WP: 109 ISSN: 1749-361 (online)

The Output Effect of Stopping Inflation when Velocity is Time Varying

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(August 2007)

Abstract

This paper explores the effect of time varying velocity in a transition to price stability. Nonstationary velocity, expressed as function of consumption, is made endogenous in Ireland's (1997) model. We find that the 'disinflationary booms' found by Ball (1994) may or may not disappear; and also that temporary output losses may be much larger than previously thought, depending on velocity. A gradual disinflation of low inflation may even be undesirable given its overall negative impact on the economy. Finally, we explore the optimal speed of disinflation.

JEL Classification: E20, E32, F32, F41

Keywords: price stability, velocity, disinflation, output boom, optimal speed of disinflation

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We are grateful to Parantap Basu for many helpful discussions. The usual disclaimer applies.

1 Introduction

This paper explores the effect of time varying velocity on the output response to a disinflationary monetary policy and on the optimal speed of disinflation. The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms and where there is rigidity in the setting of prices. The monetary policymakers are committed to price stability in the strict sense of achieving and maintaining a constant price level. This environment is familiar from the recent body of research on monetary contractions; and this research has provided a number of insights. Important amongst these insights is, firstly, the result that following a contraction in the money supply, real output initially declines below its new long run equilibrium level. Secondly, and more striking, it has been found that a gradual disinflation may actually result in a temporary output boom (after an early decline) since output is rising above its new steady state level. Furthermore, it seems that it is optimal to end high inflations quickly and low inflations gradually. Key papers that develop these results are Ball (1994), Ireland (1997), King and Wollman (1999), and Khan, King and Wollman (2003).

Since the output effects of monetary contractions and the optimal speed of disinflation are of first order policy importance, it is not surprising that there should be interest in exploring the robustness of these results to relaxation of key assumptions. For example, Nicolae and Nolan (2006) relax the assumption of perfect credibility. They demonstrate that the so-called 'disinflationary booms' may or may not disappear in an environment characterized by imperfect credibility, depending on the speed of learning relative to the speed of disinflation.

In this paper we focus on relaxing the assumption that velocity is constant over the entire time horizon of disinflation. It is well known that velocity is not a constant. As long ago as the mid 1960s, Mundell (1965) wrote that: "[t]he simplest hypothesis that velocity is constant, is clearly inadmissible when different rates of inflation are involved". The current consensus on velocity, supported by numerous empirical studies over the years, including Gould and Nelson (1974) and Friedman and Kuttner (1992), is that velocity displays nonstationary behavior.

To introduce time varying velocity to the model used here, we employ a nonlinear relation between velocity and real consumption. In doing this, we have drawn on the money and business cycle literature which endogenizes money velocity in models with shocks to the goods sector, productivity and the money supply. For example, we highlight from Cooley and Hansen (1995) that monetary aggregates and velocity are procyclical. We also draw on empirical evidence from the money demand literature that aggregate consumption, rather than income, is the preferred proxy for the scale variable (see for example Mankiw and Summers, 1986). The specific form of the relationship used here captures velocity as a nonstationary variable and nests constant velocity as a special case. This functional form has theoretical as well as empirical support (see, for example Basu and Dua (1996), Basu and Salyer (2001)).

We find that the introduction of time-varying velocity forces us to modify our thinking about policies for stopping inflation. The next section of this paper presents the model and the parameter values used in model calibration. Section 3 presents benchmark results familiar from the existing literature showing the output response to immediate and gradual disinflations when velocity is constant. Section 4 analyses the output responses to disinflation when velocity is time varying. Section 5 discusses the optimal speed of disinflation for the case of time varying velocity and section 6 concludes the paper.

2 The Model

2.1 The Representative Agent

The framework employed for this analysis extends the model developed in Ireland (1997), the component parts of which are now familiar in the literature. The economy consists of many identical consumers. Each period a representative agent makes plans for consumption and leisure/labour such that (expected) present discounted value of utility is maximised. This measure of utility is given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\alpha} - 1}{1-\alpha} - \gamma N_t \right\} \qquad \alpha, \gamma > 0, \tag{1}$$

and is separable in consumption, C_t, and labour supply, N_t; $\beta \in (0,1)$ is a discount factor and γ is the disutility of work Following Dixit and Stiglitz (1977), C_t is defined over a continuum of goods,

$$C_t = \left[\int_0^1 c_t(i)^{\frac{b-1}{b}} di \right]^{\frac{b}{b-1}} \qquad b > 0,$$

where $c_t(i)$ denotes, in equilibrium, the number of units of each good i from firm i that the representative agent consumes. b is the price elasticity of demand. The Dixit-Stiglitz aggregate price level, P_t, at time t is given by:

$$P_t = \left[\int_0^1 p_t(i)^{1-b} di \right]^{\frac{1}{1-b}},$$

where $p_t(i)$ is the nominal price at which firm i must sell output on demand during time t.

Labour supply, N_t is given by:

$$N_t = \int_0^1 n_t\left(i\right) di,$$

where $n_t(i)$ denotes the quantity of labour supplied by the household to each firm i, at the nominal wage W_t, during each period. This means that households effectively supply a portion of labour to *all* firms which, together with (2) below ensures that the marginal utility of wealth equalizes across agents.

