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Abstract—The use of AGC loops at the input and output of the FM link in an AM–FM telemetry system allows the mean-square transmitter deviation to be maintained near maximum value, even though data may be nonstationary. However, errors result because of the inability of the receiver AGC loop to track perfectly gain variations in the transmitter loop. In this paper the general characteristics of AGC are discussed, and a theoretical analysis is performed to determine the time constant, steady-state error, and tracking error for a first-order loop. Also, tracking error for for first-order and second-order loops is investigated by simulation. Curves are presented to illustrate the principal results.

Introduction

IN AN AM–FM telemetry system, data can be nonstationary, causing the mean-square value of the baseband signal to vary significantly. Since the mean-square deviation of the transmitter is proportional to this signal, the transmitter can be either over or under modulated resulting in a decreased signal-to-noise ratio or in an excessively wide RF spectrum. This difficulty can be overcome by employing an AGC loop at the FM link input to hold the rms value of the baseband signal constant and a second loop at the FM link output to restore data calibration. Hereafter, the input and output loops will be called the transmitter and receiver loops, respectively.

In an actual system the receiver loop does not perfectly track gain variations in the transmitter loop, resulting in tracking error. This error is greatest when the rms values of a large percentage of the data signals change simultaneously, as would occur in a booster telemetry system at the instant of ignition or staging. Additionally, the tracking error can have a steady-state value. Theoretical analysis and simulation will be used to study these errors.

Much excellent theoretical work has been accomplished over the past several years on AGC loops [1]–[5]. This paper is an extension to the case of two loops in cascade. The model used for the analysis is the linear model previously assumed by Schacter and Bergstein [5]. The major purpose of the work is to make the system designer aware of the types of difficulties likely to be encountered in an operational system.

Increase in Signal-to-Noise Ratio Due to AGC

A block diagram of a baseband AGC system is shown in Fig. 1. The increase in the output signal-to-noise ratio of an individual channel can be determined from a consideration of mean-square transmitter deviation. Let $S_j(t)$ represent the instantaneous value of the channel j signal so that the baseband signal $S(t)$ is given by

$$S(t) = \sum_{j=1}^{n} S_j(t). \tag{1}$$

If all the channel signals are statistically independent, the rms value of $S(t)$, S, is

$$S = [\sum_{j=1}^{n} S_j^2]^{1/2} \tag{2}$$

where S_j is the rms value of $S_j(t)$. The rms transmitter deviation D is determined by S. Therefore, maximum rms transmitter deviation D_m results when S assumes maximum value S_m so that

$$D = (S/S_m)D_m = LD_m \tag{3}$$

where $L = S/S_m$ is the load factor [6]. It follows that if a controlled amplifier with gain $\mu(t) = L^{-1}$ is placed ahead of the FM link, the rms transmitter deviation will be maintained constant at D_m. Since the individual channel signal-to-noise ratio is proportional to the mean-square transmitter deviation, this ratio is increased by a factor of $(L^{-1})^2$.

Tracking Error

The occurrence of a tracking error can be demonstrated by considering a change in baseband load factor from 0.5 to 1.0. In order for the transmitter deviation to remain constant, the transmitter amplifier gain $\mu(t)$ should change instantaneously from two to one. However, the gain actually changes as shown in Fig. 2(a) with time constant T_t dependent upon the bandwidth of the transmitter loop. It follows that the pilot amplitude at the receiver loop output is $E_p\mu(t)$, where E_p is the nominal pilot amplitude. If the gain of the receiver loop amplifier $K(t)$ were $\mu^{-1}(t)$, no error would exist since the pilot amplitude at the receiver loop output would be constant at E_p. However, this would require that the receiver loop respond instantaneously to changes in $\mu(t)$, which would necessitate an infinite loop bandwidth. Therefore, in a practical system the pilot will be returned to its proper level as illustrated in Fig. 2(b), yielding a tracking error ϵ_T as shown in Fig. 2(b) and (c).

