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Performance Degradation Due to Specular Multipath Intersymbol Interference

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Abstract

Plots of performance degradation are used to compare the effects of fading and intersymbol interference in a two-component specular multipath digital communications channel. Similar plots are then used to compare two practical receivers designed to combat the interference. Degradation plots are shown to allow easy identification of each receiver's range of usefulness, as well as identification of variance bounds demanded of channel parameter estimates which the receivers require.

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Introduction

Multipath has long been known to be the source of the severe selective fading encountered in digital microwave communications systems, and much effort has been devoted to minimizing the effect of this fading, as indicated by the extensive bibliography of Lindsey [1]. Another aspect of multipath interference, which has been less extensively considered, is multipath intersymbol interference (MISI). This aspect is becoming a more significant problem with the advent of very high data rate systems [2].

Although MISI is discussed as a special case in many papers which treat generalized types of intersymbol interference (see the bibliography of Valerdi and Simpson [3]), few of these papers have addressed themselves specifically to practical solutions to the specular multipath problem. Gonsalves [4] has found the optimum (maximum likelihood) receiver for a two-component specular reflected path case in which the channel is completely known. Aein and Hancock [5] have investigated two suboptimum receivers, again assuming a known channel.

In this paper, curves of performance degradation will first be derived and used to indicate the relative significance of MISI compared to fading, assuming a biphase signal and a standard coherent (integrate-and-dump) detector. Two practical modifications of the standard coherent detector are then compared for performance using degradation curves. Since these two receivers require estimates of the channel parameters, it is necessary to know how accurate these estimates must be. The degradation curves are used to provide the answers.

System Model and Results

The message is modeled as a binary phase-shift keyed (PSK) signal with bit energy E and bit duration T seconds, plus an identical multipath component with time delay τ, phase shift θ, and relative amplitude α, plus an additive, zero-mean, white Gaussian noise component with a double-sided spectral density of $N_0/2$; i.e.,

$$S_T(t) = S_s(t) + S_r(t) + n(t) \tag{1}$$

where

$$S_s(t) = \sqrt{\frac{2E}{T}} \sin\left[\omega_0 t + k_i \frac{\pi}{2}\right];$$

$$k_i = \pm 1, (i-1)T \leqslant t \leqslant iT \tag{2}$$

$$S_r(t) = \alpha\sqrt{\frac{2E}{T}} \sin\left[\omega_0(t-\tau) - \theta + k_i \frac{\pi}{2}\right];$$

$$\tau \leqslant T, (i-1)T \leqslant t - \tau \leqslant iT \tag{3}$$

and $n(t)$ is stationary white Gaussian noise with

$$E\{n(t)\} = 0$$

$$E\{n(t)n(t')\} = \frac{N_0}{2}\delta(t-t').$$

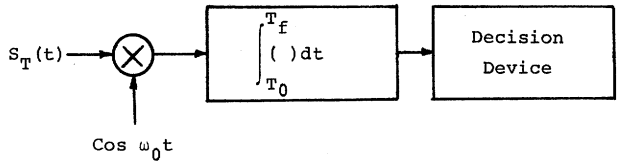


Fig. 1. Coherent receiver structure.

It is assumed that $k_i = 1$ and $k_i = -1$ are equally probable and that k_i is independent of $k_{j \neq i}$. It is also assumed that the receiver is perfectly synchronized with respect to carrier reference phase and bit period.

Standard coherent detection of the signal, as shown in Fig. 1, is performed by multiplying the received signal by a synchronized local oscillator, integrating over the bit period, and feeding the integrator output into a comparator with threshold of zero. As shown in the Appendix, the bit error probability for this receiver in the presence of the multipath component is

$$P_E = \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\left(1 + f - 2f\frac{\pi}{T}\right)\right) + \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}(1 + f)\right)\right] \quad (4)$$

where

$$\operatorname{erfc}\{x\} = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt \quad (5)$$

and

$$f \triangleq \alpha \cos(\omega_0 \tau + \theta). \quad (6)$$

Note that the parameter f is a measure of the fading experienced by the channel; negative f corresponds to fading, and the closer f is to -1, the more severe the fading becomes. This error probability was evaluated numerically as a function of signal-to-noise ratio (SNR) E/N_0, τ/T, and f. The degradation is defined as the increase in SNR (in decibels) necessary to achieve $P_E = 10^{-4}$ relative to a channel without multipath. This degradation is easily calculated graphically from plots of P_E, and the results are plotted in Fig. 2 for the case $\tau = 0.2T$ and $\tau = 0.8T$.

