01 Jan 1975

Verifications Of The Kalman Conjecture For Irrational Transfer Functions

D. Ronald Fannin
Missouri University of Science and Technology

Charles E. Connelly

Recommended Citation

D. R. Fannin and C. E. Connelly, "Verifications Of The Kalman Conjecture For Irrational Transfer Functions," *Proceedings of the IEEE*, vol. 63, no. 6, pp. 982 - 983, Institute of Electrical and Electronics Engineers, Jan 1975.
The definitive version is available at https://doi.org/10.1109/PROC.1975.9865

concentrated. In other discharge regions, D and ΔD are negligibly small and the enhancement in ionization rate is generally greater than that of the diffusion current, thus causing the detected signal to be positive. In other words, in the regions where the drift current is strong or the average electron energy is already high, the ionization rate is most susceptible to be increased as a result of incident microwave radiation. Such regions include the cathode fall where the potential gradient is at its maximum, and the anode region where the accelerating potential and ν_i are very high and dn/dx increases. Therefore, it is easy to understand result 5), obtained in the experiments of Lampert and White and those of Udelson, in addition to results 2) and 3) obtained by Chiplonkar and Gokhale. The roles of collision and spectral frequencies and gas ionization potential in the enhanced ionization process have already been discussed by Farhat [9].

In triode lamps, it is possible to observe each enhanced current separately at low discharge currents. The positive signal observed at the anode is the enhanced drift current, while the negative signal sensed at a floating electrode (internal or external) is the enhanced diffusion of electrons in all directions away from the negative space charge region. As the discharge current is increased, the volume taken up by the negative space charge region increases too, and eventually aproaches the sensing internal electrode or glass wall adjacent to a sensing external electrode [10]. Consequently, the diffusion current reaching this electrode decreases until finally the major component of the detection current picked up by the sensing floating electrode becomes the drift current. Since the diffusion current is in all directions away from the space charge region, only a small proportion of it is collected by the sensing electrode. Hence the floating electrode mode of sensing is more effective at higher discharge currents where the drift current is predominant. Thus result 1) is now explained. It is interesting to note that, in the TRJ250 and TRQ250 triode trigger tubes, the high responsitivity sensed by the floating electrode [3]-[5] can also be achieved by sensing from the anode at the same high discharge currents. This means that the novel biasing technique [5] so successful in noise suppression is not the reason for the high responsivity of these devices at millimeter wave frequencies. It rather seems that it is the very small interelectrode spacing (0.5 mm) and the consequential higher potential gradients of these devices that are responsible for their high responsivity. The effect of the high potential gradient is an enhancement of the drift current and inelastic collision rate which depend upon the local electric field strength. Thus from the point of view of electrode geometry and biasing, improving the potential gradient in a glow discharge enhances the responsivity. The ultimate limit imposed upon this improvement by the collision frequency has already been dealt with by Farhat [9]. Thus result 4) has now been explained.

Similar arguments concerning potential gradient can account for the experimental disagreements observed by Lampert and White and Udelson described above as result 6). In Lampert and White's experiment the negative glow was so close to the cathode that no cathode glow was observed, whereas in Udelson's experiment, the cathode glow was clearly observed, presumably because of the large electrode separation. Furthermore, the discharge current in the former experiment is a thousand times larger than that used in Udelson's experiment. The electrodes in each experiment were of about the same size ($\simeq 0.3$ cm). Since Lampert and White's experiment involved not only a much shorter electrode separation, but also a considerably larger discharge current, it seems reasonable to assume that the potential gradient in the former experiment must have been considerably greater than that in the latter experiment. Accordingly, on the basis of (4) and its potential gradient implications, result 6) now becomes clear.

References

[1] N. H. Farhat, "A plasma microwave power density detector," *Proc. IEEE*, vol. 52, pp. 1053-1054, Sept. 1964.

[2] D. C. McCain, "A plasma video detector," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 64-65, Jan. 1970.

[3] N. S. Kopeika, "Millimeter wave detection with glow discharge plasma and its application in holography," Ph.D. dissertation, Univ. of Pennsylvania, Philadelphia, 1971.

