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Comparison Of Lagrangian Time Correlations Obtained From Dispersion Experiments And From Space-time Correlation Functions

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D_{AB} = binary coefficient of mass diffusion in the mixture of gases A and B
g = gravity acceleration
h = height difference between the interfaces mercury-gas in the viscometer reservoirs
L; L_{Hg} = length of gas capillary and of mercury tube, respectively
n = molecular concentration
N_{Re} = Reynolds number
P; $P_{tot.}$ = experimental and total pressure of the gas, respectively
ΔP = pressure drop corresponding to height difference, h
q; q_{Hg} = volumetric flow rate of the gas and of the mercury, respectively
r; r_{Hg} = radius of gas capillary and of mercury tube, respectively
R = gas law constant
S_0 = area of the interface mercury-gas at zero inclination of the viscometer
S = area of the interface mercury-gas in the viscometer reservoirs during a run
$\bar{t}$; t^* = time required for displacing the mercury and the gas during a run, respectively. $\bar{t} = t^*$ when $P \geqq 50$ atm.
T = absolute temperature
X_i = radial distribution function at distance σ_i from the center of a molecule i having diameter σ_i
V = volume between contacts e_1 and e_2 in reservoir b of the viscometer
$V + V_I$ = gas volume in reservoir b of the viscometer at the beginning of the run
V_{II} = gas volume in reservoir a of the viscometer at the beginning of the run
$\bar{V}$ = molar volume
Z = compressibility factor of the gas, $Z = P\bar{V}/RT$

Greek Letters

ϵ_0 = distance between contacts e_1 and e_2 in reservoir b at zero inclination of the viscometer
η; η_{Hg} = viscosity of the gas and of the mercury, respectively
θ = inclination angle of the viscometer
ρ; ρ_{Hg} = density of the gas and of the mercury, respectively
σ_i = molecular diameter of specie i ($i = A, B, AB$)
Ω_d; Ω_v = generalized collision integrals for mass diffusivity and for viscosity of molecular pair AB, respectively
δ = parameter, defined by Equations (24), (25), related to the volume change of the gas during its displacement through the capillary

Subscripts

$(\ )_1$ = for viscosity and for mass diffusivity at $P = 1$ atm.
i = for component i ($i = A, B, AB$)
1; 2 = for quantities at the beginning and at the end of a run, respectively
eff = for effective displaced volume of gas through the capillary

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Comparison of Lagrangian Time Correlations Obtained from Dispersion Experiments and from Space-Time Correlation Functions

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The relation between Lagrangian and Eulerian statistics for turbulent flow has been approached only through approximations or models of the actual motion. Some of these approaches have been motivated by a purely theoretical interest in the problem (*1* to *5*) and others by need to justify the interpretation of an experimental measurement (*6* to *10*). Altogether, little progress has been made in this endeavor despite its importance in the research on turbulent dispersion, mixing, and theoretical models for turbulence.

The most direct technique for gaining information about a turbulent field in the Lagrangian sense is the measurement of dispersion rate of an injected material or of heated fluid into a turbulent stream. This was done by Baldwin and Walsh (*6*) in a pipe where they heated the flowing air with a wire and measured temperature profiles downstream. They calculated Lagrangian time correlation functions using such data by applying the Taylor theory of continuous movements (*11*). Results of this type have also been obtained by Uberoi and Corrsin (*8*), Mickelsen (*9*), and Hay and Pasquill (*10*).

Since this measurement is based on the spreading rate of material (a rate of dispersion in the lateral direction) the Lagrangian time correlation obtained applies only to that direction unless the turbulence is isotropic. Lagrangian correlations applying to the longitudinal direction could probably be derived by measuring the longitudinal spread of a pulse of material as it moved downstream, in a way very similar to the time response studies in process dynamics. Such attempts are not known to the authors.

Baldwin and Walsh attempted to relate their Lagrangian time correlations derived from lateral diffusion measurements to Eulerian space-time correlations measured with hot-wire anemometers. Their space-time correlations involved correlations over a range of time delays at several longitudinal probe separations. They showed that with the following approximations the locus of their space-time correlation maxima corresponds to a Lagrangian time correlation function.

