from the aforementioned test data; (*c*) the employment of heat conduction and stress type finite-element computer programs to evaluate temperature distributions and stress states in actual pressure vessels; and (*d*) the utilization of the results of steps (*b*) and (*c*) for evaluating the structural integrity of a given pressure vessel. This procedure has been successfully applied in the analysis of a variety of pressure vessels which are described herein. Since the development of yield and fracture models for specific materials has been reported in reference [6], this paper will concentrate on describing the function and applicability of the finite-element technique for analyzing complex pressure vessels.

Finite-Element Technique

In reviewing the possible methods that could be applied to the analysis of pressure vessels, the advantages of the finite-element method, as compared to other numerical methods, appear to be numerous. For example, the finite-element method is completely general with respect to geometry and material properties, and either stress or displacement boundary conditions can be specified at any point within the system. In addition, arbitrary thermal and mechanical loads can be applied without increasing the complexity of the problem.

The term finite element indicates the type of idealization which is used to reduce the continuous structure to a system of discrete bodies. In the finite-element approximation of axisymmetric solids, the continuous structure is replaced by a system of axisymmetric elements, which are interconnected at circumferential joints or nodal circles. In stress analysis problems the displacements of these nodal points are the basic unknown parameters. The finite-element programs used in this project

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[1] Numbers in brackets designate References at end of paper.

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utilize a linear displacement function which uniquely defines the state of displacement within each element in terms of its nodal point displacements. The strain-displacement relations and Hooke's law (for an elastic material) can be used with the displacements to obtain the element strains and stresses. By writing the total potential energy of the structure in terms of the nodal point displacements, and applying the principal of stationary potential energy, a system of equations is defined that can be solved for the unknown nodal point displacements. With these values, the element displacements, strains, and stresses are readily calculated.

If any element in the structure is stressed beyond the elastic limit, there are two approaches that can be followed to determine the resulting plastic stress state. In the first approach (deformation theory of plasticity) the computer program uses the computed strain state and the plastic portion of the material stress-strain curve to determine a new modulus of elasticity. With this information, the stiffness matrix used in determining the nodal point displacements is reformulated to obtain a second stress state. This procedure is repeated until the final computed stress state in an element is located on the material stress-strain curve. The second method (incremental theory of plasticity) involves the use of the method of successive elastic solutions as described by Mendelson in reference [8]. The incremental theory provides a more realistic assessment of the plastic stress state; however, this is accomplished at the expense of increased computer time. The reason for the increased computing time is a result of having to increase the applied loads (after reaching yield) in an incremental fashion, and iterate at each step of the load to determine the appropriate plastic strain increments.

Finite-element stress programs have been written at the University of Missouri–Rolla that are capable of handling plane and axisymmetric problems with temperature-dependent orthotropic material properties. The STRATA I program utilizes the deformation theory for obtaining plasticity solutions whereas the STRATA II program is based on the incremental theory of plasticity. The example plastic solutions presented in this paper have been obtained by using the deformation theory.

The reader that is unfamiliar with the finite-element technique may consult texts by Rubinstein [9] or Zienkiewicz [10] for a more complete discussion.

Example Problems

In order to illustrate the effectiveness of the finite-element method for evaluating stress states in pressure vessels, a summary of the results obtained from the analysis of several types of pressure vessels is presented. These example problems will demonstrate the use of the finite-element programs for treating time-dependent boundary conditions, heat conduction, nonisotropic and nonhomogeneous material properties, elastoplastic stress states, etc.

1 Thermoelastic Analysis of a Skirted Pressure Vessel. The term skirted vessel indicates that the vessel proper (usually a cylindrical vessel with domed-shaped heads) is supported in a vertical position by attaching a thin-walled cylinder (skirt) to the lower-head of the vessel. The longitudinal axis of the skirt coincides with the same axis of the vessel, and the skirt attachment is obtained via a specificially contoured circumferential weld. A typical skirted vessel is shown in Fig. 1.