This error for a step increase in baseband load factor can be used to define the AGC system settling time as illustrated in Fig. 2(c), where the tolerance is the desired system accuracy. Since tracking error only depends upon the ability of the receiver loop to track changes in the transmitter loop gain, the error can be made arbitrarily small by increasing the bandwidth of the receiver loop or decreasing the bandwidth of the transmitter loop. Thus,

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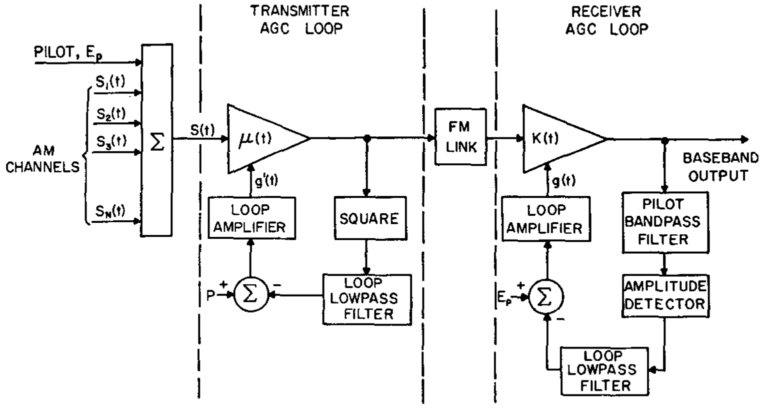

Fig. 1. Baseband AGC system.

the time constant ratio

$$\lambda = \frac{\text{transmitter loop time constant}}{\text{receiver loop time constant}} = \frac{T_t}{T_r} \qquad (4)$$

is the parameter which controls tracking error. However, λ may not be freely varied in a practical system because increasing the transmitter loop time constant can result in errors due to transmitter overdeviation, while decreasing the receiver loop time constant can result in errors due to interference from data channels adjacent to the pilot in the baseband.

Theoretical Analysis

Because of the variable gain element, an AGC loop is described by a differential equation which has time-varying coefficients. This complicates determination of the general loop response; however, an exact solution is obtainable for the first-order case [5]. This solution can be utilized to determine the loop time constant, the steady-state error, and the system tracking error. Additionally, the analysis provides insight which is helpful in understanding the operation of baseband AGC. In an analysis of tracking error, the only concern is in how well the receiver loop tracks the transmitter loop gain variations. If these gain variations are specified, only the receiver loop need be considered in the analysis. Therefore, the transmitter loop will be assumed to be first-order in the following analysis.

Since the receiver loop responds only to variations in the pilot amplitude, the pilot frequency may be translated to dc for purposes of analysis. This results in the reduction of the receiver loop in Fig. 1 to the simplified model shown in Fig. 3. The low-pass filter represents the cascade combination of the loop low-pass filter and the low-pass equivalent of the pilot bandpass filter. Usually, the pilot bandpass filter will have a much larger bandwidth than the loop low-pass filter and can be neglected in an analysis not concerned with noise and interference. Also, the position of the comparison device has been changed for simplicity. The loop input $e_i(t)$ is the assumed transmitter loop output.

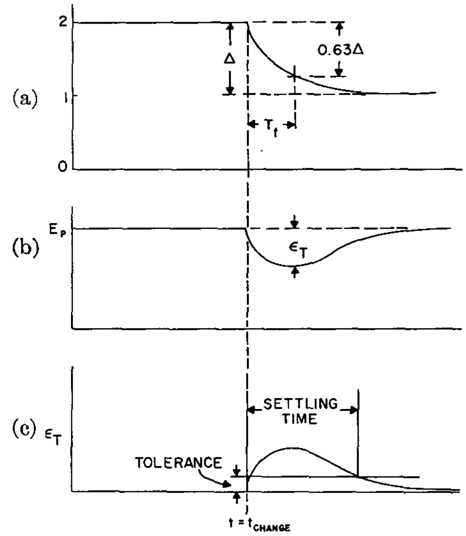

Fig. 2. Response to step change in load factor. (a) and (b) Pilot amplitude at output of transmitter AGC. (c) Tracking era.

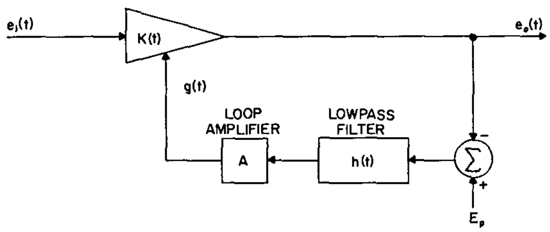

Fig. 3. Model of receiver AGC loop.

