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Computer Modeling Of The Statistical Properties Of Transionospheric Scintillation Channels

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Density \ Burst Length	1	2	3	4	5	6	7	8	9	10
1	100	71	67	66	66	66	66	66	66	66
2		29	28	26	24	24	24	23	23	23
3			5	6	6	5	4	4	4	4
4				2	3	4	3	3	3	3
5					1	1	2	2	2	2
6							1	1	1	1
7								1	1	1
8										
9										
10										

Fig. 11. Error density.

greater reliability and higher throughput (as a result of fewer retransmissions), easily justify the incremental product cost due to inclusion of the code. Implementation of the code can be accomplished by the traditional hardware approach or by the inclusion of programming routines to handle the encoding and/or decoding functions. The IBM Mobile Terminal System contains examples of both hardware and software implementations.

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Computer Modeling of the Statistical Properties of Transionospheric Scintillation Channels

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***Abstract*—This concise paper shows that a band-limited bivariate-normal time series is a useful model for the envelope of scintillating radio signals traversing the ionosphere. A software model is presented whose generated time series is compared to an actual block of scintillation data. A comparison of computer-generated and actual scintillation data shows that the fade-duration distribution of the simulated data fits that of the actual data closely through proper choice of model parameters.**

I. INTRODUCTION

Ionospheric scintillation in the polar and equatorial latitudes during local nighttime hours has been observed as a perturbation in transionospheric communications channels which causes signal envelope fades on the order of seconds. In order to predict the reliability of a transionospheric radio communications link a model of ionospheric scintillation is needed.

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The purpose of this concise paper is to present a useful computer model based on the band-limited bivariate-normal distribution that generates a time series whose fade-duration distribution approximates that of a sample of ionospheric scintillation data. The actual data were selected from a quasi-stationary portion of the received beacon signal from the Tacsat I satellite (250 MHz), recorded on Guam [1] in 1972. It is intended that this model will provide the groundwork for developing more precise models.

Model development is proceeding along two lines in the literature. Scintillation strength is studied as a function of ionospheric conditions. The statistical properties of scintillation (amplitude and spectral distributions) are studied independently. Since much data have been accumulated on scintillation strength, the modeling of scintillation strength as a function of ionospheric conditions is proceeding rapidly [2]. However, since no data are available on the amplitude and frequency distribution of the inphase and phase-quadrature components of a scintillating signal, modeling of the statistical properties of ionospheric scintillation has been directed at the amplitude distribution (i.e., the percent time signal level is below a given level) of the envelope.

The amplitude distribution of ionospheric scintillation has been approximated by the Rice–Nakagami [2], [3], log-normal [2], and bivariate-normal [2] distributions. A detailed comparison of these distributions in the literature [2] demonstrates that the bivariate-normal distribution yields the best representation of the amplitude distribution of ionospheric scintillation. The curve fitting is based on two parameters that are functions of in-phase noise power, phase-quadrature noise power, and covariance. Since these three independent parameters are represented by two control parameters, a range of each of these three parameters exists that will satisfy the restraints of the two control parameters.

While the amplitude distribution does provide useful data to the communications engineer, the fade-duration distribution would show how long fades are likely to last. The fade-duration distribution is the number of fades per unit time below a given level for a given length of time or longer. Knowledge of the amplitude and spectral statistics of the in-phase and phase-quadrature components of the scintillating signal would greatly facilitate the modeling procedure. Since this information is not available, an iterative procedure is used here to approximate the overall fade-duration distribution by manipulating the statistics of the in-phase and phase-quadrature components. In this fashion, a suitable model is obtained which fits the second-order fade-duration statistics of the envelope of the received signal in addition to the first-order intensity histograms.