Experience in the construction and use of skirted vessels has shown that failures can occur at the skirt-vessel weld junction. These failures are not totally unexpected in that an improper design of the skirt-vessel weld junction can generate stress concentrations which, when superimposed on the existing tensile biaxial normal stress state in the pressurized vessel head, would cause a premature vessel failure.

The skirt-vessel weld junction for the first weld design was simulated with the finite-element grid shown in Fig. 2. The temperature distribution within the vessel structure was determined with a finite-element heat-conduction program. (The grid used in the thermal analysis was the same as that shown in Fig. 2 except that the insulation was also included.) All of the input data to the finite-element programs is checked by a special check program and the resulting grid and boundary conditions are plotted on a Calcomp plotter. This procedure assures the designer that the structure to be analyzed by the computer is correct.

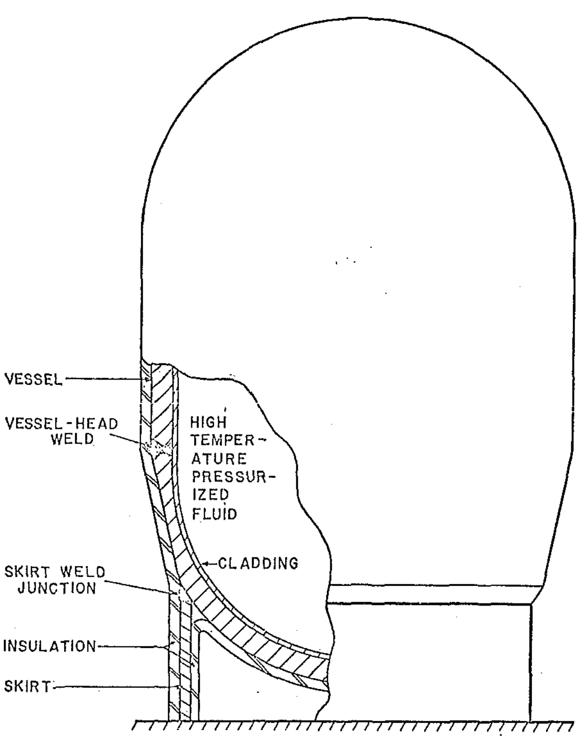

Fig. 1 Skirted vessel

The output from the STRATA program gives the nodal point displacements and element strains and stresses. In order to expedite the review of the element stresses, the output from the STRATA program is usually presented in the form of punched cards which can be fed directly into a stress region plot program. Since we use the mean stress and effective stress (these parameters correspond to the first and second invariants of stress tensor, respectively) to construct a model [6] for illustrating material behavior under arbitrary stress states, we usually construct effective stress and mean stress region plots such as the ones shown in Figs. 3 and 4. The region plots are constructed by first drawing the boundary of the structure and then placing a letter symbol at the centroid of each element which identifies the stress intensity within a certain range. A dividing line is drawn around all neighboring elements that are labeled with the same letter symbol. This technique enables the analyst to make a rapid comparison between the skirt weld design shown in Fig. 3 and the new weld design shown in Fig. 5. The final step in the design approval procedure is to plot, on the appropriate material response model [6], the effective stress—mean stress values obtained at the critically stressed points in the structure—and determine if these conditions correspond to an elastic state, plastic state, or fracture.

2 Elastic-Plastic Analysis of a Sectionally Pressurized Thick-Walled Cylinder. The overpressurization or autofrettaging of a thick-walled cylinder represents a situation where an elastoplastic analysis must be performed. If the cylinder is internally pressurized along its entire longitudinal length, the location of the elastic-plastic interface and the resulting stresses in both regions