Each period, the representative agent faces a budget constraint of the following sort:

$$\int_0^1 \left[Q_t\left(i\right) s_{t-1}\left(i\right) + \Phi_t\left(i\right) + W_t n_t\left(i\right)\right] di \geq \int_0^1 \left[p_t\left(i\right) c_t\left(i\right) + Q_t\left(i\right) s_t\left(i\right)\right] di. \tag{2}$$

$Q_t\left(i\right)$ denotes the nominal price of a share in firm i, s_t denotes the quantity of shares, $\Phi_t\left(i\right) di = D_t(i)s_t(i)$, where $D_t(i)$ is the dividend associated with a unit share, and $\int_0^1 p_t\left(i\right) c_t\left(i\right) di = P_t C_t$ denotes total nominal expenditure. We assume that for $t = 0$, $s_{-1}(i) = 1$, for all $i \in [0, 1]$ thereby assuming that each household owns an equal share of all the firms. Equation (2) says that each period income (financial plus labour) can be worth no less then the value of expenditure (on non-durable consumption plus financial investment). The household problem, then, is to choose $c_t(i), n_t(i), s_t(i)$ so as to maximize (1) subject to the sequence of constraints (2), and the relevant initial and transversality conditions. Optimal household behaviour is described by the requirement that household consumption spending must be optimally allocated across differentiated goods at each point in time (i.e., the optimal $c_t(i)$). It can be shown that the Dixit-Stiglitz preference relation requires that purchases of each good i satisfies:

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-b}. \tag{3}$$

Ireland (1997) simplifies his analysis by letting the aggregate nominal magnitudes be determined in equilibrium by a quantity-theory type relation with an assumed constant velocity of circulation. Specifically, Ireland's quantity equation is

$$M_t V_t = \int_0^1 p_t(i)\, c_t(i)\, di = P_t C_t.$$

where V_t $(= 1)$ is the velocity of circulation.

A constant velocity of circulation is, arguably, a somewhat strong assumption and one that we seek to relax here. One way to permit velocity to vary in this model is to present it as:

$$V_t = \Omega C_t^{\delta}, \qquad \delta \in [0, 1) \tag{4}$$

where δ captures the degree of velocity circulation[1]. This is a nonlinear relation between velocity and real consumption and draws on the work of, for example, Cooley and Hansen (1995), Mankiw and Summers (1986), Basu and Dua (1996) and Basu and Salyer (2001)[2]. Importantly, velocity is now nonstationary and endogenous to the model. Ireland's case of a constant velocity is nested as a special case (for $\delta = 0$). For any value of $\delta \in (0, 1)$ velocity is time varying.

The quantity-theory type relation can now be written:

$$M_t = P_t C_t^{1-\delta} \tag{5}$$

An interior optimum for the agent's problem will include (2) with equality, (3) for all i, (5) and the following conditions:

$$C_t^{-\alpha} = \lambda_t P_t; \tag{6}$$

$$\gamma = \lambda_t W_t. \tag{7}$$

$$W_t = \gamma P_t C_t^{\alpha} \tag{8}$$

And for all i

$$Q_t(i) = D_t(i) + \beta(\lambda_{t+1}/\lambda_t) Q_{t+1}(i), \tag{9}$$

where λ_t is an unknown multiplier associated with (2).

[1] For simplicity Ω is here set equal to unity.

[2] A full explanation of the microfundations of this velocity function is an interesting exercise in its own right but is beyond the scope of the current paper.

2.2 The Corporate Sector

The corporate sector is modelled as in Ireland (1997) and Nicolae and Nolan (2006). The supply-side of the economy consists of monopolistically competitive firms and there is price rigidity. There is a continuum of firms indexed by i over the unit interval, each of them producing a different, perishable consumption good. So, goods may also be indexed by $i \in [0,1]$, where firm i produces good i.

Each firm i sells shares, at the beginning of each period t, at the nominal price $Q_t(i)$, and pays, at the end of the period, the nominal dividend $D_t(i)$. The representative household trades the number of shares that it owns, $s_t(i)$, in each of the firms at the end of each period t. Under market clearing, $s_t(i) = 1, \forall i \in [0,1]$, in each period. Firms are able to change prices each period, subject to a fixed cost. As a consequence, in equilibrium, firms will not necessarily be willing to change prices in each period. The criterion for the price-setting decision at time t is to maximise the return to shareholders.

At time t we assume that firms are divided into two categories, such that firms from the first category can freely change their prices, $p_{1,t}(i)$, while the firms belonging to the second must sell output at the same price set a period before, $p_{2,t}(i) = p_{2,t-1}(i)$, unless they pay the fixed cost $k > 0$, measured in terms of labour. We may think of this cost as being associated with information collection and decision making. At time $t+1$, the roles are reversed and the first set of firms keeps prices unchanged, $p_{1,t+1}(i) = p_{1,t}(i)$ unless they are willing to pay the fixed cost k, while the second set of firms can freely set new prices.

The model assumes, then, that firms are constantly re-evaluating their pricing strategy, weighing the benefits of holding prices fixed versus the alternative of changing prices and incurring the fixed penalty. However at moment t the firms belonging to the set of firms that can freely change price are able to choose between two strategies, depending on whether the inflation rate is moderate or high. At moderate rates of inflation, or in the face of gradual changes in the monetary stance, they are more likely to keep their prices constant for two periods and hence avoid the cost k (single price strategy). On the other hand, in the case of a high inflation, or in the face of sharp changes in the monetary stance, firms are more likely to choose a new price and pay the cost k (two price strategy).

