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Momentum, Size and Value Factors versus Systematic Co-moments in Stock Returns

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Key Words: Asset Pricing, Systematic Co-Moment, Momentum, Size, Value

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Abstract

This article assesses the empirical performance of the momentum, size and book-to-market factors versus higher systematic co-moments and also examines whether the momentum factor (*WML*) proxies for higher systematic co-moments. Both the three and four-moment CAPMs have lower absolute pricing errors than the Fama and French (1973) and the Carhart (1997) models. The three-moment CAPM that incorporates coskewness well explains the cross-section of returns of size portfolios. The four-moment CAPM that further incorporates cokurtosis well explains the cross-section of momentum returns. The momentum factor, apart from *SMB* and *HML* factors, also proxies for the measures of market risk not captured by the two-moment CAPM.

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1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), theoretically underpinned by mean-variance portfolio efficiency, postulates that the market beta (systematic second co-moment scaled by the market return variance) suffices to explain expected return. However, a number of studies show that the CAPM beta does not completely measure systematic risk and that the cross-section of stock returns is strongly associated with return momentum (Jegadeesh and Titman, 1993), market capitalization (Banz, 1981) and book-to-market ratio of companies (Fama and French, 1992; thereafter, F-F). F-F argue that these non-market risk factors are priced and propose a three-factor model that includes a size factor, *SMB* (the monthly return difference between the returns on the small and big size portfolios), and a value factor, *HML* (the monthly return difference between the returns on the high and low book-to-market-ratio portfolios) in addition to the market factor. Carhart (1997) further includes a momentum factor constructed by the monthly return difference between the returns on the high and low prior return portfolios, to capture the cross-sectional return patterns.

Motivated by the non-normality of asset return distributions, the higher systematic co-moment models suggest that, due to the simplifying assumption of return normality, the CAPM does not completely capture non-diversifiable risk beyond the second co-moment, and thus, results in its empirical failures[1]. Jean (1971), Rubinstein (1973) and Scott and Horvath (1980) show that if returns are not normally distributed, moments of returns higher than variance matter in maximizing investors' expected utility. In addition, higher-order co-variations in returns between

[1] Other approaches in identifying systematic risk without restrictions of return normality include Ross's arbitrage pricing theory (1976) and the colower partial moments of Bawa and Lindenberg (1977).

risky assets and the market portfolio should also be priced. Kraus and Litzenberger (1976) provide evidence for the pricing of the third systematic co-moment (coskewness) for stocks that were continuously listed on the NYSE from 1926 through 1935. Barone-Adesi (1985) and Lim (1989), among others, also show evidence for the pricing of coskewness. Harvey and Siddique (2000), Smith (2005) and Errunza and Sy (2005) find evidence that conditional coskewness helps explain the cross-section of stock returns. Fang and Lai (1997) further document evidence for the pricing of the fourth systematic co-moment (cokurtosis) for stocks that were continuously listed on the NYSE from 1969 through 1988.

Recently, Chung, Johnson and Schill (2006) show that Fama and French's (1993) *SMB* and *HML* factors succumb to (in term of *t*-statistics) the presence of systematic co-moments 3 through 10 in explaining the cross-section of returns of size and book-to-market sorted portfolios. This article assesses the empirical performance of these empirical factors versus higher systematic co-moments and also examines whether the momentum factor, the 'winner minus loser' (*WML*) hedge portfolio return, proxies for higher systematic co-moments. I find evidence that the momentum factor, apart from *SMB* and *HML* factors, also proxy for the measures of market risk not captured by the CAPM.

I first examine return characteristics of momentum, size and book-to-market portfolios and find evidence that the return distributions of these portfolios are significantly different from normal[2]. In addition, these portfolios exhibit significant market coskewness and cokurtosis. To investigate whether the empirical factors of momentum, size and book-to-market are priced in the data, I examine whether these factors explain the cross-section of returns and find evidence that these factors have

[2] Chung, Johnson and Schill (2006) documents return non-normality for size and book-to-market portfolios.

significant explanatory power for returns of momentum, size and book-to-market sorted portfolios.

