Rehearsing in the margins:
Mathematical print and mathematical learning in the early modern period

Benjamin Wardhaugh

Arithmetical computation and proportional reasoning, algebraic reasoning and geometrical proof: these were all experienced in the early modern period – as they still are today – as special kinds of performance. They were transmitted by processes involving demonstration by a teacher, private rehearsal, and specimen performance by the student. Printed and manuscript texts recorded the traces of successful (or unsuccessful) performances, and learners used slates, waste paper and the margins of printed texts as rehearsal space in which to perfect their own performances. The result was a pedagogy and a mathematical culture that used the written word – printed or manuscript – in unique, highly distinctive ways. This chapter examines the evidence for that culture and the influences on it of ancient and early modern examples.

1. The fragmented text

Early modern readers experienced mathematical text as fragmented and non-linear, as negotiable and malleable, and as a model for imitation and from which to assimilate praxis (Raphael 2013, 2015, 2016, 2017; Oosterhoff 2015). Consider the famous story related by John Aubrey about Thomas Hobbes (Aubrey 1898, 332).

> He was (vide his life) 40 yeares old before he looked on geometry; which happened accidentally. Being in a gentleman's library in ..., Euclid's Elements lay open, and 'twas the 47 El. libri I. He read the proposition. 'By G––,' sayd he, 'this is impossible!' So he reads the demonstration of it, which referred him back to such a proposition; which proposition he read. that referred him back to another, which he also read. Et sic deinceps, that at last he was demonstratively convinced of that trueth. This made him in love with geometry.

(Broadly similar stories were told for instance about Newton: see (Whiteside 1967, 6), citing an account by Abraham de Moivre.) The story is plausible in general terms, and part of its plausibility is the familiarity to early modern ears of the model of engagement with the Euclidean text it narrates. Hobbes did not begin at the beginning of the text, with the meticulous logical basis of axioms, postulates and common notions. Nor did he begin at the beginning of the propositions in Book 1 – simple constructions and proofs to do with circles and triangles – and work up from there. He began *in medias res* with the forty-seventh proposition in the *Elements* ('Pythagoras' theorem') and having formed his initially skeptical response he continued by working through the text selectively and in reverse order, guided presumably by the printed cross-references that adorned the main text or the margins of most early modern editions, jumping from proposition to proposition until he arrived, finally, at the most basic material, and the definitions and axioms. Compared with our usual images and assumptions about the reading of a printed book this is a rather extraordinary way to proceed; but nothing Aubrey says suggests that it was unusual. As we will see, the evidence we have indicates that for mathematical books it was the norm.

The nonlinear experience of the text produced by readerly rearrangement and selection of course responds directly to the textual conventions which governed mathematical print in this period; Hobbes would hardly have been able to work as he did had not the printed text been reasonably dense with cross-references. The division of mathematical knowledge into discrete theorems, for which a serial order is necessarily somewhat unhelpful, and its transformation into a meaningful network of logical relationships by printed (or manuscript) cross-references, surely invited selective, non-linear styles of reading, and this even at the most elementary level of arithmetic primers and practical manuals.

That many readers were thus guided by the cross-references printed in mathematical books is clearly indicated by the fact that so many of them added supplementary cross-references or corrected those that were printed when they found them to point to the wrong place.[1] That they read selectively is also indicated by the frequency with which they adorned the text with marks of attention applied to selected parts: selected theorems, selected chapters or sections, selected exercises or examples. The hand-written, marginal cross indicating 'I studied this' or 'I will study this' or 'this is worthy of attention' – and implicitly that everything not so marked was less worthy of attention – is an absolutely ubiquitous device in early modern mathematical books as marked by their early modern readers (Wardhaugh 2019a).[2] Some readers of mathematical books showed a concern to make the text as complete as possible within the sections they studied, remedying omissions in the operational instructions, however slight; recopying diagrams to bring them into better proximity with the text to which they related; supplementing the proofs with the definitions or the definitions with examples of their use. Some simply wrote ‘done’ by the exercises they had completed. Many readers went further in their restructuring of the text, numbering the sections to be studied, marking up the contents page or index or supplying a manuscript contents page or index to direct attention exactly where they wanted it.[3] For example, late seventeenth-century users of a copy of the *Elements* of Euclid in the popular teaching edition by Christoph Clavius, at the Queen’s College, Oxford, went through the contents list, picking out and numbering 23 of the propositions from Books 1–4 of the text. Users of another copy of the same text at Trinity College, Cambridge did the same, marking a total of 43 propositions in Books 1, 3, 4 and 6.[4]

