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Effects of Cold Working Under Pressure on Subsequent Yield

A method utilizing high pressure fluid environments is described whereby a three-dimensional subsequent yield surface was determined for 304 stainless steel. Cylindrical parent specimens of this material were prestrained in axial compression under fluid pressure and then small subspecimens were sectioned from these parent specimens. Finite element techniques were used to optimize the parent specimen size so that a zone of uniform axial stress would result during the prestraining. Longitudinal strains in this zone were monitored during the prestraining and the subspecimens were cut from this region in a manner that did not allow the machining to appreciably affect the properties of the specimens. Following this, conventional tension and compression tests were performed on the subspecimens in various fluid pressure environments to determine the yield strengths for the cold-worked material in the direction of the principal axis of prestrain and the two transverse axes. These data are used to construct the three-dimensional subsequent yield surface which clearly illustrates the effects on 304 stainless steel, of cold working under pressure.

Introduction

A yield criterion specifies a state of stress which, when reached, will result in plastic flow in a material. When a virgin material is subject to loads that impose only one stress component, the formulation of a yield criterion is straightforward. The problem becomes more complicated, however, when such a material sustains a more complex state of stress. In this case, parameters must be selected to predict yielding and the effect of each stress component on these parameters must be evaluated. Many such criteria have been formulated to predict yielding in virgin materials subjected to triaxial stresses. For example, in the von Mises yield condition, the second deviatoric stress invariant is chosen as the governing parameter and, as a consequence, the hydrostatic stress component is assumed to be insignificant in determining whether or not yielding has occurred. Similarly, the Tresca yield condition assumes that the intermediate principal stress has no effect on yielding since the Tresca theory considers the maximum shearing stress as the parameter governing yielding. All of these yield criteria can be categorized as either including, or not including, the effects of hydrostatic stresses. Currently, only yield criteria that neglect the effects of hydrostatic stresses enjoy widespread use. The effect of the hydrostatic stress component on yielding in virgin materials is dependent upon the material under consideration and, indeed, this effect can be significant. Further, this effect can be of even greater significance when yielding in a prestrained, or cold worked, material is under study.

The reason for studying yielding in prestrained materials is that most load carrying members are nonvirgin. Frequently, structural members are plastically deformed during the forming process, or as the result of service use. The literature survey section of this paper discusses yield theories which attempt to account for previous material yielding. Despite the increasing sophistication appearing in more recent theories, theory is not yet in line with experimental data. Because of the widespread application that would be available to a subsequent yield criterion, and because of the scarcity of documented information concerning such a criterion, the authors have undertaken a research project aimed at the development of a yield model that will accommodate some of the more complicated effects associated with coldworking a material in ambient and high pressure environments.

Review of Literature

The two most frequently used yield theories—those of von Mises and Tresca—assume that the result of plastic straining is an isotropic expansion of the yield surface. The initial and subsequent yield surfaces as predicted by the von Mises criteria are shown in Fig. 1. The utility of the assumption of isotropic expansion lies in mathematical tractability rather than inherent accuracy. A simple compression test of a specimen prestrained

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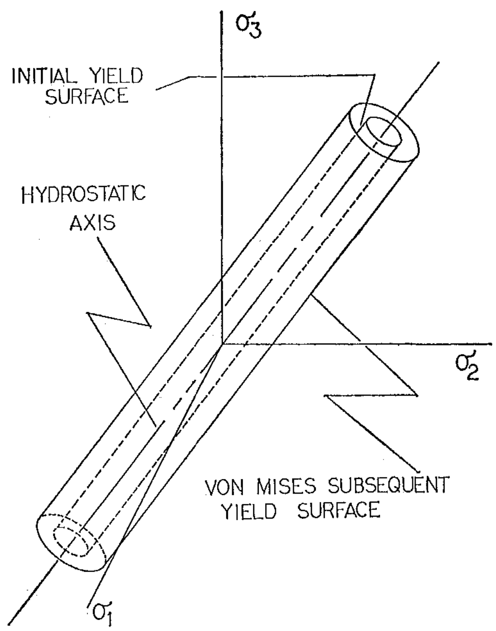

