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Free-convection Heat Transfer From An Inclined Heated Flat Plate In Air

S. C. Yung

R. B. Oetting
Missouri University of Science and Technology, oettingr@mst.edu

Δx, depth Δz, and height $\delta(x)$. The volume V of this element may be written as

$$V_{\text{ABCD}} \simeq (\Delta x)(\Delta z)(\text{average height})$$

If the height AD is taken equal to δ, then

$$V_{\text{ABCD}} \simeq (\Delta x)(\Delta z)\left(\delta + \frac{\partial \delta}{\partial x}\frac{\Delta x}{2}\right) \quad (5)$$

We also write

$$Q_{\text{AD}} = \frac{\tau_0 \delta^2}{2\mu_L}\Delta z, \quad (6)$$

$$Q_{\text{BC}} = Q_{\text{AD}} + \frac{\partial Q_{\text{AD}}}{\partial x}\Delta x$$
$$= \left\{\frac{\tau_0\delta^2}{2\mu_L} + \Delta x\left(\frac{\delta^2}{2\mu_L}\frac{\partial \tau_0}{\partial x} + \frac{\tau_0 \delta}{\mu_L}\frac{\partial \delta}{\partial x}\right)\right\}\Delta z \quad (7)$$

where τ_0 is the shear stress at x_{AD}. If the local rate of vaporization is proportional to shear and if the average stress along AB is

$$\tau_{\text{av}} = \tau_0 + \frac{\partial \tau_0}{\partial x}\frac{\Delta x}{2}$$

we write for Q_{DC}

$$Q_{\text{DC}} = k \cdot \tau_{\text{av}} \cdot \Delta x \Delta z = k\left(\tau_0 + \frac{\Delta x}{2}\frac{\partial \tau_0}{\partial x}\right)\Delta x \Delta z \quad (8)$$

where k is a constant and Q_{DC} is the amount of liquid evaporating due to the action of τ_{av} on the liquid surface. Setting

$$\frac{\partial V}{\partial t} = Q_{\text{AD}} - Q_{\text{BC}} - Q_{\text{DC}} \quad (9)$$

and simplifying we obtain

$$\frac{\partial \delta}{\partial t} + \frac{\partial^2 \delta}{\partial x \partial t}\frac{\Delta x}{2} + \frac{\delta^2}{2\mu_L}\frac{\partial \tau_0}{\partial x} + \frac{\tau_0\delta}{\mu_L}\frac{\partial \delta}{\partial x} + k\tau_0 + k\frac{\partial \tau_0}{\partial x}\frac{\Delta x}{2} = 0 \quad (10)$$

In the limit as $\Delta x \to 0$, equation (10) becomes

$$\frac{\partial \delta}{\partial t} + \frac{\delta^2}{2\mu_L}\frac{\partial \tau_0}{\partial x} + \frac{\tau_0 \delta}{\mu_L}\frac{\partial \delta}{\partial x} + k\tau_0 = 0 \quad (11)$$

The proper choice of μ_L must be such that the second and third terms of equation (11) are vanishingly small. In this case we have

$$\frac{\partial \delta}{\partial t} \simeq -k\tau_0 \quad (12)$$

Let us consider the case of laminar adiabatic flat plate flow; this case is chosen just to illustrate the evaluation of μ_L. Since the shear stress is assumed continuous at the liquid-air interface and the liquid film is in Couette flow, τ_0 and $\partial\tau_0/\partial x$ may be evaluated using the Blasius solution:

$$\frac{u}{u_\infty} = F'(\eta); \quad \eta = y\sqrt{\frac{u_\infty}{\nu x(2-\beta)}}$$

and

$$\frac{\partial u}{\partial y} = \frac{u_\infty^{3/2}}{\sqrt{(2-\beta)\nu x}}F''(0) \quad (13)$$

