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Detection Of Slowly Fading Targets With Frequency Agility

Kenneth L. Horn

Norval D. Wallace

Missouri University of Science and Technology

polarization is also seen. One should also note the absence of a shadow boundary or penumbra region contribution for either polarization. as has been suggested for the case of parallel polarization [8].

Our primary interest in the impulse response waveforms presented in this letter is in the nature of the creeping wave contribution for perpendicular polarization.

Note that the onset of the creeping wave contribution occurs at $(1+\pi/2)t_0$, as would be expected for a wavefront which traverses the rear of the cylinder at the free space velocity. This can also be shown from the time-dependent creeping wave contribution derived analytically from the high-frequency creeping wave term [5]. However, the high-frequency creeping wave terms or geometrically diffracted terms do not always yield such a result, as in the case of the conducting sphere or certain targets with edges such as the flat-based right circular cone [5], [9]. There is then no clear onset in the time response corresponding to a traversal of the shadowed surface via a geodesic path, but evidence of precursors which arrive at earlier times. In such cases, the geodesic path is not a minimum-time path.

DAVID L. MOFFATT
Dept. of Elec. Engrg.
ElectroScience Lab.
Ohio State University
Columbus, Ohio 43212

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Detection of Slowly Fading Targets with Frequency Agility

Abstract—Detection probabilities for a noncoherent pulsed radar are presented as a function of maximum possible pulse-to-pulse frequency change (transmitter bandwidth) and target depth for frequency agility applied to Swerling's case I target. The transmitted frequency is a uniformly distributed random variable.

This letter presents some preliminary results of an extended analysis of frequency agility techniques applied to target detection in noncoherent pulsed radar systems. The majority of previous results have been concerned with the reduction of scintillation in tracking radars [1], [2]. Recently, a few limited results have been given for frequency agility applied to target detection in pulsed radars [3], [4]. Most of these results assumed pulse-to-pulse frequency shifts sufficient for complete decorrelation of target returns, for which detection probabilities may be found from Swerling's cases II and IV, for specified target fluctuations [5]. In realistic cases, complete decorrelation is seldom achieved with frequency agility, except for systems which integrate a small number of pulses.

The receiver considered in this letter consists of the standard linear predetection stage, square law detector, linear integrator, and threshold decision circuit. The transmitter is assumed to transmit signals of constant amplitude, independent of frequency. In addition, it is assumed that the transmitter frequency is a uniformly distributed random variable with probability density function given by $g(f)$, as follows:

$$g(f) = \frac{1}{\Delta f} \qquad f_0 - \frac{\Delta f}{2} \le f \le f_0 + \frac{\Delta f}{2} \tag{1}$$
$$= 0 \qquad \text{otherwise.}$$

The model for the return signal fluctuations is described by Swerling's case I [5]. The echo power for each pulse is constant for the time on target during a single integration period but fluctuates independently between integration periods and follows the probability density given by

$$g(x, \bar{x}) = \exp(-x/\bar{x})/\bar{x} \tag{2}$$

where

x = signal-to-noise power ratio at the input to the square law detector
$\bar{x}$ = average signal-to-noise power ratio at the input to the square law detector.

It is well known [1], [3] that the correlation function for the voltage at the input to the square law detector is a function of the frequency change between pulses and the target depth D for a distributed target of the type assumed by Muchmore [6]. Using this model, the correlation coefficient for the amplitude of the ith and jth pulses is given by

$$C_{i,j}(y) = \sin y/y$$
$$y = 2\pi D(f_i - f_j)/V \tag{3}$$

where V is the velocity of light and f_i is the frequency of the ith pulse.

The probability density function for $f_i - f_j$ is easily obtained from (1). Multiplying (3) by this density function and taking the expected value with respect to the frequency difference yields the correlation coefficient $C_{i,j}(z)$ for the case under consideration:

$$C_{i,j}(z) = \frac{2}{z}\left\{Si(z) + \frac{1}{z}\left[\cos(z) - 1\right]\right\} \tag{4}$$

where

$$z = 2\pi D \Delta f / V$$

and

$$Si(z) = \int_0^z \frac{\sin y}{y}\, dy.$$

Swerling's generalized correlation analysis [7] is now applied to obtain detection probabilities, utilizing the correlation coefficient given by (4). Obtaining these detection probabilities involves finding the eigenvalues of the correlation matrix whose elements are $C_{i,j}(z)$, transforming the characteristic function to obtain the probability density function for the integrator output, and integrating this density function above the threshold.

Figs. 1 and 2 give the probability of detection for $N=10$ and 100, respectively. (N is the number of pulses integrated.) The notation first utilized by Marcum [8], and almost universally adopted since then, was employed to enable direct comparison with other results. The range parameter R/R_0 is inversely proportional to the one-fourth root of the average signal-to-noise power ratio $\bar{x}$. (R_0 is the range at which $\bar{x}=1$.) P_d is the probability of detection, while U is the product of target depth D, in meters, and Δf, the maximum possible frequency shift from pulse to pulse (transmitter bandwidth) in megahertz. The probability of false alarm is given along with n, another standard parameter defined by Marcum. Note that the curves $U=0$ and $U=\infty$ correspond to Swerling's cases I and II (complete correlation and decorrelation), respectively. Also note that the product of target depth and maximum possible pulse-to-pulse frequency change should be increased as N increases, in order to fully

