Foreword to the Special Issue in honor of Prof. Luigi Preziosi "Nonlinear mechanics: the driving force of modern applied and industrial mathematics"

Giuseppe Saccomandi[a,b,*], Chiara Giverso[c], Alfio Grillo[c]

[a] *Dipartimento di Ingegneria, Università di Perugia, Via Goffredo Duranti, 93, 06125 Perugia PG, Italy*
[b] *NUI Galway, University Road, Galway, Ireland H91 TK33*
[c] *Dipartimento di Scienze Matematiche (DISMA) "G. L. Lagrange", Politecnico di Torino, C.so Duca degli Abruzzi, 24, 10129 Torino TO, Italy*

Figure 1: Prof. Luigi Preziosi, in the occasion of his 60$^{\text{th}}$ birthday.

Mathematical modelling is a discipline pledged to identify problems, which may arise from virtually any branch of the human knowledge, and formalise them in the language of mathematics by developing methodologies of investigation framed within the appropriate rational framework. The problems addressed by mathematical modelling may stem from biology or physics, or from the study of anthropic phenomena, such as those pertaining economics, medicine, pharmaceutics, social sciences, industry as well as the more recent contexts of big

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*Corresponding author
Email address: `support@elsevier.com` (Giuseppe Saccomandi)

data and artificial intelligence.

To pursue its goals, modelling must build connections with other mathematical sciences and, in the last decades, the accessibility to always richer computational resources has given the thrust for a tight combination of modelling and numerics. Indeed, on the one hand, mathematical models have become more descriptive, since they can rely on more efficient and robust numerical methods, which permit to account for several effects simultaneously. On the other hand, numerics has advanced under the stimulus of increasingly involved models, and has led to a better understanding of the investigated problems by improving the precision and the accuracy of simulations, and granting more realistic visualisations of the simulated problems.

While all this has applied to almost all fields of scientific and technical interest, the root of this intellectual viewpoint is to be considered in the framework of mechanics. Leonardo da Vinci (1452-1519) says (Codice E, f. 8 v)

> *"La Meccanica è il paradiso delle scienze matematiche, perché in quella si viene al frutto matematico."*[1]

Two major examples of the efficiency of the combination of modelling and simulations and of their tight relationship with nonlinear mechanics are industrial mathematics and the mathematics in biology and biomechanics.

At first sight, industrial mathematics is just a branch of applied mathematics focusing on problems that come from industry and it aims at determining solutions relevant to manufacturing, including the determination of the most efficient ways for solving those problems. However, it is clear that the increasing complexity and sophistication of modern industry involve technical issues that cannot be solved by just using simulation models. Indeed, they need true mathematical models grounded on well established *general laws*. Some relevant examples are petroleum engineering and hydrogeology, in which no real ad-

[1] "Mechanics is the paradise of the mathematical sciences, because by means of it one comes to the fruits of mathematics."

vances are possible without a detailed knowledge of the constitutive modelling of non-Newtonian fluids, or environmental sciences, in which one has to handle multi-phase flow problems in the presence of phase transitions and/or coupled phenomena, or the detailed description of sand dynamics in the neighbourhood of railways in desert zones.

Although sprouting in the completely different context of biology, the adoption of mathematics to formalise problems of biological relevance has attracted many scientists all through the years. Besides the models typically encountered in biomathematics, such as those addressing population dynamics, epidemiology and related fields, a strong impact on the scientific community has been given by the combination of mathematical modelling with the mechanics of biological tissues, thereby giving rise to biomechanics. This branch of science grew considerably in the nineteenth and in the twentieth centuries, and especially in the second half of the latter one, when it gathered together a considerably big number of scientists coming from continuum mechanics. In the last thirty years, particular attention has been drawn on the problems of tissue growth and remodelling, especially in the context of the mechanics of bone and of "soft tissues", such as tendons, ligaments, articular cartilage, blood vessels, organ tissues, cell cultures, etc. A huge step forward, in addition, was provided by the introduction of the mechanics of porous media and the theory of mixtures and multi-phasic materials in the realm of tissue biomechanics, thereby providing the ground for formulating new models in which the mechanics of tissues and cellular aggregates could be studied in connection with fluid dynamics and chemistry. All this, in turn, suggested to develop new branches of research in which once again the driving force of the innovation is mechanics. Few examples in this respect are the mechanics of cell motion and migration, which necessarily relate to kinetic theories, the mechanics of the interactions of cells with the extracellular matrix, the conversion of mechanical signals into chemical stimuli, and the vast field of research referred to as mathematical oncology.

