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A Determination of the Speed of Light by the Phase-Shift Method

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A low-frequency, phase-shift method for the measurement of the speed of light has been developed. This technique gives results commensurate with other advanced laboratory methods. One advantage of this technique is that the apparatus is of reasonable size and most of the circuitry involves widely known amateur radio techniques. Furthermore, our use of a low modulating frequency permits use of the solid-state, electro-optical light shutter. This eliminates the rather dangerous liquid Kerr cell, thus making our apparatus more acceptable to application in the undergraduate advanced laboratory. From a pedagogical point of view, the student is allowed to use and become acquainted with the lock-in amplifier which is so commonly found in the modern research laboratory. Two rather novel techniques were employed in this apparatus. These were phase multiplication and mixing of the rf signal in the last stages of the photomultiplier. Our value for the speed of light in air was 2.937×10^8 m/sec. The accepted value of the speed of light in air to the same number of significant figures is 2.997×10^8 m/sec.

INTRODUCTION

Several methods for the determination of the velocity of light suitable for use in the advanced optics laboratory have been developed. The Kerr cell method,[1] measurement of electrical constants,[2] and the Foucault rotating-mirror method[3] are perhaps the most familiar.[4,5]

An improvement of the traditional Kerr cell approach is presented here. It is felt that the new method has several distinct advantages over the traditional Kerr cell procedure while retaining its desirable features. The traditional setup requires modulation of a liquid Kerr cell at rf frequencies and the shifting about of a pair of mirrors to obtain a signal "null" in the detector. In the new procedure herein described, a solid-state, electro-optical light modulator is employed at the rather low frequency of 510 kHz, thereby obviating the tedious requirements of rf shielding and permitting direct use of a commercial solid-state light shutter which is a safe and simple device. The phase signal obtained at this low frequency is multiplied electronically by a factor of 9 making the phase measurement equivalent to an experiment carried out at 4.59 MHz. Data consist of precise measurements of the actual phase relation between a reference beam and a signal beam. The use of a sensitive phase detection system allows a very sharp phase determination.

I. EXPERIMENTAL PROCEDURE

In the measurement of the speed of light, the equipment was arranged as indicated in Fig. 1.

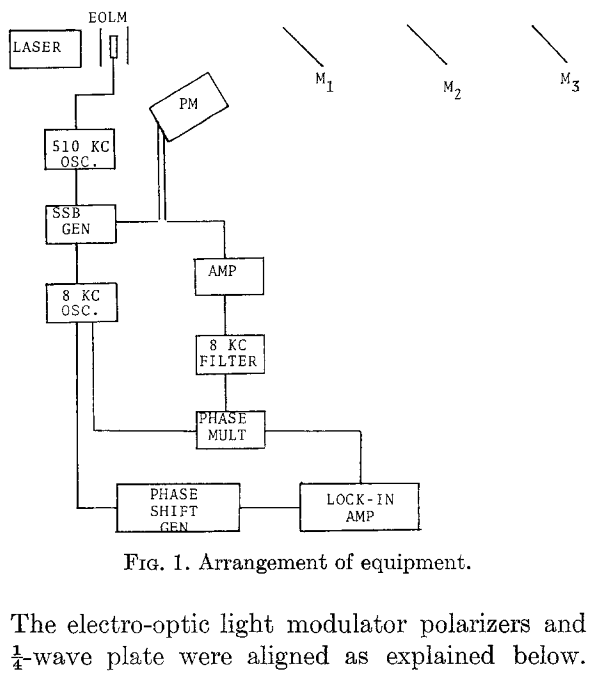

FIG. 1. Arrangement of equipment.

The electro-optic light modulator polarizers and $\frac{1}{4}$-wave plate were aligned as explained below. The crystal was modulated at 510 kHz with a 1000-V peak-to-peak sine wave. A light signal from a laser was sent through the modulator and was reflected from a good-quality, front-surfaced mirror and collected into a 9558Q EMI photo-

[1] C. H. Palmer and George S. Pratt, Amer. J. Phys. **22**, 481 (1954).

[2] George W. Clark, Amer. J. Phys. **24**, 189 (1956).

[3] L. T. Dillman, Amer. J. Phys. **32**, 567 (1964).