We assume a simple linear production technology $y_t(i) = l_t(i)$, where $y_t(i)$ and $l_t(i)$ are the output of firm i and the labour used to produce it, respectively. Let us denote aggregate output as Y_t. Equilibrium profits at time t for firm i are given by

$$D_t(i) = [p_t(i) - W_t\,(i)] \left(\frac{p_t\,(i)}{M_t}\right)^{-b} C_t^{1-b(1-\delta)} - I_t(i)W_t(i)k. \tag{10}$$

while, in equilibrium, the units of labour supplied to each firm at nominal wage W_t are given by:

$$n_t(i) = Y_t^{1-b(1-\delta)} \left(\frac{p_t\,(i)}{M_t}\right)^{-b} + I_t(i)W_t(i)k,$$

where

$$I_t(i) = \begin{cases} 1, & \text{if the firm pays the cost of price adjustment } k \text{ at moment } t; \\ 0, & \text{if the firm does not pay the cost } k \text{ at moment } t. \end{cases}$$

Ireland's paper makes a clear distinction between the two price strategies agents follow, each depends on the level of inflation. When inflation is low, the single price strategy is employed, which means that agents are happy to keep prices unchanged for two consecutive periods of time. When inflation is high, they follow the two price strategy which means that they are changing prices every period, as these are getting eroded by the high inflation.

2.3 Single price strategy

Under this strategy we may think of firm i choosing $p_t(i)$ so as to maximize the following expression:

$$\Pi_t(i) = D_t(i) + \beta \left(\frac{\lambda_{t+1}}{\lambda_t}\right) D_{t+1}(i), \tag{11}$$

which follows from (9), and implies that prices are set to maximize market value. We then substitute (6), (7), the quantity theory type equation (5) and goods market equilibrium (3) into (10). It then follows that the price for firm i that will be used for two consecutive time periods is:

$$p_t(i) = \frac{b}{b-1}\gamma \frac{M_t^b Y_t^{1-b(1-\delta)} + \beta M_{t+1}^b Y_{t+1}^{1-b(1-\delta)}}{M_t^{b-1} Y_t^{2-b(1-\delta)-\alpha-\delta} + \beta M_{t+1}^{b-1} Y_{t+1}^{2-b(1-\delta)-\alpha-\delta}}. \tag{12}$$

This equation is familiar from the New Keynesian economics literature. It basically says that the optimal price is a function of current and future anticipated demand and costs conditions, and that in steady state, price is a fixed mark-up over marginal costs. As is familiar in models of monopolistic competition based on Dixit-Stiglitz preferences, the markup is constant and determined by the elasticity of demand (that is, it is tied down via the preference side of the model): the lower the elasticity, the higher the mark-up.

2.4 Two Price Strategy

In this case the firm chooses the price $p_t(i)$ to maximise profits in each period

$$\Pi_t(i) = D_t(i). \tag{13}$$

The optimising price in this case is given by:

$$p_t(i) = \frac{b}{b-1}\gamma\frac{M_t}{Y_t^{1-\alpha-\delta}}. \tag{14}$$

Here we see that prices are a mark-up as before but now only current period demand and cost conditions are relevant.

2.5 The Steady-State

The steady state equilibria exist when money growth is constant ($\mu_t = \mu$), for all t. Real money balances are also constant, so from (5) we have that ($C_t = C$), for all t. The steady-state velocity is constant too, $V = \Omega C^{\delta}$. Steady-state output is not a function of velocity either for the single price strategy

$$Y = \left(\frac{b-1}{b}\frac{1}{\gamma}\frac{1+\beta\mu^{b-1}}{1+\beta\mu^{b}}\left(\frac{2}{1+\mu^{1-b}}\right)^{\frac{1}{1-b}}\right)^{\frac{1}{\alpha}}$$

or for the two price strategy

$$Y = \left(\Omega\frac{b-1}{b}\right)^{\frac{1}{\alpha}}$$

These relations are essentially those in Ireland (1997) [3].

2.6 Monetary Policy

We define a disinflationary policy following the approach adopted by Ball (1994), Ireland (1997) and Nicolae and Nolan (2006). The monetary policy brings money growth to zero over some time horizon. Specifically, at period 0, the authorities make a surprise announcement about the path for the money supply, $\{M_t\}_{t=0}^{T}$, such that by time period T inflation will be zero. This announced path for the money supply, in turn, implies a gradual decrease in the growth rate of the money supply. Let θ_t denote the growth rate of the money stock at time t. We study, then, processes for the money growth rate of the following sort:

$$\theta_t = \theta_{t-1} - \frac{\theta-1}{T}, \tag{15}$$

[3] Setting $\gamma = 1$ and $\Omega = 1$ we obtain Ireland (1997) equations (14).

for any value of t from 0 to $T-1$, where θ_{-1} is equal to the initial rate of inflation, and where $\theta_{t>T} = 1$. So, a horizon of time $T = 1$ entails immediate disinflation, while for $T > 1$ the policymakers engineer a more gradual path towards price stability.