General Solution

In order to compute the output $e_0(t)$ of the system in Fig. 3, an expression for the gain $K(t)$ is necessary. Then $e_0(t)$ can be computed from

$$e_0(t) = K(t)e_i(t). \qquad (5)$$

The gain is assumed to be a linear function of the error signal $g(t)$ so that

$$K(t) = 1 + g(t). \qquad (6)$$

Since the analysis is for a first-order loop filter, let the loop filter impulse response be

$$h(t) = \beta \exp(-\beta t). \quad (7)$$

Since the input to the filter is $[E_p - e_0(t)]$ and the filter output is $g(t)$, the differential equation describing $g(t)$ is

$$[dg(t)/dt] + \beta g(t) = A\beta[E_p - e_0(t)]. \quad (8)$$

Substituting (5) and (6), this equation can be written in the standard form

$$[dK(t)/dt] + \beta[1 + Ae_i(t)]K(t) = \beta[1 + AE_p]. \quad (9)$$

Using the integrating factor

$$\exp \int_0^t \beta[1 + Ae_i(x)]\,dx$$

allows (9) to be solved yielding

$$K(t) \exp\left[\int_0^t \beta[1 + Ae_i(x)]\,dx\right] = \int_0^t \beta[1 + AE_p] \cdot \exp\left[\int_0^\tau \beta[1 + Ae_i(x)]\,dx\right] d\tau + C \quad (10)$$

where C is the constant of integration which is unity if the initial condition $g(0) = 0$ is assumed. Accordingly, (10) may be simplified to yield the general first-order solution

$$K(t) = \exp\left[-\int_0^t \beta[1 + Ae_i(x)]\,dx\right] + \beta[1 + AE_p] \cdot \int_0^t \exp\left[-\int_\tau^t \beta[1 + Ae_i(x)]\,dx\right] d\tau. \quad (11)$$

This expression can be used to determine the loop time constant, the steady-state error, and the tracking error.

Time Constants

The time constant of an AGC loop is defined in the conventional manner from the step-function response. To determine the time constant of the receiver loop, let

$$e_i(t) = E_p + \Delta. \quad (12)$$

Substituting (12) in (11) and performing the integrations yields the result

$$K(t) = 1/[1 + A(E_p + \Delta)] \cdot ((1 + AE_p) + A\Delta \exp\{-[\beta + \beta A(E_p + \Delta)]t\}). \quad (13)$$

From this equation, the time constant of the receiver loop is, by definition,

$$T_r = 1/[\beta + \beta A(E_p + \Delta)]. \quad (14)$$

The preceding expression illustrates that the time constant of the receiver loop is dependent upon both the magnitude and polarity of the step. It should be kept in mind that a positive Δ corresponds to a step decrease in load factor, since Δ represents the change in pilot amplitude. The dependency of T_r on the loop input signal makes it extremely difficult to specify the loop time constant or the effective loop bandwidth for a general input signal.

It follows from the similarity of the transmitter and receiver loops that if a step function is applied to the input of a first-order transmitter loop, the pilot amplitude at the loop output will be of the form

$$e(t) = E_p + \Delta[1 - \exp(-\alpha t)]. \quad (15)$$

By definition, the time constant of the transmitter loop T_t is

$$T_t = 1/\alpha. \quad (16)$$

Steady-State Errors

If the steady-state value of $e_i(t)$ is different from E_p, then $g(t)$ cannot be zero and a steady-state error will result. Assuming that $e_i(t) = E_p + \Delta$, the steady-state value of $K(t)$, i.e., $K(\infty)$, can be computed from (9). By definition of steady-state, $d/dt[K(t)]$ at $t = \infty$ is zero. Thus,

$$K(\infty) = (1 + AE_p)/[1 + A(E_p + \Delta)]. \quad (17)$$

The steady-state error ϵ_{ss} is given by

$$\epsilon_{ss} = e_0(\infty) - E_p = K(\infty)(E_p + \Delta) - E_p = \Delta/[1 + A(E_p + \Delta)]. \quad (18)$$

In a practical system this value can be made negligible by making A sufficiently large. Of course, the steady-state error in no way depends upon the loop low-pass filter if $H(0)$ is unity. Thus, unlike the general solution and the expression derived for the time constants, (18) is valid for any choice of loop filter.

Tracking Error

From Fig. 2 the tracking error ϵ_T of an AGC system is given by

$$\epsilon_T = K(t)e_i(t) - E_p. \quad (19)$$

For a first-order system ϵ_T can be obtained from (11), the general solution for $K(t)$. If $e_i(t)$ results from a step change in load factor at the input of a first-order transmitter loop, it will have the form given by (15). Substituting (15) into (11) yields

$$K(t) = \exp\left(-\int_0^t \beta\{1 + A[E_p + \Delta - \Delta\exp(-\alpha x)]\}\,dx\right) + \beta[1 + AE_p] \cdot \int_0^t \exp\left(-\int_\tau^t \beta\{1 + A[E_p + \Delta - \Delta\exp(-\alpha x)]\}\,dx\right) d\tau \quad (20)$$

which can be written as

$$K(t) = \exp\{-\beta t - A\beta(E_p + \Delta)t - (A\beta\Delta/\alpha)[\exp(-\alpha t) - 1]\} + \beta[1 + AE_p] \cdot \exp[-\beta t - A\beta(E_p + \Delta)t - (A\beta\Delta/\alpha)\exp(-\alpha t)] \cdot \int_0^t \exp[\beta\tau + \beta A(E_p + \Delta)\tau + (A\beta\Delta/\alpha)\exp(-\alpha\tau)]\,d\tau. \quad (21)$$