[4] N. R. Isenor, Amer. J. Phys. **35**, 355 (1967).

[5] J. H. Sanders, *The Velocity of Light* (Pergamon Press, Inc., New York, 1965).

multiplier. It was mixed in the photomultiplier with a 502-kHz signal to obtain an 8-kHz signal containing the phase information. This signal was amplified and filtered and then multiplied by 9 in the phase-multiplying system. The filter was from the Electronic Research Company, model 6200–1. The data reported in this paper was taken using this filter, but equally reliable data can be taken without a filter. Rf signals are attenuated naturally by the circuit. A reference 8-kHz signal from the 8-kHz oscillator, which was a standard signal generator, was multiplied by 8 and then mixed with the 72-kHz signal containing the phase angle 9ϕ to obtain an 8-kHz signal with phase angle of 9ϕ. This was then sent into the signal channel of a PAR lock-in amplifier model JB-4. A reference 8-kHz signal from the 8-kHz oscillator was sent through the phase-shift generator into the reference channel of the lock-in amplifier. The signal amplitude was monitored and set to 4 V peak-to-peak, and the phase angle dialed so as to zero the lock-in amplifier output. The reading on the phase shift generator was recorded. Next, the mirror was moved a known distance. The signal amplitude was again set to 4 V peak-to-peak, and another phase reading was taken as before. The values of the phase angles were calculated and subtracted giving the phase shift from which the speed of light can be calculated.

When the lock-in amplifier is used as a phase detector, its output is proportional to the cosine of the phase difference between the signal and the reference inputs. This can be seen by the following argument. Consider a sine wave sent into the signal channel of the lock-in amplifier. This signal goes through a selective amplifier and then into the phase sensitive detector which is a product device (a balanced mixer). This signal may be called

$$S' = A' \cos(\omega t + \phi),$$

where A' is the amplitude, ω is the angular frequency, and ϕ is the phase angle associated with a position of the mirror. At the same time a reference signal is sent into the reference channel of the lock-in amplifier. This signal passes through a phase-shift generator and thus into the phase-sensitive detector. This signal may be called

$$S'' = A'' \cos(\omega t + \alpha),$$

where A'' is the amplitude, ω is the angular frequency which is the same as for S', α is the phase angle of this signal and can be varied by using a calibrated phase-shift generator. The phase-sensitive detector, a product device, mixes the two signals giving a signal of the form

$$S''' = S'S'' = A'A'' \cos(\omega t + \phi) \cos(\omega t + \alpha).$$

Using a trignometric identity, this becomes

$$S''' = (A'A''/2) [\cos(2\omega t + \phi + \alpha) + \cos(\phi - \alpha)]$$

This signal is sent through a low-pass filter, and only the dc term, $\cos(\phi - \alpha)$, survives. Finally,

$$S^{IV} = (A'A''/2) \cos(\phi - \alpha).$$

With the mirror in the first position, as explained above, set $S^{IV} = 0$. That is, vary α so that

$$\alpha_1 = \phi_1 - \pi/2.$$

With the mirror moved to the second position, again set $S^{IV} = 0$, with the calibrated phase-shift generator as explained above. Now

$$\alpha_2 = \phi_2 - \pi/2,$$

where α_1, α_2 and ϕ_1, ϕ_2 are the phase angles associated with the reference and signal inputs, respectively, corresponding to mirror positions one and two. The phase difference $\alpha_1 - \alpha_2$ or $\phi_1 - \phi_2$ is of primary interest. It is

$$\alpha_1 - \alpha_2 = \phi = \phi_1 - \phi_2,$$

which gives the phase shift associated with the known light-path difference.

In order to modulate the light, a Baird–Atomics Electro-Optic Light Modulator (E.O.L.M.) model JW-1 was used, and it modulated the light from a University Laboratories $\frac{1}{4}$-mW model 240 He–Ne laser. The basic component of the E.O.L.M., which can be considered a crystalline analogue of the Kerr cell, consists of a Z-cut (001) plate of potassium dihydrogen phosphate (KDP). The crystal plate is placed between electrodes which allow light to pass in the same direction as the applied electric field. For normally incident collimated light, the unit has the properties of a polarization retardation plate, with the magnitude of its retardation directly proportional to the applied voltage. On placing the crystal unit between polarizers, a light beam can be intensity modulated in accordance with the voltage applied to frequencies well beyond the audio region.