For many readers, like Hobbes in his apparently private setting, these activities seem to have been autonomous. Out of about a hundred annotated early modern copies of the *Elements*, I have yet to see two in which the selection of theorems picked out for attention seems to be substantially the same, suggesting that such decisions were frequently made locally if not individually (see below for one example of the transmission of Euclidean annotations, however). On the other hand, many readers’ attention will certainly have been directed by a teacher. Individual ownership of textbooks did not become the norm in British schools until the second half of the eighteenth century, and up to – and in many places beyond – that point the typical practice was for the teacher to copy into the pupils’ exercise books selected parts of a printed text (Walkingame 1751, a2^{r}–a2^{v}; see also (Denniss 2012) and (Ellerton and Clements 2012, 2014)). Something similar can be seen happening in university teaching, where the teaching notebooks of tutors consist – on the mathematical side – almost invariably of *excerpta* from classical mathematical texts: see (Poole 2018), which discusses in particular the manuscript volumes in the Queen’s College, Oxford, MSS 425–32; on the autonomous and the teacher-directed modes of note-taking see also (Blair 2010). And in university lecturing too, the lectures that survive approach the classics of mathematics not as wholes to be worked through page by page but as sources from which to select, rearrange and reassemble in a new order, thought suitable for the particular audience at hand: see (Wardhaugh 2019b) and John Wallis’s manuscript lectures on Euclid’s *Elements*.[5] Henry Savile at Oxford, for instance, chose a fairly small selection of Euclid’s propositions for discussion in his *Prælectiones* (1621), but gave rather fuller attention to his preliminary definitions, as well as to general questions about philology and geometrical method.

The impulse to rearrange certainly influenced the production of printed mathematics too; from the seventeenth century onwards there began to appear with increasing frequency textbooks whose titles invoked Euclid or Apollonius or Archimedes but which consisted in fact of *excerpta* rearranged according to the fancy of the editor. Giovanni Alfonso Borelli's (1658) *Euclides restitutus* is typical of the type; its seven books contain the propositions of *Elements* 1–6 and 11–13, rearranged and partly re-written: compare (Pardies 1671) and (Mohr 1673), which both also appeared in English. The evidence provided by readers' annotations shows that these books too were received by readers not as wholes but as sources from which to select and rearrange.

Non-linear reading easily crossed the boundaries between one text and another. Almost as soon as the texts of the university curriculum were put into print their margins were used by students to record something of what their teachers had said about those texts (Oosterhoff 2018, Groote 2013, Grafton and Leu 2013, Grafton 1981). And only slightly later it became quite normal for the attentive student to customize a printed book not only with internal cross-references and internal excerpting and rearrangement, but also to supplement it with sections copied or even cut out from other books. A 1638 copy of John Wells' *Sciographia: or, The art of shadowes* was supplemented a generation later by an anonymous reader who copied in summaries of the principles of geometry as well as diagrams cut from other books.[6] A 1659 copy of Norwood's *Epitome* was customized in the mid-eighteenth century by owners Roger Sherman and Elijah Portter, who added extracts from the 1707 *Thesaurium Mathematicæ* of John Taylor and William Alingham and from John Love's 1688 *Geodaesia*.[7] Another similar example is a copy of John Seller's 1677 *Pocket Book*, with dense annotations indicating that despite its author's practical intentions it was studied by university students.[8] Printed mathematical books were thus customized into compendia, and nonlinear reading took on a new dimension emphasizing still further that the one thing no reader seems to have done with printed mathematics was to read it in the order in which it was printed.

