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Equalization Of QPSK Data Transmission In Specular Multipath

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Abstract

This paper gives performance results for transversal-filter equalization of quadriphase phase-shift-keyed (QPSK) signals with two-component multipath and demodulator phase error. An analytical expression for optimum, minimum mean-square-error tap weights in terms of the multipath and signal parameters is given. Probability of error results for no equalization and equalization with adaptive decision-feedback tap-weight adjustment are compared. The results show that significant improvement can be obtained with relatively simple equalizer structures.

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I. Introduction

The use of quadriphase phase-shift-keyed (QPSK) modulation is a relatively simple, yet efficient, means of digital data transmission which is becoming increasingly common with the tendency to high data rates. Since QPSK modulation basically consists of multiplexing two binary data streams on quadrature carriers, a dispersive channel will introduce both intersymbol interference (ISI) and crosstalk between channels.

This paper is concerned with the probability of error P_E for QPSK data transmission through a two-component multipath channel. The receiver, shown in Fig. 1, consists of coherent demodulators with static phase error α for each quadrature channel, transversal-filter equalizers for the channel-induced ISI and crosstalk, and integrate-and-dump detectors. An expression for the optimum equalizer tap weights for two-component multipath is derived, and the improvement in P_E for equalized detection is determined through computer simulation. Adaptive adjustment of the equalizer weights with an estimate-gradient algorithm with decision feedback is also simulated.

The equalizer structure considered in this paper is patterned after that suggested by Monsen [1] for the class of so-called finite angle modulated signals of which QPSK is a special case. Equalization of digital data systems has received considerable attention in recent years. Representative papers are [2] through [4], none of which have given performance results for equalization of QPSK specifically. While the theoretically optimum receiver for dispersive channels, in the maximum likelihood sense, is nonlinear [5], attention here has been limited to the simpler linear transversal-filter structure. The data rates of interest (100 megabits per second and higher) preclude the use of complex analog structures or structures requiring digital computations, such as the Viterbi algorithm, as recently discussed by Forney [6].

II. Signal Models and Optimum Tap Weights

A. Demodulated Signals

The transmitted QPSK signal can be represented as the sum of two carriers in phase quadrature, biphase modulated by binary sequences $d_1(t)$ and $d_2(t)$ of period T_c:

$$s_{\mathrm{tr}}(t) = d_1(t)\cos\omega_0 t - d_2(t)\sin\omega_0 t. \tag{1}$$

The received two-component multipath signal plus noise is

$$\begin{aligned} y(t) &= s_{\mathrm{rec}}(t) + n(t) \\ &= s_{\mathrm{tr}}(t) + \beta s_{\mathrm{tr}}(t-\tau_m) + n_c(t)\cos(\omega_0 t + \alpha) \\ &\quad - n_s(t)\sin(\omega_0 t + \alpha) \end{aligned} \tag{2}$$

where β is the attenuation and τ_m is the delay of the multipath channel. The additive Gaussian noise $n(t)$ has