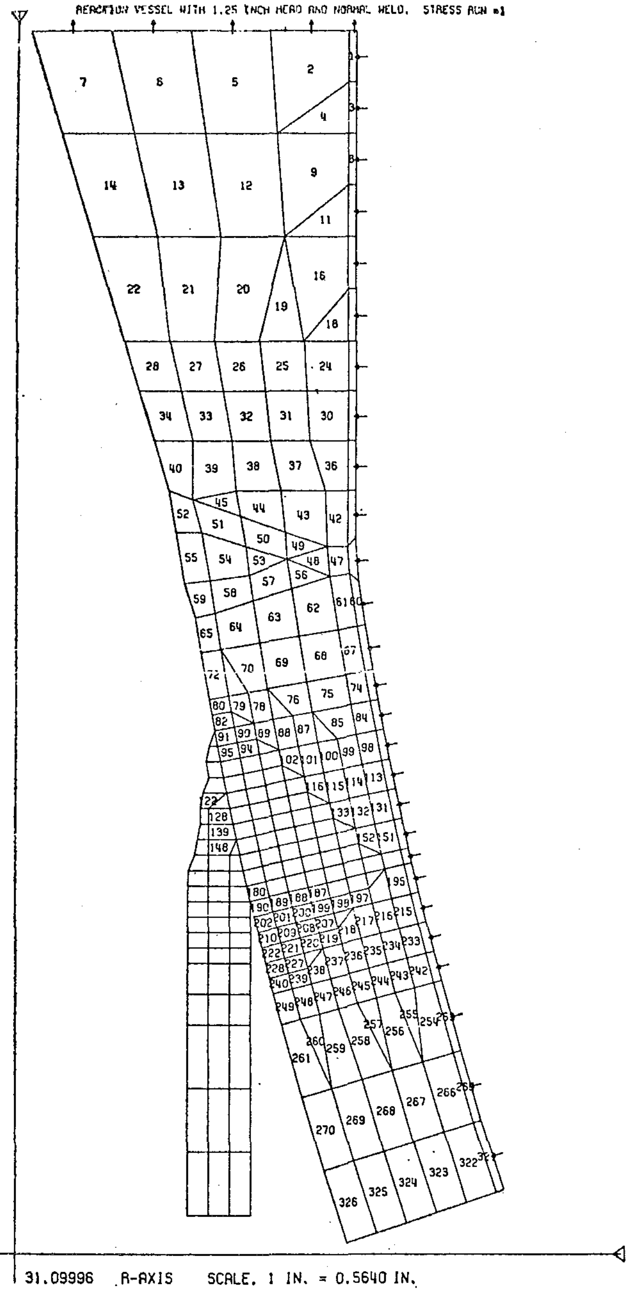

Fig. 2 Finite-element grid for skirted vessel weld junction

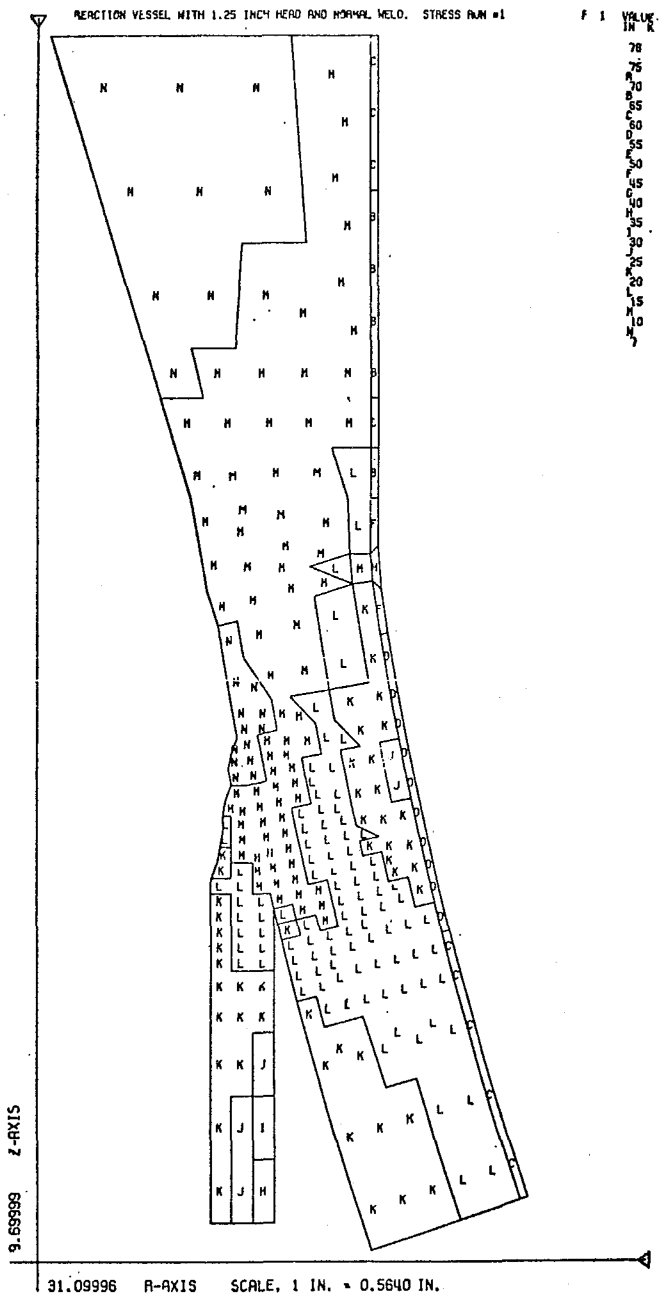

Fig. 3 Effective stress region plot

can be determined analytically as shown by Hill in reference [11]. If the region of pressurization does not extend over the full cylinder length, the analytical approach becomes exceedingly difficult whereas the finite-element technique is completely general in this respect.

Fig. 6 shows the grid used in the analysis of a thick-walled cylinder that has an autofrettage pressure acting over the central half. The input data are checked and the finite-element grid plotted as explained in the previous problem. In this problem, certain element stresses exceeded the cylinder material yield strength thus indicating that a plastic analysis must be performed. One simple instruction directs the STRATA I program to perform a plastic analysis in accordance with the deformation theory of plasticity. The location of the resulting elastic-plastic interface is shown in Fig. 6. Stress region plots similar to those shown in Figs. 3, 4, and 5 can also be constructed by the plotter for this problem.

Fatigue Analysis of Cladded Multilayer Pressure Vessels

The repeated cycling of high-temperature high-pressure process fluids through a cladded multilayer vessel can give rise to repeated stress conditions that can initiate fatigue cracks. The vessel shown in Fig. 7 is subjected to a temperature-pressure loading condition that varies in a sinusoidal fashion with time. The heat conduction and stress programs were applied at several different times to determine the stress and strain variations occurring during each complete cycle. This information is then used with fatigue data to predict the life expectancy. A partial assessment of the accuracy of this analysis is provided in Fig. 8 where the output from the finite-element heat-conduction program is compared with the response of a thermocouple which was attached to the vessel head.