2. The negotiable text

Mathematics presented special difficulties for the early modern print shop. Geometry involved a wealth of diagrams, which were expensive to commission and required either wastefully wide margins for their insertion or the complication of inserting them within the text itself. When copperplate engraving became available the new technology was soon adopted for the printing of geometrical diagrams, but this raised other problems. Diagrams were now relegated to a plates section at the back of the book, distant from the portion of text to which they related. One expedient was to print the plates on wide fold-outs, sufficiently wide that they could be folded out and remain visible while the main text block was turned back to the theorem in question: see for example to the note 'To the Bookbinder' in (Whiston 1703, $P8^v$). Another solution was for the reader to cut up the plates and paste the separate diagrams into the book where they were needed.[9]

Again, the points in these geometrical diagrams were conventially labelled using upper case letters starting from the beginning of the alphabet, and the diagrams were referred to from the main text using these labels: often over and over again. It is occasionally clear that the print shop ran out of the required upper case letters and had to substitute letters from another font. And geometry is repetitious in other ways, too. When almost every paragraph began with one of two or three fixed phrases, the supply of decorated initials could also run low: (Ratdolt 1491) uses in twenty locations a decorated S which is faulty (it is reversed left-right), and (d'Étaples 1516) has at least two uses of a similarly faulty decorated N. Algebra in its sixteenth- and seventeenth-century forms required some altogether new symbols or the adaptation of old ones, and it was easy to make mistakes that resulted in incomprehensibility.

More frequent and more serious, however, were the perfectly usual errors of the press in which a single character was omitted, inverted or substituted for another: which in prose would be easily ignored, but in mathematics could turn a comprehensible statement into nonsense, whether the wrong symbol was a number, an algebraic letter or a geometrical label. Such errors were easy to make and relatively easy to spot for a reader who was working closely through the mathematical content of a particular passage: but they were hard for proof-readers, working rapidly, to detect compared with typographical blunders of similar magnitude in printed prose or poetry. A wrong character in the midst of a geometrical proof does not leap off the page in the same way as a wrong character in a line of verse. It was, indeed, routinely acknowledged by authors, editors and printers that it was especially difficult to correct the press thoroughly for mathematical books. John Ward's

Young Mathematician's Guide, in 1707, contained the following characteristic lament (Ward 1707, Mmm2^{v}):

> If the Reader were but Sensible of the Great Care and Difficulty that unavoidably attends Correcting the Press to Books of this Nature; he would the more readily Excuse and Amend the following Errata's.

Later in the eighteenth century it would become the norm for mathematical authors to correct the press for their own and one another's books;[10] we do not know for certain what the usual practice was in the sixteenth or the seventeenth century (to my knowledge no correctors' sheets for mathematical books are known to survive, nor have I seen a publisher's 'correction copy' with interleaved blanks, although editors certainly did make an effort to collect errata in mathematical books). But, certainly, not a few errata slips were introduced by laments like Ward's, admitting at least implicitly that print shops that were printing mathematics did not feel they had the resources required to correct it adequately. Jacques Lefèvre d'Étaples, printing his great edition of Euclid's *Elements* in 1516, ended the errata leaf typically by stating that he had noted the errors he judged worthy of correction but left it to the reader to correct any others while reading (d'Étaples 1516, errata leaf).

At the other end of the scale, John Tapp introduced his *Sea-Man's Kalendar* – a small compendium of practical mathematics – in 1672

> intreating the courteous Readers to do me that favour, as to correct what they shall find amiss, either in the Printer's over-sight or mine own errour

and promised to 'endeavour the mending of them in the next Impression' (Tapp 1674, A2^{r}). A long printed errata list implicitly made the same point, and some mathematical books came with very long lists indeed.

Thus, readers of early modern mathematical print routinely found themselves faced with a direct invitation to correct faults in the text and to find others for themselves, and they were faced with a book which in order to be effectively used needed to be so corrected. They were also faced with a situation in which the very mathematical skills the book aimed to transmit must be deployed in order to determine whether the book itself was correct at a given point: whether the calculation was accurate or the algebra correct. One of the functions of the printer, therefore, was to provide the reader with an adequate writing surface on which to make corrections, and one of the underexplored functions of white space in mathematical layouts is just that.

Many printers did provide adequate such surfaces, and very many readers did indeed get involved with such negotiations about what the text should say. Correction is not unique to mathematical text – indeed, a 'culture of correction' (Grafton 2011) has rightly been identified with respect to printed and manuscript texts quite generally in the sixteenth century – but that culture persisted, and extended socially, much longer and wider for mathematical texts than for any other genre. Something like eighty percent of surviving early modern mathematical books bear readers' corrections, making this one of the most distinctive features of mathematical reading in this period (Wardhaugh 2019a).

