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Verifications Of The Kalman Conjecture Based On Locus Curvature

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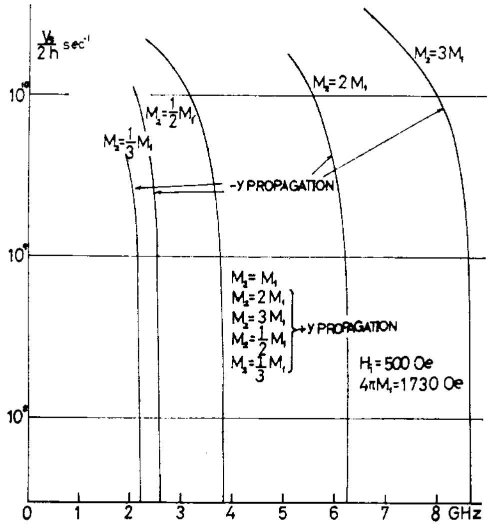

Fig. 2. Group velocity normalized with gap thickness versus $\omega/2\pi$ for four different values of $p=M_2/M_1$. $H_i=500$ Oe and $4\pi M_1=1730$ Oe.

propagating in the $-y$ direction; while for $k_y \to 0$,

$$f_{0+}=\frac{\gamma}{2}\left\{-(M_2-M_1)+\frac{1}{\pi}\sqrt{[\pi(M_2-M_1)]^2+H_i[H_i+2\pi(M_1+M_2)]}\right\}, \quad +y \text{ direction}$$

$$f_{0-}=\frac{\gamma}{2}\left\{M_2-M_1+\frac{1}{\pi}\sqrt{[\pi(M_2-M_1)]^2+H_i[H_i+2\pi(M_1+M_2)]}\right\}, \quad -y \text{ direction.}$$

If the two magnetic substrates have the same saturation magnetization (for example, a YIG–YIG system), we obtain in (4), by letting $M_1=M_2=M$,

$$e^{-4hk_y}=\frac{(\omega_s^2-\omega^2)}{(2\pi\gamma M)^2}. \qquad (5)$$

It is interesting to note that the dispersion relation of (5) completely coincides with the dispersion relation of the magnetostatic surface wave in a ferrite slab having a thickness of $2h$, which was described previously by Damon and Eshbach [1].

It is well known that the characteristics of magnetostatic surface waves propagating in a ferrite slab are extremely dependent on the dimension of the slab thickness [1], [2]. Slab thickness is not easily varied, since grinding destroys the surface quality. However, for the magnetic structure considered here, the propagation characteristics can be controlled simply by adjusting the thickness of the air gap. In this case, even if the dielectric spacer occupies the air gap region, the propagation characteristics are not affected in the limit of magnetostatic waves [3].

The solution of (4) is evaluated numerically for $H_i=500$ Oe, $4\pi M_1=1730$ Oe, and various ratios of saturation magnetization of substrate 2 to saturation magnetization of substrate 1 ($p=M_2/M_1$). The dispersion diagram is presented in Fig. 1. The $+y$ directed propagation (k_y^+) is possible for $f_{0+}<(\omega/2\pi)<f_{s1}$ and the permitted region for the $-y$ directed propagation (ky^-) is $f_{0-}<(\omega/2\pi)<f_{s2}$. Fig. 1 is the diagram for the $\pm y$ direction. It is found that the unidirectional propagation occurs when the two substrates have a different saturation magnetization (for example, a GaYIG–YIG system). From the numerical results calculated here, it seems that the magnetostatic surface wave in this structure does not have a negative group velocity which appeared at layered magnetic structures [4]. The group velocity is derived from (4) as

$$v_g=2h\frac{(\omega_{s2}\pm\omega)(\omega_{s1}\mp\omega)}{\omega\pm\pi\gamma(M_2-M_1)}. \qquad (6)$$

Fig. 2 gives the frequency dependence of the group velocity as a function of the normalized gap thickness. This figure shows that the group velocity is highly dispersive and, over a frequency range of 3.5 GHz to 3.8 GHz, approaches a constant independent of p. To obtain a delay time of 1 μs/cm, the thickness of the air gap must be chosen to be about 10 μm.

In conclusion, it has been shown that the magnetostatic surface wave in the two substrates separated by a distance of $2h$ is similar to the magnetostatic surface wave in a ferrite slab of thickness $2h$. Also, highly dispersive behavior is confirmed on the basis of numerical computation of group velocity. As a result, the magnetostatic surface wave in the two substrates may form the basis for nonreciprocal components and delay line devices at microwave frequency [6], with transmission characteristics controlled by adjusting the thickness of the air gap.

