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Methods for Evaluating the Performance of Small Acoustic Filters*

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Experimental methods are needed for determining the characteristics of small acoustic filters used in systems with pulsating gas flows. These characteristics could then be used to predict the performance of proposed filter designs in a particular system, according to plane-wave acoustic theory. Dependence on trial-and-error experimentation in solving noise-control problems would thus be minimized. A literature survey revealed only a few earlier attempts to evaluate the acoustic performance of small filters and filter elements. Three methods for determining reflection and transmission factors are described, evaluated, and compared. A method employing a standing-wave tube of unique design is recommended for determination of reflection factors. Transmission factors are obtained from reflection factors and pressure measurements at the filter inlet and outlet. Use of an anechoic termination simplifies the calculations and increases accuracy; design and evaluation of such a termination is described.

INTRODUCTION

Small acoustic filters are often used as suction and discharge mufflers on pulsating-flow equipment such as refrigeration compressors. Their purpose is to reduce the transmission of fluid-borne noise into other system components such as evaporators and condensers. The design of these mufflers has been based primarily on trial-and error experimentation and on previous experience with similar equipment. These methods, applied over several decades, have resulted in many systems that are quite in operation.

However, a general design method combining theory with experiment is needed to reduce the dependence on prototype filters for satisfactory noise control. Of course, the desired performance characteristics could be predicted solely from acoustic theory if mathematical models representing the proposed system and muffler were available, but these are not known except for the simplest cases. Therefore, an accurate experimental method for measuring the reflection and transmission characteristics of acoustic filters and filter elements is essential, particularly if effective noise control is to be incorporated into the initial design of new products.

I. LITERATURE SURVEY

The current state of the art relating to measurement of the acoustical characteristics of porous materials and filter elements was investigated primarily by using the Engineering Index for the period 1946–1967. Numerous textbooks, handbooks, reports, patents, and older journal articles dating from 1913 were also reviewed.

Although only a few authors consider the measurement of transmitted sound,[1–3] many have investigated the measurement of reflected sound or acoustical impedance. Table I summarizes previous experimental work to determine sound reflected from, or absorbed by,

* The research reported in this article was performed at The Ray W. Herrick Laboratories, Purdue Univ., Lafayette, Ind. 47905.

[1] D. D. Davis, G. M. Stokes, D. Moore, and G. L. Stevens, "Theoretical and Experimental Investigation of Mufflers with Comments on Engine-Exhaust Muffler Design" NACA Rep. 1192 (1954).

[2] J. Igarashi and M. Toyama, "Fundamentals of Acoustical Silencers (I)," Aeron. Res. Inst., Univ. of Tokyo, Rep. No. 339 (December 1958).

[3] T. Miwa and J. Igarashi, "Fundamentals of Acoustical Silencers (II)," Aeron. Res. Inst. Univ. of Tokyo, Rep. No. 344 (May 1959).

TABLE I. Summary of methods for measurement of reflected sound.

Investigator	Reference	Apparatus	Frequency range	Microphone	Sample or unknown termination	Source	Type of measurement	Quantity obtained[a]
Mawardi Loye and Morgan Cook	5 6 7	Rigid cavity	Wavelength large relative to cavity dimensions	Fixed, near source	Fixed, opposite source	Fixed, infinite impedance	Comparative cavity wall versus sample	α or R
Loye and Morgan	8	3½-in.-diam tube, ∼5 ft long	200 Hz and higher	On sliding piston with source	Fixed, at end of tube	Plane wave, infinite impedance on sliding piston	Comparative, steel cap versus sample	α or R, θ (plane waves assumed)
Beranek	10	3-in.-diam tube, 6 ft long	100–3000 Hz[b]	Fixed, near source	On sliding piston	Fixed	Comparative, rigid cap versus sample near resonance point	α or R, θ (plane waves assumed)
	4, 8, 9, 11–14	Standing-wave tubes, cross-section 2–11½ in.	100–3000 Hz	Movable, along tube axis	Fixed, at end of tube	Fixed	Absolute	α or R, θ (plane waves assumed)
Hall	15	Rectangular groove, 2.81 cm², formed in 270° arc	270–6000 Hz	Flush mounted movable in circular path	Fixed, at end of tube	Fixed	Absolute	α or R, θ (plane waves assumed)
Eijk, Kosten, and Kok	16	Two fixed tubes, separated by outer sliding tube ∼½ in. diam	Up to 6000 Hz	Flush mounted at midpoint of sliding tube	Fixed, at end of tube	Fixed	Absolute	α or R, θ (plane waves assumed)
	17–20	Two small closely-spaced microphones, signal-processing equipment	Depends on microphone spacing	Phase and amplitude matched	Fixed, at end of tube	Fixed	Absolute	α or R, θ (plane waves assumed)

[a] α is the absorption coefficient; R, the reflection factor magnitude; θ, the angle between incident and reflected waves.
[b] A smaller diameter tube is used for higher frequencies.

test samples or other termination.[4–20] Note that only the last three methods, each of which employs a flush sidewall microphone probe, are practical for measuring the characteristics of acoustic filters with ½-in. or smaller openings.

The following comments supplement and more fully describe the methods summarized in Table I.

1. Beranek[4] measures the influence of a microphone on the sound field being investigated; others demonstrate noninterference by measuring the sound reflected from a rigid termination and comparing it with the value predicted from plane-wave acoustic theory. In