By placing a $\frac{1}{4}$-wave-retardation plate adjacent

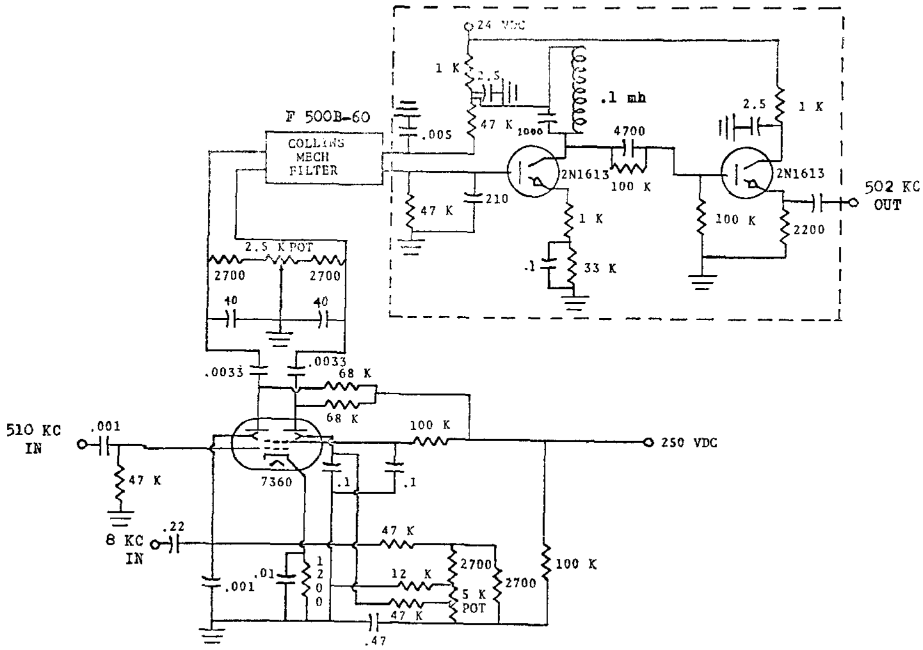

FIG. 2. 510-KHz oscillator and power amplifier. Resistances are in ohms. Capacitive values greater than 3 are in picofarads. Capacitive values equal to or less than 3 are in microfarads.

to the crystal, the transmission is biased to 50% of the maximum. Now impressed modulating signals are reproduced linearly. The $\frac{1}{4}$-wave plate was a mica sheet.

The maximum transmission of the E.O.L.M. is determined by the transmission of the crystal and the two polaroids and by reflection losses. Since the transmission of the parallel polaroids is about 35%, the maximum for the E.O.L.M. will be 10%–25%, depending on the wavelength and the number of reflection surfaces in the optical system.[6]

The minimum transmission is determined by the efficiency of the crossed polarizers and the angular field used. Since the crystals are naturally birefringent, there will be some retardation and, therefore, transmission for light that is not passing exactly along the optic axis. For a 100/1 ratio between maximum and minimum transmission, the angular field must be limited to a few degrees. Use of a laser light source is then highly desirable. We were easily able to modulate the Universities Laboratory He–Ne laser at 50% with a 1000 VPP sine wave.

The alignment procedure for the modulation system is critical. Optimum alignment is required to maximize the percent modulation. It is necessary to orient the polarizing axes of the polaroids

[6] Baird Atomics, Inc., Technical Data, Bulletin RD-501-1.