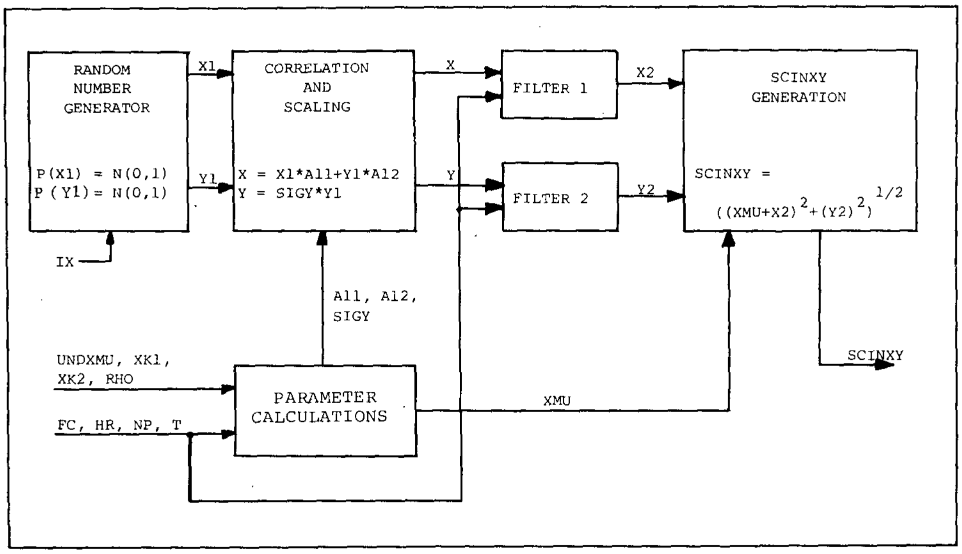

Fig. 1. Block diagram of model of ionospheric scintillation.

TABLE I

DEFINITIONS OF SUBROUTINE *SCINT* VARIABLES

Model Input Variables	
FC	*FC* is the cutoff frequency of Filter 1 and Filter 2 in hertz.
HR	*HR* is the desired time interval between data points in seconds.
IX	*IX* is the seed value used in the random number generator, GAUSS. The value for *IX* is chosen to give GAUSS a nearly normal distribution and to produce a long string of random numbers without repetition.
NP	*NP* is the number of poles of Filter 1 and Filter 2. *NP* can specify a 1, 2, 3, or 4 pole Butterworth filter.
RHO	*RHO* is the correlation coefficient between the in-phase and phase quadrature components, *X*1 and *Y*1, respectively.
T	*T* is the time corresponding to each data point. At $T = 0$ the model initializes itself and generates one data point. At $T > 0$, the model generates data points.
UNDXMU	*UNDXMU* is the undisturbed mean magnitude (rms voltage level) of the signal before scintillation starts.
*XK*1	*XK*1 is the ratio of phase quadrature noise power to total noise power.
*XK*2	*XK*2 is the ratio of noise power to total signal plus noise power.
*E*1	*E*1 is an error parameter that tells the calling program that $NP(E1 = 1)$, $FC(E1 = 2)$, or $HR(E1 = 3)$ has been changed without resetting *T* to zero.
SCINXY	*SCINXY* is the magnitude of the signal level as it scintillates: $SCINXY = [(XMU + X2)^2 + Y2^2]^{1/2}$.

II. COMPUTER MODEL DESCRIPTION

A functional block diagram of the proposed model is shown in Fig. 1. The variables *FC*, *HR*, *IX*, *NP*, *RHO*, *T*, *UNDXMU*, *XK*1, and *XK*2 are the external variables that control the model. *SCINXY* and *E*1 are the output variables. These variables are defined in Table I.

The model begins with a Gaussian random number generator, subroutine GAUSS. Each call to GAUSS yields two uncorrelated random numbers, *X*1 and *Y*1. For this model, the time series of *X*1 and *Y*1 each has a mean of zero and a variance of unity. Random numbers *X* and *Y* are obtained by a linear transformation of *X*1 and *Y*1 that yields the specified variance and correlation corrected for the attenuation of Filter 1 and Filter 2. Filter 1 and Filter 2 are Butterworth filters obtained by the bilinear *Z* transform [4]. Their order and cutoff frequency are specified by *NP* and *FC*, respectively. Random numbers *X*2 and *Y*2, the outputs of these filters, are the in-phase and phase-quadrature components of the received signal. Choosing the sample time *HR* to be small compared to $1/FC$ will cause *X* and *Y* to be essentially white noise sources in the passband of the filters. Thus, *X*2 and *Y*2 then have the power spectral density provided by Filter 1 and Filter 2. *XMU* is the steady-state component of the received signal. The instantaneous received signal level *SCINXY* is calculated according to

$$SCINXY = ((XMU + X2)^2 + (Y2)^2)^{1/2} \quad (1)$$

where

$$XMU^2 = UNDXMU^2\ (1\text{-}XK2) \quad (2)$$

$$\sigma_{x2}{}^2 = (1\text{-}XK1) * XK2 * UNDXMU^2 \quad (3)$$

and

$$\sigma_{y2}{}^2 = XK1 * XK2 * UNDXMU^2. \quad (4)$$

Since *UNDXMU* is the mean rms signal level, $UNDXMU^2$ is the mean rms power level. *XK*2 is the fraction of the total signal power that is diverted into the scintillating portion of the signal. Therefore, $(1\text{-}XK2) \times UNDXMU^2$ is the power in the nonscintillating portion of the signal as stated in (2). *XK*1 is the fraction of the noise power that resides in the out-of-phase scintillating component so that (1-*XK*1) is the fraction of the noise power that resides in the in-phase scintillating component. Since the total noise power is $XK2 \times UNDXMU^2$ and the signal variance is the signal power, (3) and (4) are written directly as above.