[4] L. L. Beranek, "Some Notes on the Measurement of Acoustic Impedance," J. Acoust. Soc. Amer. **19**, 420–427 (1947).
[5] O. K. Mawardi, "Measurement of Acoustic Impedance," J. Acoust. Soc. Amer. **21**, 84–91 (1949).
[6] D. P. Loye and R. L. Morgan, "A Small Acoustical Tube for Measuring Absorption of Acoustical Materials in Auditoriums," J. Acoust. Soc. Amer. **17**, 326–328 (1946).
[7] R. K. Cook, "A Short Tube Method for Measurement of Impedance," J. Acoust. Soc. Amer. **19**, 922–23 (1947).
[8] D. P. Loye and R. L. Morgan, "An Acoustic Tube for Measuring the Sound Absorption Coefficients of Small Samples," J. Acoust. Soc. Amer. **13**, 261–64 (1942).
[9] E. C. Wente and E. H. Bedell, "The Measurement of Acoustic Impedance and the Absorption Coefficient of Porous Materials," Bell System Tech. J. **7**, 1–10 (January 1928).
[10] L. L. Beranek, "Precision Measurement of Acoustic Impedance," J. Acoust. Soc. Amer. **12**, 3–13 (1940).
[11] H. O. Taylor, "A Direct Method for Finding the Values of Materials as Sound Absorbers," Phys. Rev. **2**, 270–287 (1913).
[12] R. D. Berendt and H. A. Schmidt, Jr., "A Portable Impedance Tube," J. Acoust. Soc. Amer. **37**, 1049–1052 (1963).
[13] H. J. Sabine, "Notes on Acoustic Impedance Measurement," J. Acoust. Soc. Amer. **14**, 143–150 (1942).
[14] R. A. Scott, "An Apparatus for Accurate Measurement of the Acoustic Impedance of Sound-Absorbing Materials," Proc. Phys. Soc. (London) **58**, 253–264 (1946).
[15] W. M. Hall, "An Acoustic Transmission Line for Impedance Measurement," J. Acoust. Soc. Amer. **11**, 140–46 (1939).
[16] J. v.d. Eijk, C. W. Kosten, and W. Kok, "Sound Absorption by Porous Materials," Appl. Sci. Res. Sec. B, 50–62 (1947).
[17] P. K. Baade, "Instrumentation Trends for Noise Reduction Work, as Seen by a Mechanical Engineer," Audio Eng. Soc. preprint no. 429 (October 1965).
[18] R. H. Bolt and A. A. Petrauskas, "An Acoustic Impedance Meter for Rapid Field Measurements," J. Acoust. Soc. Amer. **15**, 79 (1943).
[19] H. Grote, "Akustiches Reflektometer," Acustica **14**, No. 5, 296–297 (1964).
[20] T. J. Schultz, "Acoustic Wattmeter," J. Acoust. Soc. Amer. **28**, 693–699 (1956).

other cases, the small size of the microphone relative to the tubing diameter is assumed sufficient to demonstrate that interference is negligible.

2. Several authors have considered variations in source impedance. White[21] presents a method for measuring source impedance and tube attenuation in which a connecting tube is terminated by a microphone that is assumed to be perfectly reflecting. Kosten[22] correlates the electrical impedance of a loudspeaker with the acoustical impedance at the speaker diaphragm; measurements of electrical impedance are then used to calculate the impedance and absorption characteristics of test samples at frequencies below 1000 Hz. Smith[23] analyzes a method for measuring the half-wavelength of sound by observing changes in speaker impedance as a slideable piston is moved.

3. The effects of attenuation and microphone area on standing-wave measurements have been investigated by several authors. Sabine[13] and Scott[14] analyze the influence of attenuation on the locations and magnitudes of standing-wave minima; effects are most pronounced when frequency is low and attenuation is high. Hall[15] and Eijk, Kosten, and Kok[16] show that pressure variations across the microphone diaphragm do not affect the true value of the ratio $p_{\max}/p_{\min}$.

4. Several authors[10,12–14] have published experimental results that set limits on tube diameters for plane-wave propagation. The first transverse mode occurs when the acoustic wavelength (λ) is less than about 3.5 times the tube radius (r). In practice, however, the first mode is not pronounced, so the practical limit becomes $\lambda \simeq 1.8r$.

The simplified plane-wave equation can be used to describe the propagation of sound in tubes when the sound pressure is much less than static pressure, when density changes produced by the sound wave are much less than static density, and when both the particle and steady-flow velocities are much less than the propagation velocity. In most small systems, sound-pressure levels (SPL's) at a given frequency rarely exceed 150 dB (0.095 psi or 0.0063 atm); since sound propagation is nearly adiabatic[24] in virtually all practical cases, the perfect gas law demonstrates that density variations are much less than the static density. The particle velocity produced by a free progressive wave of 150-dB amplitude traveling in refrigerant 22, for example, is approximately 62 in./sec, which is less than 1% of the propagation velocity.

In most small systems, steady-flow velocities do not usually exceed 40 ft/sec. Trimmer[25] shows that when the steady-flow velocity is much greater than the particle velocity, the standing-wave pattern is altered by the factor $1-(V/c)^2$, where V is the steady-flow velocity and c, the sound-propagation velocity. If the medium is refrigerant 22, the effect is $1-(40/700)^2=0.997$.

From these considerations, it can be concluded that the simplified plane-wave equation adequately describes the propagation of sound in tubes 1 in. or less in diameter. However, these same velocities and SPL's may produce significant three-dimensional or nonlinear effects at plane discontinuities, orifices, and side-branch elements[26–34]; simple plane-wave theory cannot be applied in such cases. Suitable modification of the theory may account for these effects in a few instances; for most elements, including those whose mathematical descriptions are unknown, actual measurement of reflection and transmission characteristics appears to offer the only practical alternative.

II. METHODS FOR MEASUREMENT OF REFLECTION FACTORS

The following requirements were established for measurement of reflection factors:

1. Measuring equipment must have a negligible effect on the sound field being investigated.

2. Reduction of measured data to desired quantities must be reasonably simple and straightforward.

3. Calculated values for reflection and transmission factors should be accurate to within 5% (0.4 dB) for magnitude and 6° for phase angle. If these limits are exceeded, significant errors can result in some cases. Furthermore, unless errors in individual measurements are held to a minimum, the accumulated error resulting from the combination of elements into a muffler may be excessive.

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[21] J. E. White, "A Method for Measuring Source Impedance and Tube Attenuation," J. Acoust. Soc. Amer. **22**, 565–67 (1950).

[22] C. W. Kosten, "A New Method for Measuring Sound Absorption," Appl. Sci. Res. Sec. B, 35–49 (1947).

[23] P. W. Smith, Jr., "Systematic Errors in Indirect Measurements of the Velocity of Sound," J. Acoust. Soc. Amer. **24**, 687–695 (1952).

[24] L. L. Beranek, *Acoustics* (McGraw-Hill Book Company, Inc., New York, 1954), Chaps. 1 and 2.

[25] J. D. Trimmer, "Sound Waves in a Moving Medium," J. Acoust. Soc. Amer. **19**, 162–164 (1937).