I next examine the four-moment CAPM (Fang and Lai, 1997) that restricts investors' preference to depend only on the first four moments of returns. The three-moment CAPM that incorporates coskewness well explains the cross-section of returns of size portfolios. The four-moment CAPM that further incorporates cokurtosis well explains the cross-section of momentum returns. In contrast, both the Fama and French (1973) and the Carhart (1997) models show significant intercepts in pricing these portfolio returns. On the other hand, although coskewness and cokurtosis are significant in explaining the cross-section of book-to-market portfolio returns, both the three and four-moment CAPMs have significant intercepts while both the Fama and French (1973) and the Carhart (1999) models do not.

I further compare pricing errors of the four-moment CAPM with those of Fama-French three-factor model and Carhart's (1997) four-factor model that further includes a momentum factor in addition to the F-F's factors. I find evidence that both the three and four-moment CAPMs have lower absolute pricing errors than the Fama and French (1973) and the Carhart (1999) models in explaining the cross-section of returns of momentum and size portfolios.

I finally relax the restriction of preference to higher-order moments of returns and examine whether momentum, size and book-to-market factors are proxies for higher-order systematic co-moments in the spirit of Chung, Johnson and Schill (2006). I combine higher-order systematic co-moments together with momentum, size and book-to-market in the cross-sectional tests to investigate the competitive roles of higher-order co-moments with these empirical factors. I find that the momentum factor reduces the significance of the FF's size and book-to-market for momentum returns to insignificant in explaining average returns of momentum returns. Furthermore, including co-moments 3 through 10 cause the momentum, size and book-to-market factors to become insignificant in the three portfolio sorts. Finally, this study uses all stocks in the universe of the CRSP NYSE/AMEX and Nasdaq for the period between January 1926 and December 2005. Thus, this paper provides out-of-sample tests of higher-order co-moments and also avoids the survivorship bias contained in prior research that used companies continuously listed through sample periods.

I outline higher-order systematic co-moments models in the next Section. Section 3 presents data and portfolios. Section 4 describes empirical tests. Section 5 presents results. Section 6 concludes.

2. Higher-Order Co-Moment Models

Rubinstein (1973) is the first to derive a theorem, which links expected returns to all moments of returns. Consider an investor who constructs a portfolio *p* by investing, respectively, q_i and q_f of his current wealth W_0 in risky asset *i* and the risk-free asset. The investor maximizes the expected utility of end-of-period wealth $U(\tilde{W})$ by choosing investment holdings on assets, but subject to the budget constraint that all investments must sum to the investor's current wealth. Approximate the investor's expected utility by a Taylor series expansion around mean wealth $\overline{W}$ and ignore terms of order higher than *n*. Assuming that the investor's utility function is continuously differentiable and measurable and that the first *n* moments of terminal wealth exist and are finite, the investor's expected utility is:

$$E\left[U\left(\tilde{W}\right)\right] = \sum_{n=0}^{\infty} \frac{U^{(n)}(\overline{W})}{n!} E\left[\tilde{W} - E(\tilde{W})\right]^{n} \qquad (1)$$

where $U^{(n)}(\overline{W})$ is the n^{th} derivative of $U(\tilde{W})$ evaluated at the mean of the investor's end-of-period wealth.

The expected return of an asset in excess of the risk-free rate R_f is equal to the weighted sum of co-moments with the weights reflecting measures of the investor's risk aversion as:

$$E(R_i) - R_f = \sum_{2}^{\infty} \frac{-U^n}{(n-1)!E\left[U^{'}\left(\widetilde{W}\right)\right]} \cdot \sigma_{in}\left(R_i, \tilde{W}\right) \quad (2)$$

where R_i is the return on risky asset i. The n^{th} co-moment of risky asset i is the contribution of a marginal increase in the holdings of the security to the corresponding central moments of the investor's future wealth,

$$\sigma_{in} \equiv E[R_i - E(R_i)][\tilde{W} - E(\tilde{W})]^{n-1} \text{ for } n \geq 2 \quad (3)$$

At the aggregate market level, assuming homogeneous subjective probability beliefs and separable cubic utility, Rubinstein (1973) shows that the expected return of an asset is expressed as:

$$E[R_i] - R_f = \lambda_2 Cov_{i,M} + \lambda_3 Cos_{i,M} \quad (4)$$

where λ_2 and λ_3 are market measures of risk aversion. $Cov_{i,M}$ and $Cos_{i,M}$ are covariance and coskewness of asset i with the market portfolio m.