Fig. 1 Von Mises initial and subsequent yield surfaces

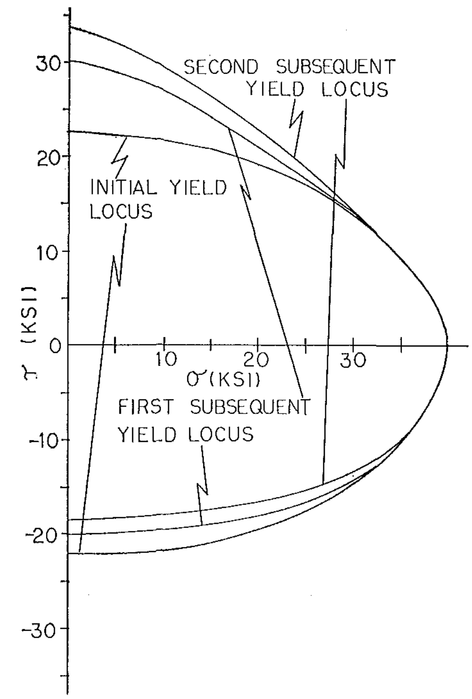

Fig. 2 Results of Naghdi, et al.

plastically in tension often will show that the yield surface does not always expand in an isotropic manner. A compressive yield strength somewhat lower than that for the virgin material could be observed—a phenomenon known as the Bauschinger effect.

Prager [1][1] has devised a kinematic hardening model which describes the subsequent yielding phenomenon in a manner which, at least, partially accounts for the Bauschinger effect, but this model also has its shortcomings. According to this hypothesis, the virgin yield surface will shift without deforming as the material undergoes plastic straining. The inability of the yield locus or yield surface to change shape as it is being displaced puts the model in conflict with experimental data.

A major departure from these two theories resulted from the work of Naghdi, Essenburg, and Koff [2]. These researchers determined that the yield curve in a particular two-dimensional case does not displace but, instead, expands outward in the direction of loading with a consequent inward contraction of other areas of the curve. By preloading thin-walled aluminum tubes in torsion, followed by reloading with various ratios of torsion and tension, they obtained the subsequent yield curves shown in Fig. 2. Unfortunately, it is difficult to express the type of data obtained by Naghdi, et al., in principal stress space. In cases such as torsion, where the principal stress axes do not remain fixed, it is necessary to know the amount of shift of the current principal stress axes with respect to some arbitrary stationary reference so that the stress vector at a point can be properly located in the stationary reference system.

Despite this drawback, tests involving the combined torsion and tension of thin-walled cylinders are widely used for work directed toward developing subsequent yield criteria. Minor variations of the Naghdi experiments have been performed on thin-walled tubes of aluminum, by Ivey [3] and Smith and Almroth [4]; copper, by Mair and Pugh [5]; and nickle, by Iagn and Shishmarev [6]. The data resulting from the tension-torsion tests of thin-walled cylinders are almost invariably presented in terms of a shear stress parameter and a normal stress parameter in a manner similar to that shown in Fig. 2. Despite the fact that this data presentation is not in principal stress coordinates, the presentation is still adequate enough to convey the idea that the observed phenomenon is neither purely isotropic nor purely