[26] D. A. Bies and O. B. Wilson, Jr., "Acoustic Impedance of a Helmholtz Resonator at Very High Amplitude," J. Acoust. Soc. Amer. **29**, 711–714 (1957).

[27] U. Ingard and D. Pridmore-Brown, "Propagation of Sound in a Duct with Constrictions," J. Acoust. Soc. Amer. **23**, 689–694 (1951).

[28] F. C. Karal, "The Analogous Acoustical Impedance for Discontinuities and Constrictions of Circular Cross Section," J. Acoust. Soc. Amer. **25**, 327–334 (1953).

[29] R. F. Lambert, "Acoustic Filtering in a Moving Medium," J. Acoust. Soc. Amer. **28**, 1054–1058 (1956).

[30] R. F. Lambert, "Side Branch Insertion Loss in a Moving Medium," J. Acoust. Soc. Amer. **28**, 1059–1063 (1956).

[31] E. Meyer, F. Mechel, and G. Kurtze, "Experiments on the Influence of Flow on Sound Attenuation in Absorbing Ducts," J. Acoust. Soc. Amer. **30**, 165–174 (1958).

[32] J. W. Miles, "The Reflection of Sound due to a Change in Cross-Section of a Circular Tube," J. Acoust. Soc. Amer. **16**, 14–19 (1945).

[33] J. W. Miles, "The Analysis of Plane Discontinuities in Cylindrical Tubes (Parts I, II)," J. Acoust. Soc. Amer. **17**, 259–284 (1946).

[34] G. B. Thurston, "Nonlinear Acoustic Properties of Orifices of Varied Shapes and Edge Conditions," J. Acoust. Soc. Amer. **30**, 452–455 (1958).

These requirements eliminated many of the methods developed for relatively large diameter tubes, such as those using a longitudinal probe tube or a cavity. Those methods requiring an infinite impedance source were not considered because of errors that could result if the assumption were violated. Methods using two microphones, while potentially attractive, were rejected because of the difficulty and expense involved in obtaining two matched systems. The only feasible methods for tubing $\frac{1}{2}$ in. or less in diameter appeared to be those using a small microphone inserted into the tube wall. While a few such methods are described in the literature, little comment is available as to their accuracy or their effect on the sound field being measured.

A major problem in measuring sound pressures in small tubes is minimization of the disturbance produced by the pressure transducer. Even a $\frac{1}{4}$-in.-diam condenser microphone inserted into the wall of a tube with 0.300 in. i.d. can produce a significant disturbance, particularly in short tubes under resonant conditions. This phenomenon was investigated by connecting a 6-in. length of $\frac{3}{4}$-in. i.d. tubing, capped on one end, to a small acoustic driver. Flat and contoured plugs were machined to fit a probe tube soldered to the tubing wall. These plugs were sized so that the tube cross section with the flat plug inserted was geometrically similar to the cross section of $\frac{3}{8}$-in.-diam refrigeration tubing with a $\frac{1}{4}$-in.-diam microphone inserted in the tube wall. Each plug was then drilled to accommodate a $\frac{1}{4}$-in. Brüel & Kjær (B&K) condenser microphone (Fig. 1). In this way, measurements of an undisturbed sound field could be taken with the contoured plug and compared to those obtained with the flat plug. Because of similar geometry, the results could then be applied to a $\frac{1}{4}$-in. microphone inserted in the wall of $\frac{3}{8}$-in.-diam refrigeration tubing.

Flat and contoured plug data from 100 to 4500 Hz were in agreement to within 7% or less except at 4000 Hz, where a sharp resonance of the air column in the tube occurred. However, if resonances were avoided or the tube was long enough so that attenuation from losses at the tube wall became significant, the presence of a microphone could be tolerated.

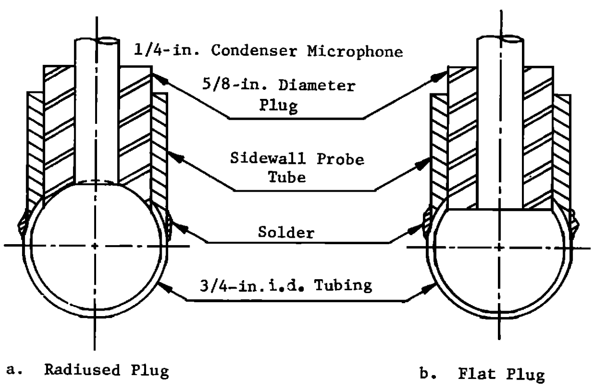

FIG. 1. Flat and radiused plugs inserted in tubing. Geometry (b) is identical to that of $\frac{1}{4}$-in. microphone inserted in $\frac{3}{8}$-in. refrigeration tubing.

TABLE II. Data from three-pressure method.[a]

Test No.	Frequency (Hz)	p_1 (mV)	p_2 (mV)	p_3 (mV)	θ (degrees)	R_0
1	2800	350	365	129	−0.4	0.92
2	2800	348	365	129	−1.0	1.08

[a] p_i is the rms microphone output in millivolts at Station 1, 2, or 3; θ, the phase angle at termination; R_0, the amplitude of reflection factor at termination.

In the following Sections, three methods for measuring reflection factors are analyzed and compared on the basis of accuracy, ease of use, and complexity of equipment required. The measurement of transmission factors is an extension of each method and is also discussed.

A. Three-Pressure Method

To evaluate the three-pressure method, three microphone locations were built into the sidewall of a length of refrigeration tubing $\frac{3}{8}$-in. o.d. with 0.032 in. wall thickness. The rms sound pressure was recorded at each location with the other two locations sealed by contoured aluminum plugs. The sound source was assumed invariant for the three measurements. From the sound pressures and distances between locations, the phase angle and magnitude of an unknown reflection factor were calculated, using plane-wave theory.

A rigid ("perfectly reflecting") termination was used for experimental measurements, so that calculated values could be compared with the (assumed) true reflection factor of 1.0, angle 0°. Measurements were made at frequencies ranging from 300 to 5000 Hz. Unfortunately, the calculations required squaring and subtracting the rms pressures; thus a small change in pressure caused appreciable changes in phase angle and magnitude of the reflection factor, particularly if two or more of the pressures were of about the same amplitude. Furthermore, the phase angle had to be determined and used to find the amplitude; this interdependence was undesirable from the standpoints of accuracy and ease of calculation. In two identical tests with a rigid termination, for example, the results shown in Table II were calculated:

Note that a variation of only 2 out of 350 mV at one location had a negligible effect on phase angle, yet changed the reflection factor magnitude from 0.92 to 1.08. For these reasons, the three-pressure method was not acceptable.