Fang and Lai (1997) and Christie-David and Chaudhry (2001) consider a four-moment CAPM that includes coskewness and cokurtosis. The expected return of an asset is linearly associated with the contributions of an asset to the variance, skewness and kurtosis of the market portfolio,

$$E[R_i] - R_f = \eta_\beta \beta_i + \eta_\gamma \gamma_i + \eta_\delta \delta_i \quad (5)$$

where R_f is the risk-free rate. η_β, η_γ and η_δ are market prices of beta, gamma and delta. The beta, gamma (coskewness scaled by the market return skewness) and delta (cokurtosis scaled by the market return kurtosis) of risky asset i with the market portfolio measure systematic risks[3]:

$$\beta_i = E\left[\left(R_i - \overline{R}_i\right)\left(R_M - \overline{R}_M\right)\right] / \sigma_M^2$$
$$\gamma_i = E\left[\left(R_i - \overline{R}_i\right)\left(R_M - \overline{R}_M\right)^2\right] / s_M^3$$
$$\delta_i = E\left[\left(R_i - \overline{R}_i\right)\left(R_M - \overline{R}_M\right)^3\right] / k_M^4 \qquad (6)$$

where $R_{i,}$ and R_M are returns on risky asset i and the market, respectively. The notations of $\overline{R}_M$ and $\overline{R}_i$ are mean returns on the market and the asset; σ_M , s_M and k_M are the standard deviation, skewness and kurtosis of the market portfolio.

Recently, Chung, Johnson and Schill (2006) extend the market relation to the case of n co-moments and the expected return of an asset is:

$$E\left[R_i\right] - R_f = \sum_{n=2}^{N} \lambda_n b_i^n \qquad (7)$$

where b_i^n is the nth-order co-moment of asset i with the market portfolio, and λ_n is the market measure of risk aversion for the nth-order co-moment.

3. Data and Summary Statistics of Portfolio Returns

For the empirical analysis, I use monthly common equities of all NYSE, AMEX and NASDAQ firms in the CRSP (Center for Research in Security Price) database from January 1926 to December 2005. Book value data are obtained from Compustat from 1962 to 2005. The market portfolio is the CRSP value-weighted index and the risk-free rate is the one-month Treasury bill rate. The monthly risk factor returns for *SMB*, *HML* and *WML* are obtained from the data library of Kenneth French.

[3] β_i , γ_i and δ_i are additive and equal to unit for the market portfolio.

This paper follows Jegadeesh and Titman (1993) for constructing the representative overlapping momentum strategies. At the end of each month, all sample stocks with prices equal to or higher than $5 are ranked into 40 portfolios based on their past 6-month compounded returns. Portfolios are equally weighted and are held for the following six-month period[4]. With the six-month holding period, the composite portfolio position in each month comprises of past six ranking strategies, and that the monthly portfolio return is a combination of one-sixth of each of the six strategies.

For size sorted portfolios, stocks are sorted into 40 portfolios based on their market capitalizations at the time of portfolio formation. I also form 40 value portfolios by sorting stocks based on their book-to-market ratios at the end of June each year as Fama and French (1973). Both size and value portfolio are equally weighted and are rebalanced every 12 months. I compute time-series of equally-weighted returns for both momentum and size portfolios during the 942 months from January 1927 to June 2005 and for book-to-market portfolios during the 504 months from July 1963 to June 2005. Portfolio beta (β_{pt}), gamma (γ_{pt}) and delta (δ_{pt}) are calculated in each month t, by using portfolio returns from $t = \tau - 60$ to $t = \tau - 1$ as:

$$\beta_{pt} = \left[\sum_{\tau=t-60}^{\tau=t-1} \left(r_{p\tau} - \bar{r}_{pt} \right) \left(r_{M\tau} - \bar{r}_{Mt} \right) \right] / \sum_{\tau=t-60}^{\tau=t-1} \left(r_{M\tau} - \bar{r}_{Mt} \right)^2$$

$$\gamma_{pt} = \left[\sum_{\tau=t-60}^{\tau=t-1} \left(r_{p\tau} - \bar{r}_{pt} \right) \left(r_{M\tau} - \bar{r}_{Mt} \right)^2 \right] / \sum_{\tau=t-60}^{\tau=t-1} \left(r_{M\tau} - \bar{r}M_{Mt} \right)^3$$

$$\delta_{pt} = \left[\sum_{\tau=t-60}^{\tau=t-1} \left(r_{p\tau} - \bar{r}_{pt} \right) \left(r_{M\tau} - \bar{r}_{Mt} \right)^3 \right] / \sum_{\tau=t-60}^{\tau=t-1} \left(r_{M\tau} - \bar{r}_{Mt} \right)^4 \quad (8)$$

