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# Transients Analysis Of Distributed Systems

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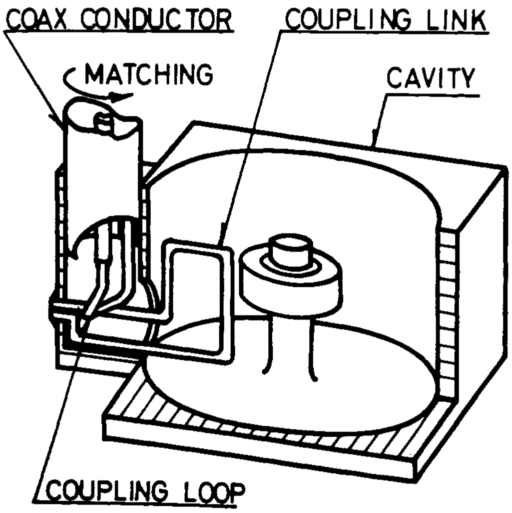


Fig. 1. Details of the coupling to the lower cavity.

to couple to the lower cavity, particularly if the coax line to the cavity should be impedance matched, by turning its coupling loop.

The double-loop coupling shown in Fig. 1 offers a solution to the problem. The coax line is terminated by a coupling loop as usual, but the loop is coupled to a second loop of copper wire which serves to guide a part of the microwave field out of the cavity. By turning the coupling loop of the line with respect to the loop of the cavity it is possible to impedance match the coax line to the cavity.

This system has been used with success in our laboratory in experiments with 3 GHz phonons.

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## Transient Analysis of Distributed Systems

**Abstract—An isomorphic relation exists between the transient response of systems whose transfer functions are of the form $B(s)$ and the transient response of systems whose transfer functions are of the form $H(s)=B(\sqrt{s})$. Several interesting conclusions are drawn from this relationship concerning the transient response of distributed systems.**

In several tables of integral transforms [1], [2] it is noted that if $b(t) = \mathscr{L}^{-1}[B(s)]$ and $h(t)=\mathscr{L}^{-1}[H(s)]$, and if

$$H(s) = 2^{n/2}\pi^{\frac{1}{2}}s^{(n-1)/2}B(\sqrt{s}), \tag{1}$$

then

$$h(t) = t^{-(n+1)/2}\int_0^\infty e^{-y^2/4t}He_n(2^{-\frac{1}{2}}yt^{-\frac{1}{2}})b(y)dy \tag{2}$$

in which

$$He_n(x) = (-1)^n e^{\frac{1}{2}x^2}\frac{d^n}{dx^n}(e^{-\frac{1}{2}x^2}). \tag{3}$$

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One can envision $B(s)$ as the transfer function of a lumped system possibly with time delay and $H(s)$ as the transfer function of a distributed system.

Suppose that

$$H(s) = B(\sqrt{s}), \tag{4}$$

then with the aid of (1)–(3) we can prove the following theorems:

*Theorem 1:* If the transfer functions of two systems are related as in (4), then the impulse responses are related by the equation

$$h(t) = \frac{t^{-3/2}}{2\sqrt{\pi}}\int_0^\infty ye^{-y^2/4t}b(y)dy \tag{5}$$

$$= \frac{2}{\sqrt{\pi t}}\int_0^\infty \xi e^{-\xi^2}b(2\sqrt{t}\xi)d\xi, \tag{6}$$

where $h(t)=\mathscr{L}^{-1}[H(s)]$, and $b(t)=\mathscr{L}^{-1}[B(s)]$.

*Theorem 2:* If the transfer functions of two systems are related as in (4), then the unit step responses are related by the equation

$$v_h(t) = \frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-y^2/4t}v_b(y)dy \tag{7}$$

$$= \frac{2}{\sqrt{2\pi}}\int_0^\infty e^{-x^2/2}v_b(\sqrt{2t}x)dx \tag{8}$$

where $v_h(t)=\int_0^t h(\tau)d\tau$, and $v_b(t)=\int_0^t b(\tau)d\tau$.

Thus we have shown that an interesting relationship exists between the impulse response of the system $B(s)$ and the impulse response of the system $B(\sqrt{s})$, and between the step response of the system $B(s)$ and the step response of the system $B(\sqrt{s})$.