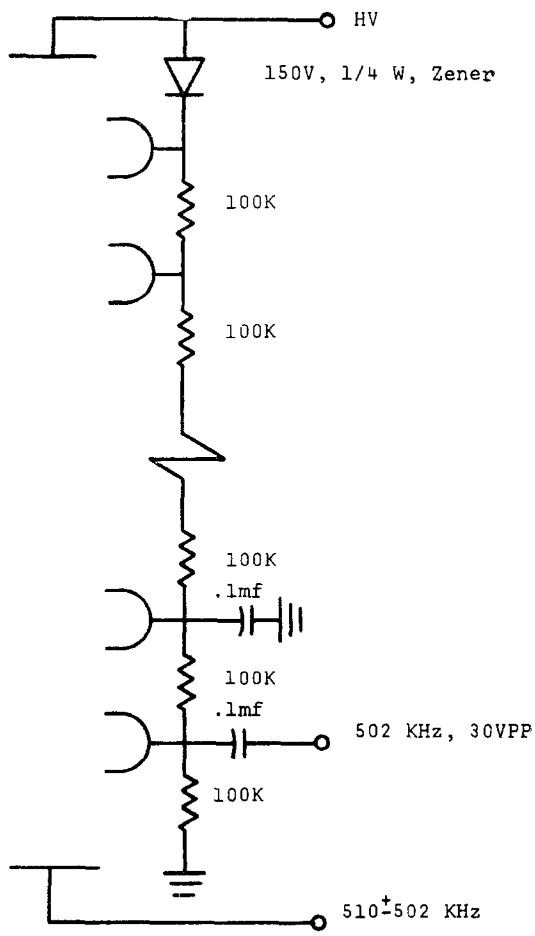

FIG. 3. 9558Q EMI photomultiplier used as combination mixer and detector.

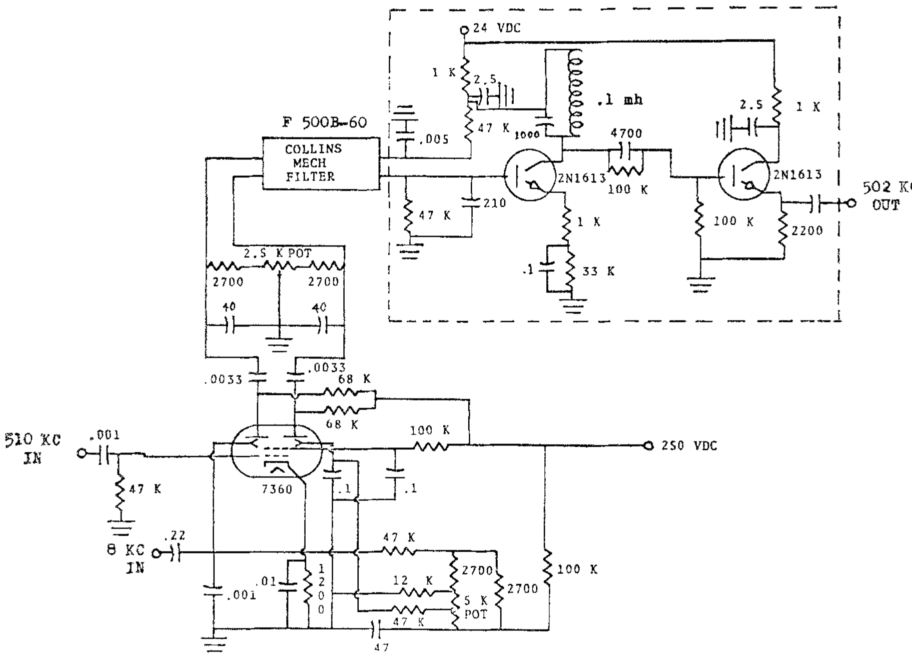

FIG. 4. Single side-band generator. Resistances are in ohms. Capacitive values greater than 2.5 are in picofarads. Capacitive values equal to or less than 2.5 are in microfarads.

parallel to the sides of the square-crystal plates. Placing a piece of white paper in the light beam after the second polarizer, the characteristic polarization interference pattern of a uniaxial crystal is observed. This pattern is a black cross defining the polarizer axes and a series of concentric dark and light rings. This enables one to determine whether the colliminated light is parallel to the optic axis of the crystal (source centered in the pattern), and whether the size of the source is such as to be within the useful angular field of the E.O.L.M. Further increase of the source area will not increase the amount of modulated light. There will only be a reduction in modulation percentage.

A 510-kHz oscillator and power amplifier were built to operate the E.O.L.M. as shown in Fig. 2. A 1000-V peak-to-peak sine wave which allowed 50% modulation of the laser beam was produced.