1. terms higher than second order in a series expansion are negligible.
2. the turbulence is homogeneous in the longitudinal direction.
3. the turbulence is stationary.
4. Burger's approximation of the Lagrangian time derivative is applicable $[(du/dt) = (\partial u/\partial t) + U(\partial u/\partial y)]$.
5. and $u'_{La} = u'_{Eu}$

They compared experimental results from the diffusion experiments and from the longitudinal space-time correlations even though the former experiment yields correlations based on lateral dispersion and in the latter all the information is in the longitudinal direction. With complete isotropy this may have been a valid comparison, but the pipe flow conditions involved do not yield isotropy. As shown by Figure 1 the function shapes and magnitudes for the two approaches are significantly different.

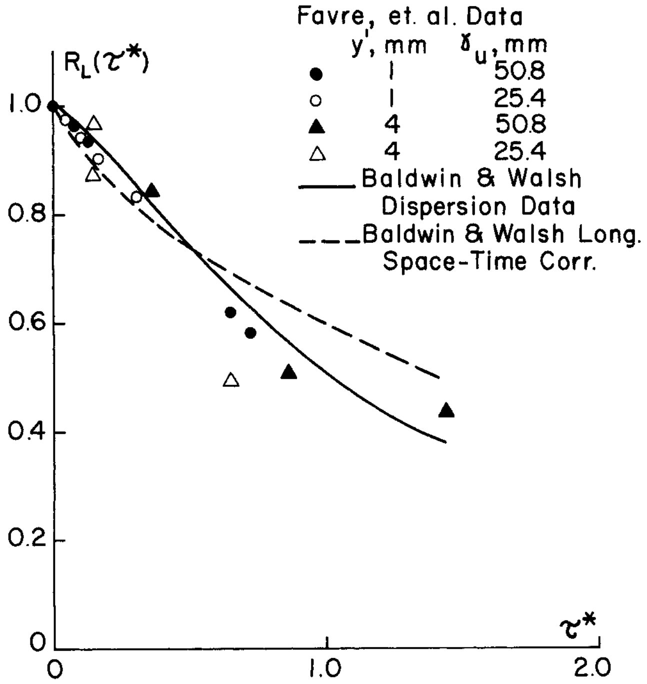

Fig. 1. Comparison of Lagrangian correlation approximations.

Corrsin (*3*) and Phillip (*5*) developed relations between the Lagrangian time correlation $R_L(\tau')$ and the space-time correlation $R(\zeta, \gamma, \tau')$, where γ is the longitudinal separation in turbulence with no mean velocity and ζ is the lateral separation. The form used by Phillip is

$$R_L(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \theta(\zeta, \gamma, \tau') R(\zeta, \gamma, \tau') d\zeta d\gamma \qquad (1)$$

where $\theta(\zeta, \gamma, \tau')$ is the probability of a particle released at $(0, 0, 0)$ being at (ζ, γ, τ'). To test the relation Phillip assumed a Gaussian distribution for $\theta(\zeta, \gamma, \tau')$ and used a model equation for $R(\zeta, \gamma, \tau')$ having good integral properties. With known probability density distributions and a space-time correlation map of isotropic turbulence this relation could be tested by numerical integration then comparison with an $R_L(\tau')$ derived from dispersion measurements in the same isotropic turbulence. For a turbulent field with a mean flow, γ must be expressed as $(\gamma_U - U\tau')$, as shown by Phillip.

With the absence of such data, an attempt was made to develop a good approximation to the Lagrangian time correlation using only space-time correlation data. It is the purpose of the following argument to show that space-time correlations with lateral as well as longitudinal probe separations are necessary for derivation of a function comparable to the Lagrangian time correlation derived from dispersion results. The development is similar to that used by Baldwin and Walsh for the longitudinal space-time correlation.

