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Comments On “A Counterexample To Harmonic Linearization”

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Proceedings Letters

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Comments on "A Counterexample to Harmonic Linearization"

D. R. FANNIN

In the above letter,[1] the author presents the following system as a counterexample to harmonic linearization:

$$\frac{d^3y}{dt^3} + \epsilon b_1 \frac{d^2y}{dt^2} + (1+\epsilon b_2)\frac{dy}{dt} + \epsilon b_1 y + \epsilon \frac{dy^3}{dt} = 0,$$

$$0 < \epsilon << 1, \quad b_1 > 0, \quad b_2 > 0. \quad (1)$$

Assuming

$$y = a_1 \sin \omega t \quad (2)$$

the conditions for harmonic balance of frequency ω are obtained as

$$\epsilon b_1 a_1 - \epsilon b_1 \omega^2 a_1 = 0$$

$$-\omega^3 a_1 + \omega a_1 + \epsilon b_2 \omega a_1 + \frac{3\epsilon\omega a_1^3}{4} = 0. \quad (3)$$

Applying the usual procedures for harmonic linearization Singh lets

$$\omega^2 = 1 + \epsilon\beta \quad (4)$$

and determines that an oscillation is predicted for $a_1 \neq 0$ when, in fact, an exact analysis shows the system to be globally asymptotically stable. While the system does constitute a counterexample to harmonic linearization, it appears that this technique is not the natural one to apply to the problem in the first place.

Rather than apply the relation (4), a more exact analysis may proceed from (3). From the first equation, a nonzero value of a_1 is obtained if and only if $\omega = 1$. For this ω the second equation of (3) is satisfied only when $a_1 = 0$, since a_1 is nonnegative. Thus no oscillation is predicted. It is straightforward to show that the foregoing analysis is exactly equivalent to applying the describing function analysis to a single-loop system with

$$G(s) = \frac{\epsilon s}{s^3 + \epsilon b_1 s^2 + (1+\epsilon b_2)s + \epsilon b_1}, \quad f(e) = e^3. \quad (5)$$

A Routh array indicates no crossing of the imaginary axis for any allowable value of the real describing function. Hence no oscillation is predicted. Since both of the analyses presented here involve a lesser degree of approximation than represented by harmonic linearization and (4), one wonders at Singh's choice of analysis technique. Granted that a counterexample has been established, but is this particularly surprising? The describing function, though closely related to harmonic linearization, appears to be a better approximation. Yet it is well-known that the describing function fails in some cases [1]. It is hardly surprising that harmonic linearization fails occasionally, too.

Singh is correct in pointing out that it is unfortunate that we have relied rather heavily on harmonic linearization to disprove Aizerman's conjecture [2]. But is he seriously suggesting the conjecture may be true in general?

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[1] V. Singh, *Proc. IEEE* (Lett.), vol. 63, p. 1610, Nov. 1975.

References

[1] J. C. Hsu and A. U. Meyer, *Modern Control Principals and Applications.* New York: McGraw-Hill, 1968.
[2] R. E. Fitts, "Two counterexamples to Aizerman's conjecture," *IEEE Trans. Automat. Contr.*, vol. AC-11, pp. 553–556, July 1966.

Reply[2] by Vimal Singh[3]

The discussion by Fannin is valuable. His observation that we may possibly construct a certain better approximation (or even a certain exact analysis) avoiding the difficulty is correct. This is precisely the observation which I myself convey through my letter. My statement that we have relied rather heavily on harmonic linearization to disprove Aizerman's conjecture should not be taken to mean that the conjecture may be true in general.

As a convincing analysis of the example in question in the context of harmonic linearization, we may assume the solution

$$y = a_1 \sin \omega t + a_2 \cos \omega t, \qquad (\omega^2 = 1 + \epsilon\beta).$$

Then the conditions for harmonic balance of frequency ω are obtained as

$$\omega^3 a_2 - \omega a_2 - \epsilon b_1 \omega^2 a_1 + \epsilon b_1 a_1 - \epsilon b_2 \omega a_2 - \frac{3\epsilon\omega a_2(a_1^2 + a_2^2)}{4} = 0$$

$$-\omega^3 a_1 + \omega a_1 - \epsilon b_1 \omega^2 a_2 + \epsilon b_1 a_2 + \epsilon b_2 \omega a_1 + \frac{3\epsilon\omega a_1(a_1^2 + a_2^2)}{4} = 0.$$

Employing $\omega^2 = 1 + \epsilon\beta$ and ignoring the terms multiplied by $\epsilon^p, p \geqslant 2$, we realize the presence of oscillation if

$$\beta = b_2 + \frac{3(a_1^2 + a_2^2)}{4}.$$

However, a better approximation (that is, describing function) predicts no oscillation, as Fannin has correctly shown.

[2] Manuscript received December 8, 1975.
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Transient Behavior of a Transistor-Tunnel Diode Circuit

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***Abstract*—The transient behavior of a transistor with tunnel diode feedback has been studied using the charge-control model for the transistor and empirical power functions for the tunnel diode. Though an**

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