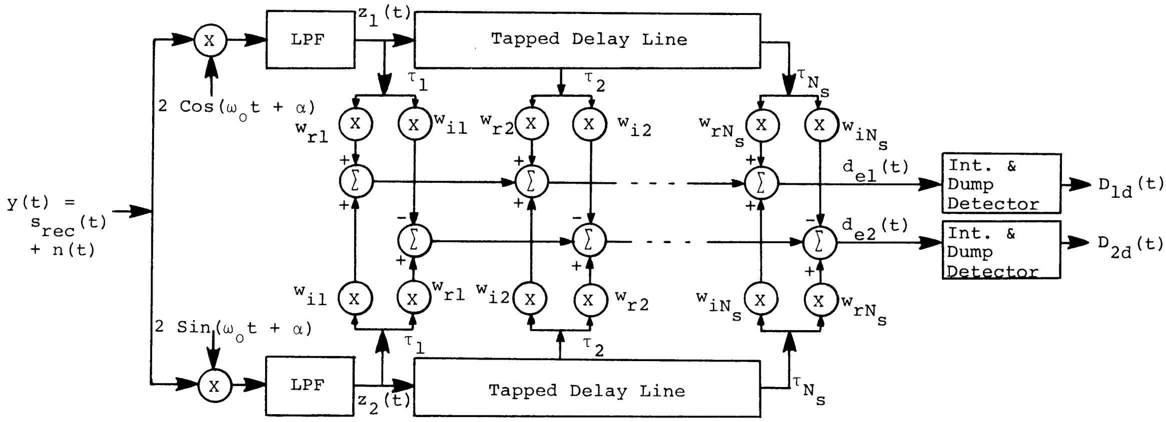

Fig. 1. Coherent demodulator, equalizer, and integrate-and-dump detector for QPSK.

double-sided power spectral density $N_0/2$ for $|f-f_0| \leqslant B$, and zero otherwise, B being the signal bandwidth. Thus, the quadrature noise components $n_c(t)$ and $n_s(t)$ each have double-sided power spectral densities equal to N_0 for $|f| \leqslant B$.

From (1) and (2), it follows that the outputs of the quadrature phase detectors in Fig. 1 are

$$z_1(t) = d_1(t) \cos \alpha + d_2(t) \sin \alpha + \beta d_1(t-\tau_m) \cos(\omega_0 \tau_m + \alpha) + \beta d_2(t-\tau_m) \sin(\omega_0 \tau_m + \alpha) + n_c(t) \quad (3)$$

and

$$z_2(t) = d_2(t) \cos \alpha - d_1(t) \sin \alpha + \beta d_2(t-\tau_m) \cos(\omega_0 \tau_m + \alpha) - \beta d_1(t-\tau_m) \sin(\omega_0 \tau_m + \alpha) + n_s(t). \quad (4)$$

Equations (3) and (4) can be combined into a single complex equation as

$$z(t) = z_1(t) + jz_2(t) = [d_1(t) + jd_2(t)]e^{-j\alpha} + \beta[d_1(t-\tau_m) + jd_2(t-\tau_m)] \exp[-j(\omega_0 \tau_m + \alpha)] + \tilde{n}(t) \quad (5)$$

where

$$\tilde{n}(t) = n_c(t) + jn_s(t). \quad (6)$$

From (3) and (4), it follows that the effect of a non zero phase error at the demodulator is to introduce crosstalk between channels. For zero phase error, note that the multipath component gives only ISI if $\omega_0 \tau_m$ is an integer multiple of π, whereas crosstalk alone results if $\omega_0 \tau_m$ is an odd integer multiple of $\pi/2$. Thus, for arbitrary multipath delays and demodulator phase errors, it is clear that both ISI and crosstalk must be equalized.

B. Equalizer Structure

The equalization approach chosen in this study is the use of feed-forward transversal filters. Two are required, one for each data channel, as well as two to compensate the crosstalk. In Fig. 1, the ISI-compensating filters consist of the tapped delay lines and the weighting coefficients w_{rk}, $k = 1, 2, \cdots, N_s$. To compensate for crosstalk from one channel in the other, the weighting coefficients w_{ik}, $k = 1, 2, \cdots, N_s$, are required. Thus, from Fig. 1, the equalized data streams can be written as

$$d_{e1}(t) = \sum_{k=1}^{N_s} [w_{rk} z_1(t-k\Delta) + w_{ik} z_2(t-k\Delta)] \quad (7)$$

and

$$d_{e2}(t) = \sum_{k=1}^{N_s} [w_{rk} z_2(t-k\Delta) - w_{ik} z_1(t-k\Delta)] \quad (8)$$

where $\tau_k = k\Delta$. Letting $\tilde{w}_k = w_{rk} + jw_{ik}$ and $d_e(t) = d_{e1}(t) + jd_{e2}(t)$, we can write (7) and (8) as the single complex equation

$$\tilde{d}_e(t) = \sum_{k=1}^{N_s} \tilde{w}_k^* \tilde{z}(t-k\Delta). \quad (9)$$