B. Transient Method

This method required long lengths (at least 24 ft for frequencies from 300 to 5000 Hz) of tubing, with a stationary microphone located about midway between the sound source and unknown termination. A *tone-burst* *generator* was used to gate a preselected number of cycles of oscillator output to the sound source, an acoustic driver. The number of cycles selected was sufficient to permit the driver output to reach a steady sinusoidal amplitude; however, the tone burst terminated before reflected sound waves (from the unknown termination) arrived at the microphone. In this way, incident and reflected sound waves were displayed individually on an oscilloscope and photographed (Fig. 2). The phase angle between incident and reflected waves at the microphone was measured by adjusting the oscilloscope to display first the incident wave and then the reflected wave, by delayed triggering at a preselected point on the oscillator output voltage. The procedure required use of the delayed single sweep (with variable triggering) and the external trigger features of a Tektronix model 564 storage oscilloscope. Each wave was photographed on the same negative by double exposure; the relative phase angle between the waves at the microphone was then found by direct measurement. The validity of this technique required that the phase relationship between oscillator output (voltage) and driver output (sound pressure) be invariant after the driver reached a steady-state condition. Apparently this assumption was valid, since repeated recording of phase-angle data under identical conditions yielded the same results.

The phase angle measured was the total shift of the reflected signal relative to the incident signal at the microphone location; the phase shift resulting from the additional distance ($2L$) traveled by the reflected wave was subtracted to obtain the phase angle of the unknown termination. The phase shift could be determined if τ, the period of the signal, and T_0, the time required for the signal to propagate through distance L, were known. The accuracy to which τ and T_0 must be measured increases with frequency. For example, at a frequency of 5000 Hz, T_0 must be known to six decimal places for an accuracy of 0.1 rad in phase shift. This can be accomplished by using a digital counter that is gated by voltages from two microphones, one located at the measuring station and the other at the unknown termination. For accuracy of $\pm 6°$, however, signal conditioning circuits are required.

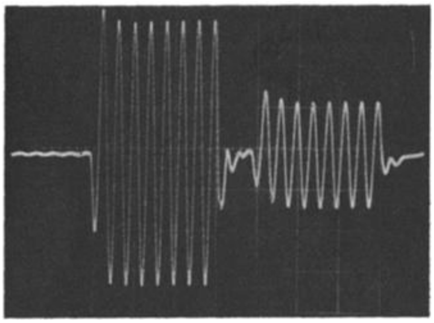

FIG. 2. Incident and reflected sound pressure waves for rigid termination (500 Hz)—transient method.

The incident and reflected amplitudes measured by the microphone were corrected for attenuation that occurred as the wave traveled to and returned from the unknown termination; the incident-wave amplitude at the termination was reduced and the reflected-wave amplitude was increased relative to the amplitudes recorded by the microphone. Attenuation of the sound waves results from viscous and thermal conduction losses in the boundary layer at the tube walls.[35–37] This attenuation is of the form $e^{-\alpha L}$, where α is a function of tube radius, gas properties, and frequency.

The actual attenuation in an aluminum tube of 0.415-in. i.d. was measured at 300, 500, 1000, 3000, and 5000 Hz using a single microphone at two locations 12 ft. apart. Sound-source output was assumed invariant, which was verified by repetition of the measurements. The medium was air at a temperature and pressure of approximately 750 mm Hg and 85° F. Assuming that the measured attenuation is of the form $e^{-\alpha L}$, α was calculated for each frequency. Results are shown in Table III, which also lists measured and corrected reflection factors for a rigid (reflecting) termination determined by the transient method. The attenuation constants are within the $\pm 10\%$ range of deviation from the theoretical values that have been reported by other investigators.[38–39]

TABLE III. Attenuation and reflection factors—transient method.

Frequency (Hz)	α (in.$^{-1}$)	Reflected wave / Incident wave (at microphone)	(at termination)
300	0.002335	0.483	0.920
500	0.00310	0.400	0.940
1000	0.00447	0.2895	0.994
3000	0.00768	0.117	0.973
5000	0.01033	0.0549	0.955

When the termination is a three-dimensional acoustic filter, a finite time is required (for internally reflected waves to reach steady-state amplitudes) before the reflection factor can be determined. This means that the microphone location and tube lengths must be chosen so that both incident and reflected waves reach con-

[35] D. P. Bogert, "Classical Viscosity in Tubes and Cavities of Large Dimensions," J. Acoust. Soc. Amer. **22**, 432–437 (1950).
[36] O. K. Mawardi, "On the Propagation of Sound Waves in Narrow Conduits," J. Acoust. Soc. Amer. **21**, 482–486 (1949).
[37] D. E. Weston, "The Theory of the Propagation of Plane Sound Waves in Tubes," Proc. Phys. Soc. (London) **66**, 695–709 (August 1953).
[38] M. C. Henderson and G. J. Donnelly, "Acoustic Resonance Tube for High Pressures and Low f/p," J. Acoust. Soc. Amer. **34**, 779–784 (1962).
[39] G. T. Kemp and A. W. Nolle, "The Attenuation of Sound in Small Tubes," J. Acoust. Soc. Amer. **25**, 1083–1086 (1953).

stant amplitudes before additional reflections arrive at the microphone.

Transmission factors can be measured by the transient method if a second measuring station is located beyond the muffler. It is assumed that the muffler is terminated anechoically or by tubing of a length such that no reflections arrive at the microphone before the desired measurements are obtained. Measurements of incident and transmitted waves can be obtained with a single microphone if the triggering and sweep-delay techniques previously described are used. However, if matched recording systems are available, simultaneous measurements can be taken and photographed directly from a dual-trace oscilloscope.