The generality and complexity of the above scenario are today under an intellectual "onslaught". As pointed out by Coveney et al. [1], the growing appeal exerted by disciplines like machine learning or the relatively new subjects pertaining to data sciences, like data analytics and big data, seems to suggest that the classical scientific method of investigation is no longer necessary. For this reason we need to emphasise once again the relationship between mathematical modelling and nonlinear mechanics. To understand this relationship, we first need a deep reflection about the "complementary" difference between a *model* and a *simulation*. The simulation of a real world phenomenon increases in usefulness with the quantity of specific data incorporated. For this reason, big data revolution and statistical methods may be very useful in this framework. On the other hand, the mathematical models should find the correct balance between the maximal level of complexity and the minimal level of detail that are necessary to preserve the essential outline of a given phenomenon. In this respect, modelling and situation are complementary. Whereas a simulation is a concretely descriptive approach that aims at the visualisation of one case at a time, a mathematical model is abstract and universal and it allows our intellectual activity to go over the actual set of observations and realisations. Mathematical models and simulations are fundamental tools in our system of knowledge for producing true innovation, because they can be used to *dissecare naturam* in the Baconian sense. However, in doing this, one should bear in mind that mathematical models are grounded on *laws* and not on *correlations*. Clearly, nowadays, we are far away from the reductionism paradigm, but, because we are far away from this approach, we are aware that it is not possible to *dissecare naturam* if we are not trained in a concrete way in mathematical modelling. It is well-known, in this respect, that the model *par excellence* in the humankind system is mechanics.

To date, we are still far away from an axiomatic *metatheory of models*[2], although it is possible to be trained implicitly by operating in a tangible way in this activity. By slightly paraphrasing Piero Villaggio's view of elastic structures

[2]See also `https://writings.stephenwolfram.com/2021/09/charting-a-course-for-complexity-metamodeling-ruliology-and-more/`.

[2], nonlinear mechanics *"is, by definition, the collection of all reasonable models* [and mathematical methods]*, proposed during almost three centuries, concerned with simplifying the solution of problems involving* [material] *bodies"*, regardless of whether one considers the *"vibration of bars or plates, and the stability of columns"*, the motion of a fluid or the dynamics of a rigid body. The progress of nonlinear mechanics has been possible by the complete blend of experimental and theoretical ideas adopted by the human kind. nonlinear mechanics is a theoretical discipline where your testing activity is the rule. Nonlinear mechanics is concrete because it addresses problems of interest in everyday life, with a widespread range of applications, at your home, in a factory or in a scientific laboratory. These are the reasons why nonlinear mechanics is the driving force of mathematical modelling.

Since it is not possible to present a theoretical corpus of all that, our idea is to present a special issue in which remarkable examples of the cited relationships are given by the solution of explicit problems. The aim of the present special issue is to put together a list of outstanding scientific papers giving clear connections among nonlinear mechanics, industrial mathematics, biomathematics, biomechanics and kinetic theories, in different fields of interest.

With reference to the general and industrial aspects of mathematical physics, this Special Issue presents five contributions, whose topics range from the characterisation of non-Newtonian fluids and viscoelastic materials to the thermodynamics of gases, from the modeling of swelling as the origin of "morphing" to traffic dynamics. In particular, the work by Fusi et al. [3] addresses the flow of *"shear-thinning fluids"* exhibiting Bingham-type behavior. Farina et al. [4] study the phenomena of *"creep, recovery and vibration of an incompressible viscoelastic material"* and consider, in particular, the response of the material in simple tension. Arima et al. [5] study the *"gases with internal structure"* in the context of Rational Extended Thermodynamics and investigate the moments that are more suitable for their characterisation. The contribution by Colorado Cervantes et al. [6] deals with the *"morphing of soft structures"*, assuming this process to be guided by *"active swelling"*. Finally, Chiarello et al.

[7] study *"macroscopic limits of car-following traffic dynamics"* by means of tools of statistical mechanics.