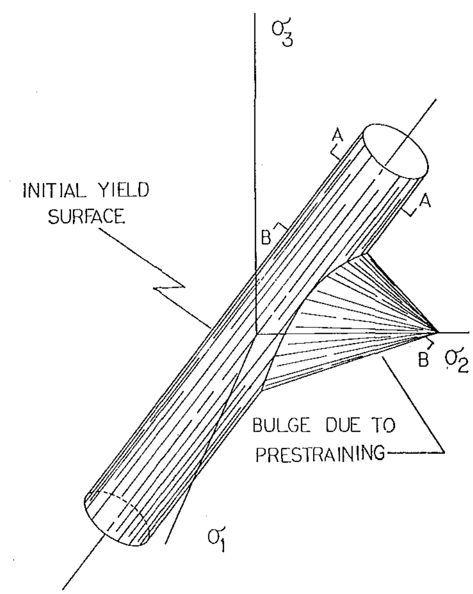

Fig. 3 Subsequent yield model proposed by Hu

kinematic in nature. Thin-walled aluminum cylinders were also used by Hu and Bratt [7] in tests that involved axial tension and internal pressure. After axial prestraining, the specimens were subjected to combined axial tension and internal pressure in order to determine the subsequent yield loci. These data also indicate that subsequent yield phenomena cannot be categorized as isotropic or kinematic.

In an attempt to generalize the method of presentation of yield data for prestrained materials, Hsu [8] has developed a method which transforms data such as that of Naghdi, et al., into the deviatoric plane. The most significant problem with Hsu's method is that some of the torsion-tension data for thin-walled cylinders transform into yield loci, in the deviatoric plane, that show concavities. In addition, the method involves a method of describing local principal stress axis orientations that does not seem particularly well suited to handling states of stress which

[1] Numbers in brackets designate References at end of paper.

are more complex than those encountered in the experiments of Naghdi, et al.

No known information was published on three-dimensional subsequent yield surfaces until Hu [9] described a series of experiments in which the expansive deflections of yield surface elements were found to be a function of the proximity of a surface element to the loading vector tip. An example of a subsequent yield surface as proposed by Hu is shown in Fig. 3. The author's literature survey has shown this to be the only published work connected with three-dimensional, subsequent yield surfaces and this statement is supported by a recent survey paper by Prager [10].

Discussion

A. Experimental Approach. Since most existing subsequent yield theories are in poor agreement with experimental data, an experimental effort which would generate a true subsequent yield surface seemed desirable. Most of the current subsequent yield data deals with two-dimensional stress states since this condition is relatively easy to produce in the laboratory. *In order to develop* a three-dimensional yield surface for a nonvirgin material, conventional tension and compression tests of prestrained specimens can be conducted in a hydrostatic pressure environment. By varying the pressure environment from test to test, a significant portion of the yield surface then comes within experimental reach.

For reasons of economy of effort and material, compressive prestraining of a parent specimen that would yield several smaller subspecimens was selected as the prestraining method for this program. The problem of prestraining a material, and then sectioning specimens from this for the tests to follow, involves several compromises. If the prestraining is done in an atmospheric environment, the loads can be applied in a conventional compression testing machine. Since high compressive loads are easily achieved, the cross section of the parent specimen can be large which will allow, in turn, large subspecimens to be cut from a transverse axis. Opposing the obvious advantages of reasonably sized subspecimens is the fact that the subsequent yield surface cannot be examined in detail below the point on the hydrostatic axis at which the prestraining was done, if this point is that of ambient pressure. Prestraining under a pressure environment will permit investigation of the subsequent yield surface below the pressure region of prestraining (which would be the situation after metalforming under pressure) but serious limitations are placed on the degree of prestrain to be achieved and the size of the parent specimens by the physical limitations of the environmental containment vessel and the preloading device. The second approach, i.e., prestraining under pressure, was chosen by the authors since it is extremely desirable to investigate the subsequent yield surface at pressures above and below the pressure region where the prestraining was done. Also, the obstacles posed by the small size of the subspecimens were not insurmountable. Ideally, the prestraining should be carried out under several different pressure environments. This would allow the influence of the prestraining environment on the subsequent yield surface to be studied. In order to provide at least fragmentary information in this area, a single parent specimen was prestrained in atmospheric pressure.