A 9558Q EMI photomultiplier was used as both a signal detector and as a mixer as illustrated in Fig. 3. The laser beam (modulated at 510 KHz) was detected by the photomultiplier where a 450-V potential was applied across the tube, and a 150-V quarter-watt Zener was placed between the photocathode and first dynode. When this small voltage and Zener are used, the tube operates as a photodiode. Use of the photomultiplier in this mode was possible, since use of a laser made high-intensity, light-signal levels possible. This was a desirable circumstance since in this operational mode, the photomultiplier yielded a quiet, stable analog signal that was easily capable of being phase multiplied with minimal loss of information in processing. The last dynode of the photomultiplier was modulated by a 502-kHz signal from the single side-band generator shown in Fig. 4. As far as the electrons are concerned, the last two dynodes and the plate of the photomultiplier constituted a triode. A mixing effect is seen and can be quantitatively analyzed as in any mixing tube.[7] It results in a high-performance, low-noise mixer. The advantage of simplicity is obvious.

The phase multiplier which was built is shown schematically in Fig. 5. It is similar to the one built by Noble and Cook, except that it was designed for 8-kHz signals rather than 10 kHz.[8] The phase multiplier consists of two frequency multipliers; one provides a factor multiplication of 8 and the other a factor of 9. Base multiplication factors of 2 and 3, respectively, are chosen, since frequency multipliers of greater multiplica-

[7] F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill Book Co., New York, 1955), 4th ed., Chap. 16.

[8] F. W. Noble and P. W. Cook, Rev. Sci. Instr. **36**, 971 (1965).