Acknowledgment

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Verifications of the Kalman Conjecture Based on Locus Curvature

D. RONALD FANNIN AND ALLEN J. RUSHING

Abstract—**The Kalman conjecture is verified for certain transfer functions with only negative real poles and no numerator dynamics. The results are based on the off-axis circle criterion of Cho and Narendra and a consideration of the curvature of the Nyquist locus of the transfer function.**

Introduction

The conjectures of Aizerman and Kalman [1] are of importance because, if true, they allow the stability of nonlinear systems to be determined from the stability analysis of a linear system. Unfortunately, the conjectures are not true in general [2]. This letter presents a method which allows the class of systems known to satisfy the Kalman conjecture to be significantly broadened.

Method

The class of systems to be considered is shown in Fig. 1. It is assumed that the nonlinearity is single-valued, time-invariant, and monotonic and satisfies

$$u(0)=0, \qquad 0\le\frac{du}{de}\le k. \qquad (1)$$

The linear part described by $G(s)$ is assumed to be time-invariant and output stable. For such systems the criterion of Cho and Narendra states that the system is globally asymptotically stable if the $G(j\omega)$ locus, $\omega\ge 0$, lies entirely to the right of a straight line through the point $(-(1/k+\delta), 0)$, $\delta>0$ and arbitrarily small [3]. If the point $(-1/k, 0)$ through which the line passes is also the leftmost real axis crossing point of the $G(j\omega)$ locus (i.e., k is the maximum gain for a linear characteristic as determined by the Nyquist criterion), then the system satisfies the Kalman conjecture. The following theorem gives sufficient conditions for the existence of the desired geometrical configuration.

Theorem

Given a function $G(j\omega)$ such that:

i) $G(j\omega)$ is a rational function of $j\omega$;
ii) $|G(j\omega)|$ is bounded and monotonically decreasing for all $\omega\ge 0$;
iii) the curvature of $G(j\omega)$ exists and is in the same nonzero sense for all $\omega\ge 0$.

Then:

1) There exists a leftmost point at which the locus of $G(j\omega)$ intersects the real axis (the point $(-1/k, 0)$, where $1/k$ is finite).

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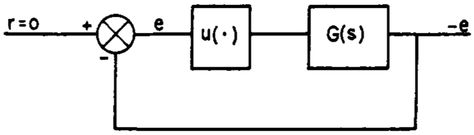

Fig. 1. Class of nonlinear system considered.

2) A line can be drawn through the point $(-(1/k+\delta), 0)$, $\delta>0$ and arbitrarily small, such that the locus of $G(j\omega)$ lies to the right of the line for all $\omega \geq 0$. This line is parallel to the tangent line of the locus of $G(j\omega)$ at the point $(-1/k, 0)$.

Proof

From i) and ii) it follows that $G(0)$ and $\lim_{\omega\to\infty} G(j\omega)$ are finite real numbers. Hence there are at least two intersections of $G(j\omega)$ and the real axis. Other intersections may exist, but from ii) none may be further to the left than $-|G(0)|$, and only one may be farthest to the left. Denote this intersection by $(-1/k, 0)$. Then $1/k$ is finite and proposition 1) is proved. Now consider the tangent line to the locus of $G(j\omega)$ at the point $(-1/k, 0)$. From ii) and iii) it follows that in the vicinity of $(-1/k, 0)$ the locus of G must move away from the tangent line and to the right as ω is varied from its value at $(-1/k, 0)$. But once it has moved away it cannot approach the tangent line again. To do so would mean either that the curvature changes sign or does not exist at a point, in violation of iii), or that the locus makes a loop, closing on itself, and then approaches the tangent line, violating ii) in making the loop. Therefore, a line drawn through $(-(1/k+\delta), 0)$ and parallel to the tangent line at $(-1/k, 0)$ has the property that the locus of $G(j\omega)$ lies to its right for all $\omega \geq 0$, and proposition 2) is proved.

Systems satisfying the assumptions of the theorem then satisfy the Kalman conjecture. Assumptions i) and ii) are easily checked, but iii) requires more effort. The following examples illustrate the method and give pertinent results.