Consider, now, the π-plane (deviatoric plane) shown in Fig. 4. This is a view of an assumed von Mises yield surface for a virgin material as seen by looking down the hydrostatic axis. The projected principal axes and their extensions cut the circle into equal sectors. By assuming symmetry about a principal axis, say the axis of prestraining or σ_1, it is evident that at least four tests are required to sense the yield surface at a particular station along the hydrostatic axis. These tests are tension and compression along the direction of prestraining and tension and compression along an axis normal to the direction of prestraining. By performing this sequence of four tests in various pressure environments, sufficient data can be obtained to generate six lines which lie in the subsequent yield surface. This was the technique used in this project.

B. Selection of Material and Preparation of Specimens. Electing to do the parent specimen prestraining under a pressurized fluid environment presented several additional questions concerning parent specimen size, material selection, and the environmental pressure level to be used for prestraining. Since the available fluid pressure generation facility was limited to 80,000 psi, it was decided to conduct the prestraining at half of that level—40,000 psi. This would permit the subsequent yield surface to be investigated at environmental pressures ranging to levels of 40,000 psi above and below the prestraining pressure level.

Because of the size of the pressure chamber which was available (3 in. dia by 11 in. length), a two in. diameter cylindrical compression slug was chosen for the parent specimen. This would permit transverse specimens of up to two inches in length to be sectioned from the slugs after prestraining. It was desirable to have the parent specimens as short as possible to avoid any buckling problems that might arise during plastic compressive prestraining, but it was also necessary to make them long enough so that a 2.5 in. long center portion would exist in which the axial stress distribution would be constant over the cross section during the prestraining. In an elastic situation where the loads are applied as point loads, St. Venant's principal suggests that a 6.5-in. length would suffice. For this project, however, the parent specimen would be squeezed between a lower support block and a movable upper platten, or adapter. The adapter fits on the end of a ram which protrudes through the pressure vessel and is used to distribute the preload over the parent specimen face.

In order to determine, with some certainty, an appropriate parent specimen length, a finite element stress analysis program was employed. At this point, the specimen was assumed to have a yield strength in compression of 35,000 psi and elastic and plastic moduli which conformed to those of 304 stainless steel. The point at which compressive prestraining was to cease was at a uniform axial stress of 60,000 psi. The required load was assumed to be evenly distributed over the ram-platten interface and the bottom of the support block was assumed to be axially constrained. The results of this analysis indicated that a 5.0 in. long parent specimen, subjected to an average compressive stress of 60,000 psi, would have a uniformly stressed center section 2.5 in. in length.

The problem of material selection was further complicated by the fact that the total prestraining load and pressure force on