To show that the equalizer structure of Fig. 1 is capable of compensating for distortion in both quadrature data channels, consider a received signal of the form

$$s_{rec}(t) = B(t) \cos[\omega_0 t + \gamma(t)] \quad (10)$$

where $B(t)$ is amplitude distortion and $\gamma(t)$ is the phase modulation distorted by the channel. Synchronous detection with the quadrature references results in

$$z_1(t) = B(t) \cos[\gamma(t) + \alpha] \quad (11)$$

and

$$z_2(t) = B(t) \sin [\gamma(t) + \alpha] . \quad (12)$$

The equalizer must compensate $B(t)$ and $\gamma(t) + \alpha$ in both (11) and (12). From (7), we obtain

$$d_{e1}(t) = \sum_{j=1}^{N_s} B(t - k\Delta) \left\{ w_{rk} \cos [\gamma(t - k\Delta) + \alpha] + w_{ik} \sin [\gamma(t - k\Delta)] \right\}$$

$$= \sum_{j=1}^{N_s} | \tilde{w}_k | B(t - k\Delta) \cos [\gamma(t - k\Delta) + \alpha - \underline{/\tilde{w}_k}] \quad (13)$$

and, from (8), we have

$$d_{e2}(t) = \sum_{k=1}^{N_s} B(t - k\Delta) \left\{ w_{rk} \sin [\gamma(t - k\Delta) + \alpha] - w_{rk} \cos [\gamma(t - k\Delta) + \alpha] \right\}$$

$$= \sum_{j=1}^{N_s} | w_k | B(t - k\Delta) \cdot \sin [\gamma(t - k\Delta) + \alpha - \underline{/w_k}] . \quad (14)$$

Thus, it is apparent that the amplitude and phase distortion can be compensated in both data channels by proper choice of the w_k.

C. Optimum Tap Weights for the Fixed Case

The optimum weights, according to a minimum mean-square-error criterion, can be readily found [1]. The desired nondistorted complex modulation is

$$D(t) = (1 + \beta)[d_1(t) + jd_2(t)] \quad (15)$$

where the factor $1 + \beta$ is included because we are considering the total received signal. A minimum mean-square-error equalizer is one which minimizes

$$I = E\left\{ | d_e(t) - D(t) |^2 \right\} \quad (16)$$

where $E(\cdot)$ denotes expectation.

Defining the column vectors

$$w = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{w}_{N_s} \end{bmatrix}$$

and

$$Z = \begin{bmatrix} z(t) \\ z(t - \Delta) \\ \cdot \\ \cdot \\ \cdot \\ z[t - (N_s - 1)\Delta] \end{bmatrix} , \quad (17)$$

we may write (16) as

$$I = E\left\{ | w'Z - D(t) |^2 \right\} \quad (18)$$

where the prime denotes complex conjugate transpose. The optimum w is found by setting the gradient to zero. The result, given in [1], is

$$\mathbf{w}_{\text{opt}} = [E(\mathbf{ZZ}')]^{-1} E[\mathbf{Z}D^*(t)] . \quad (19)$$

An explicit expression for $\mathbf{w}_{\text{opt}}$ in terms of the signal parameters, channel parameters, and noise power spectral density is derived in the Appendix.

Considering first the matrix $\mathbf{A} \triangleq E[\mathbf{ZZ}']$, it is shown that

$$A_{kl} = 2(1 + \beta^2) R_d[(l - k)\Delta] + 2\beta R_d[(l - k)\Delta + \tau_m] e^{j\omega_0 \tau_m} + 2\beta R_d[(l - k)\Delta - \tau_m] e^{-j\omega_0 \tau_m} + 2R_n[(l - k)\Delta) \quad (20)$$

where $R_d(\tau)$ is the autocorrelation function of $d_1(t)$ or $d_2(t)$ (assumed statistically identical) and $R_n(\tau)$ is the autocorrelation function of $n_c(t)$ [or $n_s(t)$].

