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An Examination Of The Adaptive Random Search Technique

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An Examination of the Adaptive Random Search Technique

Random search procedures have recently been successfully applied to the optimization of a variety of chemical engineering problems, including optimization of chemical processes by flow sheet simulation. These procedures represent the independent variables as random variables described by probability distributions. The adaptive random search procedure centers the distribution for each variable about the best search point found and examines this region for a better point. Thus, this technique has the ability of continuously moving the search region toward the optimum, which is particularly advantageous in following constraints.

This study examines the efficiency of the adaptive random search technique as applied to six different problems, which have been previously solved by various other techniques.

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SCOPE

Recently, random search techniques have been applied to the optimization of chemical processes (Luus and Jaakola, 1973*a*; Gaines and Gaddy, 1976). A random search procedure is characterized by random determination of the independent variables in a given search region. Unlike many techniques, the random search does not require approximations, auxiliary functions, or an extensive knowledge of the problem under study in order to obtain an optimum solution. The success of a random procedure is not jeopardized by a nonlinear objective function or nonlinear implicit constraints.

The adaptive or pseudo random search selects new search points by sampling a probability distribution centered about the best known value of the objective func-

tion (Gall, 1966). This method was found to have attractive constraint following capabilities in an earlier study of optimization by flow sheet simulation (Gaines and Gaddy, 1976). The objective of this study was to evaluate the search method by applying it to example problems which had previously been solved by other methods so that a direct comparison of efficiency was possible. The examples chosen were the same as those solved by Luus and Jaakola (1973*a*) using a different random search procedure. Variations of the technique and search parameters were studied in order to insure good efficiency and reliability while maintaining the general applicability to any type of optimization problem.

CONCLUSIONS AND SIGNIFICANCE

The adaptive random search procedure was found effective in solving a variety of optimization problems. As expected, the efficiency of the adaptive random technique was not always as good as for other methods, for some of the example problems. The procedure is reasonably efficient and very reliable in reaching 0.1% of the global optimum, although multiple optima exist in many of the examples. Convergence to tighter tolerances such as 0.01% of the optimum is slow and not recommended. Many of the examples were heavily constrained, and the adaptive random search was found to be effective in following constraints to the optimum.

Variations such as reducing the search region and skewing the probability distribution as the search proceeds are effective means of improving both the efficiency of the search and convergence to tight tolerances. The number of evaluations of the objective function during the search, used as an indication of efficiency, was not affected significantly by the choice of starting values for the search or by the choice of an initial search region.

The adaptive random search technique presented herein may find wide application in process optimization problems. The method is easily programmed and applied on the computer, and convergence to 0.1% of the optimum can be achieved for many problems with 1 to 5 s of computation time on an IBM 370/165 computer. Perhaps the most promising feature of the method is its good reliability, due in part to an ability to effectively handle nonlinear constraints. Although not demonstrated in this paper, the method is also applicable to problems with discrete variables, discontinuous functions, or stochastic inputs.

In recent years, significant strides have been made in applying optimization techniques to complicated process optimization problems. Optimization of chemical processes represents a difficult test for any optimization procedure. The relationships involved are almost always nonlinear, often discontinuous or discrete, and sometimes even stochastic.

In many cases, the success of process optimization studies has depended upon modification of the search procedure to suit a particular problem (Friedman and Pinder, 1972; Umeda and Ichikawa, 1971) or upon modifications (such as linearization) of the problem to suit a particular search algorithm (Dibella and Stevens, 1965; Sauer et al., 1964). In fact, the application of optimization to industrial problems by the practicing engineer has probably been limited by the lack of a general optimization procedure suitable for all problems.

Several recent studies (Luus and Jaakola, 1973*a*; Gaines and Gaddy, 1976; Fan and Mishra, 1974) have successfully applied random search procedures to optimization of chemical processes. Random search procedures represent the independent variables as random and search for the optimum by random sampling. Such a procedure would appear to be very inefficient and inappropriate by comparison with techniques that move in a definite pattern toward improving an objective function (Himsworth, 1962). This comparison may be valid for problems that exhibit unconstrained and well-behaved surfaces. However, a random search has specific advantages for process optimization studies as discussed below.