Two interesting conclusions may be drawn from Fig. 2:

1) When $f < 0$, multipath intersymbol interference (MISI) is negligible, since variations in τ/T have no significant effect on the degradation; the effects of fading dominate.

2) When $f > 0$, the degradation shows a strong dependence on τ/T, indicating that MISI is playing a significant role in performance degradation. Specifically, when τ/T is greater than about 0.5, performance deteriorates rapidly with increasing f. Note that this delay represents a path differential on the order of a meter with the high

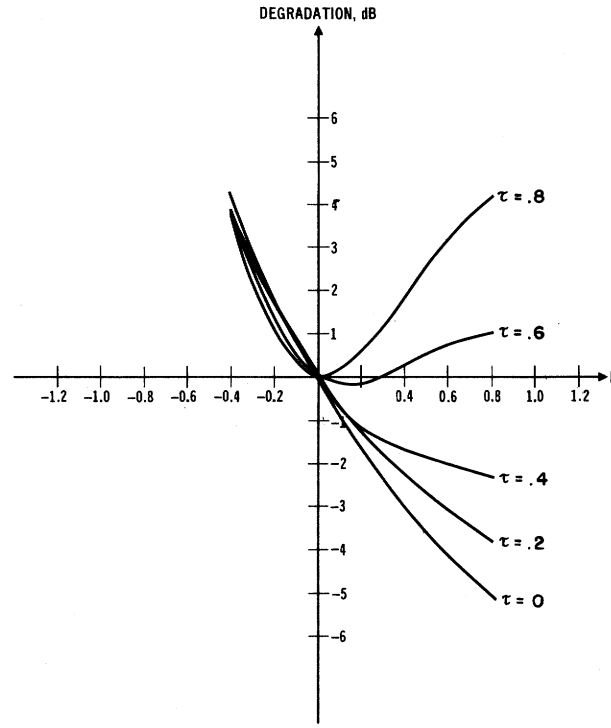


Fig. 2. Degradation versus f for standard coherent detector.

data rate systems now being developed (100 M bit/s and higher [2]). This differential could easily be incurred over typical microwave channels.

Besides being useful for identifying when MISI is detrimental, the graphical presentation of degradation is helpful in comparing receivers designed to combat the interference. Two examples are now presented.

The simplest receiver structure to combat MISI, both conceptually and practically, is to begin integrating only after the multipath tail of the previous bit has terminated. Clearly, this technique is valuable only when the relative multipath amplitude α is near 1, since part of the direct signal component is sacrificed. The error probability for this receiver is easily shown to be

$$P_E = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E(T-\tau)}{N_0 T}}(1 + f)\right), \quad 0 \leqslant \tau \leqslant T. \quad (7)$$

Degradation is caluclated using the same reference as before, the standard coherent detector with no multipath. The results for $\tau = 0.2T$ and $0.8T$, respectively, are presented in Figs. 3 and 4. Note that this receiver requires knowledge of (or an estimate of) the multipath delay τ.

Figs. 3 and 4 also show the performance of the tail cancellation receiver of Aein and Hancock [5]. This receiver may be viewed as the optimum Bayes detector if the previous bit is assumed known with certainty. It may also be regarded as a truncated version of Gonsalves' optimum detector [4], and, as such, its performance will be indicative of the optimum receiver's performance. Its structure is identical to the standard coherent detector with the