Let $R(\zeta, \gamma_U, \tau_T) = R(\zeta, \tau'; \gamma_U, \tau)$, the space-time correlation function where ζ is the lateral separation distance and τ' is the difference between τ_T, the optimum delay time at (ζ, γ_U), and τ, the optimum delay time at $(0, \gamma_U)$, and γ_U is the longitudinal separation distance with a mean velocity U. Assume at large values of γ_U and small values of ζ that $R(\zeta, \tau')$ and $R(\gamma_U, \tau)$ are independent, where $R(\zeta, \tau')$ at any large value of γ_U is $R(\zeta, \tau'; \gamma_U, \tau)/R(\gamma_U, \tau)$. Then:

$$R(\zeta, \tau') \approx R(0, 0) + \zeta \left.\frac{\partial R(\zeta, \tau')}{\partial \zeta}\right|_{\substack{\zeta=0\\ \tau'=0}} + \tau' \left.\frac{\partial R(\zeta, \tau')}{\partial \tau'}\right|_{\substack{\zeta=0\\ \tau'=0}} + \frac{\zeta^2}{2}\left.\frac{\partial^2 R(\zeta, \tau')}{\partial \zeta^2}\right|_{\substack{\zeta=0\\ \tau'=0}} + \zeta\tau' \left.\frac{\partial^2 R(\zeta, \tau')}{\partial \zeta \partial \tau}\right|_{\substack{\zeta=0\\ \tau'=0}} + \frac{\tau'^2}{2}\left.\frac{\partial^2 R(\zeta, \tau')}{\partial \tau'^2}\right|_{\substack{\tau'=0\\ \zeta=0}} \qquad (2)$$

Assuming homogeneity in the lateral direction, the second and third terms are zero (only approximate in a boundary layer). $R(0, 0)$ is, of course, unity. Also:

$$\frac{\partial^2 R(\zeta, \tau')}{\partial \zeta^2} = \frac{1}{\overline{u^2}} \left.\frac{\partial^2 \overline{u(0)u(\zeta)}}{\partial \zeta^2}\right|_{\zeta=0} = \frac{1}{\overline{u^2}} \left.\overline{u(0)\frac{\partial^2 u(\zeta)}{\partial \zeta^2}}\right|_{\zeta=0}$$

$$= \frac{1}{\overline{u^2}}\left[\frac{\partial}{\partial \zeta}\left(\overline{u\frac{\partial u}{\partial \zeta}}\right) - \overline{\left(\frac{\partial u}{\partial \zeta}\right)^2}\right]$$

$$= -\frac{1}{\overline{u^2}}\overline{\left(\frac{\partial u}{\partial \zeta}\right)^2} \qquad (3)$$

$$\frac{\partial^2 R(\zeta, \tau')}{\partial \tau'^2} = -\frac{1}{\overline{u^2}} \overline{\left(\frac{\partial u}{\partial \tau'}\right)^2} \quad (4)$$

$$\frac{\partial^2 R(\zeta, \tau')}{\partial \zeta \partial \tau'} = -\frac{1}{\overline{u^2}} \overline{\frac{\partial u}{\partial \zeta} \frac{\partial u}{\partial \tau'}} \quad (5)$$

So:

$$R(\zeta, \tau') \approx 1 - \frac{\zeta^2}{2\overline{u^2}} \overline{\left(\frac{\partial u}{\partial \zeta}\right)^2} - \frac{\zeta\tau'}{\overline{u^2}} \left(\overline{\frac{\partial u}{\partial \zeta} \frac{\partial u}{\partial \tau'}}\right) - \frac{\tau'^2}{2\overline{u^2}} \overline{\left(\frac{\partial u}{\partial \tau'}\right)^2} \quad (6)$$

Assume that $\zeta = V\tau'$, where V is some lateral velocity scale:

$$R(\zeta, \tau') \approx 1 - \frac{\tau'^2}{2\overline{u^2}} \left[V^2 \overline{\left(\frac{\partial u}{\partial \zeta}\right)^2} + 2V \overline{\frac{\partial u}{\partial \zeta} \frac{\partial u}{\partial \tau'}} + \overline{\left(\frac{\partial u}{\partial \tau'}\right)^2} \right] \quad (7)$$

Let $(du/d\tau') \approx (du/d\tau') + V(du/d\zeta)$, a form of the Burgers approximation to the Lagrangian derivative. Then:

$$R(\zeta, \tau') \approx 1 - \frac{\tau'^2}{2\overline{u^2}} \overline{\left(\frac{du}{d\tau'}\right)^2} \quad (8)$$