One of the foremost difficulties in process optimization is the handling of implicit nonlinear constraints. Such constraints are responsible for the reported failure of many optimization algorithms to successfully converge (Keefer, 1973; Friedman and Pinder, 1972; Gaines and Gaddy, 1976). Since most search procedures are designed to follow a definite pattern in improving the objective function, repeated constraint violations occur when a constraint is encountered. Without knowledge of the shape or location of the constraint, the procedure cannot readily follow the constraint to the optimum. The use of penalty functions helps this problem with some algorithms (Gottfried et al., 1970; Keefer, 1973), but collapse and failure are often the results.

A random search procedure is not limited to a fixed pattern of movement. When a constraint is encountered, sampling in the valid region is still probable, and movement toward the optimum is possible. In the region of a constraint, a random move may be more efficient than a predetermined or programmed move.

Random search procedures have other advantages. They are easy to program and to apply. Discrete variables occur frequently in chemical processes and also in reliability and synthesis problems. Discrete variables can be readily accommodated with a random search by sampling from a discrete distribution (Briggs and Evans, 1964) or by an adjustment in the significant figures of random numbers used with continuous distributions. Discontinuous functions often occur with chemical process equipment, and discontinuities pose no threat for a random sampling procedure. Also, treating the independent variables as random variables seems an appropriate approach for stochastic optimization problems (Berryman and Himmelblau, 1971). Since a broader region is examined by random search, this method is more likely to locate the global optima than methods that follow a fixed direction (Luus and Jaakola, 1973*a*).

The purpose of this paper was to evaluate the properties of the adaptive random search technique, proposed originally by Gall (1966). The specific objectives were to determine the success of this method in solving several different problems and to compare the efficiency of this method with other procedures.

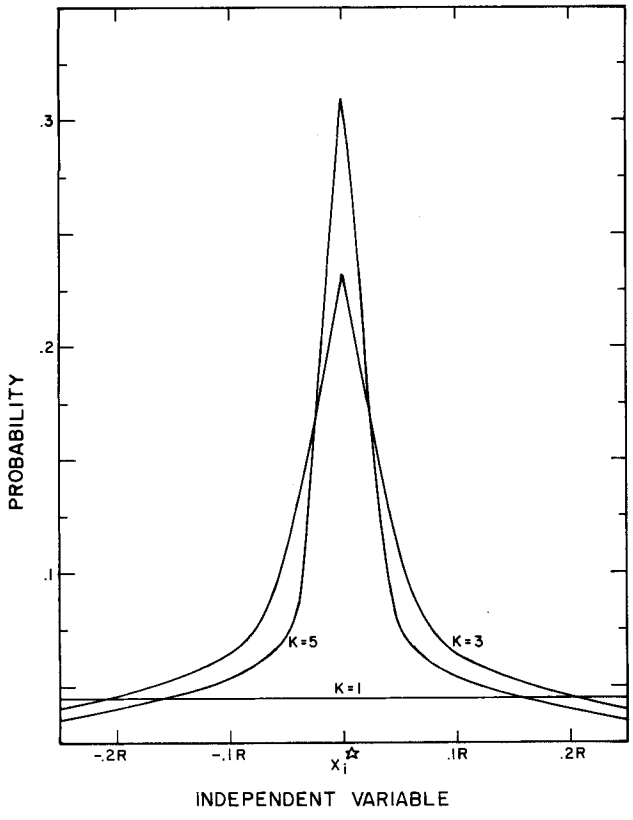

Fig. 1. Probability distributions of independent variables.