2.7 Model Calibration

In this section we calibrate the model in order to explore the output effect of active monetary policy designed to bring down inflation. To facilitate comparison with the existing literature, we set parameter values used in that broader literature and adopt the model calibration used in Ireland (1997). For ease of reference, Table 1 sets out the parameter values used in the calibration. As is evident from the table, we allow δ to take a number of different values in order to explore the effect of time varying velocity on output (Ireland's case ($\delta = 0$) is a special case of the work carried out here).

Parameter	Value	Description
α	0.1	intertemporal elasticity of substitution; (as in Ball, Mankiw and Romer, 1988)
b	6	price elasticity of demand; (as in Rotemberg and Woodford, 1992)
k	0.1075	cost of price adjustment; (as in Ireland, 1997)
β	0.97	discount factor; each interval of time corresponds to 6 months; (as in Ball and Mankiw, 1994)
γ	1	degree of disutility from work; (as in Nicolae and Nolan ,2006)
δ	$[0, 1)$	degree of velocity of circulation

Table 1. Parameter values used in the model calibration.

In the following section, we present benchmark results from the existing literature. These describe the behaviour of output during immediate and gradual disinflations starting from both low and high initial inflation rates; and where velocity is assumed constant. The subsequent section presents the behaviour of output for all of these same cases but when velocity is assmed to be time varying.

3 Benchmark Results

This section presents results familiar from the literature for the specific case where velocity is assumed constant.

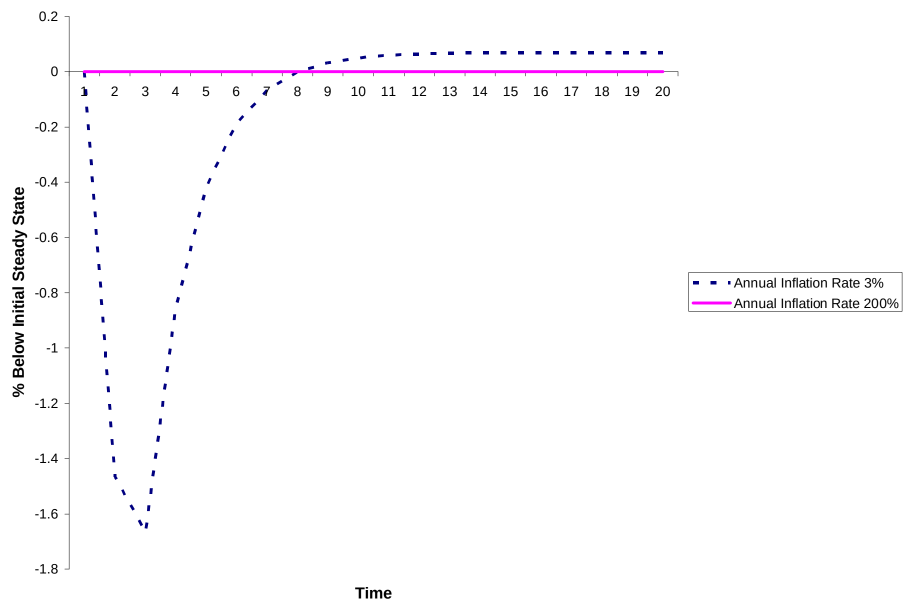

Figure 1: Output Effect of Ireland's (1997) Immediate Disinflation of 'Small' (3%) Initial Annual Inflation Rate and 'Big' (200%) Initial Annual Inflation Rate.

Figure 1 demonstrates the results: i) that immediate ($T = 1$) disinflation from a low (3%) inflation rate brings about a significant early output loss (some 1.47% in the first period and 1.67% in the second period) before reaching its new steady state level; and ii) that immediate disinflation from a high (200%) inflation has no output effect.

Figure 2 sets out the case where disinflation is gradual ($T = 6$) and focuses on disinflating from a low (3%) initial inflation rate. There are two important features to note: i) the early output loss is less than that under the immediate disinflation (now 0.2% in the first period); and ii) after the early fall in output, there is a substantive (compensatory) output boom before a new steady state is reached[4].

[4]Such disinflationary booms are typically understood as follows. Under perfect

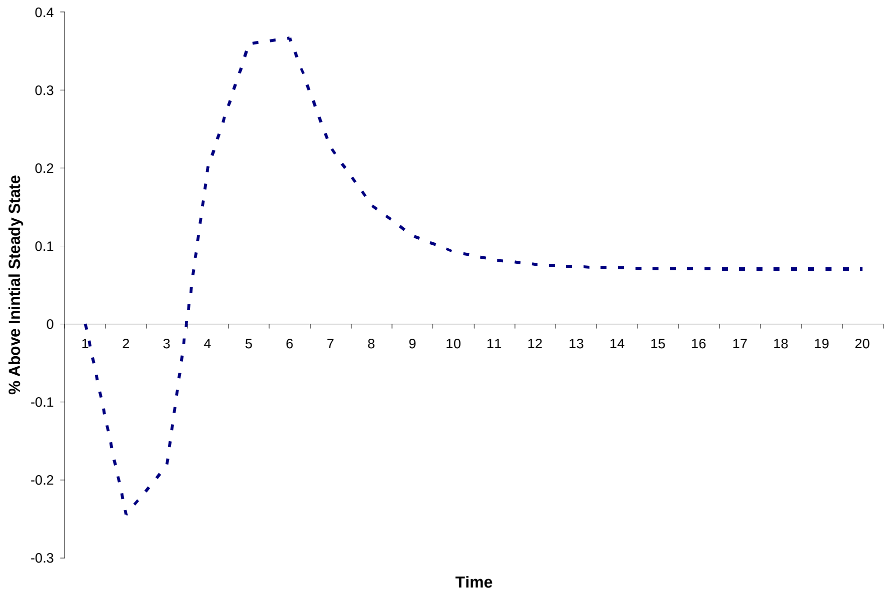

Figure 2: Output Effect of Ireland's (1997) Gradual Disinflation from a 'Small' (3%) Initial Annual Inflation Rate.