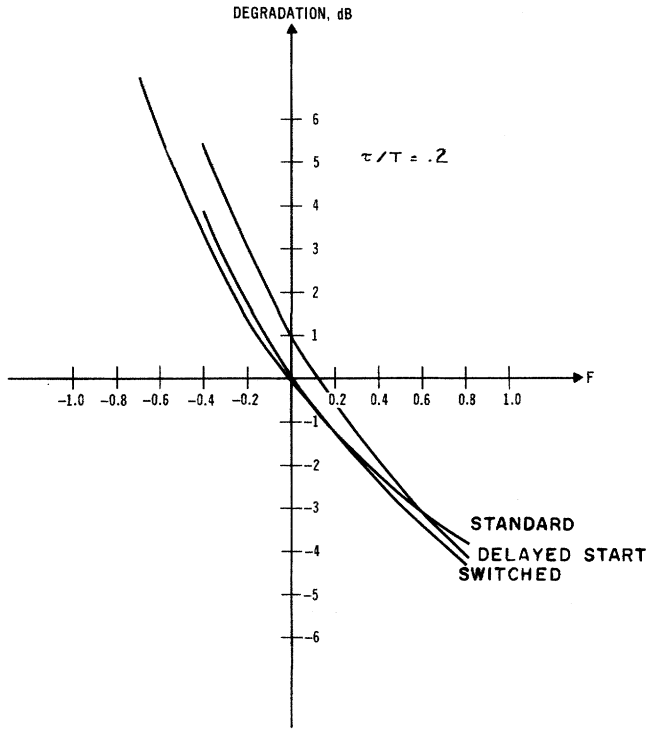


Fig. 3. Receiver comparison; $\tau = 0.2T$.

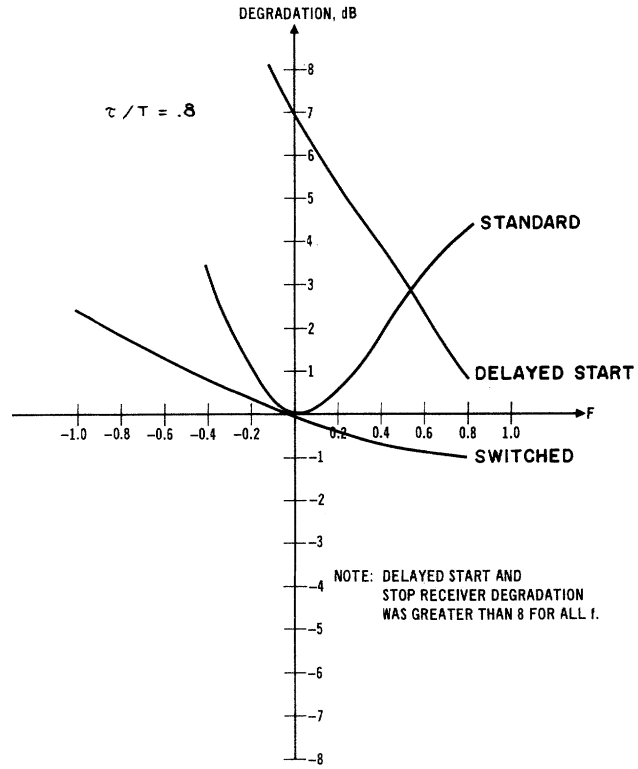


Fig. 4. Receiver comparison; $\tau = 0.8T$.

Fig. 5. Degradation for delayed start detector with channel estimate error. $\tau = 0.2T$ and $0.8T$.

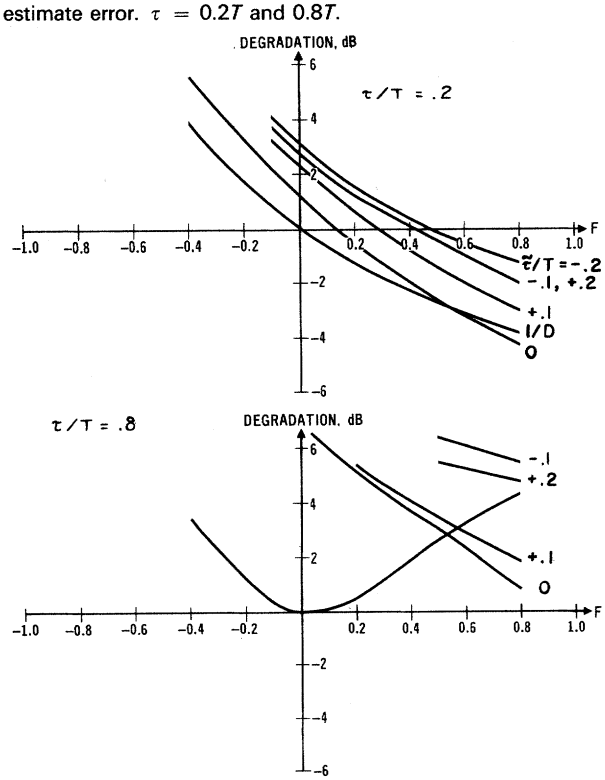
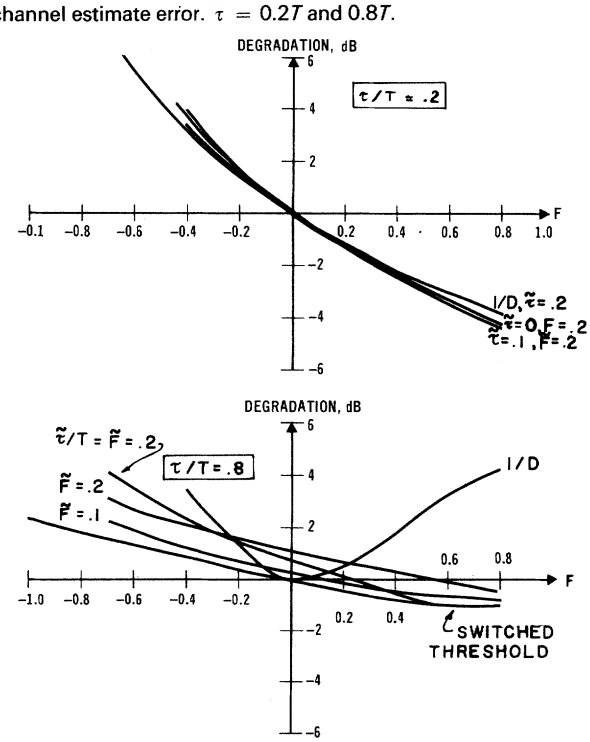