ADAPTIVE RANDOM SEARCH PROCEDURE

The adaptive random search procedure, suggested by Gall (1966) for multifactor optimization, chooses new search points in the vicinity of the best known objective function. The basic equation for determining new values of the independent variables is

$$x_i = x_i^* + R_i(2\theta - 1)^k \qquad (1)$$

The search is conducted by determining new values of the independent variables from Equation (1) and by evaluating the objective function. When a value of the objective function is found to be better than the last y^*, the x_i^* are replaced in Equation (1) with the improved values and the search continued. Thus, the adaptive random search moves toward the optimum in much the same manner as a hill climbing technique.

The quantity R_i represents the range of allowable positive or negative deviations about x_i^*. Since x_i is random, determined by θ, Equation (1) corresponds to a stochastic (Monte Carlo) sampling from a probability distribution of x_i. This distribution is centered about x_i^* and is determined by the quantity $R_i(2\theta - 1)^k$.

Figure 1 shows typical distribution functions of x_i. The variance of the distributions are seen to be dependent upon k. A uniform distribution results with $k = 1$[†], and the variance about x_i^* decreases with increasing k. Thus the sampling region can be controlled by the selection of k and, of course, the range R_i. It should be noted that the independent variable always retains some probability of achieving extreme values $x_i^* \pm R_i$, although this probability diminishes with increasing k. With an exponent larger than one, the probability density is a maximum at x_i^*; consequently, the sampling is concentrated in the region that has produced the best objective function. Since the search probability density can be shifted as the search proceeds, the method is accurately described as a pseudo random search, with an exhaustive random search performed only when the exponent k is one.

The search is begun by specifying the range R_i, initial values of the x_i^*, the corresponding value of y^*, and the exponent k. The steps in the search procedure are described as follows:

1. Obtain a random number θ.
2. Calculate the value of x_i from the Equation (1).
3. Check the explicit constraints (maximum and minimum bounds) of x_i. If x_i exceeds its bounds, return to step 1.
4. When valid x_i have been generated for each variable, check the implicit constraints g_j. If a constraint is violated, return to step 1.
5. Evaluate the objective function y.
6. If the value of the objective function y is not better than y^*, return to step 1.
7. Replace the x_i^* with the last x_i values and replace y^* with the improved y. Continue the search for y^*_{opt} with step 1.

Execution of step 4 is counted as an iteration, and execution through step 5 is counted as a function evaluation.

The strategy of the search is controlled by the choice of the exponent k. With no knowledge of the search region, the best approach is to start with an exhaustive random search and successively narrow the region as the optimum is approached. Thus k would generally be unity initially, increasing to 3, 5, 7, etc., as the search proceeds and y^* improves. Therefore, the basic strategy is to increment k either with the number of iterations (step 4) or with the number of improvements in the objective function (step 7).

There is a hazard in this strategy when multiple optima are present. If the search region is narrowed too soon, convergence to a local optimum may occur. Periodic reduction of k would enable the search to move away from a local optimum. This procedure would decrease the efficiency of the search, however, when local optima are not present. A number of strategies were employed and are explained later.

EXAMPLE PROBLEMS

The six examples used in this study have each been solved previously and reported in the literature, most recently by Luus and Jaakola (1973*a*). Consequently, a direct comparison of results can be made, in some cases. Since a detailed description of these problems is available elsewhere, only a brief description is included here.

Power Plant Fuel Allocation

The objective of this problem is to minimize the cost of fuel for a power plant. The objective function is nonlinear, with four independent variables, one equality constraint, and a nonlinear inequality constraint. Hovanessian and Stout (1963) solved this problem by linearization and the simplex method.

Concave Programming Problem

This is a mathematical example with four independent (unbounded) variables and three implicit inequality constraints. This problem was proposed by Rosen and Suzuki (1965) as a test for nonlinear programming algorithms and has also been solved by Gould (1971).

Chemical Species Allocation

The objective of this problem is to minimize the free energy of a mixture of ten gases at equilibrium, resulting from the combustion of hydrazine and oxygen. There are ten independent variables and three equality constraints.