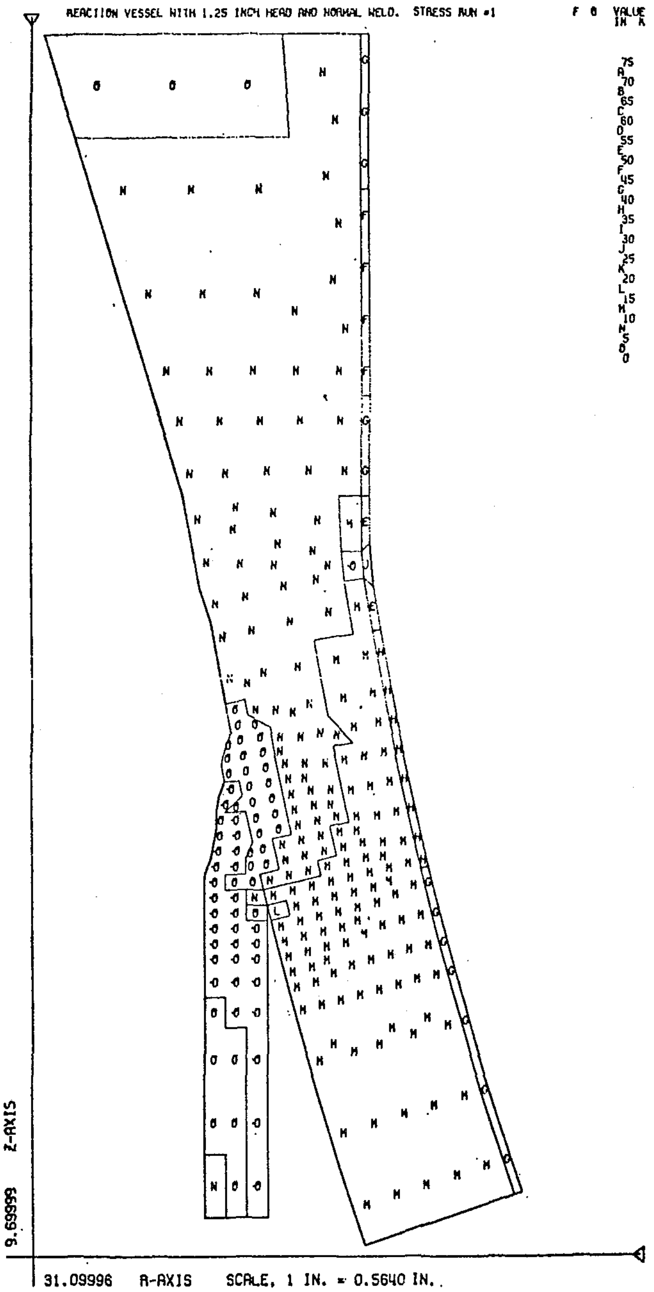

Fig. 4 Mean stress region plot

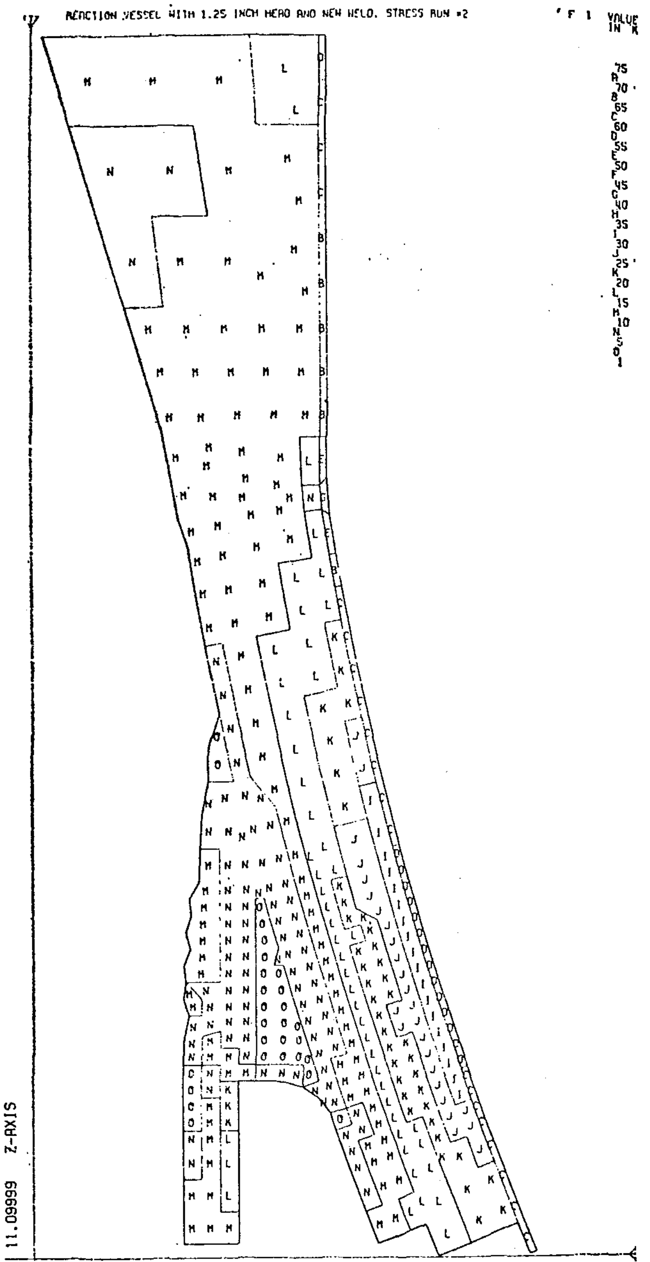

Fig. 5 Effective stress region plot for a new skirt-weld design

Other Applications

A variety of other pressure vessel problems have been analyzed by the finite-element technique with considerable success. Among these are the analysis of: Circumferential Welds in Multilayer Pressure Vessels [12], High-Pressure Heat Exchangers, Dual Chamber Pressure Vessel for Testing Deep Submersible Vehicles, and others. In each of these cases a solution by conventional techniques would not have been possible without considerable simplification.

Summary and Conclusions

The application of finite-element heat-conduction and stress analysis programs to the analysis of heated and pressurized nonhomogeneous pressure vessels has been shown to provide useful data. The ease in which input data to the finite-element programs can be prepared, checked, and graphically displayed, and the manner in which the stress output is presented in region plots, makes the finite-element technique a most powerful tool for the analysis of pressure vessels. There are many package finite-element programs available for purchase which would allow the design engineer to utilize them in the analysis of practical problems without having to have a full understanding of the theory behind such a program.

Acknowledgment

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References

1 Pugh, H. Ll. D., "Hydrostatic Extrusion," Ninth Commonwealth Mining and Metallurgical Congress, Paper No. 18, 1969.
2 Bobrowsky, A., "Description of Some Methods for Deforma-