Fig. 6. Degradation for switched threshold receiver with channel estimate error. $\tau = 0.2T$ and $0.8T$.



exception of the comparator threshold which is $\pm\Lambda_0$, where the sign depends upon the decision of the previous bit, and the value of $\Lambda_0 = \Lambda_0(f, \tau/T)$ is adjusted to effectively cancel the predicted multipath component of the previous bit. The bit error probability for this receiver is calculated in the Appendix using the parameters f and τ/T, and differs from Aein and Hancock's work only in the choice of parameters. Clearly, this receiver requires estimates of both f and τ/T in order to choose Λ_0.

The degradation curves in Figs. 3 and 4 allow two con-

clusions to be drawn for the large delay considered:

1) The delayed start receiver affords relative performance improvement for large f (greater than about 0.6), but degrades the performance drastically when f is smaller.

2) The tail cancellation receiver not only alleviates the multipath ISI, but also improves performance (relative to the standard coherent receiver) under conditions of fading.

Finally, the degradation curves may be used to indicate the receiver sensitivity to errors in the channel estimate. Thus, the graphs will indicate some practical minimum accuracy bounds required of the channel estimating device. The Appendix indicates how bit error probabilities are calculated, assuming some specified error in the channel estimate. The resulting expression for the delayed start receiver is

$$P_E = \begin{cases} \frac{1}{2}\operatorname{erfc}\left(\left[\frac{E}{N_0 T}(T-\tau-\tilde{\tau})\right]^{1/2}(1+f)\right), & \hat{t} > \tau \\ \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{E}{N_0 T(T-\tau-\tilde{\tau})}} \cdot [(1+f)(T-\tau) - f\tilde{\tau}]\right) + \operatorname{erfc}\left(\sqrt{\frac{E}{N_0 T(T-\tau-\tilde{\tau})}}[(1+f)(T-\tau) + f\tilde{\tau}]\right)\right], & \hat{t} < \tau \end{cases} \tag{8}$$

where $\hat{t}$ is the estimate of τ and $\tilde{\tau} = \hat{t} - \tau$. Similarly, for the tail cancellation receiver,

$$P_E = \frac{P_{E|C}}{1 + P_{E|C} - P_{E|I}} \tag{9}$$

where

$P_{E|C}$ = probability of a decision error given that the previous decision was correct

$$= \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\left[1 + f + \frac{(f+\tilde{f})(\tau+\tilde{\tau})}{T}\right]\right) + \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\left[1 + f - 2f\frac{\tau}{T} + \frac{(f+\tilde{f})(\tau-\tilde{\tau})}{T}\right]\right)\right] \tag{10}$$

and

$P_{E|I}$ = probability of a decision error given that the previous decision was incorrect

$$= \frac{1}{4}\left[\operatorname{erfc}\sqrt{\frac{E}{N_0}}\left[1 + f + \frac{(f+\tilde{f})(\tau+\tilde{\tau})}{T}\right] + \operatorname{erfc}\sqrt{\frac{E}{N_0}}\left[1 + f - 2f\frac{\tau}{T} - \frac{(f+\tilde{f})(\tau-\tilde{\tau})}{T}\right]\right]. \tag{11}$$