III. FITTING SCINTILLATION PARAMETERS

In order to select parameters for the model that will represent the envelope of ionospheric scintillation, actual scintillation data were obtained from tests conducted by the Naval Electronics Laboratory, San Diego, Calif. The instantaneous envelope of the 250-MHz beacon signal of Tacsat I was recorded on FM analog tape for various site configurations on Guam. Further details on the recorded data and subsequent statistical analyses results are available [1].

The analog records were then sampled at 0.2-s intervals. A stationary region of 4096 data points was selected using the run and trend tests [5] at the 10-percent level of significance. These data were analyzed and normalized such that the total signal power would be unity. Thus, the undisturbed nonscintillating signal should have a mean rms magnitude of unity. This mean in-phase magnitude drops during scintillation due to the fact that power from the signal is di-

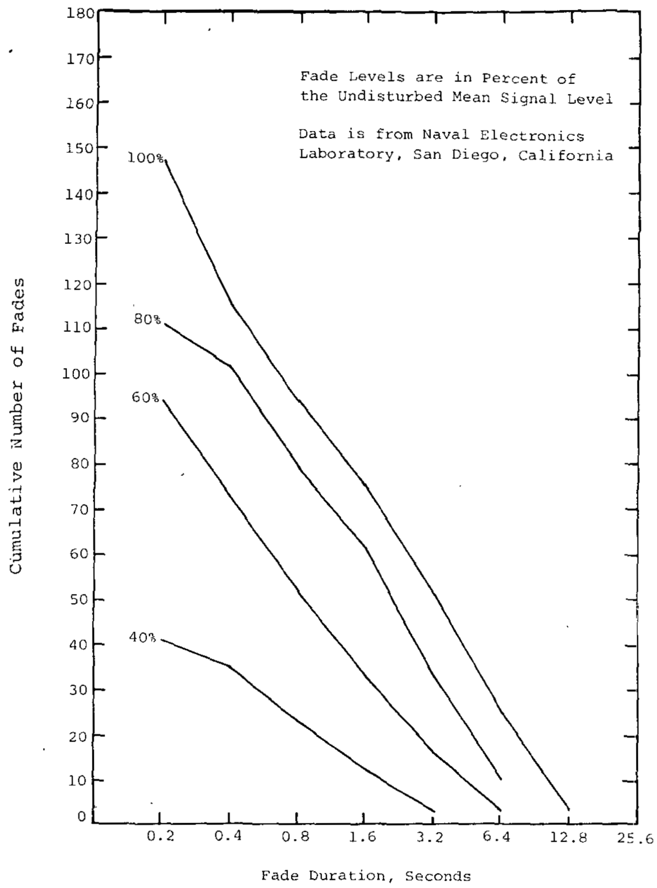

Fig. 2. Fade-duration distribution for 4096 scintillation data points spaced at 0.2-s intervals.