[4] N. S. Kopeika and N. H. Farhat, "An economical, fast, and sensitive millimeter wave video detector," *European Conf. Electrotechnics, Eurocon 1974, Conf. Dig.*, pp. C 7-6(1)-C 7-6(2).
——, "Video detection of millimeter waves with glow discharge tubes," submitted for publication.

[5] N. H. Farhat and N. S. Kopeika, "A low-cost millimeter-wave glow-discharge detector," *Proc. IEEE*, vol. 60, pp. 759-760, June 1972.

[6] N. S. Kopeika, "The influence of external electrodes in millimeter wave video detection with glow discharge plasmas," in *1974 IEEE Nat. Telecommunications Conf. Rec.*, pp. 1069-1073, Dec. 1974.

[7] G. D. Lobov, "Gas discharge detector of microwave oscillations," *Radiotekh. Elektron*, vol. 5, pp. 152-165, 1960.

[8] P. J. W. Severin, "The interaction of microwaves with the cathode fall and negative glow in a glow discharge," Ph.D. dissertation, Univ. of Utrecht, Utrecht, The Netherlands, 1964; also *Philips Res. Rep. Suppl.*, no. 2, 1965.

[9] N. H. Farhat, "Optimization of millimeter-wave glow-discharge detectors," *Proc. IEEE*, vol. 62, pp. 279-281, Feb. 1974.

[10] N. S. Kopeika, "The applicability of glow discharge plasma theory to the TRJ250 glow tube for millimeter wave detector applications," *Proc. 1st Symp. Joint Chapter of Microwaves, Antennas, and Electron Devices, IEEE—Israel Section*, Apr. 1974.
N. S. Kopeika and N. H. Farhat, "Langmuir probe analysis of the TRJ250 glow tube for millimeter wave detector applications," submitted for publication.

[11] V. T. Chiplonkar and V. D. Gokhale, "Effect of weak r.f. signal on the electron energy distribution in a gas discharge," *Int. J. Electron.*, vol. 25, pp. 251-255, 1973.

[12] M. A. Lampert and A. D. White, "Microwave techniques for studying discharges in gases," *Electron. Commun.*, vol. 30, pp. 124-128, June 1953.

[13] B. J. Udelson, "Effect of microwave signals incident upon different regions of a dc hydrogen glow discharge," *J. Appl. Phys.*, vol. 28, pp. 380-381.

[14] S. C. Brown, *Introduction to Electrical Discharges in Gases.* New York: Wiley, 1966, pp. 43-50.

[15] L. Goldstein, "Gaseous electronics," in *Advances in Electronics and Electron Physics*, vol. VII. New York: Academic Press, 1955, pp. 416-426.

Verifications of the Kalman Conjecture for Irrational Transfer Functions

D. RONALD FANNIN AND CHARLES E. CONNELLY

Abstract—**The sufficient conditions for the Kalman conjecture to be satisfied given by Fannin and Rushing [1] are relaxed in such a way as to allow their approach to be applied to irrational transfer functions. Specific results for a class of systems containing transportation lag are presented.**

Recently, Fannin and Rushing [1] have presented a theorem setting forth sufficient conditions for a stability criterion, due to Cho and Narendra [2], to be satisfied in such a way that the Kalman conjecture [3] is also satisfied. The approach is based on a consideration of the curvature of the Nyquist locus of the linear part of the system, and has been used to extend the class of systems known to satisfy the Kalman conjecture. This letter shows that it is possible to relax the conditions of the Fannin-Rushing theorem in such a way that it may be applied to irrational transfer functions.

Consider a system of the form shown in Fig. 1. It is assumed that the nonlinearity is single-valued and time-invariant, and satisfies

$$u(0) = 0, \qquad 0 \leq du/de \leq k. \tag{1}$$

The linear part described by $G(s)$ is assumed to be time-invariant and output stable. Condition 1) of the Fannin-Rushing theorem further restricts $G(s)$ to be a rational function of s. However, the only properties of rational functions which are used in their proof are that $G(0)$ and $\lim_{\omega\to\infty} G(j\omega)$ are finite and real. Since the only conditions imposed by the Cho and Narendra criterion are that $G(s)$ be time-invariant and output stable, condition 1) of the theorem may be relaxed as follows.