Again, degradation is referenced to the standard coherent detector without multipath. Figs. 5 and 6 show the results for $\tau = 0.2T$ and $0.8T$. These figures allow two conclusions for the large delay considered:

1) The delayed start receiver is quite sensitive to errors in delay estimates. In the case $\tau = 0.8T$, average error in $\hat{t}$ should be less than about 5 percent or the improved performance is virtually lost, even in the limited range of the receiver's usefulness; i.e., $f > 0.6$.

2) The tail cancellation receiver allows improved performance even with significant errors in the channel estimate if the magnitude of f is large. On the other hand, if the relative amplitude of the multipath component is small, degradation on the order of 0.5 dB can be expected unless the channel estimate can be made extremely accurate. This requirement would demand a fairly elaborate estimating device, particularly if either θ, the phase shift, or τ is subject to rapid fluctuation.

Conclusions

It was shown that multipath intersymbol interference is a major source of performance degradation if the multipath delay is on the order of a bit period and the relative multipath amplitude is near unity, and if the multipath component is in phase with the direct signal component. While this result is intuitively obvious, a less obvious result is the fact that MISI plays a relatively insignificant role when fading occurs.

It was further shown that a coherent detector which integrates over only the part of the bit which is free from MISI is beneficial only in a highly specialized channel. Moreover, this receiver is fairly sensitive to errors in delay estimates, making it an undesirable structure. On the other hand, degradation curves indicate that the tail cancellation receiver performs well in spite of significant channel estimate errors if the parameter f has a magnitude on the order of unity. If $|f|$ is small, the receiver requires an accurate channel estimate to avoid degradation on the order of 0.5 dB relative to the standard coherent detector.

Appendix

Bit error probability is calculated first for an ideal correlation receiver for the PSK signals defined by (2) and shown in Fig. 1. After the result is obtained for ar-

bitrary start and stop times on the integrator, simplifications are made for the special cases of interest.

We first consider the expected value of the output of the integrator due to the signal component. Note that this is the expected value averaged over the noise and conditioned upon the bit sign, k :

$$\bar{S}_{s0} \triangleq \int_{T_0}^{T_f} S_s(t) \cos(\omega_0 t)\, dt \tag{12}$$

$$= \sqrt{\frac{2E}{T}} [k_i(T - T_0) + U(T_f - T)k_{i+1}(T_f - T)] \tag{13}$$

where $(i - 1)T \leqslant T_0 \leqslant iT$,

$$iT \leqslant T_f \leqslant (i + 1)T, \tag{14}$$

and $U(t)$ is the unit step. The expected value of the integrator output due to the multipath component is now considered. It is given by

$$S_{r0} \triangleq \int_{T_0}^{T_f} S_r(t) \cos(\omega_0 t)\, dt \tag{15}$$

$$\begin{aligned} &= f\sqrt{\frac{E}{2T}} [U(\tau - T_0)k_{i-1}(\tau - T_0) \\ &\quad + U(\tau - T_0)k_i(T - \tau) + U(T_0 - \tau)k_i(T - T_0) \\ &\quad + U(T + \tau - T_f)U(T_f - T)(T_f - T)k_i \\ &\quad + U(T_f - T - \tau)(k_i\tau + (T_f - T - \tau)k_{i+1})] \end{aligned} \tag{16}$$

where $f = \alpha \cos(\omega_0\tau + \theta)$, as defined previously. Consequently, the (conditional) expected value of the total output of the integrator is

$$\bar{S}_{T0} = \bar{S}_{s0} + \bar{S}_{r0} \tag{17}$$

$$\begin{aligned} &= \sqrt{\frac{E}{2T}} [fk_{i-1}(\tau - T_0)U(\tau - T_0) + k_i(T - T_0) \\ &\quad + f(T - \tau)U(\tau - T_0) + f(T - T_0)U(T_0 - \tau) \\ &\quad + f(T_f - T)U(T + \tau - T_f)U(T_f - T) \\ &\quad + f\tau(U(T_f - T - \tau)) + k_{i+1}(f(T_f - T - \tau) \\ &\quad \cdot U(T_f - T - \tau) + (T_f - T)U(T_f - T))]. \end{aligned} \tag{18}$$