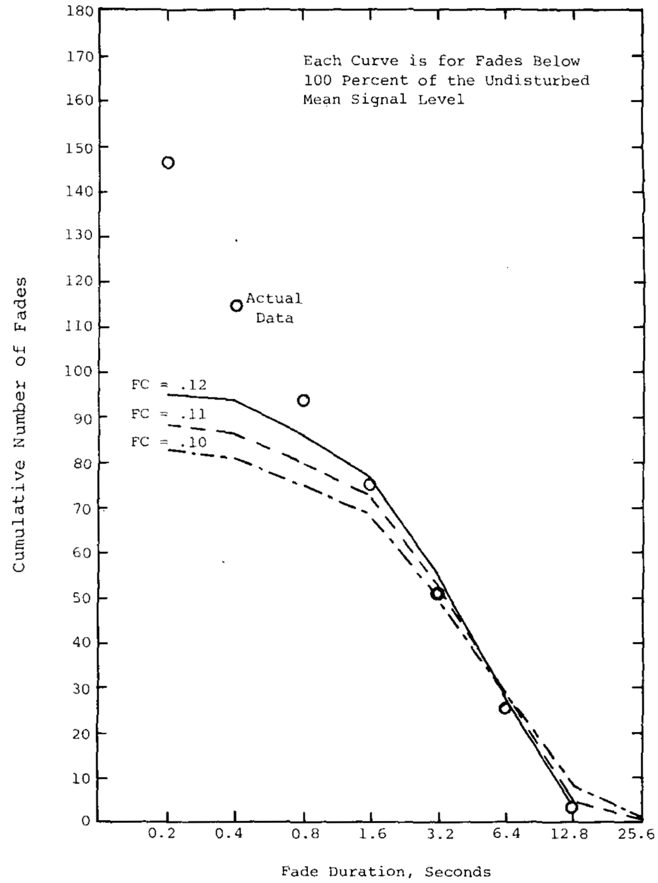

Fig. 3. Comparison of FC for $NP = 2$.

verted into the in-phase and phase-quadrature scintillating components. The fade levels chosen for the fade-duration distribution were 100, 80, 60, 40, and 20 percent of the undisturbed mean signal level. Fig. 2 shows the fade-duration distribution for this sample of scintillation data. The leftmost tip of each curve indicates the total number of fades below that level. The normalized mean and variance are 0.9352 and 0.1254, respectively. No fades were observed below the 20-percent level.

The first model parameters determined were the filter order NP and the cutoff frequency FC. Rough information in the literature [1], [2], [6] on the spectral distribution of the envelope of ionospheric scintillation served as a guide to the choice of FC. Figs. 3 and 4 show the fade-duration distribution below 100 percent of the undisturbed signal level for a second-order and a first-order Butterworth filter, respectively, at selected frequencies along with the actual data. The fade-duration distribution for fades below 100 percent of the undisturbed signal level is a measure of spectral distribution. That is, no matter how strong the fades, their mean level crossing depends on the spectral distribution of the random process. In each of these cases, care was taken to match the data at long fade durations, where communications systems are very likely to be susceptible. Fig. 3 shows that the second-order filter has far too few short fades due to the lack of high frequency content in its output. Alternatively, as shown on Fig. 4, the first-order filter has too many short fades. The control parameters $XK1$, $XK2$, and RHO, were varied to compensate for the imperfect spectral distribution and to achieve the best possible fit to the relative curve spacing.

Table II shows the effect of the control parameters $XK1$, $XK2$, and RHO on the fade-duration distribution. The effects of $XK1$ and $XK2$ were consistant regardless of cutoff frequency, filter order, and correlation coefficient. However, the effects of the correlation coefficient depend on the other conditions, as shown in Table II.

The model using the second-order filter did not yield good results for short fades. The best results for this model are shown in Fig. 5.

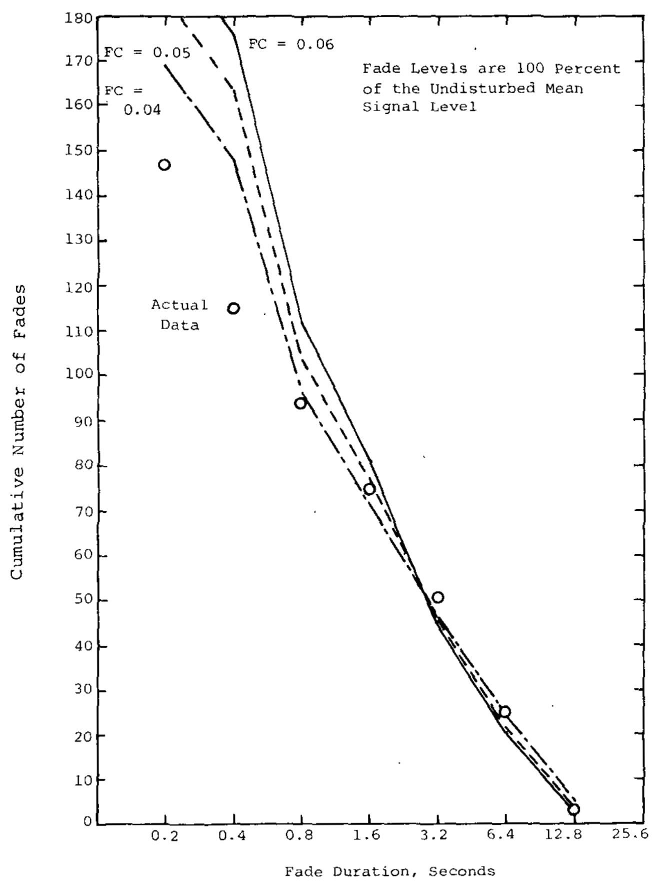

Fig. 4. Comparison of FC for $NP = 1$.