Theorem: Given a function $G(j\omega)$ such that

1) $G(0)$ and $\lim_{\omega\to\infty} G(j\omega)$ are finite and real;
2) $|G(j\omega)|$ is bounded and monotonically decreasing for all $\omega \geq 0$;
3) the curvature of $G(j\omega)$ exists and is in the same nonzero sense for all $\omega \geq 0$.

Then

a) there exists a left-most point at which the locus of $G(j\omega)$ intersects the real axis (the point $(-1/k, 0)$, where $1/k$ is finite);

Manuscript received August 23, 1974.
The authors are with the Department of Electrical Engineering, University of Missouri, Rolla, Mo. 65401.

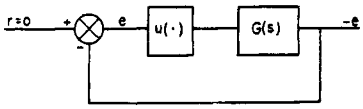

Fig. 1. Class of systems under consideration.

b) a line can be drawn through the point $(-(1/k+\delta), 0)$, $\delta > 0$ and arbitrarily small, such that the locus of $G(j\omega)$ lies to the right of the line for all $\omega \geqslant 0$; this line is parallel to the tangent line of the locus of $G(j\omega)$ at the point $(-1/k, 0)$.

Proof: From the discussion it is seen that the proof is identical to the proof given in [1].

This simple relaxation of condition 1) allows the theorem to be applied to irrational transfer functions meeting the conditions of the theorem. Conditions 1) and 2) are not difficult to check, but 3) promises to be rather tedious for most irrational transfer functions. However, considerable simplification may result if $G(s)$ is composed of a rational transfer function followed by pure delay.

Let $G(s)$ be given by

$$G(s) = G_1(s)\, e^{-sT}, \qquad T \geqslant 0 \tag{2}$$

where

$$G_1(j\omega) = R(\omega)\, \underline{/\theta(\omega)}. \tag{3}$$

Then

$$G(j\omega) = R(\omega)\, \underline{/\theta(\omega) - \omega T}. \tag{4}$$

The formula for curvature of a plane curve in polar coordinates is

$$C = \frac{R^2 + 2\,(dR/d\phi)^2 - R\,(d^2R/d\phi^2)}{[R^2 + (dR/d\phi)^2\,]^{3/2}}. \tag{5}$$

Only the numerator need be considered, and using (4) it may be put in the form

$$NC = \frac{R^2 + 2(dR/d\theta)^2 - R(d^2R/d\theta^2)}{((\theta' - T)/\theta')^3} + \frac{TR^2(3\theta'^2 - 3T\theta' + T^2)}{(T - \theta')^3} + \frac{T(2R'^2 - RR'')}{(T - \theta')^3} \tag{6}$$

where the primes indicate differentiation with respect to ω. The numerator of the first term in (6) may be recognized as the numerator of the curvature of $G_1(j\omega)$, denoted NC1. Hence

$$\mathrm{NC} = \frac{\mathrm{NC1}}{((\theta' - T)/\theta')^3} + \frac{TR^2(3\theta'^2 - 3T\theta' + T^2)}{(T - \theta')^3} + \frac{T(2R'^2 - RR'')}{(T - \theta')^3}. \tag{7}$$

Equation (6) or (7) must be checked for sign invariance if condition 3) is to be verified for systems with transportation lag. In some cases, however, further simplification is possible.

Suppose $G_1(j\omega)$ has curvature always in the same sense, such that

$$\mathrm{NC1} > 0, \qquad \forall\, \omega \geqslant 0. \tag{8}$$

Further, suppose that $G_1(s)$ has only left-hand plane poles (requiring the constraints of controllability and observability, in addition to output stability) and no numerator dynamics. Then $\theta' < 0$ and the first two terms of (7) are positive. If

$$2R'^2 - RR'' > 0, \qquad \forall\, \omega > 0 \tag{9}$$

then the curvature is always positive.