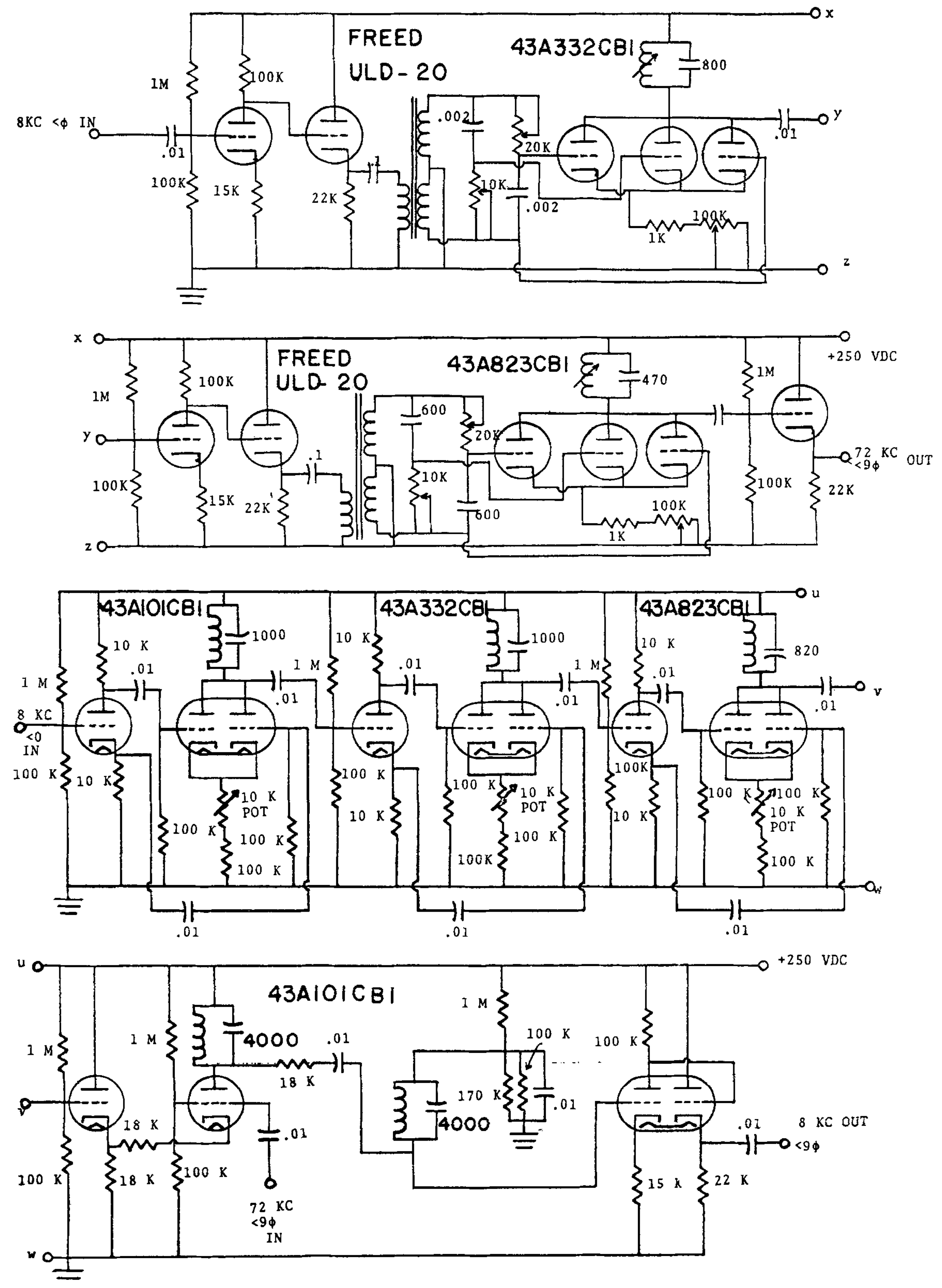

FIG. 5. Phase multiplier. Resistances are in ohms. Capacitive values greater than 1 are in picofarads. Capacitive values less than 1 are in microfarads.

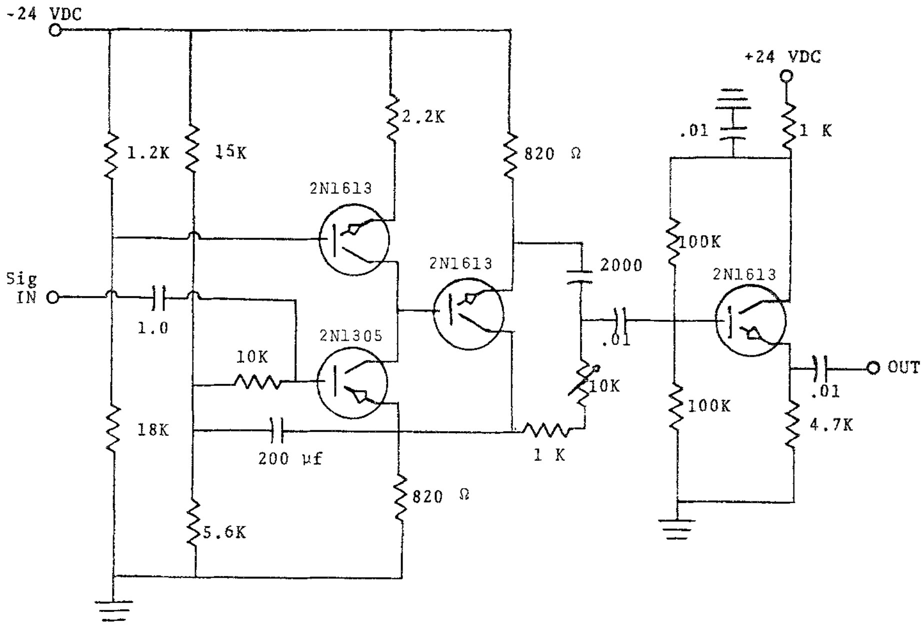

FIG. 6. Phase-shift generator. Resistances are in ohms. Capacitive values greater than 1 are in picofarads. Capacitive values less than 1 are in microfarads unless otherwise designated.

tion can be shown to be less stable. Multiplication by 8 is obtained by three multipliers of 2 in series with one another. The multiplication of 9 was obtained similarly using two multipliers of 3. The reference 8-kHz signal with phase angle 0° is multiplied in frequency by 8. The information 8-kHz signal with phase angle ϕ is multiplied in frequency by 9. These two signals are then mixed and filtered giving a signal of 8 kHz and phase angle of 9ϕ. Small phase-angle differences associated with low-frequency modulation in the phase-shift method could then be measured.