[†] With $k = 1$, this search is similar to the procedure used by Luus and Jaakola (1973).

Table 1. Optimum Value of Objective Function for Example Problems

Example No.	Minimum or maximum	Optimum by adaptive random	Optimum reported by Luus and Jaakola (1973a)	Optimum reported by another author
1	min.	3.0519	3.05	3.17 (Hovanessian and Stout, 1963)
2	min.	−43.999	−44.0000	−44.03 (Gould, 1971)
3	min.	−47.761	−47.7611	−47.7611 (White, Johnson, and Danzig, 1958)
4	max.	1 162.0	1 162.009	1 162.94 (Sauer et al., 1964)
5	max.	172.49	172.49	172.5 (Chung, 1972)
6	max.	121.53	121.534	121.33 (Adelman and Stevens, 1972)

White, Johnson, and Danzing (1958) solved this problem by steepest descent and sectional linearization. Bracken and McCormick (1968) also report solution of this example.

Alkylation Process

The objective of this nonlinear example is to maximize the profit from an alkalation process. There are ten independent variables, seven of which can be eliminated by equality constraints. This problem was proposed by Payne (1958) and has been solved by Sauer et al. (1964) and Keefer (1973).

Drying Process

Chung (1972) presented this problem, which is concerned with maximization of the moisture removal rate in a dryer. This nonlinear example has only two independent variables but three inequality constraints.

Williams-Otto Process

This process model was originally introduced for investigation of computer control by Williams and Otto (1960) and has been studied extensively by numerous investigators, including Dibella and Stevens (1965), Adelman and Stevens (1972), and Findley (1974). Luus and Jaakola (1973*b*) showed that not all of the equations describing the process are independent; thus there are eight equations in thirteen unknowns, leaving five independent variables.

RESULTS AND DISCUSSION

The first question to be considered in evaluating a new optimization technique is whether convergence to the optimum value will be achieved. Table 1 presents the results of the optimum objective function for each of the six examples found by previous investigations compared to the results of this study. The optimum values found by the adaptive random search procedure are essentially the same as those found by the other investigators.

The beginning values of the independent variables used in this study were the same as those used by the other authors. The values of the independent variables at the optimum were nearly identical with those found earlier. In example 6, it was found, as previously shown by Luus and Jaakola (1973*b*), that at the optimum the values of the independent variables are not uniquely determined.

These results show that the adaptive random search procedure is effective in locating the optimum of a variety of different problems.

Efficiency of the Basic Search Method

The computational requirements to reach the optimum are a primary concern in evaluating the effectiveness of a search method. The efficiency of a search procedure can be best measured by the number of iterations or function evaluations required to achieve the optimum.

Table 2 represents the minimum number of iterations and function evaluations to reach 0.1% of the optimum y^* for each sample (as shown in Table 1) by using the basic adaptive random search procedure. It should be noted that the computational requirements of a random search procedure are also random. Therefore, the number of iterations and function evaluations varies from one run to another, even though the starting points are the same.

Table 2. Computational Requirements of the Adaptive Random Search

	Basic adaptive random search (to 0.1% of optimum)			Adaptive random search with range reduction (to 0.1% of optimum)			
Example No.	Min. iterations	Min. function evaluations	Avg. function evaluations	Min. interations	Min. function evaluations	Avg. function evaluations	Function evaluations other studies
1	820	519	9 533	280	154	9 394	1 952*
2	728	353	3 332	310	162	1 948	1 759*
							912†
3	210	98	688	132	52	387	394*
4	342	82	607	331	95	462	919*
							3 487**
5	241	120	509	22	15	484	336*
6	87	85	1 819	100	97	607	1 244*
							124††
							343***

* Luus and Jaakola (1973*a*).
† Gould (1971).
** Keefer (1973).
†† Adelman and Stevens (1972).
*** Findley (1974).