The transient method has the advantage that amplitudes and phase angles can be separated and measured directly. Another particularly attractive application is the testing of other measuring apparatus (specifically, standing-wave tubes) for internal reflection. This capability provides a direct method for determining the interference of a proposed measuring system with the sound field being investigated. However, the transient method also has several disadvantages that make it relatively undesirable for the direct measurement of reflection and transmission factors:

1. The measurement of T_0 to the required accuracy is difficult.
2. Long tube lengths cause severe attenuation of reflected (and transmitted) waves, particularly above 3000 Hz.
3. The use of a filter to eliminate harmonics (introduced by the driver) from the microphone signal may not be possible at frequencies below 3000 Hz, unless the filter has a rapid transient response.

C. Standing-Wave Method

The third method considered, and the one best-suited to measurement of reflection factors, employed a standing-wave tube designed specifically for measurements in small-diameter tubing (Fig. 3). The apparatus consisted of an inner tube slotted along part of its length and an outer sliding tube with a microphone probe located at its midpoint. The tubes were lapped to fit in order to eliminate any leakage of sound between them. A slotted configuration was selected, rather than one with two tube segments connected by an outer sliding tube, because it offered less change in cross-sectional area and therefore potentially less interference with the sound field.

Using the coordinate shown in Fig. 3,

$$p(x,t)=e^{j(\omega/c)x}(A_0e^{\alpha x}+B_0e^{j[\theta-2(\omega/c)x]}e^{-\alpha x})e^{j\omega t}, \quad (1)$$

from which

$$|p(x,t)|^2=A_0^2e^{2\alpha x}+B_0^2e^{-2\alpha x}+2A_0B_0\cos[\theta-2(\omega/c)x], \quad (2)$$

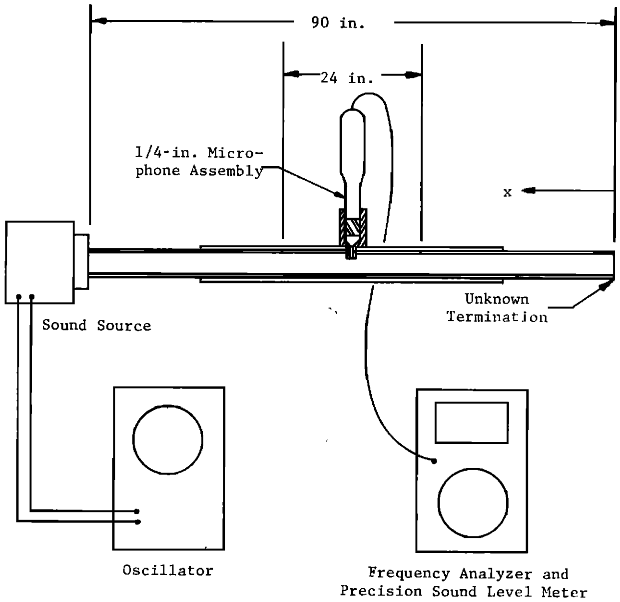

FIG. 3. Standing-wave apparatus. Inner tube is 0.500-in. o.d.×0.430-in. i.d. aluminum tubing with 24-in. slot. Outer tube is $\frac{7}{16}$-in. o.d.×0.502-in. i.d. brass tubing, 54 in. long.

where $p(x,t)$ is the instantaneous sound pressure at location x; x, the distance from unknown termination; α, the attenuation constant; ω, the circular frequency; A_0, the incident-wave amplitude at $x=0$; B_0, the reflected wave amplitude at $x=0$; and θ, the phase angle between incident and reflected waves.

After $x_{\min}$, the distance from the unknown termination to a pressure minimum, had been measured, θ was obtained by differentiating the expression for $|p(x,t)|^2$ and equating the result to zero. This operation gave

$$\sin\left[\theta-2\frac{\omega}{c}x_{\min}\right]=\frac{\alpha(R_0e^{-2\alpha x_{\min}}-(1/R_0)e^{2\alpha x_{\min}})}{2(\omega/c)}, \quad (3)$$

where $R_0=B_0/A_0$, the reflection factor magnitude at $x=0$. For most practical cases,

$$\sin[\theta-2(\omega/c)x_{\min}]\simeq 0,$$

and

$$\theta=2(\omega/c)x_{\min}-N\pi, \quad N=1,3,5,\cdots, \quad (4)$$

where N is the largest odd integer giving a positive value for θ. Note that

$$2\frac{\omega}{c}x_{\min}=\frac{2\pi x_{\min}}{\lambda/2}=\frac{2\pi x_{\min}}{|x_{\min_1}-x_{\min_2}|}, \quad (5)$$

where λ is the wavelength of sound in medium, and $x_{\min_1}-x_{\min_2}$ is the distance between adjacent standing-wave minima. If attenuation is large or R_0 is small, significant errors in calculation of R_0, θ, and λ may result. However, for frequencies above 300 Hz, errors

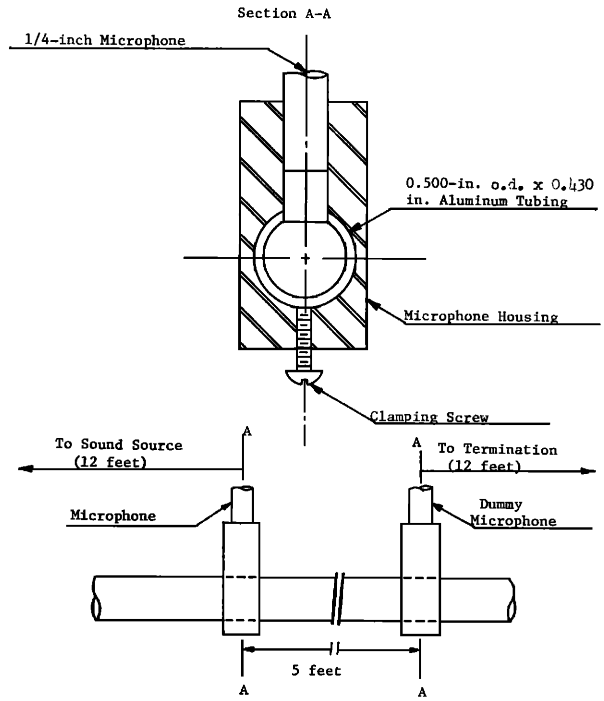

FIG. 4. Flush-mounted microphone assembly.

are negligible when Eq. 4 is used for determining θ and R_0 is calculated according to

$$R_0=\frac{p_{\max}e^{-\alpha x_{\min}}-p_{\min}e^{\alpha x_{\max}}}{p_{\max}/e^{\alpha x_{\min}}+p^{\min}/e^{\alpha x_{\max}}}. \qquad (6)$$

The accuracy of standing-wave measurements depends also upon the degree of interference imposed on the sound field by the measuring apparatus. Measurement of sound pressures without interference is particularly difficult when small-diameter tubes are involved; furthermore, the extent of interference cannot be determined from standing-wave measurements alone. Fortunately, the transient method is well suited for determining if measurable interference is produced by the standing-wave tube.