For a liquid film 10^{-5} ft thick evaporating in 100 sec[6] we have $\partial\delta/\partial t = 10^{-7}$. In laminar flow where $\beta = 0$ and $F''(0) = 0.4696$[8], if $x = 3$ in., $u_\infty = 200$ fps, and $\nu_{\text{air}} = 1.566 \times 10^{-4}$ ft²/sec, then $\partial\delta/\partial x \simeq 10^{-4}$, $\partial u/\partial y = 1.5 \times 10^5$ and the second and third terms in equation (13) are of order $(\mu_a/\mu_L)10^{-5}$ and $(\mu_a/\mu_L)10^{-4}$, respectively. If

$$\mu_a/\mu_L \leq 10^{-3} \quad (14)$$

the second and third terms of equation (13) would at most be of order 10^{-8} and 10^{-7}, respectively, and hence can be neglected. The rate of evaporation is then proportional to the shear stress. Murphy and Smith determined the appropriate μ_a/μ_L ratio by experimenting with several fluids until the desired rate of vaporization, as determined by the validity of their final results, was obtained.

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Free-Convection Heat Transfer From an Inclined Heated Flat Plate in Air

S. C. YUNG[1] and R. B. OETTING[2]

Results of investigations of free-convection heat transfer from a heated flat plate are numerous [1–6].[3] However, most of these published results, both theoretical and experimental, were obtained with the flat plate in only two positions—vertical and horizontal. There are limited data available on free-convection heat transfer from a heated inclined flat plate.

The present investigation was undertaken as a step in fulfilling the need for more data in this area, and to gain an understanding of the effect of angular position on the free-convection heat transfer. This Brief presents the results of experimental work conducted by the authors on the free-convection heat transfer from a heated flat plate in air moved from the vertical position through three inclined positions to the horizontal position, with the hot surface facing downward for the horizontal and three inclined positions.

Experimental Method

Heat was supplied at the bottom of a 27-in-long × 7-in-wide × 11/32-in-thick aluminum flat plate in an even distribution through the use of 10 electrical resistance heating elements connected in series. A variable autotransformer was used to control the power supplied to the resistance elements and thus to con-

[1] Graduate Student, University of Illinois, Chicago, Ill.

[2] Associate Professor of Mechanical and Aerospace Engineering, University of Missouri at Rolla, Rolla, Mo.

[3] Numbers in brackets designate References at end of Note.

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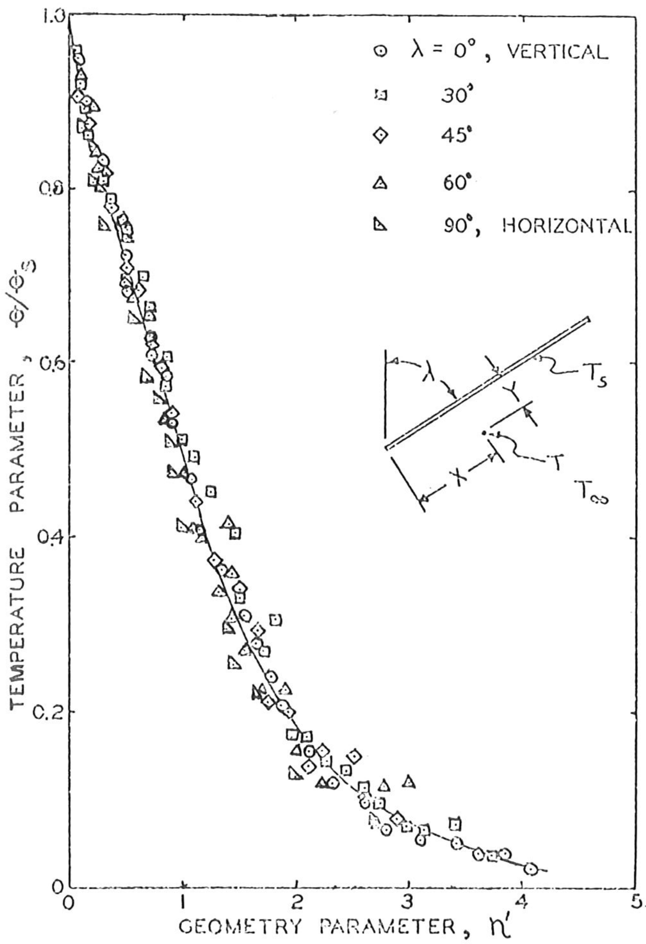

Fig. 1 Dimensionless temperature profile for laminar free convection on an inclined heated flat plate in air

trol the temperature. A $^7/_8$-in-thick insulating board was located below the electrical heating elements to limit the heat flow in that direction. Transite plates of $^1/_2$-in. thickness were used on each side (edge) of the plate to further reduce the heat loss. The exposed surface of the plate was finished to a flat tolerance of 0.002 in. Plate temperature was recorded using thermocouples located at eight positions on the heated side to determine the uniformity of the temperature distribution and the attainment of steady-state condition.