Table 3. Computational Requirements of Adaptive Random Search with Skewing

Example No.	Skewing alone: Min. iterations to reach 0.1% of optimum	Skewing alone: Function evaluations to reach 0.1% of optimum, Minimum	Skewing alone: Function evaluations to reach 0.1% of optimum, Average	Skewing with range reduction: Min. iterations to reach 0.1% of optimum	Skewing with range reduction: Function evaluations to reach 0.1% of optimum, Minimum	Skewing with range reduction: Function evaluations to reach 0.1% of optimum, Average
1	363	255	4 709	97	65	3 874
2	457	187	1 788	475	188	1 754

Consequently, several runs are required to establish the range of computational requirements.

The average number of function evaluations for convergence to 0.1% of the optimum, shown also in Table 2, was found to be five to twenty times greater than the minimum. It is this average computational requirement that should be reported and used for comparison with other algorithms.

A direct comparison with the results of the random search of Luus and Jaakola is not possible, since only one run was made in their study. Comparison of the computational requirements with other studies is difficult because, in many cases, auxiliary functions, such as gradients, are required. Computational requirements were reported for other methods only for examples 2, 4, and 6. The basic adaptive random search exhibits, on the average, a better efficiency in solving example 4 but is worse for examples 2 and 6. The computational times for the results of Table 3 ranged from 1 to 5 s on the IBM 370/165, which is comparable to times reported by other investigators. It should be noted that the adaptive random search exhibits a perfect reliability for each of the examples; that is, convergence to 0.1% of the optimum was always achieved, a characteristic not always available with other methods.

Luus and Jaakola reported the number of function evaluations to reach 0.01% of the optimum as about three times as many as required to reach 0.1%. This same level of effort was found to be required by the adaptive random search in reaching 0.01% of the optimum. Therefore, it is concluded that convergence to very tight tolerances is inefficient with random procedures at their present level of development. Of course, 0.1% accuracy is considered quite good for most problems, and convergence to closer tolerances is not considered a serious limitation of the adaptive random search. Closer convergence might be achieved more efficiently by using another type of algorithm once the search is near the optimum. Also, modifications to the basic technique, as described later, are helpful in convergence to closer tolerances.

Two methods of increasing the exponent k in Equation (1) were tried. The first involved incrementing k by 2 after a certain number of iterations between improvements in the objective function. A better procedure is to increase k with the number of better values of the objective function that are found. Thus, the procedure followed was: initially $k = 1$, after five new y^*, $k = 3$; after fifteen more y^*, $k = 5$; and after ten more y^*, $k = 7$. Values of k larger than 7 were found to slow convergence and be ineffective in reaching 0.1% of the optimum.

Another variation that proved effective was to reduce k by 2 when convergence slowed. This periodic increase in the search region helps to avoid local optima and speeds convergence.

Alterations to the Basic Search Method

There are two alterations to the basic adaptive random search method given in Equation (1) that were found to improve the efficiency: systematic range reduction and skewing.

Systematic Range Reduction

In Equation (1), the range R_i about x_i^* is constant, although the probability of achieving extreme values of x_i^* becomes less as k increases. It would seem logical that as the search gets closer to the optimum, certain portions of the search region should be excluded from further sampling. Such a reduction can be achieved by dividing the range R_i by k as shown in Equation (2):

$$x_i = x_i^* + \frac{R_i}{k}(2\theta - 1)^k \qquad (2)$$

Table 2 also presents the results of the computational requirements to reach the optimum by using the range reduction scheme of Equation (2). A reduction in the minimum number of function evaluations is noted in most every example. However, a significant improvement in the *average* number of function evaluations was achieved by range reduction. These averages are comparable to the single values found by Luus and Jaakola. By comparison with efficiencies reported for other optimization methods, the adaptive random search requires about twice as many function evaluations, except for example 4, where the random search was superior. Range reduction is considered an important extension of the basic method.