TABLE IV. Effect of standing-wave tubes on sound field.

Frequency (Hz)	Relative amplitude ratio[a] standing-wave tube/plain tube $\frac{9}{32}$ in. slot	$\frac{1}{16}$ in. slot	Phase-angle[b] difference standing-wave tube versus plain tube (degrees) $\frac{9}{32}$ in. slot	$\frac{1}{16}$ in. slot
300	1.03	0.97	−10	−6
500	1.12	1.03	+7	0
1000	1.00	1.00	−7	0
3000	0.87	1.05	+14	0

[a] Amplitude ratio is the ratio of reflected wave to incident wave.
[b] Phase angle is the angle between reflected and incident waves, at microphone location.

Two standing-wave tubes were tested. One had a $\frac{9}{32}$-in. wide slot that exposed a $\frac{1}{4}$-in.-diam microphone directly to the sound field. The other had a $\frac{1}{16}$-in. slot and a small probe tube with a 0.040 in.-diam connecting passage between the sound field and microphone. Both slots were 24 in. long, which limited the lower frequency limit for air to about 300 Hz, since calculation of reflection factors required measurement of at least one pressure maximum and one pressure minimum. The $\frac{1}{16}$-in. slot caused less reflection of sound waves, but the probe and cavity had to be designed so that pressure resonances in the cavity had a negligible effect on the sound field.

The first investigation involved only the extent of reflections from a $\frac{1}{4}$-in. dummy microphone inserted into the wall of 0.415-in. i.d. aluminum tubing, 5 ft from the recording microphone (Fig. 4). No reflections were observed at frequencies ranging from 300 to 5000 Hz.

In a second investigation, each standing-wave tube was inserted into the transient apparatus, and the reflections were recorded by a microphone located a few feet away (toward the sound source). Measureable reflections (which must be increased in amplitude to compensate for attenuation) were produced from the $\frac{9}{32}$-in. slot (Fig. 5); in a subsequent test, no observable interference resulted from the $\frac{1}{16}$-in. slot.

To further substantiate these findings, a third investigation was undertaken in which incident and reflected waves were recorded by a microphone placed alternately in each standing-wave tube (microphone location at midpoint of traverse) and at the midpoint of an unslotted tube of the same length. Results are shown in Table IV. The $\frac{9}{32}$-in. slot caused significant changes in amplitude ratio and relative phase angle at some frequencies, whereas the $\frac{1}{16}$-in. slot had a negligible effect on these quantities.

For a final check on the accuracy of the standing-wave tube with $\frac{1}{16}$-in. slot, the reflection factor for a rigid termination was calculated at various frequencies. Expected values for R are a magnitude $R_0=1.0$ and a phase angle $\theta=0°$. Calculated values for 300, 500, and 1000 Hz agreed with the theoretical values to within 0.05 for amplitude and to within 5° for angle. However, unpredictable and sometimes large (approaching 1 rad) errors were found in calculated values for phase angles at 3000 and 5000 Hz.

The source of these errors was found to be the distance (85–105 cm) between the microphone and the termination, relative to the half-wavelength at these frequencies. At 5000 Hz, the half-wavelength could be measured to an accuracy corresponding to about 1° of phase shift. The accumulated error in the total phase shift due to separation of the microphone and termination could then exceed 50°. At 3000 Hz, the error was correspondingly less, but still significant. Therefore, an additional standing-wave tube was built with a slot that was $\frac{1}{16}$ in. wide and $2\frac{1}{2}$ in. long. These dimensions permitted

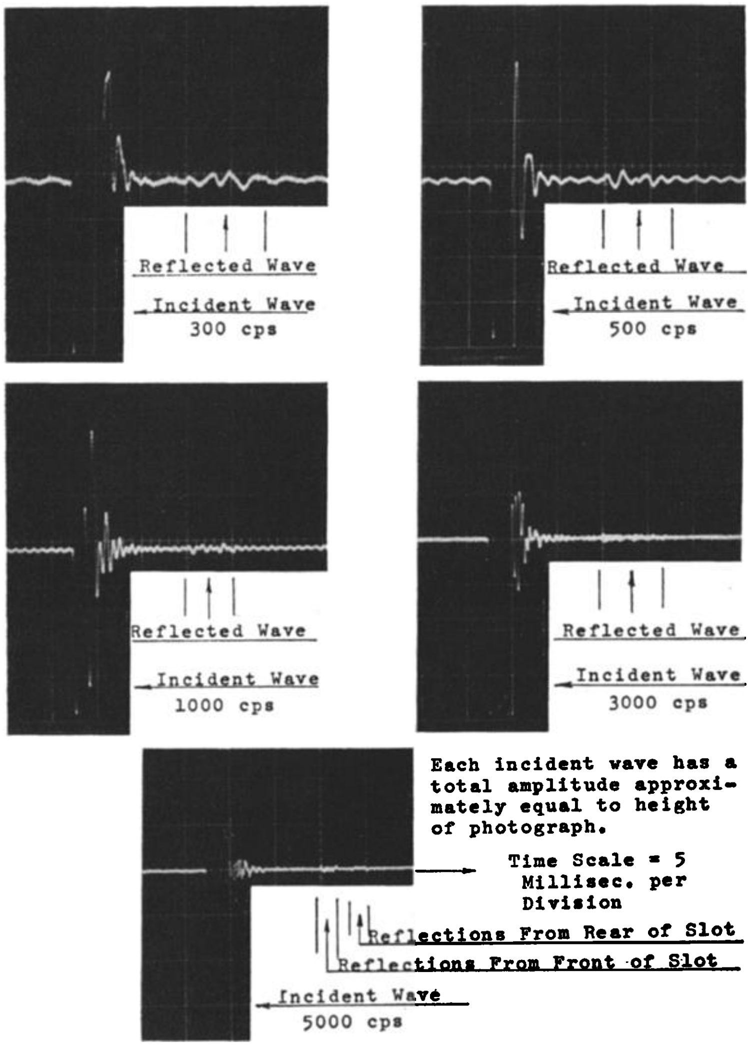

FIG. 5. Reflection of sound from $\frac{3}{32}$-in. slot.

the location of at least one pressure minimum and one pressure maximum at 3000 Hz (for air), and two pressure minima at 5000 cps. Locations of minima were about 15 cm from the unknown termination; accumulated phase-angle error was thereby reduced to 5° or less.