[4] In the case when a stock is delisted during the holding period, the liquidating proceeds are reinvested in the remaining stocks in the same decile portfolio.

where r_p and r_M are the returns on the portfolio p and the market portfolio (the CRSP value-weighted portfolio of NYSE, AMEX and NASDAQ stocks) in excess of one-month Treasury bill rate; and $\bar{r}_{pt}$, and $\bar{r}_{Mt}$ are the average excess returns in the preceding 60 months for the portfolio and the market.

Table 1 presents summary statistics for the return distributions of momentum, size and value portfolios. The average momentum portfolio return is 1.38% per month with a standard deviation of 6.95%, a positive skewness of 0.86 and a kurtosis of 14.75. Both the Jarque-Bera statistic and Kolmogorov-Smirnov statistic show that the return distribution of momentum portfolios is significantly different from normal at 1% level. All of the 40 momentum portfolios exhibit significant beta and delta at the 5% level and 97.5% of momentum portfolios exhibit significant gamma at the 5% level.

Size portfolios have an average return of 1.39% per month, a standard deviation of 7.21%, a positive skewness of 0.81 and a kurtosis of 13.42. The return distribution of size portfolios is significantly different from normal at 1% level as also shown by Chung, Johnson and Schill (2006). All of the 40 size portfolios exhibit significant beta and delta at the 5% level and 90% of size portfolios exhibit significant gamma at the 5% level. Value portfolios have an average return of 1.52% per month, a standard deviation of 5.98%, a positive skewness of 0.02 and a kurtosis of 6.44. The return distribution of value portfolios is significantly different from normal at 1% level. All of the value portfolios exhibit significant beta, gamma and delta at the 5% level.

4. Empirical Tests

4.1 Testing Whether Models Explain Expected Returns

I conduct both cross-sectional regression test and also evaluate the absolute pricing error, which is the average of the model alphas, for the two-moment, three-moment and four-moment CAPMs, the Fama and French and the Carhart models.

The Fama-French and the Carhart models

I investigate whether the Carhart (1997) model that includes a momentum factor, *WML*, in addition to the Fama-French's *SMB* and *HML* factors explains the cross-section of returns. I employ a two-pass methodology that allows for time-variation in coefficient estimates as in Chung, Johnson and Schill (2006). The factor loadings $\boldsymbol{s_p}$, $\boldsymbol{h_p}$ and $\boldsymbol{m_p}$ for *SMB*, *HML* and *WML*, respectively, are estimated by regressing portfolio returns on returns of *SMB*, *HML* and *WML* factor portfolios from $t = \tau - 60$ to $t = \tau - 1$. For each period $t = \tau$, I estimate a cross-sectional regression of portfolio returns on the loadings on *SMB*, *HML* and *WML* factor loadings as:

$$r_{pt} = \eta_{0t} + \eta_{st} s_{pt} + \eta_{ht} h_{pt} + \eta_{mt} m_{pt} + \varepsilon_{pt} . \qquad (9)$$

The parameter estimates and test statistics are obtained from the time series of monthly cross-sectional regression estimates as in Fama and MacBeth (1973). The *p*-value for testing the significance of each coefficient is the *p*-value corresponding to the *t*-statistic that is calculated by the mean of the coefficient divided by its standard error.

The four-moment CAPM

Since portfolios exhibit non-normally distributed returns with significant beta, coskewness and cokurtosis, risk-averse investors may be concerned about extreme outcomes and that higher-order systematic co-moments may be priced. In this section, I test the prediction of the four-moment CAPM about the intercept and risk premiums of market beta, gamma and delta. Having estimated portfolio beta, gamma and delta, a cross-sectional regression of the four-moment CAPM as the following form is performed in each period $t = \tau$ across portfolios to estimate risk premia $\eta_{\beta t}$, $\eta_{\gamma t}$ and $\eta_{\delta t}$ associated with β_{pt}, γ_{pt} and δ_{pt} of portfolios:

$$r_{pt} = \eta_{0t} + \eta_{\beta t}\beta_{pt} + \eta_{\gamma t}\gamma_{pt} + \eta_{\delta t}\delta_{pt} + \varepsilon_{pt} \,. \tag{10}$$

The model predicts that the intercept in the regressions is insignificantly different from zero, and the coefficients on beta, gamma and delta are significant.

