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A Novel Method For Solving The Uniform Distributed Network Analysis Problem

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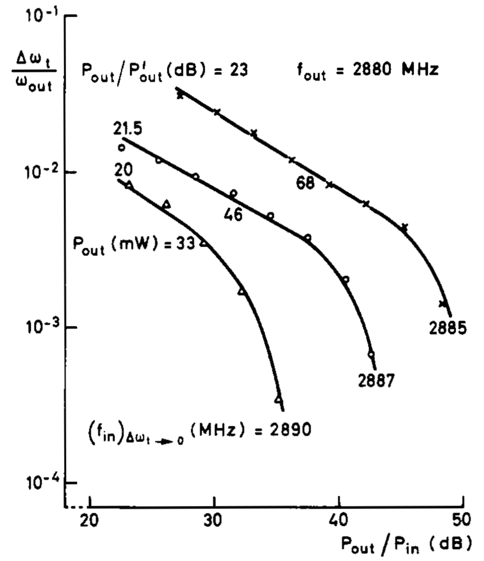

Fig. 1. Relative trigger bandwidth $\Delta\omega_t/\omega_{out}$ versus ratio of Gunn oscillator power P_{out} to triggering power P_{in} for three different power levels P_{out}.

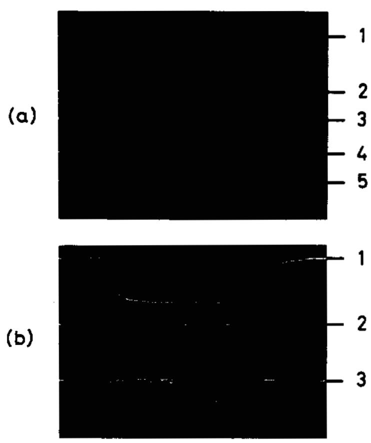

Fig. 2. (a) Dc bias voltage (trace 1), detected RF triggering signal (traces 2 and 4), and detected RF Gunn oscillation (traces 3 and 5). Different scale for traces 2 and 4, as compared to traces 3 and 5. (b) Dc bias voltage (trace 1), and, as incoherent sampling display, RF triggering voltage (trace 2) and RF voltage of Gunn oscillation (trace 3). Different scales for traces 2 and 3.
Horizontal scale: 150 ns/div. Vertical scale for bias voltage: 20 V/div.

about 10 dB lower than that for trace 2. A spectrum-analyzer display gave a clean, well-defined (sin f)/f frequency spectrum for the triggered oscillation, whereas in the pretriggered state no individual lobes at all could be distinguished in the noisy spectrum.

Measurements of the output power as a function of the load admittance established that by RF triggering it is possible to initiate coherent oscillations for load conductances $G_L > G_{L,\text{crit}}$, where $G_{L,\text{crit}}$ is an upper limit at which spontaneous oscillations heavily diminish [10]. Up to $G_{L,\text{crit}}$, the power of spontaneous oscillations increases with increasing G_L. Furthermore, the loaded Q of the external circuit showed a sharp increase with increasing G_L near $G_{L,\text{crit}}$, and the triggering bandwidth $\Delta\omega_t$ decreased with increasing Q_L. The observation of the possibility of triggering strong oscillations for $G_L > G_{L,\text{crit}}$ suggests that the trigger signal simulates a change in load conductance so as to present the load condition necessary for strong oscillations to be possible [11]. Once the oscillation has built up, the then altered sample admittance may allow the oscillation to continue even when the trigger is removed and the original load condition is restored. Since the oscillation buildup is necessarily tied to the occurrence in the sample of mature high-field domains with their associated capacitance (in the mode investigated here), the frequency of the triggered oscillation is lower than that of the weak pretriggering oscillation and that of the trigger signal. The existence of the pretriggering oscillation probably helps in achieving the triggering because some beginning of domain formation is already present. A larger pretriggering level results in a more pronounced domain formation, and consequently increases the triggering bandwidth $\Delta\omega_t$. The observed Q-dependence seems to indicate an additional influence for the oscillation buildup process by the external signal [8]. Further investigations are necessary to explain satisfactorily all the effects observed.

The triggering mechanism described here needs less triggering power than the dc operated one [2], [3] as long as no additional sensitivity-increasing control electrode is employed.

Acknowledgment

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A Novel Method for Solving the Uniform Distributed Network Analysis Problem

***Abstract*—An application of the Laplace transform method of Ogata to the solution of the uniform distributed network analysis problem is described.**

The development of thin-film, multilayered microcircuits offers important distributed parameter network analysis problems. Fig. 1 displays the n-layered uniform distributed parameter network.

As shown by Bertnolli,[1] the telegrapher's equations for the n-layered distributed parameter network can be formulated as a first-order matrix differential equation

$$\frac{\partial}{\partial x}\begin{bmatrix} \boldsymbol{V}(x, s) \\ \boldsymbol{I}(x, s) \end{bmatrix} = \begin{bmatrix} 0 & \boldsymbol{Z} \\ \boldsymbol{Y} & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{V}(x, s) \\ \boldsymbol{I}(x, s) \end{bmatrix} = \boldsymbol{K}\begin{bmatrix} \boldsymbol{V}(x, s) \\ \boldsymbol{I}(x, s) \end{bmatrix} \tag{1}$$

where $\boldsymbol{V}(x, s)$ and $\boldsymbol{I}(x, s)$ are column vectors, and $\boldsymbol{Z}$ and $\boldsymbol{Y}$ are nth-order square matrices.

Since (1), for the uniform line, is analogous to the homogenous "state variable" formulation of a linear non-time-varying system, methods used to solve the non-time-varying system problem are applicable here.