The lth component of the vector $\mathbf{B} \triangleq E[\mathbf{Z}D^*(t)]$ can be shown to be

$$B_l = 2(1 + \beta) R_d(l\Delta) e^{-j\alpha} + 2\beta R_d(l\Delta + \tau_m) \exp [-j(\omega_0 \tau_m + \alpha)] ; \quad (21)$$

Finally, by inverting A, with elements (19), and performing the matrix multiplication $A^{-1}B$, we obtain w_{opt} for a two-component multipath channel.

D. Estimate-Gradient Tap-Weight Adjustment for an Unknown Channel

Since the channel and receiver parameters required to compute the optimum weights will, in general, not be known and may be time-varying, an adaptive mechanism for adjusting the weights is required. A suitable adjustment technique is an estimate-gradient algorithm [1] with decision feedback, which updates the equalizer weight vector according to

$$\mathbf{w}(n + 1) = \mathbf{w}(n) - A\mathbf{Z}_{T_C} [d_e(t_n - T_c) - D_d(t_n)]^* \quad (22)$$

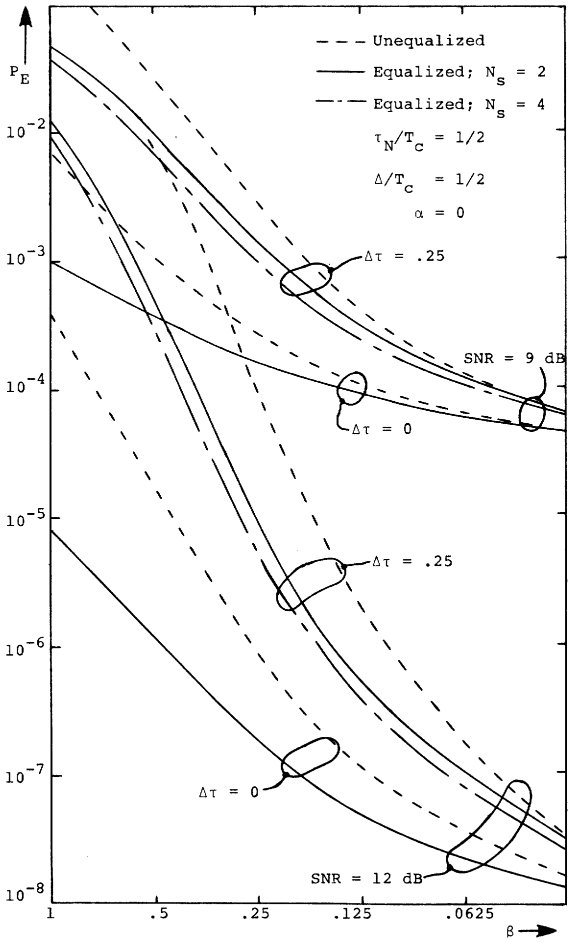

Fig. 2. Comparison of P_E for QPSK in fixed two-component multipath with and without equalization.

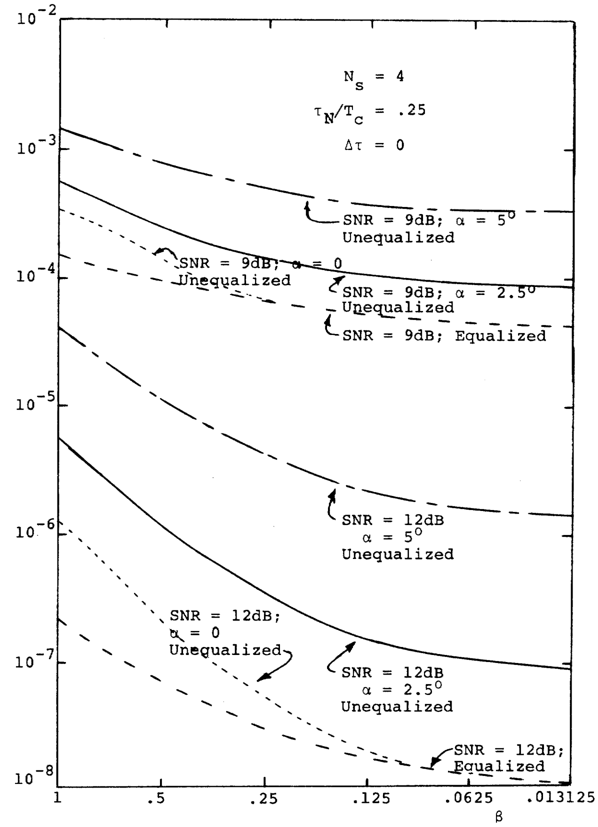

Fig. 3. P_E for two-component multipath with demodulator phase error with and without equalization.