4.2 Testing Whether Factors Proxy for Systematic Co-Moments

I further test the hypothesis that the factors based on momentum, size and book-to-market ratio proxy for the pricing of higher-order co-moments in the spirit of Chung, Johnson and Schill (2006). I calculate systematic co-moment estimates (Com_{pt}) of order 2 through 10 using past 60 months of portfolio returns:

$$Com_{n,p,t} = \left[\sum_{\tau=t-60}^{\tau=t-1} \left(r_{p\tau} - \bar{r}_{pt} \right)\left(r_{m\tau} - \bar{r}_{mt} \right)^{n-1} \right] / \sum_{\tau=t-60}^{\tau=t-1} \left(r_{m\tau} - \bar{r}_{mt} \right)^{n}, \quad n = 2, 3, \cdots 10. \tag{11}$$

where n denotes the order of co-moments and r_m is the returns on the CRSP value-weighted portfolio.

In each month t in the sample period, I estimate a cross-sectional regression of size, value and momentum factors together with higher-order co-moments on portfolio returns as in the following form:

$$r_{pt} = \eta_{0t} + \eta_{st} s_{pt} + \eta_{ht} h_{pt} + \eta_{mt} m_{pt} + \sum_{i=2}^{n} \eta_{it} Com_{i,p,t} + \varepsilon_{pt} \,. \tag{12}$$

5. Results

5.1 Cross-sectional results for the Fama-French and Carhart models

Table 2 shows results for the Fama and French (1973) and the Carhart (1999) models. Panel A, for momentum portfolios, shows that in the FF model the value factor is significant and the size factor only shows moderate significance. The CAPM beta losses its significance in explaining the cross-section of momentum returns. For the Carhart model, the momentum factor shows significance while the significance of the value factor disappears. Both models have significant intercepts. For size portfolios, Panel B shows that the size factor is the most significant factor in both models. The momentum factor does not show significance in explaining the cross-section of size returns. Again, both models exhibit significant intercepts. As reported in Panel C for the book-to-market portfolios, the Fama and French model has an insignificant intercept and both the size and value factors are significant. The Carhart model also has an insignificant intercept and the momentum factor is insignificant.

5.2 Cross-sectional results for the four-moment CAPM

Table 3 shows results for the four-moment CAPM. Panel A, for momentum portfolios, shows significance for portfolio beta in explaining the cross-section of momentum returns. The market price of coskewness is insignificant in the three-moment CAPM. The market prices of coskewness and cokurtosis are significant in the four-moment CAPM. Model intercepts are insignificant in all cases and the adjusted R^2s increase with the inclusion of higher co-moments. Panel B shows that the portfolio beta in the -moment CAPM has significant explanatory power for size portfolios. Both the portfolio beta and coskewness premia are significant in the three-moment CAPM. However, the beta, coskewness and cokurtosis premia all become insignificant in the four-moment CAPM. Model adjusted R^2s increase as coskewness and cokurtosis are included.

As displayed in Panel C for the book-to-market portfolios, model intercepts are significant and the beta premium is insignificant in all cases. The coskewness premium is significant in both model 2 and model 3 and the cokurtosis premium is significant in the four-moment CAPM. Overall, both the third and fourth co-moments show significant roles in explaining the cross-section of portfolio returns. However, both the positive coskewness and the negative cokurtosis premia for all tested assets are puzzling since the market return over the cross-section period between 1932 and 2005 has a positive skewness of 0.52 and kurtosis of 11.40.

5.3. Absolute pricing errors of models

Table 4 shows results for the absolute pricing error. For momentum portfolios, the three-moment CAPM achieves the lowest pricing error of 0.25% per month among all models while the Fama and French model has the highest pricing error of 1.35% per month. For size portfolios, again, the three-moment CAPM achieves the lowest pricing error of 0.18% per month and the Fama and French model has the highest pricing error of 0.9% per month. For book-to-market portfolios, the three-moment CAPM achieves the lowest pricing error of 0.32% per month. However, the four-moment CAPM shows the highest pricing error of 1.47% per month.