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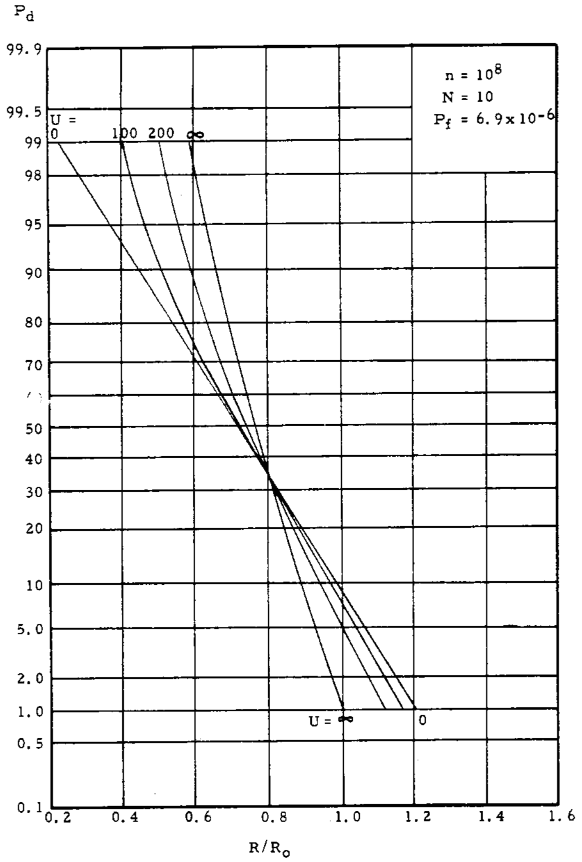

Fig. 1. Probability of detection versus R/R_0 for $N=10$, probability of false alarm $=6.9\times10^{-6}$.

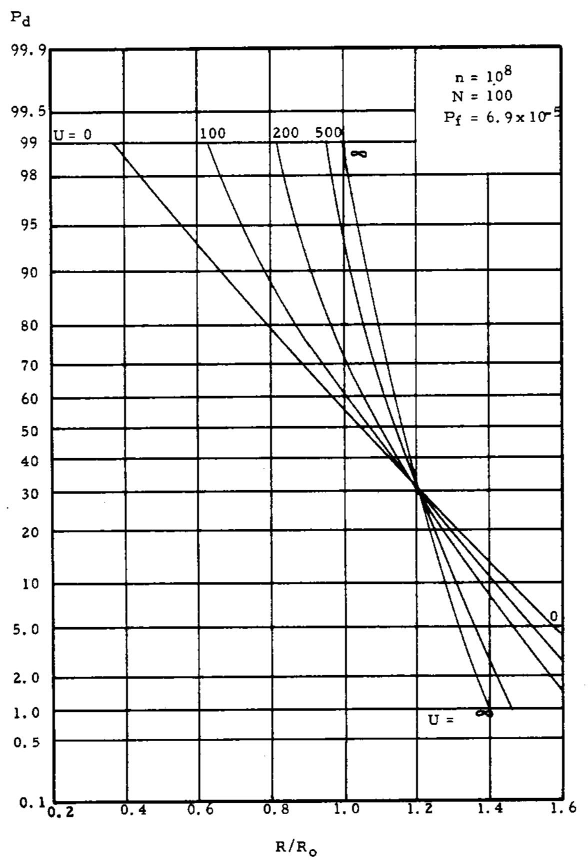

Fig. 2. Probability of detection versus R/R_0 for $N=100$, probability of false alarm $=6.9\times10^{-5}$.

exploit frequency agility for target detection in high- (greater than one half) detection cases. Complete results and details will be published at a later date.

Kenneth L. Horn
Norval D. Wallace
Space and Electronics Div.
Emerson Electric Co.
St. Louis, Mo. 63136
and
University of Missouri at Rolla
Rolla, Mo.

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On the Transverse Surface Boundary Effect in Gunn Devices

Abstract—A comparison between a rather rigorous space-charge wave theory by Kino and Robson and a simplified large-signal model proposed by the author, concerning the influence of the transverse surface boundary condition on the space-charge dynamics in thin-sheet Gunn devices, shows that the results of both approaches are equivalent. It is also concluded that the oscillation frequency in such devices may be much larger than in ordinary samples.

Recently Kino and Robson[1] calculated the dispersion relation $\omega=\omega(\alpha,\beta)$ for space-charge waves in a two-dimensional semiconductor without diffusion, which relates the frequency ω to the transverse and longitudinal wavenumbers α and β, respectively (the longitudinal direction is defined by the drifting carriers). They also showed that for a small transverse dimension d of the semiconductor, the dispersion relation can be brought into a form similar to the one-dimensional case by eliminating α using the boundary condition at the transverse semiconductor surface (infinite thickness of the dielectric on both sides of the semiconductor sheet). For waves which vary as $\exp j(\omega t-\beta x)$ this dispersion relation reads

$$\omega = v_0\beta + j\frac{\beta d}{2}\frac{\varepsilon_a}{\varepsilon_b}\omega_c \tag{1}$$

where v_0 is the dc drift velocity of the carriers, ε_a and ε_b the permittivity of the semiconductor and dielectric, respectively, and ω_c the reciprocal dielectric relaxation time; hence, the small-signal space-charge growth time is

$$\tau = -\frac{1}{\omega_c} = \frac{\varepsilon_a}{en_0(-\mu)} \tag{2}$$

where μ is the longitudinal differential mobility, n_0 the static carrier density, and e the electron charge. For the derivation of (1), $|\beta d/2| \ll 1$ with $\varepsilon_b/\varepsilon_a \geq 1$ has been assumed, a condition which may be termed "heavy dielectric

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