TABLE II
EFFECTS OF $XK1$, $XK2$, NP, AND RHO ON THE FADE-DURATION DISTRIBUTION

			Fade Levels[a] 20 percent		40 percent		60 percent		80 percent		100 percent			
Quiescent Conditions	VAR	R	SF	LF	SF	LF	SF	LF	SF	LF	SF	LF	Mean	Variance
$FC = 0.2$, $XK1 = 0.5$ $XK2 = 0.5$, $RHO = 0$	NP	$1 \rightarrow 4$	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	—	—
$FC = 0.2$, $XK2 = 0.5$ $NP = 2$, $RHO = 0$	$XK1$	$0.1 \rightarrow 0.9$	↓	↑	↓	↓	↓	↓	↑	↓	↑	—	↑	↓
$FC = 0.2$, $XK1 = 0.5$ $NP = 2$, $RHO = 0$	$XK2$	$0.1 \rightarrow 0.9$	↑	↑	↑	↑	↑	↑	↑	↑	—	—	↓	↑
$FC = 0.11$, $XK1 = 0.8$ $XK2 = 0.5$, $NP = 2$	RHO	$0 \rightarrow 0.9$	↓	↓	↓	↓	↓	↓	↑	—	↓	↑	—	—
$FC = 0.4$, $XK1 = 0.1$ $XK2 = 0.152$, $NP = 1$	RHO	$0 \rightarrow 0.9$	↓	↓	↓	—	↑	—	↑	—	↑	↓	—	—

As the variable (VAR), is extended over the range (R), in the sequence shown, the mean, variance, and the number of short fades (SF) and long fades (LF), for the fade levels shown, increase (↑), decrease (↓), or do not change significantly (—).
[a] Fade levels are in percent of the rms level of the nonscintillating signal.

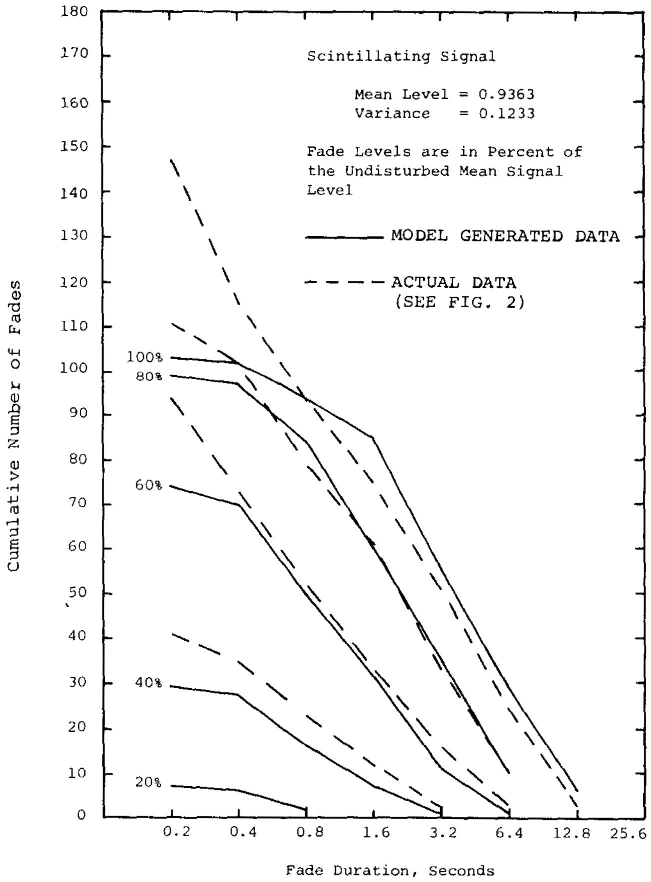

Fig. 5. Fade-duration distribution for model, $XK1 = 0.8$, $XK2 = 0.5$, $RHO = 0.3$, $FC = 0.11$, $NP = 2$.

However, the first-order filter yielded a fairly good fit without varying RHO as shown on Fig. 6 ($RHO = 0$). The data for $RHO = 0.6$, shown in Fig. 7, were further refinements based on the least-squared error between actual and model-generated fade-duration distributions. The fade-duration distribution for $RHO = 0.9$ is also included in Fig. 7 at the 100 and 20 percent levels for comparison.

A decrease in the number of fades below the 20 percent level is shown for $\rho = 0.9$ over that for $\rho = 0.6$, but a worse overall fit to the actual data results, especially for fade durations between 0.8 and 3.2 s at the 100 percent level. Fig. 8 shows the variation of fade-duration distribution as the parameter $XK1$ is varied from 0.2 to 0.7. In general, the fit is very close for the longer fade durations, but departs markedly from the measured curves at the short fade durations, thus indicating that an increase in ρ or filter order is necessary to decrease the number of short fades.