Concerning the application of mathematical models to biological and medical sciences, this Special Issue addresses the problems of cell motion [8, 9, 10], stability of viral capsids [11], neurodegenerative diseases [12], mechanical behaviour of biological tissues [13, 14] and cancer growth and invasion [15, 16, 17]. Specifically, the works of Stotsky and Othmer [8] and Chelly et al. [9] investigate the transmission of *"intra- and extracellular forces"* to the cell membrane and their influence in determining cell shape [8] and *"cell crawling on a compliant substrate"* [9], respectively. The mathematical findings in [9] are confirmed by experiments with *"PDMS stamps"* designed ad-hoc [9]. Likewise, the work by Braun et al. [10] is inspired by laboratory experiments, developed to investigate the *"cross-talk between immune and cancer cells in a confined environment given by a microfluidic chip, the so called Organ-on-Chip"*. The calibration algorithm for a system of coupled reaction-diffusion-transport equations with chemotactic terms, proposed therein [10], could be of great interest also in other experimental settings. Another technique to infer the model parameters from the medical/biological data is also proposed in Medaglia et al. [15], using *"suitable numerical methods for uncertainty quantification of the resulting kinetic equations"* applied to tumour growth and response to therapeutic protocols. Lorenzi and Painter [16] extended the standard Keller-Segel model for chemotaxis-driven cell invasions to *"account for the possibility of phenotypic heterogeneity"* in the chemotactic term and proliferation rate of different cells inside the same population. The mathematical problem of describing a cellular population composed by cell with different genotypes and phenotypes is also investigated in Chiari et al. [17], by proposing a multiscale hybrid modelling framework in which a discrete structuring variable differentiates cells and *"a specific mathematical representation (i.e., individual/pointwise vs. collective/density-based) is assigned to each individual on the basis of its phenotypic hallmarks"*. Another way *"to bridge the gap between microscopic and macroscopic approaches"* is proposed in Sampaoli et al. [12], where the theoretical framework of mixtures theory is used to derive a macroscopic Cahn-Hilliard type equation to describe the evolution of misfolded proteins inside the healthy brain, during neurodegenerative diseases. Looking at the mechanical behaviour of tissues at the macroscopic scale, the paper by Di Stefano et al. [13] provides an analytical model of focal adhesions and evaluates how the elasticity of an adhesion affects the *"transition from a ductile to a fragile decohesion regime"*. Moreover, Marzocchi and Musesti [14] propose a mathematical model of the skin, described as a hyperelastic material, and prove *"the existence and uniqueness of the solution for a general measure-valued external load"*. Finally, in [11], mathematical models for viral capsids present in the literature are reviewed, to identify *"fundamental structural units"* (i.e., either single proteins or groups of them) in *"coarse-grained mechanical models"*.

All the research themes addressed above are very dear to Prof. Luigi Preziosi, who has been brightly dedicating his scientific career to many of them. Indeed, this special issue of IJNLM is the *Festschrift* celebrating the 60th birthday of Luigi Preziosi. Luigi's recognized research is a typical example of how mechanics may be the fuel for interesting applied mathematics. Luigi's PhD thesis was about the mechanics of immiscible Newtonian fluids [18] and viscoelastic media [19]. For many years, Luigi has been engaged principally in the framework of non-Newtonian fluid mechanics [20] and in the study of heat propagation [21, 22]. Then, he moved to work on kinetic theories [23, 24] and on some industrial problems involving porous materials [25, 26, 27]. In the second half of the nineties of the last century, Luigi's skills in mechanics allowed him to focus his research activity on the study of the interactions between tumour evolution and immune system [28, 29], with some innovative and breakthrough ideas. This was the beginning of a prolific activity in biomechanics and mathematical biology, which todays allows to recognise Luigi as one of the top scientists in the field of tumour and cell mechanics [30, 31, 32, 33, 34, 35, 36, 37, 38]. Besides all this, Luigi has been involved in the development of Mathematical Physics along the Italian tradition, which once again is strictly related to classical mechanics, mainly at the *Politecnico di Torino*, but also all along Italy, where he has played a major role in the organisation of the community of applied and industrial mathematics. Recently, Luigi's scientific activity has been awarded with his membership of the *Accademia dei Lincei*.

Last but not least, it must be clear that the most important quality of Luigi is that he is a true gentleman in science and life: Luigi is a true friend, an extremely loyal and collaborative colleague, always willing to help, and proactive in resolving problems.

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