Several other interesting observations are noted from the above results.

*Theorem 3:* If the step response of a system with transfer function $B(s)$ is a monotonically increasing function, no overshoot, then the step response of a system with transfer function $B(\sqrt{s})$ is also a monotonically increasing function.

Since the integral of the impulse response of a system is its step response, we know that if $v_b(t)$ is a monotonically increasing function, then $b(t)\geq 0$ for all $t\geq 0$. From (5) we conclude that if $b(t)\geq 0$ for all $t\geq 0$, then $h(t)\geq 0$ for all $t\geq 0$. This implies that $v_h(t)$ is a monotonically increasing function.

*Theorem:* If $\int_0^\infty |b(t)|dt < M$, where $M$ is a positive number and $b(t)$ is bounded, then $\int_0^\infty |h(t)|dt \leq \int_0^\infty |b(t)|dt < M$.

The above theorem implies that if a system with transfer function $B(s)$ is bounded-input bounded-output stable, then the system with transfer function $B(\sqrt{s})$ is bounded-input bounded-output stable. Thus the stability of system $B(s)$ is a sufficient condition for the stability of $B(\sqrt{s})$, but not a necessary condition. One can construct many counter examples [3].

Often it is difficult for one to find the inverse transform of the transfer function $B(\sqrt{s})$ of a distributed system. However, the inverse transform of the transfer function $B(s)$ may be readily available. The above theorems show how the knowledge of the inverse transform of $B(s)$ can be used to draw important conclusions about the inverse transform of $B(\sqrt{s})$. An example follows.

Let us consider the RC network shown in Fig. 1. The input–output voltage transfer function is

$$\frac{E_o(s)}{E_i(s)} = \operatorname{sech}\sqrt{s}, \tag{9}$$

where it is assumed that the line is normalized, i.e., $r=c=1$ and the length of the line is one. Thus,

$$H(s) = \operatorname{sech}\sqrt{s} \quad \text{and} \quad B(s) = \operatorname{sech} s.$$

Note that $B(s)$ is the transfer function for an $\overline{\text{LC}}$ line, a pure time delay system. The impulse response of the $\overline{\text{LC}}$ system is

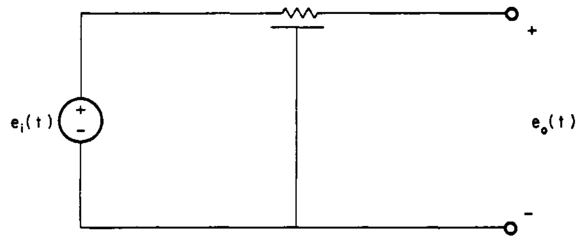


Fig. 1. Distributed $\overline{RC}$ line.

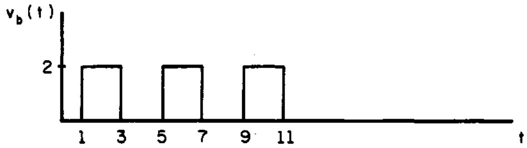


Fig. 2. $\overline{LC}$ network step response.

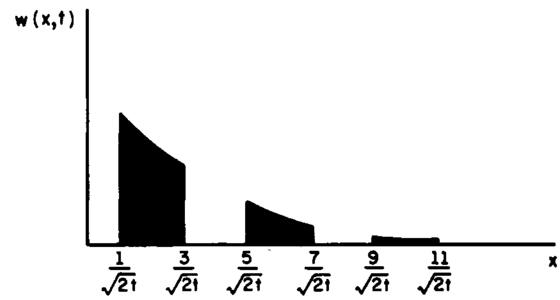


Fig. 3. Shaded area represents $v_h(t)$.

$$b(t) = \mathscr{L}^{-1}[\operatorname{sech} s] = 2 \sum_{n=0}^{\infty} (-1)^n \delta(t - (2n + 1)), \tag{10}$$

and the step response (see Fig. 2) is

$$v_b(t) = \mathscr{L}^{-1}\left[\frac{1}{s} \operatorname{sech} s\right] = 1 + (-1)^n, \quad \text{for} \quad (2n - 3) < t \leq (2n - 1), \quad n = 1, 2, \cdots \tag{11}$$