A number of studies of early modern annotation (such as Jackson 2001, 164) have remarked on the frequency of adversarial annotation, the kind that says 'nonsense!' or 'the author is a fool'. This type proves to be very rare among the responses of readers to early modern mathematical print (for some exceptions see Goulding 2005); but detailed, usually autonomous correction effectively took its place. Furthermore, those who corrected autonomously much outnumber those who carried out the corrections directed in the printed errata (Wardhaugh 2019a). Readerly autonomy here, as in the selection of material, seems to have been the norm.

Correction could of course take a variety of forms, and could go well beyond cleaning up single-character errors within mathematical working. A significant dimension of the 'mathematical classicism' that informed the response to ancient mathematical texts, particularly in the British Isles, took the form of grammatical and linguistic correction – a particle here, a spelling or a Byzantinism there – or of philological collation with another printed or manuscript version of the same text. Many are the copies of Euclid's *Elements*, in particular, that bear evidence of this kind of correction – even on every page – as well as or instead of correction of the mathematical content.[11]

Furthermore, and as some authors admitted, it was all too easy for the mathematics to be wrong in more gross ways. Ward admitted in his *Young Mathematician's Guide* (Ward 1707, Mmm3^{v}) that

> at Question 5. Page 93. the Operation or Work is all Mistaken; for the Answer should be only 13 Ounces.

And Isaac Barrow, in the 1659 impression of his translation of the *Elements*, included ten dense pages of 'annotations', in which errors and obscurities in the printed text were emended and whole sections replaced (Barrow 1659, 343–52). Just as lists of errata invited readers to find small errors, these large-scale authorial second thoughts invited readers to get involved on a similar scale, and there were those who took up the challenge, substituting on what appears to have been their own initiative alternative methods of proof or even entirely different sequences of theorems. Such work could result in frankly customized copies, such as the interleaved volumes occasionally prepared by zealous teachers, sometimes for presentation to pupils, in which a printed text was supplemented *in extenso* with alternative, supplementary or explanatory material.[12]

A culture of correction is self-perpetuating. Early modern mathematical readers were served by a brisk trade in second-hand books, and it was undoubtedly the case – sometimes the sale catalogues tell us so – that many of the books sold bore the corrections of previous readers. While 'clean' could in some cases be a selling point, for mathematical books it was also on occasion noted as a positive feature that a book for sale had been 'corrected'. Thus a substantial fraction – very possibly the majority – of readers encountered mathematical books that contained not only printed invitations to correct but the corrections of one or more previous readers. Some, inevitably, corrected the corrections, or supplemented them, perhaps choosing different parts of the text to work on. Corrections, then, could become sociable in the hands of subsequent readers, and occasionally they were copied from book to book: from teacher to student, or on occasion from scholar to scholar, say in the context of projects to re-edit a text. John Chambers, for instance, studied Euclid with Savile at Oxford, and his own copy of the book bears annotations which are in many cases copied from Savile, while others report or paraphrase Savile's discoveries, making it clear that Chambers had Savile and Savile's annotated copy of the text with him as he studied.[13] Annotations and emendations by Savile would subsequently be used by both Edward Bernard and David Gregory in their projects to re-edit the Latin and Greek texts of the *Elements*; Bernard incorporated material from other Oxford annotators including a Dr Pain of Christ Church, while Gregory incorporated Bernard's own notes after the failure of the latter's editorial and publication project.[14]

3. The imitable text

Donating a copy of the *Elements* to Corpus Christi College, Oxford in the early sixteenth century, John Claymond – president of the college – indicated that he intended it for the use of students and, specifically, so that they could copy out Euclidean theorems from it.[15] We have already heard that in a school context copying was a usual practice, carried out until the mid-eighteenth century usually by the teacher. It is also the case, of course, that a culture of commonplacing governed much of both the learned and the student response to printed (and manuscript) texts in this period: so we certainly cannot say that copying out sections in itself made for a distinctive culture of mathematical reading. Commonplacing as such did not really occur in mathematical contexts, as far as our evidence goes (indeed, such a principle of reorganization might have sat rather uneasily with what I have argued was the normal experience of mathematical text as essentially fragmented and nonlinear). But it was usual to construct, as we saw above, volumes of *excerpta*, particularly for teaching purposes.