5.4. Size, value and momentum factors with higher-order co-moments

Table 5 shows estimation results for equation (12) for the Carhart factors and higher-order co-moments for each portfolio sorts. Panel A shows results for momentum portfolios. The momentum loading remains significant when both the beta and coskewness are added, but its *t*-statistic drops significantly as higher-order co-moments are added. Similar to the result in Panel A of Table 3, the *HML* loading is insignificant in all cases. The *SMB* loading shows sporadically weak significance and becomes insignificant once co-moments greater than the sixth order are included.

Panel B presents results for size portfolios. Both the *SMB* and *HML* loadings remain significant, but become insignificant once co-moments greater than the sixth order are included. The momentum loading remains becomes insignificant once co-kurtosis and higher-order co-moments are included. Panel C presents results for book-to-market portfolios. The *SMB* loading shows sporadically significance and becomes insignificant once co-moments greater than the seventh order are included. The significance of *HML* loading reduces as cokurtosis is included and becomes insignificant once higher-order co-moments are included. The momentum loading remains insignificant in all cases. Overall, adding a set of co-moments of order 3 through 10 reduces the explanatory power of the size, book-to-market and the momentum factors to insignificance.

5.5. *Robustness of the results*

I perform empirical tests using a 120-month window in estimating factor loadings. I also use the Morgan Stanley Capital International (MSCI) index as a proxy for the market portfolio and obtain similar results from these checks and the conclusions of the paper are unchanged.

6. Conclusions

A number of studies show that the CAPM beta does not completely measure systematic risk and that the cross-section of stock returns is strongly associated with return momentum, size and book-to-market ratio of companies. Recent asset pricing literature contends that the simplifying assumption of return normality of the CAPM ignores non-diversifiable risk beyond the second co-moment, and thus, results in its empirical failures. This article first assesses the empirical performance of the momentum, size and book-to-market factors versus higher systematic co-moments and finds that both the three and four-moment CAPMs have lower absolute pricing errors than the Fama and French (1973) and the Carhart (1997) models. The three-moment CAPM that incorporates coskewness well explains the cross-section of returns of size portfolios. The four-moment CAPM that further incorporates cokurtosis well explains the cross-section of momentum returns. This paper further shows evidence that the momentum factor proxies for higher systematic co-moments that are not captured by the two-moment CAPM.

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Table 1
Summary Statistics of Portfolio Returns

Table 1 shows summary statistics of momentum, size and book-to-market portfolios. The sample uses monthly data of all CRSP NYSE/AMEX and NASDAQ common equities from 1926 to 2005 and book value data from Compustat during 1962 to 2005. The 40 momentum portfolios are constructed using 6-month ranking and holding overlapping strategies as Jegadeesh and Titman (1993). The 40 size and book-to-market portfolios are constructed by sorting all stocks based on their market capitalizations and book-to-market ratios at the time of portfolio formation, respectively. Both size and book-to-market portfolios are rebalanced every 12 months. Time-series of equally-weighted returns are computed for both momentum and size portfolios during the 942 months from January 1927 to June 2005 and for book-to-market portfolios during the 504 months from July 1963 to June 2005. Jarque-Bera statistic tests the normality hypothesis based on skewness and kurtosis. Kolmogorov-Smirnov statistic tests the hypothesis of cumulative standard normal distribution. * and ** denote statistical significance at the 5% and 1% level, respectively. The bottom three rows show the percentages of the 40 portfolios that exhibit significant beta, gamma and delta (defined in Section 3) at the 5% level, respectively.