Figure 3 presents the output effect of disinflating gradually ($T = 6$) from a high (200%) initial inflation rate. There is now a substantive early output loss (27% below the initial steady state); and again an output boom, but only part compensatory, before reaching the new steady state.

These benchmark images underlie the now well known policy conclusion that high inflations are best ended abruptly and low inflations are best ended gradually. The key issue is the impact on the real economy. Three issues are important: (1) the extent of output losses in the early periods after a monetary contraction; (2) the existence (or otherwise) of a temporary output boom (defined as output rising above the new steady state); and (3) whether early output losses are compensated over some reasonable time horizon.

This paper goes on to explore these issues when the model assumption of constant velocity is relaxed. We will see that introducing time varying velocity to the modelling framework prompts us to modify our stance on some of these issues.

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credibility, agents are responding in advance of the change in policy, lowering their relative prices, knowing that in future, inflation is going to be lower. Because agents set prices for two periods, and because inflation will be lower in the future, they set lower prices today, inducing a boom.

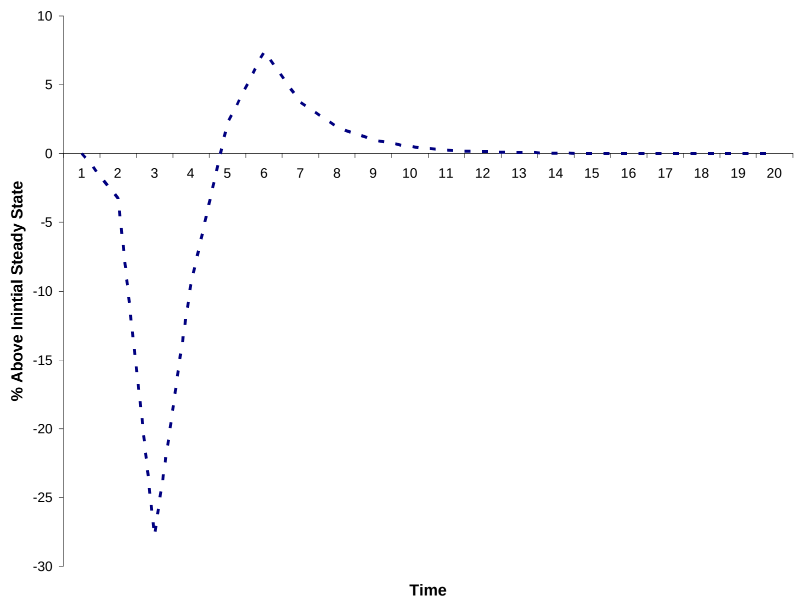

Figure 3: Output Effect of Gradual Disinflation with Constant Velocity V=1. Initial Annual Inflation Rate 200%.

4 Output Effects of Immediate and Gradual Disinflation with Time Varying Velocity

Figure 4 sets out the output effect of an immediate disinflation ($T = 1$) from a low (3%) initial annual inflation rate. Different values for δ capture different degrees of time varying velocity ($\delta = 0$ reflects the benchmark case discussed above and the (dotted) output path corresponds to that seen in Figure 1. Higher values of δ reflect higher time varying velocity. It is evident that the effect of introducing velocity is to increase the early output loss. To see why this comes about, we can refer to the price setting strategies set out in equations (12) and (14). The time varying velocity parameter (δ), enters the price setting strategies for both types of firms and serves to induce firms to make larger adjustment thereby augmenting the output effect. This process is discussed in more detail, after considering the output response to a gradual ($T = 1$) disinflation from a low initial 3% inflation rate.

In Figure 5, again, the dotted line reflects the benchmark case when velocity is constant ($\delta = 0$), as seen in Figure 2. As in the previous case of immediate disinflation we see that introducing time varying velocity to the model has induced greater output losses: the higher the value of δ, the lower the output falls below its initial steady-state level in the early