Now the impulse response for the $\overline{RC}$ network can be determined from (5),

$$h(t) = \frac{t^{-3/2}}{2\sqrt{\pi}} \int_0^{\infty} y e^{-y^2/4t} 2 \sum_{n=0}^{\infty} (-1)^n \delta(y - (2n + 1)) dy = \frac{1}{\sqrt{\pi t^3}} \sum_{n=0}^{\infty} (2n + 1)(-1)^n e^{-(2n+1)^2/4t}. \tag{12}$$

Next let us determine the step response of the $\overline{RC}$ network. Recall from (8) that

$$v_h(t) = \frac{2}{\sqrt{2\pi}} \int_0^{\infty} e^{-x^2/2} v_b(\sqrt{2t}x) dx.$$

The expression $w(x, t) = (2/\pi) e^{-x^2/2} v_b(\sqrt{2t}x)$ contained in the integral is plotted in Fig. 3 as a function of $t$ and $x$. It is an infinite sequence of pulses which decay at a rate proportional to $e^{-x^2/2}$. The shaded area represents the step response at time $t$. As $t$ increases the pulses move closer to the origin and the total area increases. As $t$ decreases the pulses move away from the origin and the total area decreases. In this case one can rewrite the equation for the step response as

$$v_h(t) = \frac{4}{\sqrt{2\pi}} \int_{1/(\sqrt{2t})}^{3/(\sqrt{2t})} e^{-x^2/2} dx + \int_{5/(\sqrt{2t})}^{7/(\sqrt{2t})} e^{-x^2/2} dx + \cdots. \tag{13}$$

TABLE I

| $t$ | 0.1 | 0.25 | 0.5 | 1.0 | $\infty$ |
|---|---|---|---|---|---|
| $v_h(t)$ | 0.0528 | 0.3172 | 0.6296 | 0.8880 | 1.0000 |

In evaluating $v_h(t)$ one can easily truncate (13) and obtain an estimate on the truncation error. In Table I four values of $v_h(t)$ have been determined to four-place accuracy by considering no more than two terms in (13). One can show that $\lim_{t \to \infty} v_h(t) = 1.0$.

Thus the above theorems establish a sometimes useful link between the transient response of lumped system which may have time delay and the transient response of distributed systems.

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[2] G. Doetsch, *Guide to the Applications of Laplace Transforms*. Princeton, N. J.: Van Nostrand, 1961, p. 229.
[3] J. J. Bourquin and T. N. Trick, "Stability of a class of lumped distributed systems," *J. Franklin Inst.*, (to be published).

## An Integrated Metal-Nitride-Oxide-Silicon (MNOS) Memory

**Abstract—A nonvolatile integrated metal-nitride-oxide-silicon (MNOS) word organized memory was fabricated. The salient features of this integrated storage array are 1) WRITE cycle 2.0 microseconds, READ cycle 500 nanoseconds, 2) WRITE voltage ± 25 V, 3) single device per storage bit, 4) MOS driving circuitry with no isolation between MOS and MNOS devices on the same chip, and 5) nonvolatile storage of information.**

Variable turn-on voltage metal-nitride-oxide-silicon (MNOS) transistors have recently been proposed[1] as storage elements for nonvolatile read-only memory. These transistors have been shown to exhibit hysteresis behavior of turn-on voltage as a function of the applied gate voltage. The storage function associated with the hysteresis characteristic has the potential advantage over conventional semiconductor memories of allowing a single transistor per storage bit. An extensive investigation[2] of charge transport and storage in MNOS structures has led to an understanding of the physical mechanisms underlying device operation, allowing optimization of device performance for digital storage applications. To demonstrate feasibility of integrated MNOS storage arrays, a nine-bit-word organized MNOS memory was fabricated. The salient features of this integrated storage array are (1) WRITE cycle 2.0 microseconds, READ cycle 500 nanoseconds, 2) WRITE voltage ±25 V, 3) single device per storage bit, 4) MOS driving circuitry with no isolation between MOS and MNOS devices on the same chip, and 5) nonvolatile storage of information.

### Storage Device Operation

A typical hysteresis characteristic of turn-on voltage $V_T$ as a function of charging gate voltage $V_C$ for a $p$-channel MNOS memory device is shown

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