The students at Corpus Christi College certainly did copy out parts from Claymond's gift; it is visible on the pages of the volume that sheets have been laid over certain of the diagrams and the diagrams traced, leaving scratches on the printed page itself (Wardhaugh 2018b). Around Oxford, other college-owned copies of similar texts bear similar marks as well as the tell-tale prick of a compass point through the centers of diagrams where copies have been made.[16] Many readers seem to have copied onto loose sheets and too have been none too careful about letting the ink dry before closing the book onto the loose sheet, leaving in some cases clearly legible impressions of diagrams or even of texts by ink transfer.[17] Actual copies made onto separate sheets very rarely survive; the few exceptions seem to show they were often reused as waste paper, such as Gerard

Langbaine's manuscript catalogue of the Savilian Library,[18] written over large geometrical diagrams that suggest group teaching, or a catalogue of coins in the Bodleian Library written over a set of small geometrical diagrams that might suggest individual study or teaching.[19] But copies into notebooks or exercise books are common, and wholesale recopying of an entire mathematical textbook remained a reasonably common practice into the second half of the eighteenth century: for example, regulations at the Royal Military Academy at Woolwich in force in the 1780s (see Jones 1895) envisaged each cadet making a fair copy of Thomas Simpson's *Algebra* (Simpson 1745 or a subsequent edition). Robert Sandham's letters from the RMA in the 1750s (Hogg 1963) also mention such activities: 'I have written all *Mr Muller's Artillery*, which is forty octavo pages'.

Such practices raise the question of what the copies were for, and take us deeper into what was distinctive about mathematical reading and the uses of mathematical text. The evidence shows that mathematics – arithmetic, proportional reasoning, algebra, and geometry – was thought of and treated as a set of practices, and that texts were received as models not just to be copied in the obvious sense but from which to learn, to assimilate those practices (cf. Jardine and Grafton 1990). This mode of use can be referred right back to the earliest evidence for mathematical texts and their construction in the Greek tradition, when they appear to have been the by-products of deduction or of *viva voce* teaching, the mathematician constructing a diagram, talking through the construction, and leaving a diagram and eventually a written description of its construction and meaning as the traces of that process (Netz 1999, 167 and *passim*).

Thus every mathematical text in this tradition is in a sense a double one. On the one hand there is the description of operations performed, whether arithmetical, geometrical, or some other kind. On the other there are the traces those operations leave: arithmetical or algebraic working, or a diagram. Mathematical books interweaved the two modes section by section, or presented them side by side: diagrams beside proofs, or calculations interspersed with the description of methods. In the era of print, and certainly from the seventeenth century, it became quite common to adopt a two-column arrangement for the two parts of the mathematical text: such as for example John Hill's (1713) *Arithmetick*, which frequently places a verbal description of an arithmetical operation side-by-side with the 'working', in a two-column layout.

Nevertheless, a printed or manuscript text cannot achieve all of the same things as *viva voce* teaching. Neither of the two parts of the text – description or traces – is really meant to be copied as such. In live teaching a diagram or written calculation is the outcome of a specimen performance that the learner(s) have observed, the imitation of which is the expected next step. In a manuscript or printed book, a diagram is the supposed outcome of a specimen performance that the learners have *not* seen, and for which the accompanying text now functions as a narrative description. The process which leads to the traces was supposed to be learned as a practice, a *habitus*. And it was to be learned by repetitive private practice whose traces were essentially inferior versions of the traces that appeared in printed books. So, almost as frequently as they corrected errors in the printed text, readers of mathematical books used the margins and other white space as places in which to re-create for themselves the diagrams, calculations or proofs as they read, using the descriptive text as a set of instructions and the printed diagrams or calculations as a model not for copying but for comparison with what they had done themselves.

Thus we find time and again the margins of printed mathematics books used as a kind of rehearsal space, bearing more or less hesitant and inadequate copies of geometrical diagrams, faulty or correct rehearsals of arithmetical working, and faulty or, less often, correct rehearsals of algebraic working. These are not mere copies of what is in the printed text; usually they differ in their detailed approach, and often they also differ in being not quite correct. It is to be presumed that a vast amount of such rehearsal also took place on surfaces that are not now available to the historian: slates for school arithmetic, waste paper for higher-level students; occasionally readers even took a sharp point and pricked or scored geometrical diagrams into the leather covers of their books. Blackboards and even sand boxes are evidenced as teaching tools in the middle ages and renaissance, and may well have been used by at least some students for these purposes of private rehearsal.[20]