Using (14) and letting $T_r = r$ to simplify notation allows (21) to be written as

$$K(t) = \exp\{-t/r - (A\beta\Delta/\alpha)[\exp(-\alpha t) - 1]\} + \beta[1 + AE_p]\exp[-t/r + (A\beta\Delta/\alpha)\exp(-\alpha t)] \cdot \int_0^t \exp[\tau/r + (A\beta\Delta/\alpha)\exp(-\alpha\tau)]\,d\tau. \quad (22)$$

This equation can be expressed as

$$K(t) = \{\exp[-t/r - b\exp(-\alpha t)]\} \cdot \{e^b + \beta[1 + AE_p]g(b;t)\} \quad (23)$$

where $b = A\beta\Delta/\alpha$ and

$$g(b;t) = \int_0^t \exp[\tau/r + b\exp(-\alpha\tau)]\,d\tau. \quad (24)$$

A series expansion for $g(b;t)$ can be obtained by making the change of variable

$$y = b\exp(-\alpha\tau). \quad (25)$$

This yields

$$g(b;t) = (1/\alpha)b^\lambda \int_{b\exp(-\alpha t)}^b y^{-(1+\lambda)}e^y\,dy \quad (26)$$

since

$$\lambda = T_t/T_r = 1/\alpha r. \quad (27)$$

Replacing e^y by its series expansion yields

$$g(b;t) = (1/\alpha)b^\lambda \int_{b\exp(-\alpha t)}^b \sum_{k=0}^{\infty} \frac{y^{(k-1-\lambda)}}{k!}\,dy. \quad (28)$$

The term in which $k = \lambda$ must be integrated separately since it yields an $\ln y$ term. Thus (28) becomes

$$g(b;t) = (1/\alpha)b^\lambda \left[\sum_{k=0;k\neq\lambda}^{\infty} \frac{y^{k-\lambda}}{k!(k-\lambda)} + \frac{1}{\lambda!}\ln y\right]_{b\exp(-\alpha t)}^b \quad (29)$$

which can be written as

$$g(b;t) = (1/\alpha)b^\lambda \left[\frac{\alpha t}{\lambda!} + y^{-\lambda}\sum_{k=0;k\neq\lambda}^{\infty} \frac{y^k}{k!(k-\lambda)}\Bigg|_{b\exp(-\alpha t)}^b\right]. \quad (30)$$

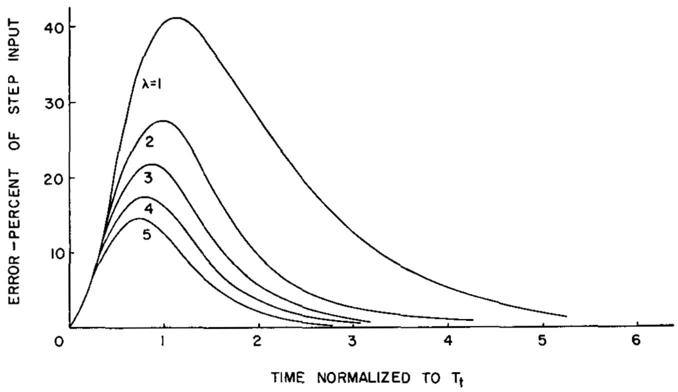

Fig. 4. Tracking error for first-order loop.

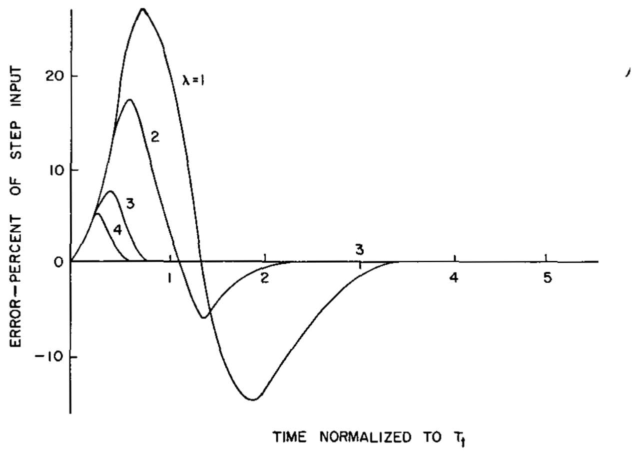

Fig. 5. Tracking error for second-order loop.