Skewing

As noted in Figure 1, Equation (1) produces a symmetrical distribution about each x_i^*. When the search has reached an inequality constraint, a large proportion of the random values of the independent variables will be invalid when we sample with a symmetrical distribution. The number of valid samples could be increased with a distribution skewed along the constraint in the direction toward improving the objective function. This procedure should accelerate constraint following. The distribution given in Equation (1) can be skewed by modifying the random numbers as follows:

$$x_i = x_i^* + R_i[B(A\theta - 1)]^k \qquad (3)$$

A number of values of A were tried on the various problems, and a value of 1.5 was found to be most efficient. Figure 2 shows the resultant distributions for $k = 1, 3, 5$ with negative skew ($B = +1$). Positive skew would be the reflection of Figure 2 and is achieved with $B = -1$.

The question of when to skew the distributions is handled quite readily. When the x_i^* produce values of any implicit inequality constraint g_j close to its bound, the distributions about the x_i^* are skewed. The direction of skew is determined by the movements of the independent variables from their initial values. If the average of x_i^* is greater than the initial value of x_i, the search is generally proceeding in the direction of increasing x_i; thus, the distribution of x_i is skewed positive (and vice versa).

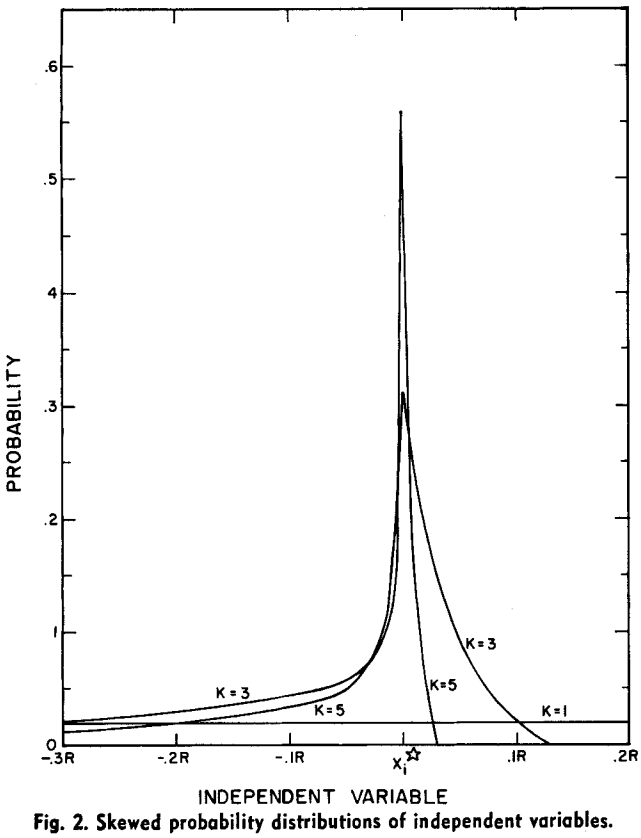

Fig. 2. Skewed probability distributions of independent variables.

Table 3 presents the results of skewing on examples 1 and 2, the two problems involving difficult implicit constraints. The minimum number of iterations and function evaluations to reach 0.1% of the optimum is significantly better than with no skewing and about the same as with range reduction. Skewing would be expected to increase the number of valid samples; thus a greater reduction is achieved in iterations than in function evaluations as noted between Tables 2 and 3. An improvement in the average efficiency is achieved with skewing, although the average number of function evaluations is still greater than with other methods, where comparable. Still further improvement is noted if range reduction is combined with skewing; however, the average number of function evaluations is not much better than with skewing alone. Further practice with skewing and range reduction will be required to define the improvements possible with these methods.

It was noted that, with the combination of skewing and range reduction, convergence to within 0.01% of the optimum was considerably more efficient in some cases. For example, a minimum of 284 function evaluations was required to reach 0.01% of the optimum in problem 2. Therefore, skewing is an attractive modification of the basic method which improves the efficiency and might prove to be an effective means for convergence to close tolerance when the optimum lies near an implicit constraint.