Example 1: Transfer functions having only negative real poles and no numerator dynamics clearly satisfy assumptions i) and ii). The Kalman conjecture is verified if it can also be shown that the locus curvature is always in the same sense. The parametric equation for the curvature of a plane curve is

$$C = \frac{\dfrac{dX}{d\omega}\cdot\dfrac{d^2Y}{d\omega^2} - \dfrac{dY}{d\omega}\cdot\dfrac{d^2X}{d\omega^2}}{\left[\left(\dfrac{dX}{d\omega}\right)^2 + \left(\dfrac{dY}{d\omega}\right)^2\right]^{3/2}} \tag{2}$$

where X and Y are the real and the imaginary parts, respectively, of $G(j\omega)$.

For the following transfer functions:

$$G(s) = \frac{A}{s+p_1} \tag{3}$$

$$G(s) = \frac{A}{(s+p_1)(s+p_2)} \tag{4}$$

$$G(s) = \frac{A}{(s+p_1)^2(s+p_2)} \tag{5}$$

$$G(s) = \frac{A}{(s+p_1)^3(s+p_2)} \tag{6}$$

where A, p_1, and p_2 are positive constants, it is found using FORMAC that the numerator polynomial (in ω) of C has coefficients which are all of the same sign. Then by the Descartes rule of signs there are no positive real roots of the numerator polynomial, and the sign of C is invariant. Results (3)–(5) are well known, but (6) is believed to be a new result.

Example 2: Somewhat more powerful results can be obtained by considering

$$G(s) = \frac{A}{(s+p)^n} \tag{7}$$

where A and p are positive constants and n is a positive integer. It is sufficient to examine the curvature of

$$G_1(j\omega) = \frac{1}{(j\omega+p)^n} = R\angle\phi \tag{8}$$

$$R = (\omega^2+p^2)^{-n/2}, \qquad \phi = -n\tan^{-1}\omega/p. \tag{9}$$

Some algebraic simplification is achieved by considering the curvature in polar coordinates, given by

$$C = \frac{R^2 + 2\left(\dfrac{dR}{d\phi}\right)^2 - R\dfrac{d^2R}{d\phi^2}}{\left[R^2 + \left(\dfrac{dR}{d\phi}\right)^2\right]^{3/2}}. \tag{10}$$

Only the numerator need be considered, and it may be put in the form

$$NC = r^{2n} + \frac{n+1}{n} r^{2(n-1)}\left(\frac{dr}{d\theta}\right)^2 - \frac{r^{2n-1}}{n}\frac{d^2r}{d\theta^2} \tag{11}$$

where r and θ are defined by

$$R = r^n \quad \text{and} \quad \phi = n\theta. \tag{12}$$

Performing the indicated derivatives and simplifying yields

$$NC = \frac{n+1}{np^2} r^{2(n-1)}. \tag{13}$$

Thus the curvature is always in the same sense, and transfer functions of the form (7) satisfy the Kalman conjecture.

Conclusions

Sufficient conditions have been given under which a system satisfies the Kalman conjecture. The conditions are based on geometrical considerations and given in terms of the curvature of the Nyquist locus. Results show that the Kalman conjecture is true for a fourth-order system consisting of three repeated poles and one isolated pole, all in the left-half plane, and for systems with n repeated left-half plane poles, both with no numerator dynamics.

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Recursive Moving Polynomial Fit of Sampled-Data Time Series

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Abstract—**A polynomial is fit to a sampled-data time series and updated at each new sample point with small memory and computation requirements. An exponentially weighted square-error criterion is used.**

For the purpose of extrapolating (predicting) or smoothing a time series, one may wish to fit a polynomial to a portion of it. Consider a sampled-data time series x_t (integer t) and suppose one wishes to find at a given time t an nth-degree polynomial that fits the time series well at points in the recent past, possibly at the cost of a bad fit in the more remote past. One criterion for the goodness of fit that places the greatest emphasis on the most recent points while it gives decreasing and ultimately vanishingly small emphasis on the more distant points is

$$\Delta = \sum_{\tau=0}^{\infty} c^\tau (x_{t-\tau} - f(t,\tau))^2, \qquad \text{with } 0 < c < 1 \tag{1}$$

where $f(t, \tau)$ is the function of τ (with t as a parameter) that is to approximate the values of x_t in the near past. When $c^\tau \ll 1$, i.e., when $\tau \gg 1/(1-c)$, the criterion effectively ignores the quality of fit at $x_{t-\tau}$.

If $f(t, \tau)$ is to be a polynomial of the nth degree, we can set

$$f(t,\tau) = \sum_{k=1}^{n} a_{k,t}\phi_k(\tau) \tag{2}$$

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