The standing-wave tubes described were suitable for measurements when the termination was a muffler, part

TABLE V. Reflection factors for rigid termination.

Frequency (Hz)	Standing-wave tube[a] A R_0	A θ	B R_0	B θ	C R_0	C θ	Maximum deviation R_0	Maximum deviation θ	Theoretical values R_0	Theoretical values θ
300	0.985	−1			0.978	−1	0.008	1	1.0	0
	0.986	0								
500	0.990	1			0.984	3	0.014	2	1.0	0
	0.978	3			0.976	1				
1000	0.990	2			0.994	−1	0.019	8	1.0	0
	0.991	−6			0.975	0				
3000			0.995	−3	0.985	3	0.023	7	1.0	0
			0.993	−3	0.972	4				
5000			0.992	−1	0.965	1	0.028	8	1.0	0
			0.993	−7						

[a] Standing Wave Tube: A—$\frac{1}{2}$-in. o.d. tubing, $\frac{1}{16}$-in.×24-in. slot; B—$\frac{1}{2}$-in. o.d. tubing, $\frac{1}{16}$-in.×2$\frac{1}{2}$-in. slot; C—1$\frac{1}{2}$-in. i.d. tubing, $\frac{3}{32}$-in.×24-in. slot; R_0 is the reflection factor magnitude; θ, the reflection factor angle in degrees.

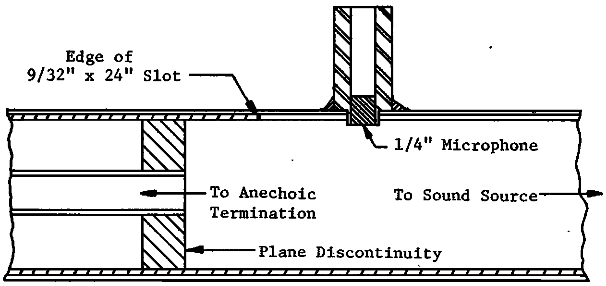

FIG. 6. Large-diameter standing-wave tube. Inner tube is 1½-in. i.d.×0.030-in. wall brass tubing. Outer tube is 1 9/16-in. i.d.×0.030-in. wall brass tubing. Tubes are lapped together for sliding fit.

of a system, or an expanding discontinuity. Evaluation of reducing discontinuities (such as those in simple expansion chambers) required construction of another standing-wave tube, shown in Fig. 6.

The three standing-wave tubes are shown in Fig. 7. Each is mounted in a supporting framework to which a meter stick graduated in millimeters is attached. With care, measurements to the nearest tenth of a millimeter can be made by roughly positioning the outer tube and then using the fine adjustment (shown on the middle standing-wave tube, Fig. 7) for a final location. In practice, the locations of pressure minima cannot be determined to sufficient accuracy from single measurements (the minima are usually not well defined relative to the accuracy required). Satisfactory results can be obtained, however, by averaging two distance measurements at equal pressures close to a minimum but on either side of it.

Results of standing-wave measurements for a rigid termination are given in Table V. Error limits, based upon repeated measurements with each tube, are also listed for θ and R_0. Note that phase angles measured with the large diameter standing-wave tube are in very good agreement with theoretical values. The reason for this is that the termination can be inserted into the inner tube and located at the slotted portion (Fig. 6). The distance between measured pressure minima and the termination is therefore minimized; correspondingly, the effect of measurement errors on phase angle is also minimized.

TABLE VI. Performance of anechoic terminations.

Frequency (Hz)	Reflection-factor magnitude ½-in. o.d. tube	1½-in. i.d. tube
300	0[a]	0.01
500	0[a]	0.03
1000	0.01	0.02
3000	0.04	0.01
5000	0.05	0.01

[a] No measurable standing-wave maxima or minima.

The standing-wave method has several advantages relative to the three-pressure and transient methods:

1. The magnitude and phase angle of sound reflected from an arbitrary termination can be measured with satisfactory accuracy.
2. Calculations required are relatively simple and straightforward.
3. The propagation velocity and wavelength of sound at a specified frequency can be determined from one set of measurements.

Disadvantages of the standing-wave method are:

1. Several standing-wave tubes of varying diameters and lengths may be required to accommodate the desired frequency ranges and types of terminations.
2. Standing-wave measurements can be tedious and time consuming. The required procedures may not be as readily adaptable to electronic calculation of reflection factors as those employing fixed microphones.[20,22]

III. MEASUREMENT OF TRANSMISSION FACTORS

The method developed for measuring the transmission characteristics of acoustic filters used reflection factors previously found from standing-wave measurements. An anechoic termination was not absolutely necessary, but its use greatly simplified the calculation of both reflection and transmission factors from measured data. (The alternative designs and method of evaluation of two anechoic terminations are presented in the next Section.)

The data required at each frequency were the reflection factor and the relative sound-pressure amplitudes and phase angle at fixed locations on either side of the

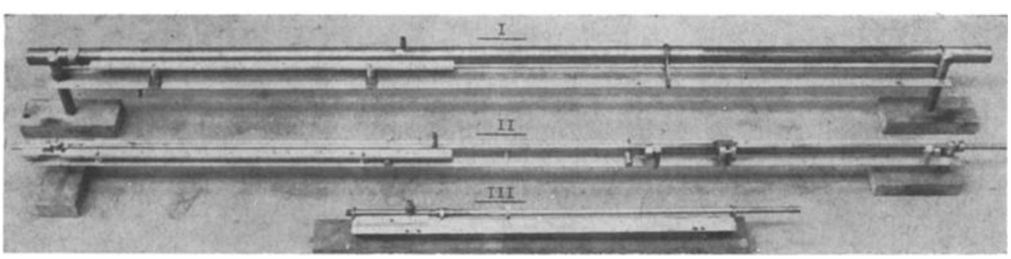

FIG. 7. Standing-wave tubes. I. 1½-in. i.d., with 9/32-in.×24-in. slot. II. ½-in. o.d., with 1/16-in.×24-in. slot. III. ½-in. o.d., with 1/16-in. ×2½-in. slot.