To summarize, the simulation results shown in Figs. 5–8 indicate that a good fit to the measured data is fairly easy to achieve for the longer fade durations, which are a more important consideration in communications system design than short fades. To obtain a good overall fit, however, requires more complete measured data than employed here.

IV. CONCLUSIONS

The model that has been developed provides data which are a reasonable approximation to the actual data in terms of fade-duration

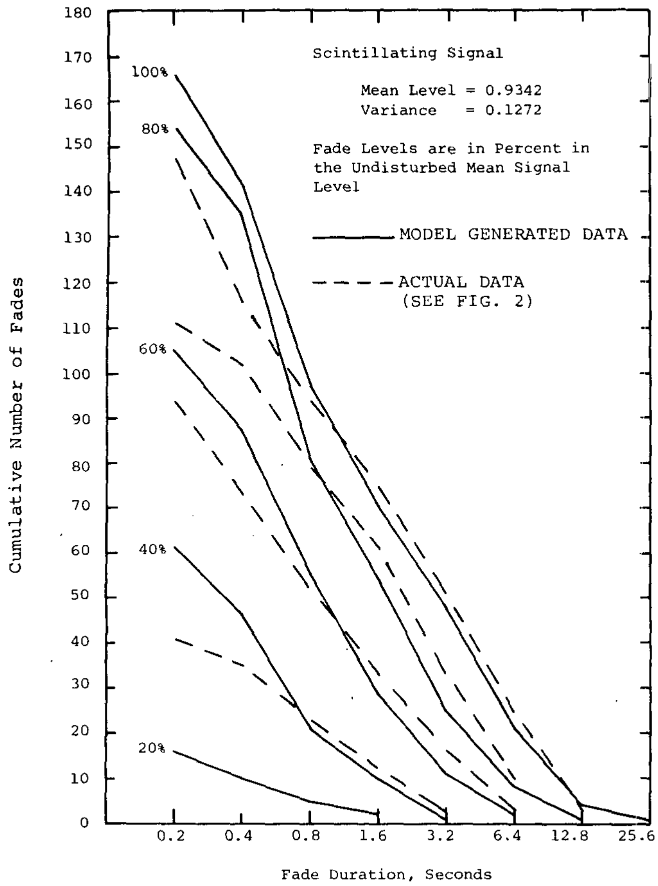

Fig. 6. Fade-duration distribution for model, $XK1 = 0.1$, $XK2 = 0.152$, $RHO = 0$, $FC = 0.04$, $NP = 1$.

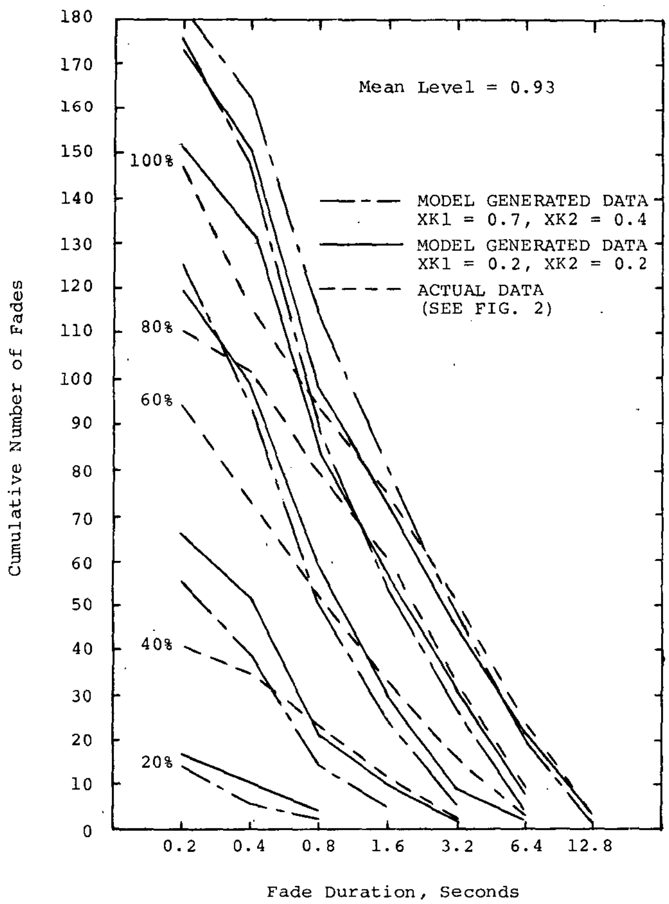

Fig. 7. Fade-duration distribution for model, $XK1 = 0.1$, $XK2 = 0.152$, $RHO = 0.6$, $FC = 0.04$, $NP = 1$.