The bit error probability consists of 8 terms, corresponding to the 8 possibilities of (k_{i-1}, k_i, k_{i+1}); however, from the symmetry of the problem, it is clear that

$$P_E(k_{i-1}, k_i, k_{i+1}) = P_E(-k_{i-1}, -k_i, -k_{i+1}), \tag{19}$$

so that the total P_E consists of four equally likely terms (since $P_r\{k_i\} = P_r\{-k_i\} = 1/2$:

$$P_E = \frac{1}{4}[P_E(1,1,1) + P_E(1,1,-1) + P_E(-1,1,1) + P_E(-1,1,-1)]. \tag{20}$$

Since the noise is Gaussian, the variance of S_{T0} is easily found:

$$\begin{aligned} \sigma^2 &= E\left[\int_{T_0}^{T_f} n(t) \cos(\omega_0 t)\, dt \int_{T_0}^{T_f} n(t') \cos(\omega_0 t)\, dt'\right] \\ &= \frac{N_0}{4}(T_f - T_0). \end{aligned} \tag{21}$$

This allows the straightforward calculation

$$\begin{aligned} P_E(k_{i-1}, 1, k_{i+1}) &= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(R - S_{T0})^2}{2\sigma^2}\right] dR \\ &= \frac{1}{2} \operatorname{erfc}\left[\frac{S_{T0}}{\sqrt{2\sigma}}\right]. \end{aligned} \tag{22}$$

Thus,

$$\begin{aligned} P_E(k_{i-1}, 1, k_{i+1}) = \frac{1}{2} \operatorname{erfc}\Bigg(\sqrt{\frac{E}{N_0}} &[k_{i-1}fU(\tau - T_0) \\ &+ (T - T_0) + fU(\tau - T_0)(T - \tau) \\ &+ fU(T_0 - \tau)(T - T_0) \\ &+ fU(T + \tau - T_f) \\ &\cdot U(T_f - T)(T_f - T) \\ &+ fU(T_f - T - \tau)\tau + k_{i+1}[f \\ &\cdot U(T_f - T - \tau)(T_f - T - \tau) \\ &+ (T_f - T)U(T_f - T)]]\Bigg). \end{aligned} \tag{23}$$

The total probability of error may now be found easily for each case of interest by substituting appropriate values for T_0 and T_f ; i.e., $T_0 = 0$ and $T_f = T$ for the standard coherent detector, and $T_0 = \tau$ and $T_f = T$ for the delayed start receiver.

The tail cancellation receiver will now be developed as the optimum receiver for the channel, assuming the previous bit is known with certainty.

If we assume that $k_i = \pm 1$ with equal probability, that either type of error in decision is equally "costly", and and that the cost of a correct decision is zero, then the Bayes risk is minimized [6] by making the decision such that

$$\Lambda(R) = \frac{P_{r|H_1,k_{i-1}}(R|H_1, k_{i-1})}{P_{r|H_0,k_{i-1}}(R|H_0, k_{i-1})} \underset{H_0}{\overset{H_1}{\gtrless}} 1 \tag{24}$$

where the hypotheses H_0 and H_1 are defined by

$$H_1 : S(t) = S_1(t)$$

$$\triangleq \begin{cases} \sqrt{\frac{2E}{T}} \sin\left(\omega_0 t + \frac{\pi}{2}\right) + \alpha \sqrt{\frac{2E}{T}} \sin\left[\omega_0 t - \tau) - \theta + k_{i-1}\frac{\pi}{2}\right] + n(t), & \text{when } (i-1)T < t \leqslant (i-1)T + \tau \\ \sqrt{\frac{2E}{T}} \sin\left(\omega_0 t + \frac{\pi}{2}\right) + \alpha \sqrt{\frac{2E}{T}} \sin\left[\omega_0 (t - \tau) - \theta + \frac{\pi}{2}\right] + n(t), & \text{when } (i-1)T + \tau < t \leqslant iT \end{cases} \tag{25}$$

and

$$H_0 : S(t) = S_0(t)$$

$$\triangleq \begin{cases} \sqrt{\frac{2E}{T}} \sin\left(\omega_0 t - \frac{\pi}{2}\right) + \alpha \sqrt{\frac{2E}{T}} \sin\left[\omega_0 (t - \tau) - \theta + k_{i-1}\frac{\pi}{2}\right] + n(t), & \text{when } (i-1)T < t \leqslant (i-1)T + \tau \\ \sqrt{\frac{2E}{T}} \sin\left(\omega_0 t - \frac{\pi}{2}\right) + \alpha \sqrt{\frac{2E}{T}} \sin\left[\omega_0 (t - \tau) - \theta - \frac{\pi}{2}\right] + n(t), & \text{when } (i-1)T + \tau < t \leqslant iT. \end{cases} \tag{26}$$