Substituting (30) into (23) yields the final result for $K(t)$

$$K(t) = \exp[-t/T_r - b\exp(-\alpha t)] \cdot \Bigg\{e^b + (1/\alpha)b^\lambda\beta[1 + AE_p] \cdot \left[\frac{\alpha t}{\lambda!} + y^{-\lambda}\sum_{k=0;k\neq\lambda}^{\infty} \frac{y^k}{k!(k-\lambda)}\Bigg|_{b\exp(-\alpha t)}^b\right]\Bigg\}. \quad (31)$$

From this expression $K(t)$ can be determined for any choice of system parameters. Tracking error is then determined using (19).

Experimental Results

In the previous section the time constant, steady-state error, and tracking error were obtained for a first-order loop. Since the analysis to determine tracking error for higher order loops is much more difficult, simulation was used to obtain results for a second-order loop. Also, for convenience and for a check on the theoretical solution, the first-order loop was simulated. The transmitter loop was replaced by a low-pass filter in the simulation to give the proper pilot response with a step function input. The receiver loop was simulated as represented in Fig. 3.

Fig. 4 shows the tracking error for a step in load factor at the transmitter loop input for a first-order system. The curves were described mathematically in the previous section. Fig. 5 gives the results for a second-order system. These families of curves provide the designer with information concerning the effect of the time constant ratio λ on the magnitude and duration of the peaks in tracking error resulting from step inputs. A comparison of the first- and second-order results indicate that the second-order loop has a lower peak tracking error than the first-order loop. However, the second-order loop has the disadvantage of requiring compensation to prevent excessive ringing and long settling times. The compensator utilized was a first-order lead network placed in cascade with the low-pass filter and was designed to yield minimum settling time.

Conclusions

The use of AGC yields an increase in the individual channel signal-to-noise ratio when the data are nonstationary, but the system introduces tracking errors which decrease system accuracy. Tracking error is a dynamic calibration error which is greatest when the rms value of the baseband is changing rapidly. The magnitude of tracking error is a decreasing function of the ratio of transmitter to receiver time constants. Additionally, steady-state errors result in the tracking process, but these can be made negligible by using sufficient loop gain.

Because the AGC loop is defined by a differential equation having time-varying coefficients, the loop time constant is a function of the input signal. This fact makes it difficult to specify the effective AGC bandwidth when different types of transients, or steps of different magnitude, are likely to be encountered.

If the transmitter loop time constant is made excessively long, the transmitter can be overdeviated for a relatively long period of time when the baseband load factor increases rapidly. This can lead to data errors resulting from the transmitter being deviated beyond its linear region or to interference with other systems because of an excessively wide RF spectrum. These problems have not been investigated in this paper but are of considerable importance in any practical application of AGC.

Baseband AGC would be particularly attractive if the majority of the channels were changing slowly in rms value so that large tracking errors could be avoided. However, from the preceeding considerations, especially tracking error, a system may be desirable in which channels carrying critical signals are not subjected to the AGC. Such a system could be easily implemented by summing together some of the channels ahead of the transmitter loop and the remaining channels after the loop.

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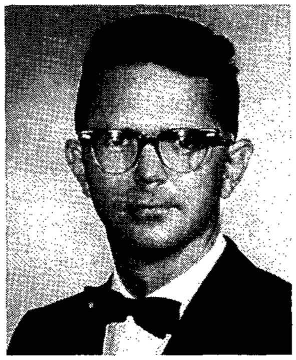

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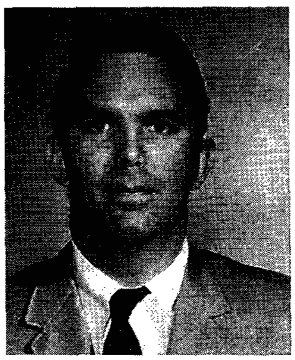

William H. Tranter (S'61–M'68) was born in Greensboro, N. C., on October 24, 1939. He received the B.S.E.E., M.S.E.E., and Ph.D. degrees from the University of Alabama, Tuscaloosa, in 1964, 1965, and 1970, respectively.

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