where $\mathbf{w}(n)$ is the equalizer weight vector after n iterations at time t_n and A is a parameter which determines the speed of convergence. In (22), $d_e(t_n - T_c)$ is the complex output of the equalizer, given by (9), delayed by T_c, and Z_{T_c} is the vector (17) with $t = t_n - T_c$. $D_d(t) = D_{1d}(t) + jD_{2d}(t)$ is the detected data, obtained by integrate-and-dump detecting the equalizer output.

The algorithm (22) seeks to adjust the equalizer weights so that the error between the equalizer output and detected data, which for low-error probability closely approximates the transmitted data, is minimized.

II. Results

A. Simulation Procedure

Because of the intersymbol interference introduced by the dispersive elements in the channel, performance characterization of the receiver requires that the error probability averaged over a typical data sequence be calculated. The procedure used to calculate error probability was to simulate the transmission of a typical noise-free data sequence, in particular, a 31-bit maximal-length PN sequence, through the channel and detect it by an integrate-and-dump receiver which may be preceded by the equalizer. At the end of each bit, the error probability for that bit is calculated using the signal component obtained through simulation and the appropriate noise variance computed from the signal-to-noise ratio desired and the equivalent noise bandwidth of the receiver. The sequence-averaged error probability is then computed.

B. Fixed Tap-Weight Results

Fig. 2 shows probability of error P_E for two- and four-stage equalization versus multipath strength β for various multipath delays, with the demodulator phase error α zero. The multipath delay is normalized to the carrier period, and is written as $\tau_N + \Delta\tau$, where τ_N is the integer part of the normalized delay and $\Delta\tau$ is the fractional part. For $\Delta\tau = 0$, only ISI is present, while, for $\Delta\tau = 0.25$, crosstalk alone is present. In Fig. 2, τ_N is equal to one-half a data character period T_c. The signal-to-noise ratio is defined as the equivalent energy per bit to single-sided noise spectral density. The improvement gained through equalization is greatest for high crosstalk situations.

Fig. 3 shows P_E versus β for $\tau_N/T_c = 1/4$, $\Delta\tau = 0$, and $\alpha = 0°$, $5°$, and $10°$. The equalizer is clearly capable of fully compensating for demodulator phase error, thus permitting less stringent specifications on the carrier acquisition loops in the receiver.

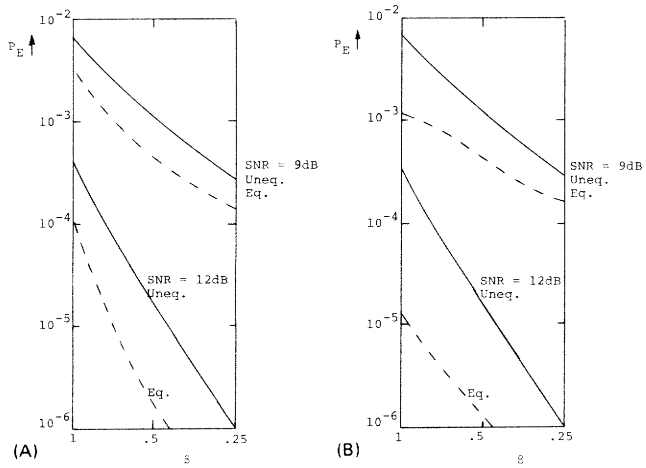

Fig. 4. P_E for adaptive equalization in two-component multipath; $\tau_N/T_c = 0.5$, $\Delta\tau = 0$, $\Delta/T_c = 0.5$, and $N_s = 2$. Time constant equal to (A) $10T_c$ and (B) $50T_c$.