Portfolio Sorts	Momentum	Size	Book-to-Market
Number of portfolio-period observations	37,680	37,680	20,160
Mean	0.0138	0.0139	0.0152
Maximum	0.7896	0.9067	0.5123
Minimum	-0.4206	-0.4286	-0.3747
Volatility	0.0695	0.0721	0.0598
Skewness	0.86	0.81	0.02
Kurtosis	14.75	13.42	6.44
Jarque-Bera Statistic	221,554**	176,852**	10,186**
Kolmogorov-Smirnov Statistic	0.4252**	0.4243**	0.4359**
% of portfolio beta at the 5% significance level	100%	100%	100%
% of portfolio gamma at the 5% significance level	97.5%	90%	100%
% of portfolio delta at the 5% significance level	100%	100%	100%

Table 2
Cross Sectional Regressions for the FF and Carhart Model

Portfolio β_{pt}, γ_{pt} and δ_{pt} are estimated on a rolling basis every month using portfolio returns for the preceding 60 months. Cross sectional regressions of excess portfolio returns on portfolio β_{pt}, γ_{pt} and δ_{pt} are performed in each of the cross sectional months. The slope coefficients and adjusted R^2 are mean values across all cross-sectional months. The *t*-statistics of the slope coefficients for each model are displayed in parentheses.

$$r_{pt} = \eta_{0t} + \eta_{st} s_{pt} + \eta_{ht} h_{pt} + \varepsilon_{pt}$$

Panel A. 40 Momentum Portfolios, 882 Months from January 1932 to June 2005						
Model	η_0	η_β	η_s	η_h	η_M	Mean Adj. R^2
FF	0.0106	0.0040	-0.0030	-0.0082		0.4898
	3.22	1.10	-1.76	-4.08		
CAR	0.0094	0.0022	0.0011	-0.0016	0.0063	0.5508
	3.95	0.90	0.84	-1.16	3.62	
Panel B. 40 Size Portfolios, 882 Months from January 1932 to June 2005						
Model	η_0	η_β	η_s	η_h	η_M	Mean Adj. R^2
FF	0.0071	0.0012	0.0063	0.0018		0.1928
	3.03	0.55	5.94	1.62		
CAR	0.0068	0.0014	0.0063	0.0021	-0.0019	0.2005
	2.91	0.62	6.00	1.82	-1.23	
Panel C. 40 Book-to-Market Portfolios, 444 Months from July 1968 to June 2005						
Model	η_0	η_β	η_s	η_h	η_M	Mean Adj. R^2
FF	0.0033	0.0030	0.0045	0.0037		0.2095
	1.01	1.10	2.73	2.28		
CAR	0.0024	0.0038	0.0042	0.0029	-0.0022	0.2167
	0.72	1.39	2.52	1.76	-1.14	

Table 3
Cross Sectional Regressions for the Four-Moment CAPM

Portfolio β_{pt}, γ_{pt} and δ_{pt} are estimated on a rolling basis every month using portfolio returns for the preceding 60 months. Cross sectional regressions of excess portfolio returns on portfolio β_{pt}, γ_{pt} and δ_{pt} are performed in each of the cross sectional months. The slope coefficients and adjusted R^2 are mean values across all cross-sectional months. The *t*-statistics of the slope coefficients for each model are displayed in parentheses.

$$r_{pt} = \eta_{0t} + \eta_{\beta t}\beta_{pt} + \eta_{\gamma t}\gamma_{pt} + \eta_{\delta t}\delta_{pt} + \varepsilon_{pt}$$

Model	η_0	η_β	η_γ	η_δ	Mean Adj. R^2
Panel A. 40 Momentum Portfolios, 882 Months from January 1932 to June 2005					
1	0.0032	0.0086			0.2479
	(1.12)	(2.80)			
2	0.0008	0.0105	0.0003		0.4145
	(0.28)	(2.15)	(0.08)		
3	0.0029	0.0233	0.0107	-0.0251	0.4813
	(1.05)	(2.99)	(2.06)	(-2.53)	
Panel B. 40 Size Portfolios, 882 Months from January 1932 to June 2005					
1	-0.0036	0.0132			0.1163
	-1.18	4.00			
2	-0.0027	0.0072	0.0044		0.1607
	-0.92	1.98	2.46		
3	-0.0013	0.0063	0.0050	-0.0012	0.3311
	-0.48	1.22	1.58	-0.22	
Panel C. 40 Book-to-Market Portfolios, 444 Months from July 1968 to June 2005					
1	0.0121	-0.0016			0.1090
	2.86	-0.43			
2	0.0133	-0.0068	0.0036		0.1401
	3.04	-1.73	2.36		
3	0.0097	0.0072	0.0075	-0.0150	0.1555
	2.42	1.29	2.60	-2.69	

Table 4
Absolute Pricing Errors of Models

Portfolio beta, gamma and delta are calculated for the entire period. The market prices of beta, gamma and delta are estimated from cross-sectional regressions.