The Lagrangian correlation may be expanded for the lateral direction as follows:

$$R_L(\tau') \approx R_L(0) + \frac{\tau'^2}{2} \left. \frac{d^2 R_{L(\tau')}}{d\tau'^2} \right|_{\tau'=0} \quad (9)$$

assuming symmetry in time. Since $R_L(\tau') = \frac{\overline{u(0)u(\tau')}}{\overline{u^2}}$, where $u(0)$ and $u(\tau')$ are the velocities of the same fluid particle, $\left. \frac{\partial^2 R_L(\tau')}{d\tau'^2} \right|_{\tau'=0} = -\frac{1}{\overline{u^2}} \overline{\left(\frac{du(\tau')}{d\tau'}\right)^2}$. Also, since $R_L(0) = 1$:

$$R_L(\tau') \approx 1 - \frac{\tau'}{2\overline{u^2}} \overline{\left(\frac{du}{d\tau'}\right)^2} \quad (10)$$

So: $R_L(\tau') \approx R(\zeta, \tau') = R(\zeta, \gamma_U, \tau_T)/R(0, \gamma_U, \tau)$, where $\tau_T = \tau' + \tau$.

The applicability of this result was tested in a crude fashion by making use of the data of Favre, Gaviglio, and Dumas (*12*). They made space-time correlations for both longitudinal and lateral probe separations in a boundary layer using several stationary-probe distances from the wall. For these comparisons four sets of data were used. Probe positions are summarized in the following table where y' is the separation of the stationary probe from the wall, δ is the boundary layer thickness, γ_U is the longitudinal probe separation, and ζ' is the lateral probe separation:

y', mm.	δ, mm.	γ_U, mm.	ζ', mm.
1.0	16.8	25.4	−0.2 to −1.8
1.0	16.8	50.8	+0.5 to −2.0
4.0	16.8	25.4	+3.3 to −2.0
4.0	16.8	50.8	+2.5 to −1.5

(+ distances are toward the wall)

U was 12.00 m./sec. in the free stream.

Since convection of velocity eddies (as shown by the loci of maxima in the space-time correlations) was not perfectly longitudinal, but tended away from the wall, the value for $R(\zeta', \tau, 0)$ with which to normalize the derived Lagrangian correlation was taken as the maximum value of the set of correlation values at their optimum delay times for each longitudinal separation distance. The value of ζ was assumed zero at that point, even though ζ' would have some value. τ' was calculated by subtracting the optimum delay time for the $R(\zeta', \tau, 0)$ value from the optimum delay time for each of the other correlation values. In order to allow comparison with data from other sources, the delay times τ' were then normalized to obtain:

$$\tau^* = \tau' U/\delta$$

Figure 1 shows the results of that procedure. The points calculated from the Favre, et al. data are compared with the result derived from lateral dispersion (indicated by a solid line) obtained by Baldwin and Walsh. Considering the diversity of the data sources and the difficulty of using the Favre, et al. data outside their intended purpose, the comparison shows good agreement both in the shape of the functions and their magnitudes at various time delays. The dotted line shows the longitudinal space-time correlation maxima of Baldwin and Walsh.

This has been only a crude test, so space-time correlation and dispersion data specifically intended for such comparisons as these should be obtained in order to more fully test the relationship proposed here and the relation of Corrsin (*3*) and Phillip (*5*). This paper was written to encourage more experimental work in this area.

NOTATION

$R(\zeta, \tau')$ = lateral Eulerian space-time correlation function
$R(\gamma_U, \tau)$ = longitudinal Eulerian space-time correlation function
$R(\zeta, \gamma, \tau')$ = Eulerian space-time correlation function
$R_L(\tau')$ = Lagrangian correlation function for time delay τ'
t = time
u = longitudinal fluctuating velocity
U = time mean longitudinal velocity
u' = root-mean-square longitudinal fluctuating velocity
V = lateral velocity scale
y' = lateral distance from wall

Greek Letters

ζ = boundary layer thickness
γ = longitudinal separation distance with no mean flow
γ_U = longitudinal separation distance with mean velocity U
δ = lateral separation distance
$\theta(\zeta, \gamma, \tau')$ = probability that a particle released at $(0, 0, 0)$ is at (ζ, γ, τ')
τ = optimum delay time at $(0, \gamma_U)$
τ' = Lagrangian correlation delay time
τ_T = optimum delay time at (ζ, γ_U)
τ^* = normalized value of τ'

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