The outcome of the rehearsal was proficiency at a practice, and it was shown to the teacher in the form of a demonstration performance whose successful enactment permitted the student to move on to the next level of material. In schools this was an absolutely continuous process, the students being called out one by one or in small groups to perform what they had just been practicing and see if they were fluent at it (Ellerton and Clements 2012); (Jones 1895, 17) shows the inspector of the Royal Military Academy in 1772 commending ‘the daily practice of calling up to their desks their respective pupils’. In advanced schools, such as the military academies, the public examinations took exactly the same form, with students performing set pieces from Euclid or their teachers’ textbooks before a public – sometimes a distinguished – audience: see (Jones 1895, 26) describing public examinations at the Royal Military Academy in 1779. It is unfortunate that we do not really have evidence which would tell us if the same kinds of mathematical demonstration performances took place in university contexts, although the same kinds of private rehearsal undoubtedly did, to judge by the evidence of marginal annotations.

Some readers, moreover, rehearsed their mathematical skills in a somewhat different way: not by writing out the material in the form in which they had read it or been shown it, but specifically by translating it into a different mathematical language. Words into algebra; algebra into words. Words into diagrams; or diagrams into algebraic equivalents. Algebra into numerical examples; or sometimes *vice versa* (Wardhaugh 2019a). An eighteenth-century reader of Ward’s *Young Mathematician’s Guide*, for example, translated verbal rules into algebra throughout the text; a century earlier a reader of Barrow’s edition of the *Elements* systematically turned the diagrams into examples by adding numbers to them.[21] The practice was so common that it seems it must have been taught or at least encouraged in schools and universities; Renee Jennifer Raphael (2015, 6–21) has suggested persuasively that transformation of this kind was an important part of scholarly and pedagogical practice. We find it at all levels from the elementary textbook to such distinguished cases as Newton reading Euclid (he worked through much of Book 10 putting the theorems about commensurability into algebraic terms).[22] Like other modes of mathematical reading this tendency to translation also found its way into the printed texts themselves; the provision of numerical examples became a feature first of editions of Euclid’s *Elements* from the later sixteenth century onwards, and subsequently of textbooks on other parts of mathematics. The editions of Christoph Clavius (1574 and subsequently) added numerical examples for instance in the preamble to Book 5; that of Henry Billingsley (1570) added them for instance to Book 2. The act of moving from the particular case to the general one would become an important part of mathematical pedagogy in the long run, as well as an important part of the early modern debate about the proper way to do mathematics and the proper way of conceiving the relationship between – say – algebra and geometry.

4. Conclusion

Throughout the early modern period, changes in the physical characteristics of print proceeded hand-in-hand with changes in the ways readers used texts, both printed and manuscript. The production and uses of mathematical print are part of this process, but they are a highly distinctive part.

Other genres of early modern print, and other kinds of text more generally, were sometimes approached non-linearly and fragmented; they were corrected, sometimes compulsively, and their contents reorganized or renegotiated; and they were sometimes used as models for more or less literal 'copying' as well as for sources of imitation at the higher level of the acquisition of the skills they embodied. Yet none was received in these ways all the time, and for none were reception, reading and marginal response so dominated by these modes. Thus we can fairly say that there was a distinctive culture of mathematical reading in the early modern period, and that mathematical print and mathematical texts were received in a distinctive way.

Mathematical knowledge was conceived primarily in terms of special kinds of performance: hybrid performances with both mental and manual aspects, producing a diagram and/or a text that served as signs of a successful performance. If educational theory seems to have valued chiefly the mental side and the notion that mathematical study would improve the mind, pedagogical practice – as witnessed by readers' marks in mathematical books – appears to have valued the manual side at least as much.

Some of the processes discussed here can be evidenced in readers' use of mathematical manuscripts before the era of print, and it seems certain that some of what was distinctive about mathematical reading carried over from older, even ancient assumptions about what mathematics was and what mathematical text was.[23] On the other hand, early modern ways of reading mathematical text and transmitting mathematical knowledge had consequences for the longer-term cultural profile of mathematics. They bear on such questions as how and why thinkers mainly associated with other disciplines attached to mathematics the particular kind of importance they did, and how mathematics came to be used as a regular source of culturally transformative ideas from the early modern period into the modern.

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[1] Copies of (Ward 1719 and 1734) in University of Michigan Library with shelfmarks QA35.W259 1719 and QA35.W259y 1734, with annotations including supplementary cross-references. Copy of (Briggs 1620) in All Souls College, University of Oxford, with shelfmark 4:SR.59.c.23, with dense annotations including many added cross-references.