Now note that correlation of the signal with the difference of the two possible transmitted signals will yield a sufficient statistic, as can easily be verified through Karhunen-Loeve expansion [6]. Thus, let

$$R = \int_{(i-1)T}^{T} \cos(\omega_0 t) S(t)\, dt. \tag{27}$$

We now proceed to calculate the required conditional a priori densities. Since all densities required are Gaussian, we require only the mean and variance of each:

$$E\{R|H_1, k_{i-1}\} = \int_{(i-1)T}^{iT} \cos(\omega_0 t) S_1(t)\, dt = \sqrt{\frac{2E}{T}}\left[(1+f)T - f(1 - k_{i-1})\tau\right]. \tag{28}$$

Similarly,

$$E\{R|H_0, k_{i-1}\} = \int_{(i-1)T}^{iT} \cos(\omega_0 t) S_0(t)\, dt = \sqrt{\frac{2E}{T}}\left[(1+f)T - f(1 - k_{i-1})\tau\right]. \tag{29}$$

The noise variances are, of course, identical:

$$\sigma^2 = \int_0^T \int_0^T \cos(\omega_0 t) \cos(\omega_0 t') E\{n(t)n(t')\}\, dt dt' = \frac{N_0 T}{4}. \tag{30}$$

Hence,

$$\Lambda = \frac{P_{r|H_1}(R|H_1, k_{i-1})}{P_{r|H_0}(R|H_0, k_{i-1})} \tag{31}$$

$$= \exp\left(\frac{4}{N_0 T}\sqrt{\frac{E}{2T}}\left[(1+f)T - f\tau\right] - \left[2R - \sqrt{\frac{2E}{T}} k_{i-1}\tau f\right]\right) \underset{H_0}{\overset{H_1}{\gtrless}} 1. \tag{32}$$

Taking the natural log of the above, the decision rule becomes

$$R \underset{H_0}{\overset{H_1}{\gtrless}} \sqrt{\frac{E}{2T}} k_{i-1}\tau f = \Lambda_0 k_{i-1}, \quad \Lambda_0 = \sqrt{\frac{E}{2T}} f\tau. \tag{33}$$

Thus, the receiver compares the correlator output to a threshold whose sign depends upon the previous bit, or, in effect, subtracts off the tail of the previous bit before making a decision.

In order to evaluate the receiver performance, it is necessary to consider the case where the previous decision was incorrect, as well as the case where it is correct:

$$P_E = P_{E|C} \cdot P_r\{\text{last bit correct}\} + P_{E|I} \cdot P_r\{\text{last bit incorrect}\} \tag{34}$$

$$= \frac{P_{E|C}}{1 + P_{E|C} - P_{E|I}}. \tag{35}$$

It remains to evaluate the conditional error probabilities above. To simplify the notation, let

$$\bar{R}_{jk} = E\{R|H_j, k_{i-1}\}, \qquad j = 0, 1, k = \pm 1 \tag{36}$$

$$= k_i \sqrt{\frac{E}{2T}}\left[(1+f)T - (1 - k_i k_{i-1}) f\tau\right] \tag{37}$$

$$P_{E|C} = \frac{1}{4}\left[P_{E|\substack{k_i=1\\k_{i-1}=1}} + P_{E|\substack{k_i=1\\k_{i-1}=-1}} + P_{E|\substack{k_i=-1\\k_{i-1}=1}} + P_{E|\substack{k_i=-1\\k_{i-1}=-1}}\right] \tag{38}$$

$$P_{E|\substack{k_i=1\\k_{i-1}=1}} = \int_{-\infty}^{\Lambda_0} \frac{1}{\sqrt{2\pi}\,\sigma} \exp - \frac{(R-\bar{R}_{1,1})^2}{2\sigma^2}\,dR$$

$$= \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{E}{N_0}}\left(1+f-f\frac{\tau}{T}\right)\right] \tag{39}$$

$$P_{E|\substack{k_i=1\\k_{i-1}=-1}} = \int_{-\infty}^{-\Lambda_0} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(R-\bar{R}_{1,-1})^2}{2\sigma^2}\right]dR$$

$$= P_{E|\substack{k_i=1\\k_{i-1}=1}}. \tag{40}$$

Similarly, it is easily shown that

$$P_{E|\substack{k_i=-1\\k_{i-1}=1}} = P_{E|\substack{k_i=-1\\k_{i-1}=-1}} = P_{E|\substack{k_i=1\\k_{i-1}=1}}. \tag{41}$$