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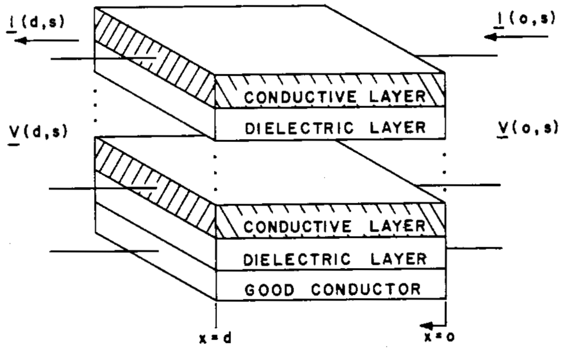

Fig. 1. The n-layered uniform distributed parameter network.

The solution to (1) is

$$\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = e^{Kx} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}. \tag{2}$$

Ogata[2] demonstrated a "Laplace transform" method for computing the matrix e^{Kx}. This method consists of taking the Laplace transform of (1) with respect to x:

$$\rho \begin{bmatrix} V(\rho,s) \\ I(\rho,s) \end{bmatrix} - \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} = K \begin{bmatrix} V(\rho,s) \\ I(\rho,s) \end{bmatrix} \tag{3}$$

or

$$(\rho I_n - K) \begin{bmatrix} V(\rho,s) \\ I(\rho,s) \end{bmatrix} = \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} \tag{4}$$

where ρ is the transform variable and I_n is the unit matrix of order n. Now (4) is written

$$\begin{bmatrix} V(\rho,s) \\ I(\rho,s) \end{bmatrix} = (\rho I_n - K)^{-1} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}. \tag{5}$$

The solution to (1), which is (2), is

$$\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = e^{Kx} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} \tag{6}$$

and the Laplace transform of this solution is

$$\begin{bmatrix} V(\rho,s) \\ I(\rho,s) \end{bmatrix} = L_x\{e^{Kx}\} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}. \tag{7}$$

Comparing (5) and (7) reveals that obviously

$$L_x\{e^{Kx}\} = (\rho I_n - K)^{-1}. \tag{8}$$

Substitution of $K = \begin{bmatrix} 0 & Z \\ Y & 0 \end{bmatrix}$ into (8) yields

$$L_x\{e^{Kx}\} = \begin{bmatrix} \rho I_n & -Z \\ -Y & \rho I_n \end{bmatrix}^{-1}. \tag{9}$$

Since this is a partitioned matrix, the formula presented by Frame[3] is helpful in calculating this inverse. The formula is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} I_n & -A^{-1}B \\ 0 & I_n \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & (D-CA^{-1}B)^{-1} \end{bmatrix} \begin{bmatrix} I_n & 0 \\ -CA^{-1} & I_n \end{bmatrix} \tag{10}$$

and applying it results in

$$\begin{bmatrix} \rho I_n & -Z \\ -Y & \rho I_n \end{bmatrix}^{-1} = \begin{bmatrix} \rho[\rho^2 I_n - ZY]^{-1} & [\rho^2 I_n - ZY]^{-1}Z \\ [\rho^2 I_n - YZ]^{-1}Y & \rho[\rho^2 I_n - YZ]^{-1} \end{bmatrix}. \tag{11}$$

The inverse Laplace transform with respect to ρ, from the Appendix, is

$$e^{Kx} = \begin{bmatrix} \cosh \sqrt{ZY}\,x & \sinh \sqrt{ZY}\,x\,(\sqrt{ZY}^{-1} Z) \\ \sinh \sqrt{YZ}\,x\,(\sqrt{YZ}^{-1} Y) & \cosh \sqrt{YZ}\,x \end{bmatrix} \tag{12}$$

which is the desired result.

Ogata remarks that the matrix $(\rho I_n - K)^{-1}$ can be calculated by several methods[2] (other than the one used here).

Appendix

Laplace Transform of a Matrix

The Laplace transform of a matrix is defined by

$$F(s) = \int_0^\infty e^{-sI_n t} f(t)dt \tag{13}$$

where I_n is the unit matrix, with the same dimensions as the matrix $f(t)$, or if $f(t)$ is a vector, I_n is equal to unity.

As an example, find the Laplace transform of e^{At}, where A is a square matrix:

$$L[e^{At}] = \int_0^\infty e^{At} e^{-sI_n t} dt = \int_0^\infty e^{-[sI_n - A]t} dt \tag{14}$$

if $sI_n > A$ (element by element).

From this it is easy to compute the Laplace transform of cosh (At), since

$$\cosh At = \tfrac{1}{2}[e^{At} + e^{-At}]. \tag{15}$$

The Laplace transform is therefore

$$\begin{aligned} L[\cosh At] &= \tfrac{1}{2}\{L[e^{At}] + L[e^{-At}]\} \\ &= \tfrac{1}{2}[[sI_n - A]^{-1} + [sI_n + A]^{-1}] \\ &= sI_n[s^2 I_n - A^2]^{-1}. \end{aligned} \tag{16}$$

Similarly, for the Laplace transform of the sinh (At),

$$L[\sinh At] = [sI_n - A^2]^{-1}. \tag{17}$$

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On Some Bonds Between Autocorrelation and Power Spectra Functions

***Abstract*—Some bonds are established between values of the autocorrelation function of a signal and some parameters of the corresponding power spectrum, and vice versa. These relations are then detailed for band-limited signals, and some expressions for a rough evaluation of the signal bandwidth are deduced.**

This letter intends to show some bonds between values of the autocorrelation function of a signal and some parameters of the corresponding power spectrum. A property of the Fourier transform is used, which was previously used[1] to establish some limitations on the signal's variations related to its spectrum. Such a property is used here to refer to correlation functions and power spectra, and some particular properties of these

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