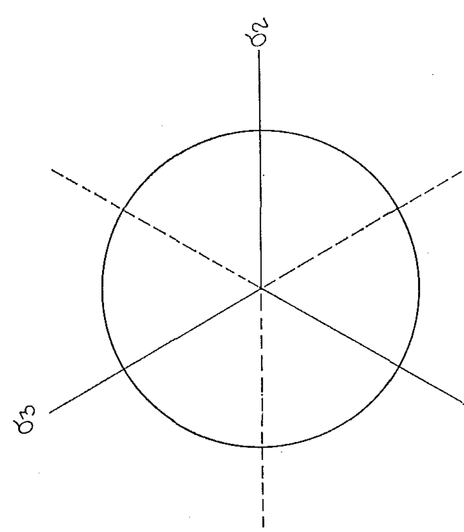

Fig. 4 Typical yield surface projected in the deviatoric plane

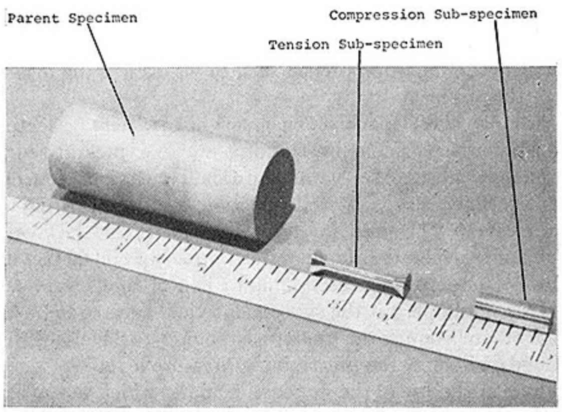

Fig. 5 Parent specimen and subspecimens

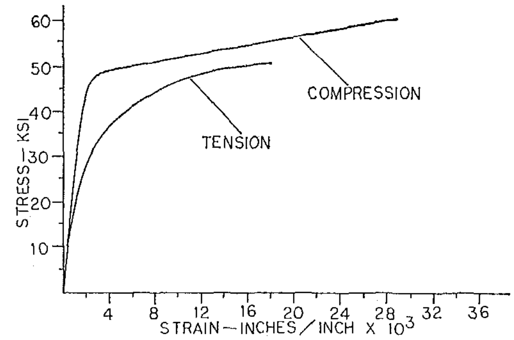

Fig. 6 Stress-strain curves for longitudinal subspecimen

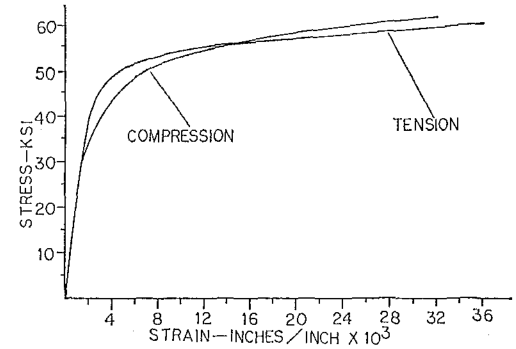

Fig. 7 Stress-strain curves for transverse subspecimens

the ram could not exceed 300,000 lb due to equipment limitations and, beyond this, by the fact that a ram of relatively small diameter had to carry this load. It was therefore necessary to use a specimen material which had a low virgin yield strength and which could be loaded to stress levels of approximately twice its yield strength—the latter requirement being imposed to insure that the effects of prestraining were, at least, observable.

Isotropy is not necessary but such a characteristic would lessen the work required to determine the virgin yield surface. Initially, three materials were under consideration—Nittany No. 2 Brass, 304 stainless steel and ultra-pure ferritic tranformer core iron. The most isotropic of these, the ferritic core iron, does not work harden to a sufficient level and consequently could not be loaded to twice its yield strength. Nittany No. 2 Brass, which was used by Hu to develop his bulge theory, also has limited work hardening capabilities and is the most likely of the three to be nonhomogeneous and nonisotropic. As a consequence, fully annealed 304 stainless steel was selected.

The austenitic stainless steels, which include 304 stainless steel, are notorious for work hardening during machining operations. Since the prestrained parent specimens had to be cut up and machined into subspecimens, great care was taken to insure that this characteristic did not enter the final data. Upon the advice of the Carpenter Technology Corporation, producer of the specimen material, all machining was planned so that the final cut was 0.005 in. in depth. This left a work hardened zone of only 0.002 in. in depth which was too shallow to be of any consequence in the subspecimens. In order to conform to the ASTM specifications for compression tests, the compression subspecimens were machined to a size of 0.5 in. in diameter by 1.5 in. in length. The tension subspecimens were 0.25 in. in dia and 2.0 in. in length. The parent specimen and the subspecimens are shown in Fig. 5.

Results

The first step in the data reduction process was to select an appropriate definition for yielding. For a material such as a low carbon steel where yielding is pronounced, this would not be a problem, but such is not the case with any austenitic stainless steel. As a result of discussions with Pugh [11], three possibilities were considered. The first was that of defining the proportional point as the yield point. This was immediately abandoned since the proportional point is extremely low, if it exists at all, for 304 stainless steel. The second definition involves extending the elastic and plastic slopes of the curve until they intersect. The resulting bilinear stress-strain curve does not even closely approximate the stress-strain curve generated by the tension subspecimens and, although the fit for the compression subspecimens stress-strain curves is somewhat better, this case is not good either.