A movable copper-constantan thermocouple of extremely fine wire at the end of its hot junction was used to record the temperature at predetermined points in the airstream on the hot side of the plate. The copper and constantan wires (28 gage) were joined to make the sensing element of a diameter less than 0.004-in. The thermocouple was attached to a small rectangular plastic bracket with the thermocouple wires extending parallel to the hot surface and the plate reference (lower) edge. This extension of $1^1/_4$ in. in the transverse direction results in negligible conduction error along the thermocouple wires. A check was made to determine the effect of radiation on the thermocouple temperature indication and a temperature error of less than 1 deg F was estimated. The thermocouple sensing element and the plastic bracket are attached at right angles to a $^3/_{16}$-in-OD stainless-steel tube. This total assembly can be located at any position in the thermal boundary layer. The vertical, Y, position of the sensing element is achieved through the use of a worm and gear combination that allows for a displacement of 0.003 in. for each revolution of a wheel on the gear assembly. Data were taken at each plate inclination, λ, of 0 (vertical), 30, 45, 60, and 90 deg (horizontal). The thermocouple was first positioned along the center line of the plate near the reference (lower) edge and moved normal to the plate (Y-direction) into the thermal boundary layer from surface contact to as high as 0.500 in. from the heated plate. Temperatures were measured with vertical spacing of 0.006 in. up to 0.050 in., then in increments of 0.015 in. up to as high as 0.500 in. (in some cases) above the plate. The thermocouple was then moved in a traverse direction parallel to the lower edge covering a span of approximately 2 in. to either side of the center line and temperature measurements made at several intermediate points. These measured temperatures were several degrees lower than those along the center line. However, for this preliminary analysis, no attempt was made to include the three-dimensional effect, assuming that the plate was sufficiently wide so that measurements made along its center line would not be significantly affected. This seems justified since the heated plate is facing down forcing the hot boundary layer upward and into the plate resulting in a thinner boundary layer than would be present if the hot side faced upward. Rich [6] noted this three-dimensional effect for a flat plate heated from below and recommended use of a modified Grashoff number (vertical plate Grashoff number multiplied by the cosine of the angle of inclination ranging from 0 to 40 deg measured from the vertical with the hot surface facing upward). The thermocouple was then returned to the center line, located at a new position a distance X from the lower edge of the plate, and the foregoing procedure repeated. A minimum of five X-positions was selected for each plate inclination starting with 1-in. (Gr $= 7 \times 10^4$) and ranging to as high as 15-in. (Gr $= 3 \times 10^8$) from the lower edge. The maximum length used in the horizontal case was 13.50 in. (Gr $= 2 \times 10^8$), one half the total plate length.

Discussion of Results

Limited data were taken with the flat plate in the vertical and horizontal positions since the purpose was only to compare results of the present investigation with previous work. Results of these tests show good agreement with previous investigations [4–6].

A solution to the continuity, momentum, and energy equations which is applicable to the free-convection thermal boundary layer has been determined [7]. This solution demonstrates that introduction of the stream function, ψ, allows a reduction of the partial differential equations to two ordinary differential equations by the similarity transformation

$$\eta = c(Y/X^{1/4}) \quad (1)$$

$$\psi = 4cx^{3/4}f(n) \quad (2)$$

where

$$c = \{g(T_s - T_\infty)/4\nu^2T_\infty\}^{1/4} \quad (3)$$

A suggested modification of equation (3), which includes the plate inclination, λ, is given by

$$B = \{(1 + \cos\lambda)/2\}\{g(T_s - T_\infty)/4\nu^2T_\infty\}^{1/4} \quad (4)$$