Another method of skewing is to modify the exponent of the distribution. An even integer produces a one-sided distribution, and noninteger values also change the shape of the distribution. These modifications were not developed in this study.

Different Starting Values

The initial values of the independent variables might have a pronounced influence upon the convergence efficiency of a search procedure. The effect of different starting values was examined in example 6. The search in this example was conducted with initial values of each of the variables at its minimum bound, maximum bound, midpoint, and even 10% above the maximum and 10% below the minimum. With each of these starting points, convergence was achieved with about the same efficiency as reported earlier. It is important to note that convergence was achieved with initial values which were invalid, a significant advantage in many applications.

Initial Range

The effect of different starting ranges (R_i) was examined in example 2. Initial ranges of 5.0 and 0.5 for the independent variables were tested. It was found that the number of function evaluations required to reach 0.1% of the optimum was essentially the same with both ranges. However, the number of iterations required for convergence with initial ranges of 5.0 was on the average five times greater than with initial ranges of 0.5. This was expected, since with a larger region of search, there would be a greater number of invalid points; consequently, a greater number of iterations was required for an equal number of function evaluations.

SUMMARY

It can be concluded from the results of this study that the adaptive random search procedure is effective in solving a variety of optimization problems. This method exhibited perfect reliability in solving the example problems; however, the efficiency was generally poorer by comparison (where possible) with other methods used for solving these same problems. The procedure results in reasonable efficiency in converging to within 0.1% of the optimum; however, convergence to closer tolerances is quite slow. Improvements in the basic method are possible with range reduction and skewing. Continued development of the method should lead to further improvements in efficiency. The starting point in one example was found to have no effect on the reliability of the procedure, even when starting in the invalid region.

In the example problems solved herein, the adaptive random method exhibited the same constraint following ability noted earlier by Gaines and Gaddy (1974) in studying a complex chemical proess. A primary advantage of this method is the ease with which it can be programmed and applied. Because of the good reliability, reasonable efficiency, constraint following capability, ease of programming, and applicability to mixed integer problems, this method may find wide application in process optimization problems.

NOTATION

A = constant controlling the amount of skew
B = ± 1, determines the direction of skew from x_i^*
g_j = implicit constraint function
k = distribution coefficient (odd integer 1, 3, 5. . .)
N = number of independent variables
R_i = allowable search region about x_i^*
x_i = new value of independent variable ($i = 1, N$)
x_i^* = value of variable x_i which has produced the highest objective function
Y = value of the objective function
y^* = highest value of the objective function
y^*_{opt} = final optimum value of the objective function
θ = random number between zero and one

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Relationships Between Velocity Profiles and Drag Reduction in Turbulent Fiber Suspension Flow

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Analytical relationships between velocity profiles and flow resistance data are developed for suspensions of papermaking fibers in turbulent shear. The relationships apply to suspensions of synthetic fibers. The wall layer appears to be unaffected by the presence of fibers, and the cause of drag reduction can be attributed to the turbulent core region.

SCOPE

The turbulent flow of fiber suspensions is possibly unique. When fibers in suspension are subjected to high shear rates, and when there is insufficient volume for the fibers to move freely, they agglomerate by mechanical entanglement. Both free fibers and agglomerates are effective in damping turbulence and reducing energy dissipated by viscous shear to below that of the suspending medium flowing alone under the same conditions. The phenomenon of drag reduction in turbulent fiber suspensions has stimulated considerable interest in recent years and is of significant practical importance to the papermaking industry.

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Most previous investigations of the drag reducing behavior of fiber suspensions have been limited to measurements of friction loss data because of the lack of suitable instrumentation for the determination of velocity profiles and turbulence parameters. An annular purge impact probe was developed by Mih and Parker (1967) to measure velocity profiles in the flow of macroscopic fiber suspensions. Their data and data obtained subsequently by Seely (1968) were limited to a small range of flow conditions.

The purpose of this work was to extend the range of velocity profiles and to investigate the relationships between flow resistance and velocity profiles for fully developed turbulent flow.