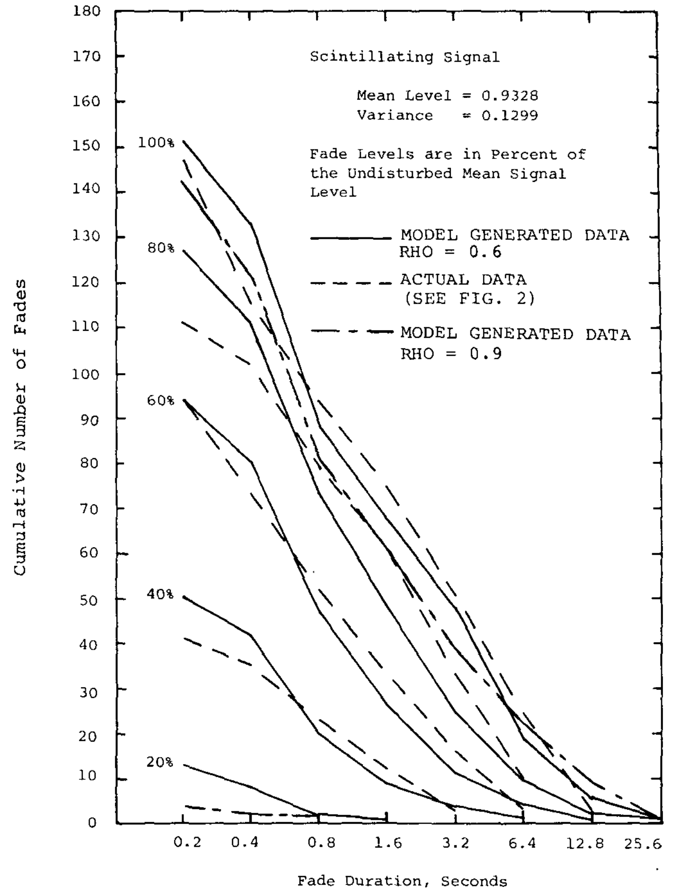

Fig. 8. Fade-duration distribution for model, $RHO = 0$, $FC = 0.05$, $NP = 1$.

distribution. From the appearance of the curves, the band-limited, bivariate-normal time series employed for the model appears to give a suitable simulation of ionospheric scintillation. If a more flexible filter had been available, a better simulation of the data may have been achieved. It is not assumed that any of the parameters that have been derived are the actual parameters of ionospheric scintillation. Instead, a combination of parameters has been chosen based on a restricted spectral distribution to form an imperfect, but suitable, fit to the data. Recent analyses by Rino *et al.* [7] suggest that the phase-quadrature Gaussian model is accurate, with more than 90 percent of the scattered power in phase quadrature with the deviated signal.

It should be stressed that this model is based on only one sample time series of ionospheric scintillation. It is possible that parameters other than $XK2$ may vary as scintillation strength varies. Work is needed in determining how the input parameters will change with scintillation strength.

The experiment suggested in Fremouw [2] would take much of the guesswork out of modeling this phenomenon. Basically, two or more signals would be transmitted through the ionosphere simultaneously with one signal at a frequency high enough to be relatively unaffected by the scintillation. This higher frequency signal would be a phase reference so that the in-phase and phase-quadrature components of scintillation could be measured separately. This would enable the analyst to design a subroutine that would closely approximate the actual amplitude and frequency distribution of each scintillation component.

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Probability of Error in Pseudonoise (PN)-Modulated Spread Spectrum Binary Communication Systems

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***Abstract*—The probability of error in a binary communication system using pseudonoise (PN) modulation to spread the transmitted spectrum is influenced by noise, bandlimiting, and synchronization errors. In this paper the average probability of error is determined, using a series expansion of the characteristic function of the intersymbol interference. It is shown that the effects of intersymbol interference are reduced by spreading and that local code synchronization errors become the dominant factor in the probability of error when the system bandwidth is large.**

INTRODUCTION

In modern digital communication systems, there are many applications for spectrum spreading, including multiple access to a single channel, private communications, and reduction of power flux density to meet statutory or treaty limitations [1]. One of the most popular techniques for spectrum spreading is modulation by a pseudonoise (PN) binary code waveform prior to transmission. Many of the applications require expansion of the transmitted signal bandwidth. In PN-modulated systems, bandwidth expansion is accomplished by using a PN code with a keying rate greater than that message sequence. The bandwidth of such a system, particularly that of the receiver, can become a critical design parameter. It is therefore of interest to analyze the performance of a PN-modulated binary communication system in the presence of bandlimiting and Gaussian noise. Many papers [2]–[7] have been written on the subject of bandlimited binary communication systems. However, the spread spectrum system has received little attention.