Consider (2) with

$$G_1(s) = \frac{A}{(s+p_1)^n\,(s+p_2)^m}, \qquad A, p_1, p_2, n, m > 0, \quad n \text{ and } m \text{ integers.} \tag{10}$$

It is straightforward (though the algebra is tedious) to show that G_1 given by (10) satisfies the Kalman conjecture, since its curvature is always in the same sense. Thus it is only necessary to verify (9). The result is

$$\begin{aligned}2R'^2 - RR'' = \{&[(n+m)^2 - n - m]\omega^6 + [p_1^2(2m^2 - 2m + 2nm + n) \\ &+ p_2^2(2n^2 - 2n + 2nm + m)]\,\omega^2 + [2p_1^2p_2^2(n + m + nm) \\ &+ p_1^4 m(m-1) + p_2^4 n(n-1)]\,\omega^2 + [mp_1^4p_2^2 + np_1^2p_2^4]\}/ \\ &[(\omega^2 + p_1^2)^{n+2}\,(\omega^2 + p_2^2)^{m+2}]\end{aligned} \tag{11}$$

which is clearly positive under the stated conditions. Thus transfer functions of the form (2) with $G_1(s)$ given by (10) satisfy the Kalman conjecture. This result subsumes all results given in [1].

References

[1] D. R. Fannin and A. J. Rushing, "Verification of the Kalman conjecture based on locus curvature," *Proc. IEEE*, vol. 62, pp. 542–543, Apr. 1974.

[2] Y. Cho and K. S. Narendra, "An off-axis circle criterion for the stability of feedback systems with a monotonic nonlinearity," *IEEE Trans. Automat. Contr.*, vol. AC-13, pp. 413–416, Aug. 1968.

[3] J. C. Hsu and A. U. Meyer, *Modern Control Principles and Applications.* New York: McGraw-Hill, 1968.

Comments on "Effect of Source Lead Inductance on the Noise Figure of a GaAs FET"

SVEIN IVERSEN

In the above letter,[1] equations (4) and (5) giving the two noise sources u and i are not strictly correct. The problem is sketched in Fig. 1. The first step may be to replace Fig. 1(a) by Fig. 2(a).

It is easily found that

$$u_1 = -\frac{y'_{22}}{\Delta y'} \cdot i_{ng} - \frac{y'_{12}}{\Delta y'} \cdot i_{ch} \tag{1}$$

$$u_2 = \frac{y'_{21}}{\Delta y'} \cdot i_{ng} + \frac{y'_{11}}{\Delta y'} \cdot i_{ch} \tag{2}$$

where

$$\Delta y' = y'_{11}y'_{22} - y'_{12}y'_{21}. \tag{3}$$

When the equivalent in Fig. 2(b) is replaced by that in Fig. 1(b), the following equations are valid:

$$u = u_1 + \frac{y_{22}}{y_{21}}\, u_2 \tag{4}$$

$$i = \frac{\Delta y}{y_{21}}\, u_2. \tag{5}$$

Putting (1) and (2) into (4) and (5) we get

$$u = \frac{y_{22}y'_{21} - y_{21}y'_{22}}{y_{21}\,\Delta y'}\, i_{ng} + \frac{y_{22}y'_{11} - y_{21}y'_{12}}{y_{21}\,\Delta y'}\, i_{ch} \tag{6}$$

$$i = \frac{\Delta y \cdot y'_{21}}{\Delta y' \cdot y_{21}}\, i_{ng} + \frac{\Delta y \cdot y'_{11}}{\Delta y' \cdot y_{21}}\, i_{ch} \tag{7}$$

where

$$\Delta y = \Delta y'/(1 + j\omega L_S \cdot \Sigma y')$$

and

$$\Sigma y' = y'_{11} + y'_{12} + y'_{21} + y'_{22}.$$

Manuscript received August 30, 1974; revised October 17, 1974.

The author is with the Teleteknisk Laboratorium, University of Trondheim, 7034 Trondheim-NTH, Norway.

[1] A. Anastassiou and M. J. O. Strutt, *Proc. IEEE*, vol. 62, pp. 406–408, Mar. 1974.