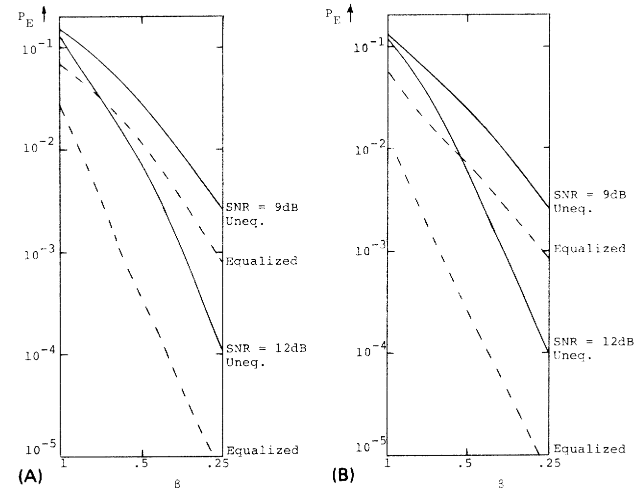

Fig. 5. P_E for adaptive equalization in two-component multipath; $\tau_N/T_c = 0.5$, $\Delta\tau = 0.25$, $\Delta/T_c = 0.5$, and $N_s = 2$. Time constant equal to (A) $10T_c$ and (B) $50\ T_c$.

C. Adaptive Weight Adjustment Results

Figs. 4 through 6 show P_E versus β for adaptive adjustment of the equalizer weights. In each figure, results are given for the adjustment constant A chosen to adapt the tap-weight values to e^{-1} of their final values in $10T_c$ [Figs. 4(A), 5(A), 6(A)] and $50T_c$ [Figs. 4(B), 5(B), 6(B)] time units. Comparing Figs. 4 through 6 with Fig. 2, it is apparent that the larger time constant gives results for P_E closer to the optimum fixed-weight case. Figs. 5 and 6 show results for identical multipath conditions, but with the number of taps equal to two and four, respectively. Overall delay-line length is the same in both cases. For the time constant equal to $10T_c$, the four-tap equalizer is clearly inferior, due to the excess mean-square error from adaptive adjustment of the tap weights. For the larger time constant, the results are essentially identical. Thus, care should be exercised in selecting the minimum number of taps consistent with the expected multipath conditions.

III. Conclusions

From the results just presented, it is apparent that considerable improvement in multipath can be provided by properly designed transversal-filter equalizers for QPSK data transmission systems. Because the equalizer can effectively compensate for demodulator reference phase errors, the design tolerances on the QPSK demodulator can be relaxed. Estimate-gradient algorithms employing decision feedback for adjusting the coefficients appear to converge satisfactorily under severe multipath conditions with time constants of the order of 50 symbol periods.

Appendix

In this appendix, the expressions (20) and (21) used to compute the optimum tap weights are derived.

Let a long string of transmitted signals be represented as

$$s_{tr}(t) \sum_{k=-\infty}^{\infty} \pi[(t-kT_c] \cos(\omega_0 t + \theta_k) \quad (23)$$

where $\pi(x) = 1, |x| = 1/2$, and is zero otherwise, and θ_k is the QPSK modulation for the kth transmission. Thus, $d_1(t)$ and $d_2(t)$ in (1) are given by

$$d_1(t) = \sum_{k=-\infty}^{\infty} \pi[(t-kT_c)/T_c] \cos\theta_k \quad (24)$$

and

$$d_2(t) = \sum_{k=-\infty}^{\infty} \pi[(t-kT_c)/T_c] \sin\theta_k, \quad (25)$$

respectively.