SYSTEM DESCRIPTION

A baseband model of a PN-modulated binary communication system is shown in Fig. 1. Bandlimiting is represented by the lowpass filter in the correlation receiver. Recovery of the signal requires that a local replica of the spreading code be synchronized to the received code and multiplied with the received signal prior to the integrate-and-dump message symbol detector [8]. It can easily be shown that the correlation receiver is an optimum receiver for *PN*-modulated binary signals in the absence of bandlimiting.

If T is the duration of one message symbol and τ is the duration of one code symbol, then the spreading ratio, r, is defined by

$$r = T/\tau \geq 1 \tag{1}$$

and the kth pulse of the code waveform is

$$\phi_k(t) = \begin{cases} (1/T)^{1/2}, & k\tau < t \leq (k+1)\tau \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

For the purposes of this paper, r is assumed to be an integer and generally much greater than one, although the results derived will be valid for $r = 1$. Then the transmitted signal can be expressed as

$$s(t) = E^{1/2} \sum_{i=-\infty}^{\infty} \sum_{k=ir}^{(i+1)r-1} M_i c_k \phi_k(t) \tag{3}$$

where E is the energy per message symbol, M_i is the transmitted message symbol (± 1 with equal probability), and c_k is the kth coefficient of the spreading code (± 1 with approximately equal probability).

If it is assumed that an exact integer number of code symbols occur during a message symbol, i.e., r is an integer, then (3) can be expressed as

$$s(t) = E^{1/2} \sum_{k=-\infty}^{\infty} b_k \phi_k(t) \tag{4}$$

$$b_k = \begin{cases} c_k, & M_i = +1 \\ -c_k, & M_i = -1 \end{cases} \tag{5}$$

and the indices i and k are related by $ir < k \leq (i+1)r - 1$.

At the receiver, the output of the filter is

$$\begin{aligned} \tilde{r}(u) &= \tilde{s}(u) + \tilde{n}(u) \\ &= E^{1/2} \sum_k b_k \tilde{\phi}_k(u) + \tilde{n}(u) \end{aligned} \tag{6}$$

where the tilde (~) indicates filtered waveforms. Because of time delay and distortion in the filtering process, the time origin ($u = 0$) has been adjusted to minimize the probability of error. Also, it is assumed that the integrate-and-dump detector is synchronized to the local code generator in such a manner that any timing error of the local code, say $\Delta\tau$, appears in the timing of the detector, i.e., the interval of integration for the jth symbol is from $(jr + \Delta)\tau$ to $[(j+1)r - 1 + \Delta]\tau$. Since the message symbols are assumed to be equally likely, the problem can be confined to the zeroth symbol without loss of generality. The output of the integrator at the sampling instant is

$$\begin{aligned} e_0 &= \int_{\Delta\tau}^{T+\Delta\tau} \tilde{r}(u)\, du \\ &= E^{1/2} \int_{\Delta\tau}^{T+\Delta\tau} \sum_j c_j \phi_j(u - \Delta\tau) \sum_k b_k \tilde{\phi}_k(u)\, du \\ &\quad + \int_{\Delta\tau}^{T+\Delta\tau} \tilde{n}(u) \sum_j c_j \phi_j(u - \Delta\tau)\, du \\ &= E^{1/2} M_0 (e_S + e_I + e_N) \end{aligned} \tag{7}$$

where e_S, e_I, and e_N are the outputs due to desired signal, intersymbol interference, and Gaussian noise, respectively.

It can be shown in a straightforward exercise that the variance of the noise output is unchanged by the code multiplier if the detector and local code are synchronized. If the noise process prior to filtering is white Gaussian noise with spectral density $N_0/2$, then the variance of e_N is

$$\text{var}\,(e_N) = \sigma = N_0 B_N / 2E \tag{8}$$

where

$$B_N = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2(\omega\tau/2)}{(\omega\tau/2)^2} |H(\omega)|^2\, d\omega. \tag{9}$$

By interchanging the order of integration and summation, (7) can be manipulated to show that

$$e_S = F_0(\Delta) \tag{10}$$

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