Finally, the conventional offset method was selected. It was assumed that the material would unload along a path parallel to the initial elastic slope of the stress-strain curve and such a path was constructed from a point on the abscissa which represents the amount of allowable permanent plastic strain. The intersection of this unloading path and the stress-strain curve was assumed to define the yield strength. Although the amount of allowable plastic strain is quite arbitrary, convention dictates the use of 0.2 percent plastic strain with longitudinal stress-longitudinal strain curves.

The 0.2 percent offset method was applied to the classical longitudinal stress-longitudinal strain curves that were constructed for each subspecimen test. If it is assumed that Poisson's ratio is constant up until yielding occurs, this method yields results identical to those obtained by using an effective stress-effective strain plot with the allowable permanent effective strain reduced to $^2/_3(1 + \mu)$ of the amount permitted in the case of the longitudinal stress-longitudinal strain curves. The effective stress is defined as

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad (1)$$

and effective strain is defined

$$\bar{\epsilon} = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2} \qquad (2)$$

The elastic modulus was found by determining the slope of the stress-strain curve at the origin. The slope value was found to be approximately 30×10^6 psi for all cases, and this value did not change with pressure environment. Further, the shape of the stress-strain curves for each of the particular types of tests, i.e., longitudinal tension, transverse compression, etc., did not vary with the pressure environment changes. Typical stress-strain curves for longitudinal tension and compression are shown in Fig. 6, and the typical curves for transverse tension and compression are shown in Fig. 7. It should be noted that these curves do not extend to fracture.

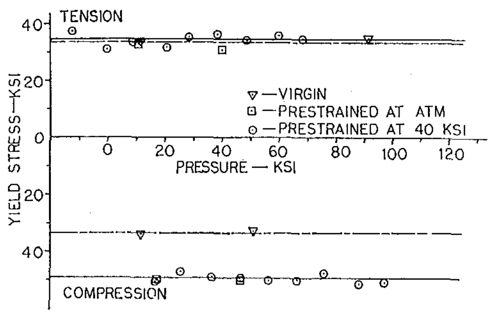

Fig. 8 Longitudinal subspecimen yield data

The data representing the tests of all the longitudinal subspecimens appear in Fig. 8. Each point represents a conventional yield strength plotted against the pressure level present in the material when yielding occurred. For this purpose, pressure is defined as the negative of the spherical component of the stress state, or

$$P = -\frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3}. \tag{3}$$

In effect, this plot represents an expanded view of the intersection of the subsequent yield surface and the σ_1-hydrostatic axis plane. The straight lines representing the yield surface are drawn parallel to the hydrostatic axis through the numerical average of the yield strengths. Attempts to fit straight lines to the data points via the least-squares technique shows this to be a reasonable representation of the data. The lines between the hydrostatic axis and the subsequent yield data, represent the virgin yield conditions. It is this plot that best illustrates the independence of the yield strength of 304 stainless steel on pressure. It is evident that the prestraining resulted in a large gain in yield strength in the direction of the prestraining while in a direction opposite to that of the prestraining very little was gained. The increases in yield strengths for these two cases are, respectively, 16,300 psi and 1,300 psi.

The four points that represent data obtained from specimens that were prestrained in atmosphere give good agreement with the remaining points. This indicates that 304 stainless steel has the same subsequent yield surface for a given type and degree of prestrain for at least two different prestraining pressure environments. In light of these data, it is reasonable to expect that the compressive pressure environment used for the prestraining operations has no influence on the subsequent yield surface. The shape of the virgin yield surface in the region of high tensile pressure environments has not been determined by experiments at this date, hence, omission of this region in the discussion is understandable. The prime reason for the lack of a complete spectrum of tensile data is the unavailability of suitable triaxial tension test specimens.