We wish to express the various factors in (19) in terms of signal, multipath, and noise parameters. The matrix

$$\mathrm{A} \triangleq E[\mathbf{ZZ}'] \quad (26)$$

will be considered first. Its klth element is

$$A_{kl} = E[z(t-k\Delta)z^*(t-l\Delta)], \quad (27)$$

which follows from the definition of $\mathbf{Z}$ given by (17). Letting $t_k = t - k\Delta$ and $t_l = t - l\Delta$ in (3) and (4), respectively, and substituting into (27), we obtain

$$\begin{aligned} A_{kl} = {} & [R_{d_1}(t_l - t_k) + R_{d_2}(t_l - t_k)](1+\beta^2) \\ & + \beta[R_{d_1}(t_l - t_k + \tau_m) + R_{d_2}(t_l - t_k) + \tau_m)] \\ & \cdot e^{j\omega_0\tau_m} \\ & + \beta[R_{d_1}(t_l - t_k - \tau_m) + R_{d_2}(t_l - t_k - \tau_m) \\ & \cdot e^{-j\omega_0\tau_m} \\ & + R_{\tilde{n}}(t_l - t_k) \end{aligned} \quad (28)$$

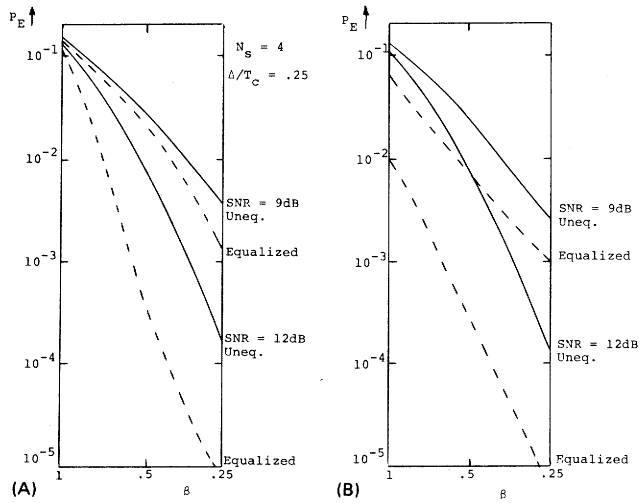

Fig. 6. P_E for adaptive equalization in two-component multipath; $\tau_N/T_c = 0.5$, $\Delta\tau = 0.25$, $\Delta/T_c = 0.25$, and $N_s = 4$. Time constant equal to (A) $10T_c$ and (B) $50T_c$.

where independence of $d_1(t)$, $d_2(t)$, and $\tilde{n}(t)$ has been assumed. In (28), the message autocorrelation functions are defined as

$$R_{d_i}(\tau) = E[d_i(t)d_i(t+\tau)], \quad i = 1, 2 \tag{29}$$

and the autocorrelation of the complex noise envelope is defined as

$$\begin{aligned} R_{\tilde{n}}(\tau) &= E[\tilde{n}(t)\tilde{n}^*(t+\tau)] \\ &= E[n_c(t)n_c(t+\tau)] + E[n_s(t)n_s(t+\tau)] \\ &= R_{n_c}(\tau) + R_{n_s}(\tau), \end{aligned} \tag{30}$$

which follows because $n_c(t)$ and $n_s(t)$ are independent for noise which is bandpass symmetric about $f = f_0$. Now, $R_{n_c}(\tau) = R_{n_s}(\tau)$, and if $d_1(t)$ and $d_2(t)$ are statistically identical sequences, (28) reduces to (20).

If $d_1(t)$ and $d_2(t)$ are random sequences, their autocorrelation function is [2]

$$R_D(\tau) = (1/2)\Lambda(\tau/T_c) \triangleq \begin{cases} 1/2(1 - |\tau|/T_c), & |\tau| \leq T_c \\ 0, & \text{otherwise} \end{cases} \tag{31}$$

where the 1/2 results because $d_1(t)$ and $d_2(t)$ take on the values $\pm 1/\sqrt{2}$. Since $n_c(t)$ and $n_s(t)$ are assumed to have power spectral densities equal to N_0 *for* $|f| \leq B/2 \gg T_c^{-1}$ and zero otherwise, their autocorrelation function is

$$R_{n_s}(\tau) = R_{n_c}(\tau) = BN_0 \operatorname{sinc} B\tau \tag{32}$$

where $\operatorname{sinc} x = (\sin \pi x)/\pi x$.