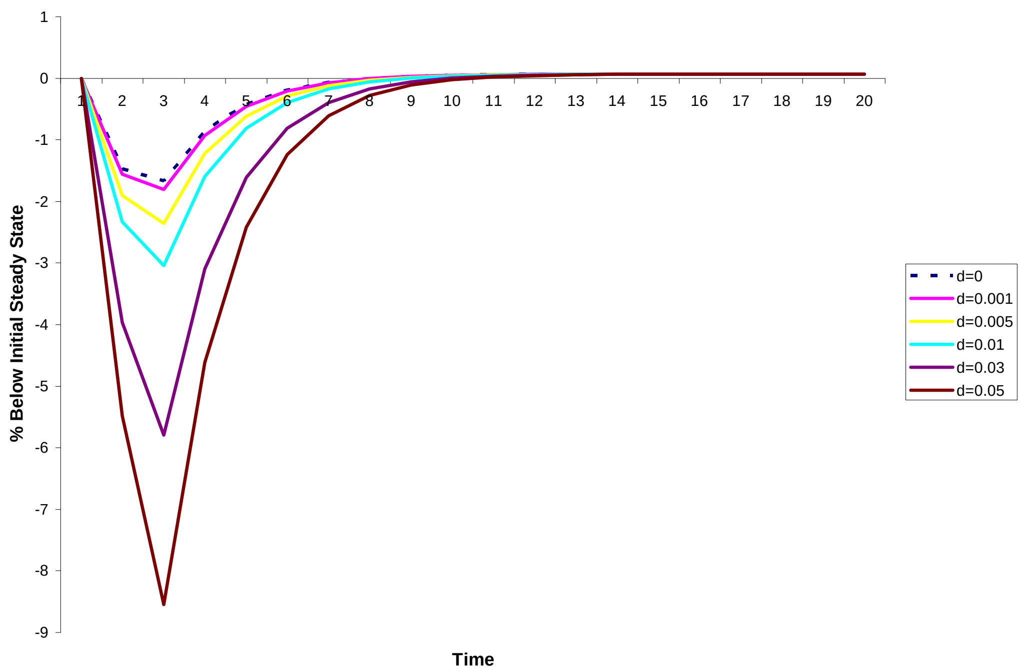

Figure 4: Output Effect of Immediate Disinflation with Velocity $V_t = \Omega C_t^{\delta}$. Initial Annual Inflation Rate 3%.

period. However, in this case, velocity seems to have one more effect. In the benchmark case of gradual disinflation and constant velocity, we saw that, after the initial fall, output not only picked up but also rose above its new steady state level, staying above for some time before returning to its new steady-state equilibrium (the output boom). However, for low velocity levels $\delta^* \in (0.01, 0.02)$ we see that, after the initial fall, output recovers but never rises above the new steady state level. Moreover, this is so for all yet higher values of delta. For any $\delta > \delta^*$ output fails to rise above the new steady-state level. Although output reaches its new steady-state at about the same time (4-5 years) regardless of the velocity parameter value (δ), the higher is velocity the greater is the output loss and the greater is the possibility that there is no output boom. This raises a key question about whether gradual disinflation is beneficial. With greater output losses for relatively high values of δ, there is the possibility that they might not be compensated.

We obtain a crude measure of the overall impact on output by projecting forward 50 years. Table 2 provides the value of the area between the 'output path' and the x axis: below the axis gives the output loss,

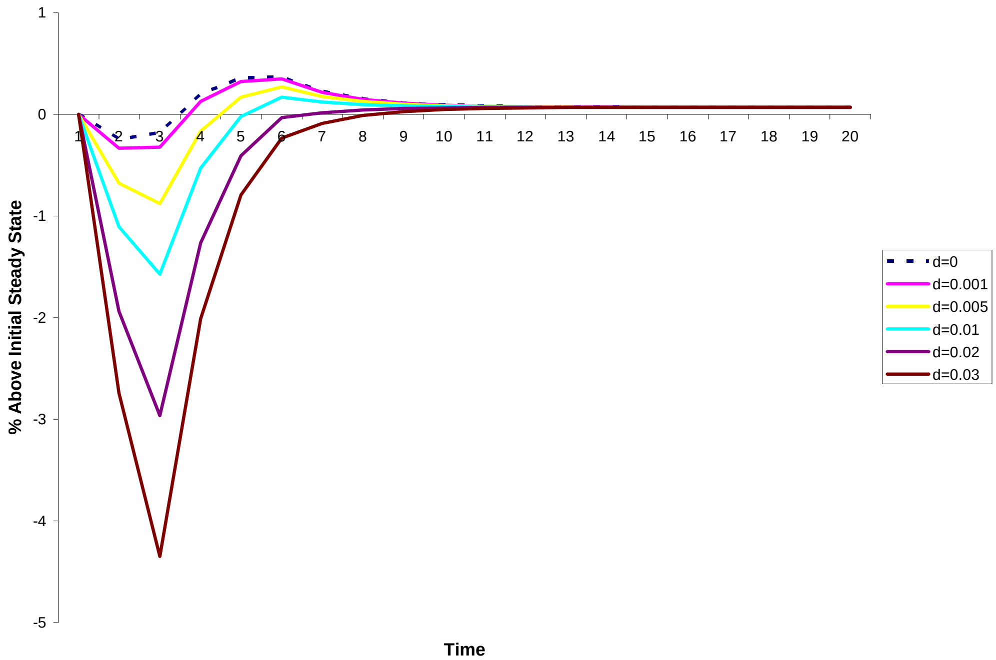

Figure 5: Output Effect of Gradual Disinflation with Velocity $V_t = \Omega C_t^\delta$. Initial Annual Inflation Rate 3%.

and above the axis gives the outptu gain. The absolute size of the overall impact is noted in the final column. From Table 2, we can see that for sufficiently high values of δ the overall impact on output is negative. (If we were to calculate the present values, overall net losses would arise at lower levels of δ).

δ	**Loss**	**Gain**	**Overall Output**
0	-0.42	7.79	7.36
0.001	-0.65	7.63	6.98
0.005	-1.72	7.19	5.47
0.01	-3.22	6.81	3.58
0.02	-6.60	6.42	-0.17
0.03	-10.22	6.30	-3.92
0.05	-17.56	6.21	-11.35

Table 2. Impact on output of a gradual disinflation from an initial 3% rate for different values of the velocity parameter (δ).