Portfolios	CAPM	3M-CAPM	4M-CAPM	FF	CARHART
Momentum	0.47	0.25	0.37	1.35	1.22
Size	0.23	0.18	0.20	0.90	0.68
Book-to-Market	1.33	0.32	1.47	0.96	0.81

Table 5

Cross Sectional Regressions for the Carhart Model and Comoments

Portfolio co-moments are estimated on a rolling basis every month using portfolio returns for the preceding 60 months. Cross sectional regressions of excess portfolio returns on portfolio co-moments are performed in each of the cross sectional months. The slope coefficients and adjusted R^2 are mean values across all cross-sectional months. The *t*-statistics of the slope coefficients for each model are displayed in parentheses.

Panel A. 40 Momentum Portfolios, 882 Months from January 1932 to June 2005				
Comoments	η_s	η_h	η_M	Mean Adj. R^2
2^{nd} to 3^{rd}	0.0025	-0.0002	0.0062	0.5563
	1.70	-0.11	2.97	
2^{nd} to 4^{th}	0.0033	-0.0019	0.0048	0.5633
	1.84	-0.87	1.57	
2^{nd} to 5^{th}	0.0037	-0.0030	0.0046	0.5690
	1.84	-1.17	1.33	
2^{nd} to 6^{th}	0.0009	-0.0025	0.0043	0.5718
	0.34	-0.72	0.94	
2^{nd} to 7^{th}	0.0016	-0.0030	0.0044	0.5739
	0.57	-0.71	0.88	
2^{nd} to 8^{th}	0.0019	-0.0040	0.0068	0.5789
	0.56	-0.84	1.19	
2^{nd} to 9^{th}	0.0021	-0.0022	0.0091	0.5830
	0.50	-0.31	1.08	
2^{nd} to 10^{th}	0.0047	0.0046	0.0028	0.5855
	0.94	0.61	0.31	

Panel B. 40 Size Portfolios, 882 Months from January 1932 to June 2005				
Comoments	η_s	η_h	η_M	Mean Adj. R^2
2^{nd} to 3^{rd}	0.00603	0.00681	-0.0037	0.2205
	4.58	4.99	-1.74	
2^{nd} to 4^{th}	0.00397	0.00638	-0.0037	0.2245
	2.46	3.67	-1.20	
2^{nd} to 5^{th}	0.00366	0.00376	-0.001	0.2278
	2.06	1.74	-0.32	
2^{nd} to 6^{th}	0.00365	0.00523	-0.0027	0.2324
	1.55	1.70	-0.67	
2^{nd} to 7^{th}	0.00387	0.0016	0.00068	0.2374
	1.42	0.42	0.15	
2^{nd} to 8^{th}	0.00436	-0.0007	0.00458	0.2449
	1.36	-0.13	0.72	
2^{nd} to 9^{th}	0.00481	0.00093	0.00443	0.2467
	1.29	0.17	0.67	
2^{nd} to 10^{th}	0.00462	-0.0012	0.00017	0.2528
	1.00	-0.19	0.02	

Table 5 (*continued*)

Panel C. 40 Book-to-Market Portfolios, 444 Months from July 1968 to June 2005

Comoments	η_s	η_h	η_M	Mean Adj. R^2
2nd to 3rd	0.0034	0.0033	-0.0027	0.2182
	1.96	2.03	-1.36	
2nd to 4th	0.0040	0.0034	-0.0011	0.2209
	1.98	1.57	-0.44	
2nd to 5th	0.0034	0.0041	-0.0012	0.2226
	1.48	1.43	-0.44	
2nd to 6th	0.0068	0.0016	-0.0004	0.2231
	2.34	0.44	-0.11	
2nd to 7th	0.0057	0.0036	-0.0021	0.2248
	1.78	0.72	-0.48	
2nd to 8th	0.0019	0.0030	0.0031	0.2245
	0.42	0.54	0.63	
2nd to 9th	0.0042	-0.0010	-0.0007	0.2293
	0.85	-0.15	-0.11	
2nd to 10th	0.0034	0.0012	0.0037	0.2288
	0.57	0.16	0.55	