Use of (31) and (32) in (20) results in

$$\begin{aligned} A_{kl} = (1+\beta^2)\Lambda[(l-k)\Delta/T_c] &+ 2\beta\Lambda[(l-k)\Delta/T_c + \tau_m/T_c]e^{-j\omega_0\tau_m} \\ &+ 2\beta\Lambda[(l-k)\Delta/T_c - \tau_m/T_c]e^{j\omega_0\tau_m} \\ &+ 2N_0B \operatorname{sinc}[(l-k)\Delta B], \end{aligned} \tag{33}$$

which is the result used to obtain the curves shown in Figs. 2 and 3.

Next, consider the vector

$$\mathbf{B} \triangleq E[\mathbf{Z}D^*(t)] \tag{34}$$

in (19). Its lth component is

$$\begin{aligned} B_l &= E[z(t-l\Delta)D^*(t)] \\ &= (1+\beta)E\{[(d_1(t_l) + jd_2(t_l)]e^{-j\alpha} \\ &\quad + \beta[d_1(t_l-\tau_m) + jd_2(t_l-\tau_m)] \\ &\quad \cdot \exp[-j(\omega_0\tau_m + \alpha)] \\ &\quad + [n_c(t_l) + jn_s(t_l)][d_1(t) - jd_2(t)]\} \end{aligned} \tag{35}$$

where $t_l \triangleq t - l\Delta$. Again, assuming that $d_1(t)$, $d_2(t)$, and $\tilde{n}(t)$ are independent, we obtain (21). Assuming data and noise statistics as above, we obtain

$$\begin{aligned} B_l = (1+\beta)\ \Lambda[l\Delta/T_c]e^{-j\alpha} \\ + \beta\Lambda[(\tau_m + l\Delta)/T_c] \exp[-j(\omega_0\tau_m + \alpha)] \end{aligned}, \tag{36}$$

which was the expression used to obtain the data of Figs. 2 and 3.

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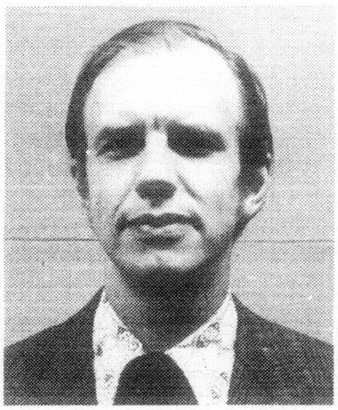

Rodger E. Ziemer (S'60–M'66–SM 70) was born in Sargeant, Minn. on August 22, 1937. He received the B.S., M.S.E.E., and Ph.D. degrees from the University of Minnesota, Minneapolis, in 1960, 1962, and 1965, respectively.

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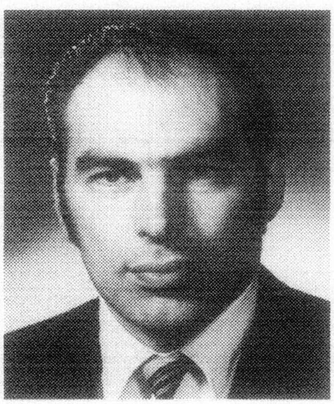

Carl R. Ryan (M'62) was born in Gateway, Ark. on March 3, 1938. He received the B.S.E.E. degree from the University of Arkansas in 1962, the M.S.E.E. degree from Iowa State University in 1963, and the Ph.D. degree from the University of Missouri-Rolla in 1969.

Since 1963 he has been employed by the Government Electronics Division of Motorola Inc., Scottsdale, Ariz., where he has been instrumental in developing their wideband signal processing technology. He has published various technical articles on system design and signal processing techniques which are capable of processing signals in excess of one gigabit per second. He is now primarily involved in high data rata, low-power, PCM system design and construction. He holds three patents.

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