[2] Among many examples a copy of (Fisher 1731) in the British Library with shelfmark 8506.bb.11, with marginal exclamation marks; a copy of (Fisher 1767) in the National Library of Scotland with shelfmark YY.8/2, with marginal Xs; and a copy of (Ward 1771) in Regent's Park College, Oxford, with shelfmark 24.g.33, with pencil ticks, crosses and the note 'done' throughout.

[3] Copy of (Ward 1758) in the British Library with shelfmark 08535.a.70, with index marked up (Qqq1^{v}–2^{r}, 3^{r}–4^{r}) by Charles Moore and/or William Canter RN, showing sections studied; copy of (Ward 1771) in the University of Michigan with shelfmark QA35.W259 1771, with supplements to contents list on A4^{v} and partial indexing on the rear end-paper.

[4] Copy of (Clavius 1607) in Trinity College, University of Cambridge with shelfmark T.38.13–14; copy of (Clavius 1654) in the Queen's College, University of Oxford with shelfmark 40a.A.18–19, donated for the use of the 'taberdars' of that college.

[5] Bodleian Library, University of Oxford, MS Don. d. 45.

[6] Copy of (Wells 1635) in the Huntington Library with shelfmark RB 271828, described in (Sherman 2008, 9, n. 29).

[7] Copy of (Norwood 1659) in the Smithsonian Library with shelfmark QA33.N67 1659, described in (Bedini 2001); information from Smithsonian online catalogue entry and from Yelda Nasifoglu (pers. comm.).

[8] Copy of (Seller 1677) in the London Science Museum with shelfmark O.B. SEL SELLER (1), described in (Tracey 2019).

[9] Copy of (Whiston 1765) in the National Library of Ireland with shelfmark Dublin 1765(23).

[10] For example, Cambridge University Library, RGO 14/5, p. 344 (Board of Longitude minutes, 6 March 1779), with a mention of Charles Hutton correcting the press for a volume of tables by Bernoulli.

[11] For example copies of Grynäus 1533 in Merton College, University of Oxford with shelfmark 40.J.15, marked probably by Henry Savile, and in Balliol College, University of Oxford with shelfmark St Cross 0625 d 06, apparently marked at that college.

[12] For example a copy of (Brown 1753) with interleaving by the teacher Samuel Davis, in a private collection: see (Wardhaugh 2019a). For other examples of interleaved and extended books see (Woolf 2000, 87 with note 19) and (Grenby 2011, 246).

[13] A copy of (d'Étaples 1516) in Oxford, Bodleian Library with shelfmark Savile W 12, annotated by John Chambers; information from Renee Raphael, pers. comm.

[14] See the heavily annotated printed copies of the *Elements* in the Bodleian Library with shelfmarks Auct. S 1.12–15, 8° B 16 Linc. and 8° C 134 Linc., and a similarly annotated 1625 copy of the *Data* with shelfmark Auct. S 2.22; also (Beeley 2019).

[15] Copy of (d'Étaples 1516) in Corpus Christi College, University of Oxford, Rare Books Collection, with shelfmark Δ.10.1, inscription on title page: 'in usum discipulorum + ut inde exscriberent theoremata euclidis'.

[16] Copy of (Rudd 1651) in Trinity College, University of Cambridge with shelfmark NQ.7.82, with prick-marks indicating copying using a compass (very likely by Isaac Newton) throughout pp. 1–40.

[17] Copy of (Fine 1536) in Trinity College, University of Cambridge with shelfmark S.10.72, p. 17: a probable ink transfer of a diagram on a loose leaf.

[18] Oxford, Bodleian Library, MS Savile 107.

[19] Oxford, Bodleian Library, MS Lister 39.

[20] Mordechai Feingold, presentation at Research workshop on 'Teaching mathematics in the early modern world', Oxford, December 2016.

[21] Copy of (Ward 1752) in a private collection; copy of (Barrow 1659) in University College, London with shelfmark Strong Room Euclid Octavo 1659 (2).

[22] Copy of (Barrow 1655) in Trinity College, University of Cambridge with shelfmark NQ.16.201[1], with algebraic explanations added in books 2, 5 and 10; see also (Warwick 2003, 31).

[23] Erfurt, Biblioteca Amploniana F. 377, f. 23^{r} has an instance of a reader adding a translation of the text into diagrammatic form, in the context of Alfonsine astronomical material of the fourteenth century.