Thus equation (1) becomes

$$\eta' = B(Y/X^{1/4}) \quad (5)$$

and now accounts for the plate inclination angle, λ, as well as the position from the plate leading edge, X, and the location within the thermal boundary layer normal to the surface, Y. Equation (4), suggested by Yung [8], includes a modification that results in close agreement with the vertical and horizontal (hot side down) flat-plate solutions suggested by Jacob [9] and McAdams [10], respectively. These two solutions for air are of the form

$$\overline{\text{Nu}} = C(\text{Gr}_L)^{1/4} \quad (6)$$

where

$$C = 0.51, \text{ heated vertical plate}$$

$$C = 0.25, \text{ horizontal plate, hot side down}$$

It should be noted that the horizontal plate solution is applicable for square plates. However, data correlations are good when the thermal boundary layer is observed along the plate center line from the reference edge up to the center or stagnation point. This corresponds to a limiting Grashoff number of approximately 10^8.

The temperature distribution in the free-convection boundary layer is determined by the function $\phi(\eta')$ where ϕ is given as $\theta/\theta_s = (T - T_\infty)/(T_S - T_\infty)$. Thus, since $\eta' = B(Y/X^{1/4})$, then the nondimensional temperature distribution, θ/θ_s, is a function of $Y/X^{1/4}$ and the plate inclination, λ. With a Prandtl number of 0.733 for air, the experimental results for all inclined angles are shown in Fig. 1.

Plate temperature during the tests was maintained at 165 ± 5 deg F. Room temperature was maintained at 85 ± 5 deg F. Temperature changes were due to day-to-day variations with data being taken at constant room temperature conditions.

The temperature profiles for all plate inclinations are in good agreement with the theory as shown in Fig. 1. Thus it is possible to develop an expression to determine the average value of the laminar free-convection Nusselt number, $\overline{\text{Nu}}$, based on the theoretical approach [7]. This equation then is

$$\text{Nu} = 0.48\{(1 + \cos\lambda)/2\}(\text{Gr}_L)^{1/4} \quad (7)$$

where

$$\text{Gr}_L = g(T_s - T_\infty)L^3/T_\infty\nu^2 \quad (8)$$

In equation (8), L is the length of plate measured from the reference (lower) edge over which the average Nusselt number is being determined. Grashoff numbers for use in equation (7) range from 10^5 to 10^9.

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Analysis of Metal Ammonia Solutions as Heat Transfer Fluids to −185 deg C[1]

S. NAIDITCH[2] and E. B. GRAPER[3]

The low freezing temperatures and high thermal conductivities of metal ammonia solutions make them attractive as heat transfer fluids in the range from +60 to −185 deg C, offering a potential substitute for the cryogenic boiling heat transfer usually required below −50 deg C. Although chemically metastable and highly corrosive, some of these solutions have been studied at 190 deg C for an hour without measurable decomposition and have been stored for years at room temperature.

Concentrated solutions of metals in liquid ammonia act as "liquid metals." Where the properties are known, these solutions have the usual properties of metals, including the thermal properties which make metals such as lithium such good heat transfer fluids.

Most of the elements in period 1 and 2 of the periodic table are soluble in ammonia; see Table 1. The solutions of the alkali metals are of particular interest due to their low freezing temperatures. These solutions have been studied intensively for 100 years but have not been explored as heat transfer media due at least in part to the fact that they have been thought to be unstable even at −30 deg C. Recent work has shown that the metastable sodium ammonia solutions are effectively stable indefinitely at room temperature (20 deg C) and for at least an hour at +190 deg C [1].[4]

Table 1 Principal alkali metal ammonia solution [2]

Metal	Atomic percent of metal	Eutectic temperature, deg C
Li	22	−185
Na	17	−110
K	15	−157
Cs	. . .	−118

Metal Ammonia Solutions

Concentrated metal ammonia solutions are bronze colored and look exactly like a molten metal. Their viscosities are of the order of ammonia and their electrical conductivities of mercury. From a heat transfer point of view, some of the properties of metal ammonia solutions are unknown. In particular, there are no specific heat data. Table 2 provides an indication of the expected performance of a lithium ammonia solution compared to that of water, lithium, and ammonia as heat transfer fluids.

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[2] Unified Science Associates, Inc., Pasadena, Calif.

[3] Unified Science Associates, Inc., Pasadena, Calif.; now at Sloan Instrument Co., Santa Barbara, Calif.

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