In the light of these results, Ireland's (1997) conclusion that small inflations are best ended gradually may need to be qualified: it seems that even disinflating a low inflation gradually may be undesirable since the 'overall impact' on the real economy is negative. This shift in potential policy conclusion is solely attributable to the introdution of time varying velocity so it is helpful to discuss its role in the (behavioural) context of the model. After the disinflation is announced at $t = 0$, at $t = 1$ the firms that changed price last period now keep their price fixed, but the other set of firms respond by adjusting their prices. When they solve their optimization problem to maximize their profits (see equations (11) or (13) depending on the strategy they follow), firms take the aggregate level of real money balances $\left(\frac{M_t}{P_t}\right)$ as given (since the nominal money supply M_t and the aggregate general level of prices P_t are taken as given). In equilibrium, we know that $\left(\frac{M_t}{P_t}\right)$ has to be consistent with the individual firm choice. Thus, given $\left(\frac{M_t}{P_t}\right)$ each optimal price $p_t(i)$, for whatever strategy they chose to follow, has to be optimal for firm i, and given each optimal price $p_t(i)$, C_t must equal $\left(\frac{M_t}{P_t}\right)^{\frac{1}{1-\delta}}$ (see equation (5)). The role of δ is important. When $\delta > 0$ we can see from a simple manipulation of this expression how consumption responds to disinflation. Taking logs one gets:

$$\ln C_t = \frac{1}{1-\delta}\left(\ln M_t - \ln P_t\right).$$

Partially differentiating with respect to M_t, yields

$$\frac{d \ln C_t}{d \ln M_t} = \frac{1}{1-\delta} > 1. \qquad (16)$$

In a disinflationary period, the induced fall in C_t is even greater than when $\delta = 0$. The greater output loss is attributable to time varying velocity. The price stickiness makes the velocity effect on output more persistent.

We now turn to consider the case where disinflation is from a high (200%) initial inflation rate. Figure 6 sets out the output path from both an immediate disinflation and a gradual disinflation. There is no impact of time varying veocity in the case of an immediate disinflation ($\delta = 0$ and $\delta = 0.05$ shown). At very high inflation rates, both sets of firms are following the two price strategy because the costs of adjustment are outweighed by the benefits. Not only is inflation ended abruptly but also, adjustment is so fast that there is no scope for velocity to have an impact.

More interesting is the case of gradual disinflation. From Figure 6, the output path with time varying velocity ($\delta = 0.05$) looks very

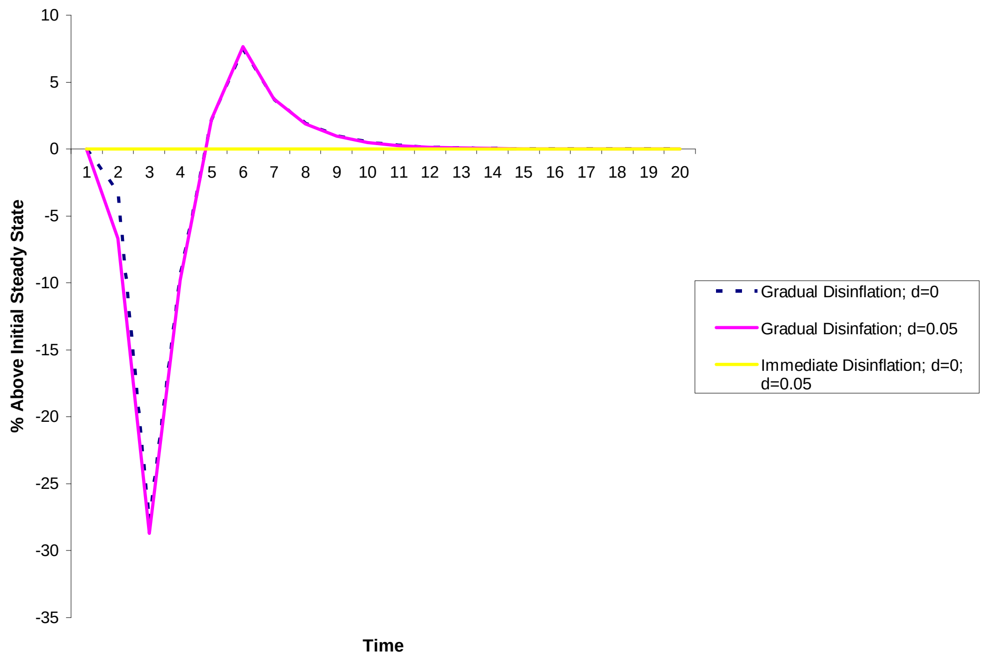

Figure 6: Output Effect of Immediate and Gradual Disinflation with Velocity $V_t = \Omega C_t^{\delta}$. Initial Annual Inflation Rate 200%.

similar to the benchmark case. However, in the first period, the output loss is more marked. The reason for this is akin to the output effect we have seen when disinflation was gradual from a low initial inflation rate. We have seen that when disinflation is gradual, δ has a role to play and its role is to reduce output more (see (16)). This reinforces Ireland's conclusion that gradual disinflation from a high initial rate is not to be recommended. We therefore turn our attention to consider gradual disinflation from a range of lower inflation rates in more detail. Specifically, we quantify the effect of time varying velocity on the optimal speed of disinflation

5 Optimal Speed of Disinflation

An important choice for policy makers is the time horizon over which they bring about price stability. Ireland (1997) gave clear guidance on this issue: a high inflation is best disinflated immediately, whilst a low inflation is best disinflated gradually. Furthermore, he calculated optimal speeds of disinflation for a wide range of inflation rates. Here, we calculate the optimal speed of disinflation in our model which allows for time varying velocity. We focus attention on a range of inflation rates at the lower end of the range (1%-20%). We abstract from very high inflation rates because we know that immediate disinflation is the preferred policy response to a high inflation rate and we have found that time varying velocity has no output impact in the case of an immediate disinflation from high inflation rates. Furthermore, gradual disinflation from a high initial inflation rate is not to be recommended.