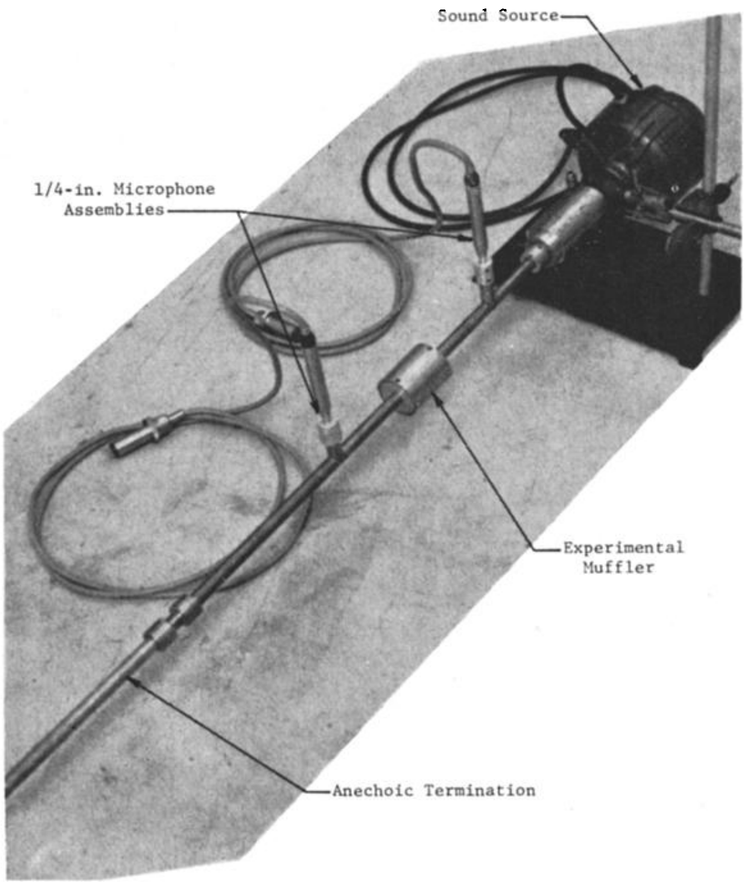

FIG. 8. Equipment for measuring transmission factors.

filter or filter element being investigated (Fig. 8). Two microphones, either matched or calibrated for amplitude and phase response, are needed for simultaneous recordings; one microphone and a suitable plug can be used for sequential measurements if an oscilloscope with the necessary delay and triggering features is available. Since the input parameters were phasors, transmission factors were most easily calculated by employing either graphical techniques or a digital computer. The reflection factor was used to determine the amplitude and phase angle of the incident wave; these quantities plus the magnitude and phase angle of the transmitted wave were then combined to yield the transmission factor.

IV. DESIGN AND EVALUATION OF ANECHOIC TERMINATIONS

Development of satisfactory anechoic terminations was essential for the determination of reflection and transmission factors for mufflers and muffler elements. Various designs were evaluated by observing reflections (as evidenced by a standing-wave pattern) in a standing-wave tube. (A true anechoic termination produces no reflections, and therefore no standing waves are present.) Several lengths of tubing packed with cotton were investigated and found unsatisfactory. A successful design (Fig. 9) was found by packing a 3-ft length of fine steel wool in a 6-ft length of tubing of appropriate diameter.[40] A sliding piston was inserted into the free space, to a location that produced minimum reflections at each frequency.[19] The particular location for each frequency was determined by trial and error.

Table VI lists the standing-wave ratios and approximate reflection-factor magnitudes for the two anechoic

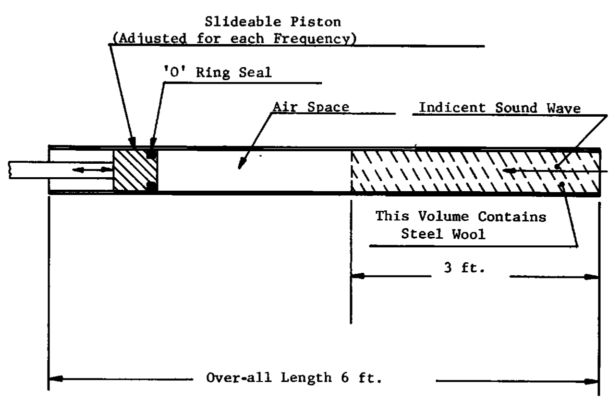

FIG. 9. Anechoic termination.

[40] D. A. Bies, "Acoustic Properties of Steel Wool," J. Acoust. Soc. Amer. 35, 495–499 (1963).

terminations used in the experimental measurements. Note that reflected sound is 5% or less of incident sound at the frequencies listed.

V. CONCLUSIONS

- A survey of pertinent literature revealed that little information was available concerning the measurement of reflection and transmission factors for small acoustic filters and filter elements.

- Of the three methods investigated, the standing-wave method was the most satisfactory from the standpoints of accuracy and ease of use. However, the transient method was an excellent tool for evaluating other methods, specifically the standing-wave method. The limits on accuracy for standing-wave determinations of reflection factors were approximately 5% for magnitude and 6° for phase angle, at frequencies ranging from 300 to 5000 Hz.

- Determination of transmission factors within approximately the same accuracy limits was possible if the reflection factor and the relative magnitudes and phase angle of sound pressures on either side of the filter were known, when the filter was terminated anechoically.

- Satisfactory anechoic terminations, with reflection factor of 0.05 or less, were constructed by packing steel wool into tubes with adjustable air spaces.

- The ability to determine reflection and transmission characteristics for acoustic filters makes possible several investigations of importance to systematic filter design: (a) determination of the effect of transverse dimensions on results predicted from simple (plane-wave) acoustic theory; (b) determination of the effects of steady flow and temperature changes on acoustic characteristics of filters; (c) formulation of a general method for predicting the performance of a given filter in a given system, or for designing a filter to meet specific performance requirements.

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