The phase shifter consists of an RC network with a precision variable resistance as shown in Fig. 6. It is similar to those of Rollefson[9] and Ware.[10] The phase angle ϕ corresponding to a given resistance setting of the phase shifter is calculated from $\tan(\phi/2)=2\pi f'RC$. When $f'=8$ kHz, this is $\tan(\phi/2)=0.10048R$, where R is measured in kΩ.

II. RESULTS AND DISCUSSION

The phase shift was measured 48 times over a two-week period when the mirror was moved a

[9] E. A. Bailey and G. K. Rollefson, J. Chem. Phys. **21**, 1315 (1953).

[10] W. R. Ware, J. Amer. Chem. Soc. **83**, 4374 (1961).

TABLE I. ϕ is the phase-angle difference measured with a distance change of 2 m.

ϕ (deg)	ϕ (deg)
1.228	1.349
1.361	1.240
1.313	1.393
1.228	1.330
1.172	1.202
1.396	1.209
1.173	1.244
1.194	1.286
1.150	1.269
1.161	1.228
1.167	1.373
1.492	1.018
1.367	1.486
1.258	1.354
1.193	1.327
1.447	1.141
1.357	1.208
1.217	1.255
1.161	1.258
1.096	1.159
1.183	1.241
1.148	1.227
1.330	1.345
1.312	1.227

ϕ av $=1.258\pm0.0142$

TABLE II. Individual values for the measured speed of light in units of 10^8 m/sec.

2.990	2.722
2.912	2.961
2.797	2.636
2.990	2.761
3.133	3.055
2.636	3.037
3.130	2.952
3.075	2.855
3.193	2.894
3.163	2.990
3.147	2.674
2.461	3.607
2.686	2.471
2.919	2.712
3.078	2.767
2.538	3.218
2.706	3.040
3.017	2.926
3.163	2.919
3.350	3.168
3.104	2.959
3.199	2.933
2.761	2.730
2.799	2.993

distance of two meters. The results of these measurements are seen in Table I. For each value of ϕ in Table I, a value for the speed of light was calculated from

$$C = 360°fd/\phi,$$

where f is the frequency, and d is the distance in meters. These values are shown in Table II. The average value for the speed of light was found to be $\bar{c}_{\text{mes}} = (2.937 \pm 0.033) \times 10^8$ m/sec in air. The value 0.033 is given by $(n^{\sigma})^{1/2}$, where σ is the standard deviation, and $n = 48$. This represents a deviation of 2.04% from the accepted value of 2.997×10^8 m/sec for the speed of light in air.

A 0.95 confidence interval was constructed as follows.

$$\bar{c}_{\text{mes}} \pm 2.012 \times 0.033,$$

or

$$\bar{c}_{\text{mes}} \pm 0.066,$$

which is

$$2.871 \times 10^8 < \bar{c}_{\text{mes}} < 3.003 \times 10^8.$$

The value of 2.997×10^8 m/sec certainly falls within this interval.

III. CONCLUSION

Although individual measurements varied substantially, the average value was close to the accepted value of the speed of light in air. It is felt that the speed of light in air was measured to an accuracy and with a confidence comparable with the best results obtained in other advanced laboratory speed-of-light experiments. The phase-shift method of measuring the speed of light has an additional advantage that the apparatus does not occupy a large amount of space. The entire setup can be made on a regular-sized laboratory bench. The apparatus used in this experiment is conventional amateur radio equipment and laboratory devices, with the exception of the lock-in amplifier, the optional Electronics Research crystal filter, and the photomultiplier. We have shown that the filter is not needed. The photomultiplier was used in the photodiode mode and incidentally as a mixer. A simple phototube and transistorized mixer would do equally as well. In this particular setup, the EMI 9558Q was used simply because it was readily available and had the desired infrared response. A lock-in amplifier is involved in all repetitive weak-signal detection problems, and experiments involving this instrument should be incorporated in advanced undergraduate laboratories. A lock-in amplifier may be easily constructed.[11]

[11] R. M. St. John, C. C. Lin, R. L. Stanton, H. D. West, J. P. Sweeney, and E. A. Rhinehart, Rev. Sci. Instr. **33**, 1089 (1962).