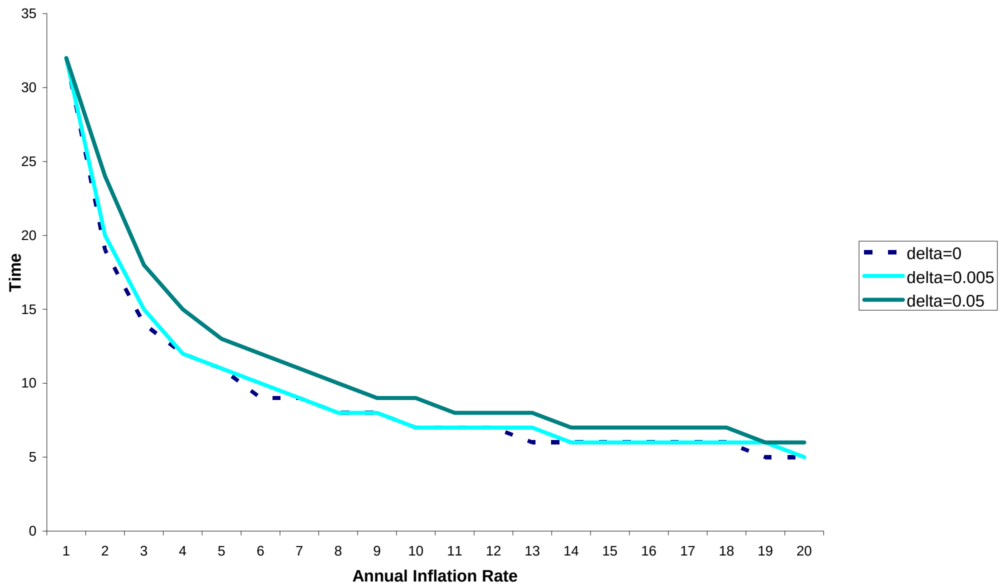

Figure 7: Optimal Speed of Disinflation

Figure 7 presents the optimal speed of disinflation for (a selected subset of) different values of δ. For each initial level of inflation we calculate the level of utility associated with different speeds of disinflation. Plotted in Figure 7 is the 'benchmark case' ($\delta = 0$) and two other $\delta > 0$ values[5]. For low values of δ, the optimal speed of disinflation is hardly changed from the benchmark case. Yet for the higher velocity case shown, the optimal speed of disinflation is decreased by approximately a year for inflation rates up to around 12%, and a half year for inflation in the teens. More generally, we can say that a more gradual period of disinflation is optimal for the entire range of inflations from 1% to 20%. What seems to be happening here is that higher velocity requires the policy makers to disinflate more slowly in order to put a 'brake' on the economy, moderating the greater output loss that we know would otherwise

[5]The calculations have been carried out for some 7 different values of δ: the general story stays the same.

result. This moderation serves to maximise utility.

6 Conclusion

Perhaps the most dramatic finding from recent research on monetary contractions is that a gradual disinflation may bring about a 'disinflationary output boom'. These disinflationary output booms were first recorded in the much cited paper, Ball (1994); and the more recent literature (in which firms are monopolistically competitive and there is rigidity in prices) consistently finds such booms (see for example, Ireland (1997), King and Wollman (1999), Khan, King and Wollman (2003)). Common to the models used in these papers are the two assumptions: perfect credibility and constant velocity. Nicolae and Nolan (2006) relax the assumption of perfect credibility and find that, whilst imperfect credibility may make these booms disappear, it is not a sufficient condition: their (dis)appearance depends on the speed of learning relative to the speed of disinflation. In this paper, we have relaxed the assumption of constant velocity and we also find that disinflationary booms may disappear, but now this is a result of time varying velocity: output boom (dis)appearance depends on velocity.

This is not the only effect of relaxing the constant velocity assumption. For example, we find that the early output loss that follows a disinflationary policy announcement is considerably larger when time varying velocity is introduced to the model; and this output loss may not be compensated by later output gains. As a result, we find that, we cannot unconditionally endorse Ireland's policy recommendation that small inflations are best disinflated gradually. We find that a gradual disinflation from a small inflation may result in an overall output loss, bringing into question the desirability of any disinflationary policy action in some cases. Finally, having introduced time varying velocity to the model, calculations of the optimal speed of disinflation show that, relative to Ireland's calculations, a more gradual period of disinflation is appropriate for those cases when gradual disinflation seems relevant. It seems that some of the familiar results and policy implications from influential work on stopping inflations are not robust to some modifications of the modelling framework and further research is needed.

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