Thus,

$$P_{E|C} = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{E}{N}}\left(1+f-f\frac{\tau}{T}\right)\right]. \tag{42}$$

We now need to consider the probability of error, assuming the previous bit was incorrect. The only difference which this introduces is that the k_{i-1} appearing in the threshold, $\Lambda_0 k_{i-1}$, is the negative of the k_{i-1} appearing in $\bar{R}$. In the following, the k_{i-1} subscript on P_E will refer to the k_{i-1} appearing in the threshold, so that

$$P_{E|\substack{k_i=1\\k_{i-1}=1}} = \int_{-\infty}^{\Lambda_0} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(R-\bar{R}_{1,-1})^2}{2\sigma^2}\right]dR$$

$$= \frac{1}{2}\operatorname{erfc}\left(\left[(1+f)-3f\frac{\tau}{T}\right]\sqrt{\frac{E}{N_0}}\right) \tag{43}$$

$$P_{E|\substack{k_i=1\\k_{i-1}=-1}} = \int_{-\infty}^{-\Lambda_0} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(R-\bar{R}_{1,1})^2}{2\sigma^2}\right]dR$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\left[1+f+f\frac{\tau}{T}\right]\right). \tag{44}$$

Since the other two cases again reflect the symmetry of the problem, there are only two distinct terms in the expression:

$$P_{E|I} = \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\left[1+f-3f\frac{\tau}{T}\right]\right) + \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\left[1+f+f\frac{\tau}{T}\right]\right)\right]. \tag{45}$$

The total error probability may now be calculated as in (35).

To calculate the bit error probabilities, assuming an error in the channel estimate, it is necessary to repeat the above analysis using the parameter estimates, rather than the actual parameter values, in the receiver implementation. Thus, in the delayed start detector, the lower limits on the integrals (12), (15), and (22) will become $\hat{\tau}$, the estimate of τ. Note that two cases need to be considered: one where $\hat{\tau} > \tau$ and another where $\hat{\tau} < \tau$. Similarly, with the tail cancellation receiver, the upper limits on (39), (40), (43), and (44) become $\hat{\Lambda}_0$, the comparator threshold based on the parameter estimates. Notice that, in this case, the $\bar{R}_{jk}$ in (44) and σ^2 in (34) remain dependent upon the actual parameter values. The results of performing the indicated analysis are presented in (8) through (11).

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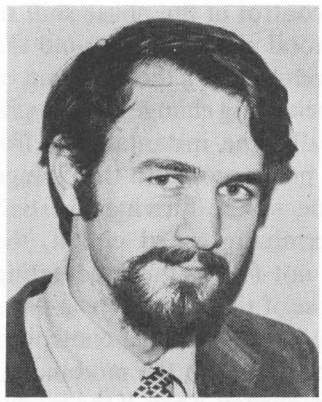

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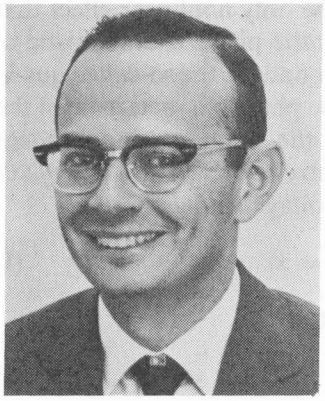

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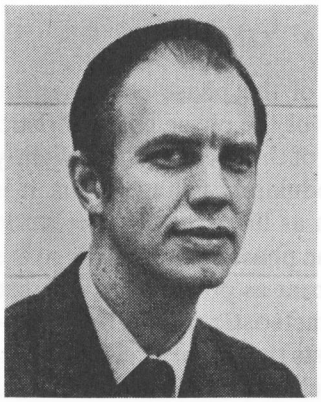

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