The data representing the tests of all the transverse subspecimens appear, similarly, in Fig. 9. This plot represents an expanded view of the intersection of the subsequent yield surface and either the σ_2-hydrostatic axis plane or the σ_3-hydrostatic axis plane. Once again, the lines drawn parallel to the hydrostatic axis through the data are in close agreement with the least-squares predictions. This plot shows that the longitudinal prestraining enhanced the yield strength in both the direction of transverse tension and the direction of transverse compression. Increases in these directions are 14,200 psi and 9300 psi, respectively.

Finally, the results of Figs. 8 and 9 are combined to form a view of the yield surface as seen by looking down the hydrostatic axis. This view is shown in Fig. 10. The lines drawn through the data points in Figs. 8 and 9 now appear as points in the deviatoric plane. Although the nonisotropic characteristics generated by the prestraining have caused the yield surface to shift in the direction of prestraining, the new yield surface still completely contains the virgin surface since the surface has simultaneously expanded outward. The von Mises subsequent yield surface for this case completely contains the empirical surface.

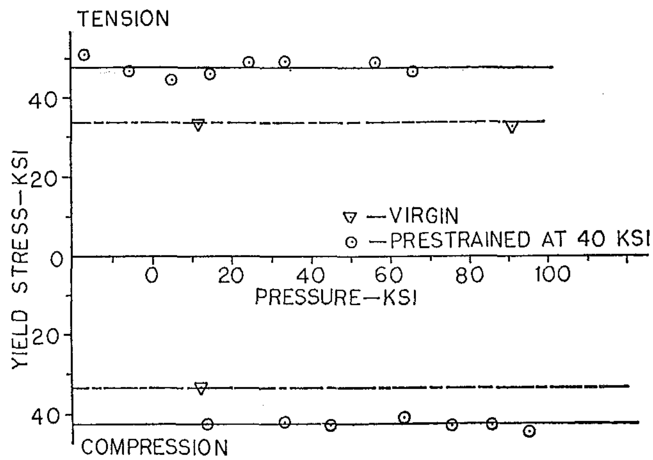

Fig. 9 Transverse subspecimen yield data

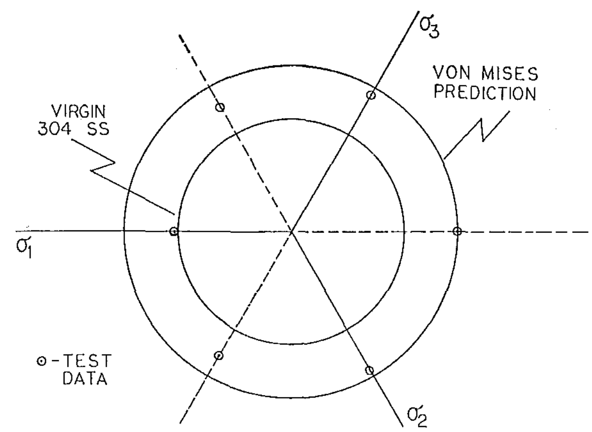

Fig. 10 Test results seen in deviatoric plane

Conclusions

The shape of the subsequent yield surface for 304 stainless steel for a particular degree of prestrain has been found to differ with the predictions of the subsequent yield theories currently in existence. Although the experimentally determined yield surface does not conform to the predictions of any particular subsequent yield theory, the surface can be thought of as a hybrid which embodies the features of both the von Mises isotropic hardening theory and the Prager kinematic hardening theory.

The subsequent yield surface has been found to be independent of the compressive pressure environment (for pressures up to 80,000 psi) associated both with the initial coldworking, or prestraining, operation and the subsequent specimen testing. The same is not necessarily true for tensile pressure environments. This work would suggest that metalforming under pressure would not complicate the problem of defining the subsequent yield surface; however, as in conventional forming processes, the prestrain history must be known.

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