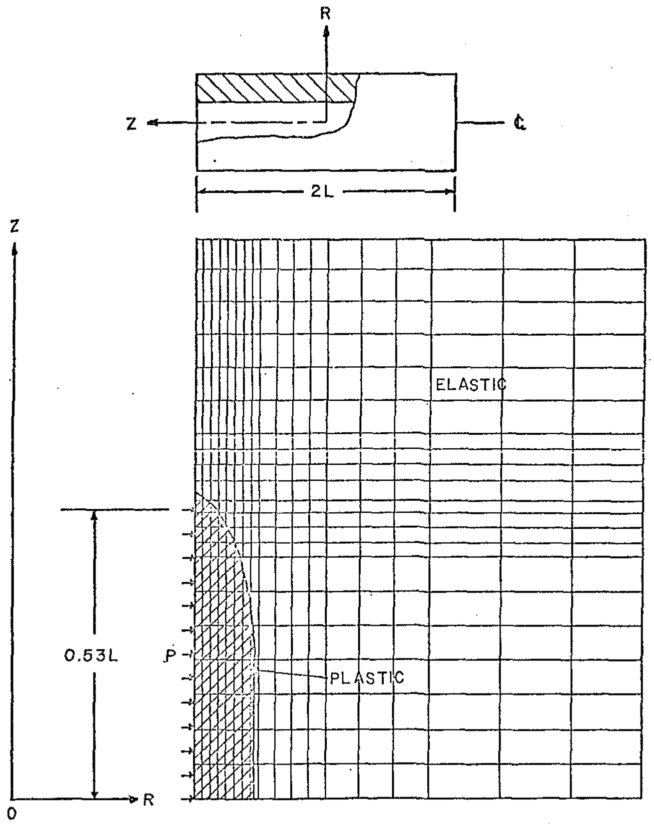

Fig. 6 Sectionally pressurized thick-walled cylinder

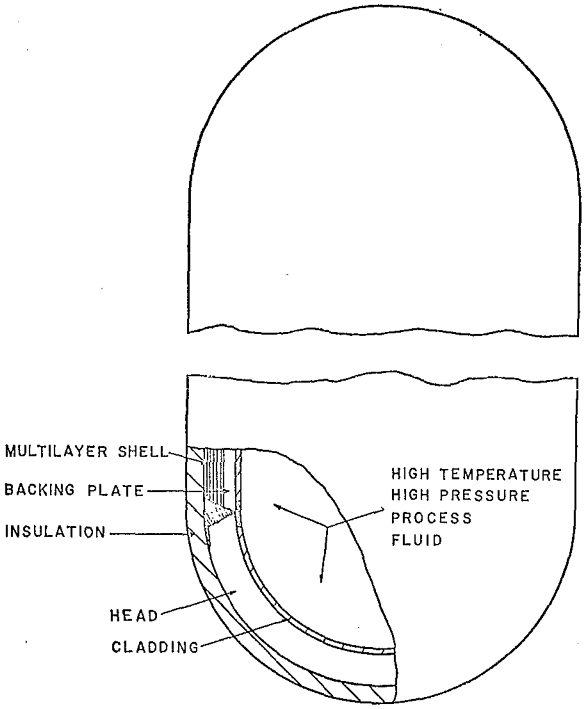

Fig. 7 Cladded multilayer pressure layer

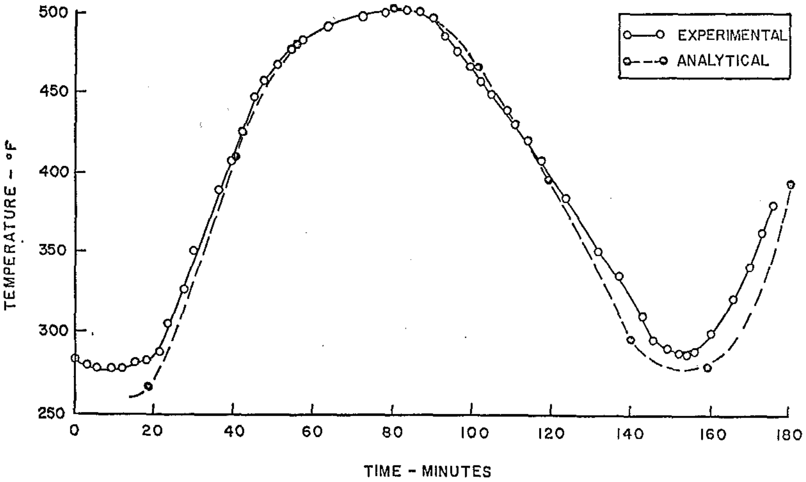

Fig. 8 Temperature response at vessel head thermocouple station

tion of Solids Under Hydrostatic Pressure," ASME Paper No. 64-WA/PT-29.

3 Coffin, L. F., Jr., "The Flow and Fracture of a Brittle Material," *Journal of Applied Mechanics*, Vol. 17, No. 3, TRANS. ASME, Vol. 72, Sept. 1970, pp. 233–248.

4 Stassi-D'asis, F., "A Limiting Condition of Yielding and Its Experimental Confirmation," *Industria Grafica Nazionale*, Palermo, Italy.

5 Bobrowsky, A., "Formability Criteria for Metals—Russian and American," American Society of Toll and Manufacturing Engineers, Paper No. 792.

6 Davis, R. L., "A Macroscopic Model of Material Behavior Under Variable Pressures," *International Colloquium on the Physics of Solids Under Pressure*, Grenoble, France, 1969.

7 Keith, H. D., and Davis, R. L., "Effects of Elevated Pressure and Temperature on Notched Tensile Specimens," SESA Paper No. 1735, 1970.

8 Mendelson, A., *Plasticity: Theory and Application*, Chapter 9, Macmillan Co., New York, 1968.

9 Rubinstein, M., *Structural Systems—Statics, Dynamics, and Stability*, Prentice Hall, Englewood Cliffs, N. J., 1970.

10 Zienkiewicz, O., *The Finite Element Method in Structural and Continuum Mechanics*, McGraw-Hill, New York, 1968.

11 Hill, R., *The Mathematical Theory of Plasticity*, Oxford Press.

12 Davis, R. L., "Circumferential Welds in Multilayer Pressure Vessels," ASME Paper No. 70-WA/PVP-6.