Imperfectly-Credible Disinflation of Small Inflations

Lynne Evans *[*] Anamaria Nicolae [†]
Newcastle University Durham University

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Abstract

In this paper we study and quantify the effects of a disinflationary policy on output and welfare. Our focus is the policy question of the optimal response to low inflation.The analysis takes place in the context of a DSGE model with sticky prices, time varying velocity and imperfect credibility. The nonlinear solution method reveals that early output losses may be more pronounced and more prolonged than previously suggested in the literature, and there may be insufficient compensation from a subsequent higher steady state to justify taking any disinflationary policy action in some cases.

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[*]Lynne Evans is at Newcastle University Business School and is an Honorary Fellow at Durham University. E-mail: Lynne.Evans@ncl.ac.uk.

[†]Anamaria Nicolae is at Durham University, Department of Economics, 23/26 Old Elvet, Durham, DH1 3HY, UK, tel. +441913346358. E-mail: Anamaria.Nicolae@durham.ac.uk. Corresponding Author.

1. Introduction

There is a general consensus in the economics profession that 'high' rates of inflation have significant adverse consequences and that these adverse consequences justify the sacrifices in employment and output that are generally needed to reduce inflation. Furthermore, it is now well established that the appropriate policy response to a high inflation is to disinflate immediately (see, for example, Ireland (1997)). However, there is no corresponding widespread agreement about the appropriate policy response to a low inflation, despite the fact that estimating potential gains in welfare from reducing inflation has been addressed in a long line of research stemming from the classic contributions of Bailey (1956) and Friedman (1969). This research embraces a number of modelling set ups ranging across partial equilibrium analyses, in which the preferred functional form of the demand for money function is central (see, for example, Chada et al (1998), Lucas (2000), Ireland (2009), Serletis and Yavari (2004) – through developments giving explicit consideration to tax distortions (Cooley and Hansen (1991), Feldstein (1997)) – to general equilibrium set ups such as Dotsey and Ireland (1996), Abel (1997) and Dibooglu and Kenc (2009).

Although much of this literature tends to find moderate estimates of the welfare cost of inflation, the policy questions of whether and how policy makers should respond to even a small inflation persist. This is partly because inflation is deeply unpopular with the general public (see, for example, Shiller (1997)) and this generates some political imperative to do something about it: but also, because the magnitude of the welfare effects of even small inflations may be economically non-trivial.

In this paper, we explore the dynamic impact of disinflating from low values of inflation using a dynamic stochastic general equilibrium model which incorporates time-varying velocity and allows for imperfect credibility. More precisely, we study the effects of a disinflationary monetary policy when policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price level. The analysis takes place in a New Keynesian environment where the supply-side of the economy is characterised by monopolistically competitive firms, and where there is rigidity in the setting of prices. The model builds on Ireland (1997), endogenising time-varying velocity in that framework (as in Evans and Nicolae, 2009) and allows for the possibility that policy makers may not enjoy complete credibility (as in Nicolae and Nolan, 2006)[1]. We employ a non-linear solution method which allows us to explore output responses to disinflationary monetary policies extending the analysis beyond the early output loss to explicit consideration of the changing steady state.

Within this set-up we explore issues of first order policy importance. Central is Ireland's clear policy conclusion - that small disinflations are best disinflated gradually – and the finding that, when time-varying velocity is allowed for, this conclusion cannot be unconditionally endorsed (see Evans and Nicolae, 2009). In this paper we explore this issue further. Not only do we explore the output effects of disnflationary policies, we explicitly consider the effects on welfare and also calculate the optimal speed of disinflation when velocity is time-varying.

The following section of this paper presents the model and the parameter values used in the calibration. Section 3 presents benchmark results familiar from the existing literature showing the output response to immediate and gradual disinflations (from a small inflation) when velocity is assumed constant and there is perfect credibility. Section 4 analyses the output response to gradual disinflation

[1]Almeida and Bonomo (2002) and Bonomo and Carvalho (2009) analyse the output cost of disinflation under imperfect credibility and endogenous state-dependent and time-dependent pricing strategies respectively. The learning schemes they adopt involve Bayesian updating. The model developed in this paper adopts an exogenous, simple learning scheme and embraces both pricing strategies.

in the full model (when velocity is time-varying and there is imperfect credibility). Section 5 discusses the optimal speed of disinflation, and section 6 concludes the paper.

2. The Model

The framework employed for this analysis extends the perfect foresight model developed in (Ireland, 1997), and (Evans and Nicolae, 2009). The component parts of this model are now familiar in the literature.

2.1. The Representative Agent

Each period, the representative agent makes plans for consumption and leisure/labour to maximize the expected present discounted utility:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\alpha} - 1}{1-\alpha} - \gamma N_t \right\} \qquad \alpha, \gamma > 0, \tag{1}$$

which is separable in consumption and labour supply. $\beta \in (0,1)$ is a discount factor, α is the intertemporal elasticity of substitution and γ is the disutility of work. Consumption, C_t, is defined over a continuum of goods

$$C_t = \left[\int_0^1 c_t(i)^{\frac{b-1}{b}} di \right]^{\frac{b}{b-1}} \qquad b > 0,$$

where $c_t(i)$ is, in equilibrium, the number of units of each good i from firm i that the representative agent consumes and b is the price elasticity of demand. Labour supply, N_t, is

$$N_t = \int_0^1 n_t(i)\, di,$$

where $n_t(i)$ denotes the quantity of labour supplied by the household to each firm i, at the nominal wage W_t, during each period.

Households face an aggregate price level, P_t, given by:

$$P_t = \left[\int_0^1 p_t(i)^{1-b} di\right]^{\frac{1}{1-b}},$$

where $p_t(i)$ is the nominal price at which firm i must sell output on demand during time t. Households supply labour to *all* firms, which, together with the budget constraint below (equation (2), ensures that the marginal utility of wealth equalizes across agents.

Each period, the representative household faces a budget constraint of the following form:

$$\int_0^1 \left[Q_t\left(i\right) s_{t-1}\left(i\right) + \Phi_t\left(i\right) + W_t n_t\left(i\right)\right] di \geq \int_0^1 \left[p_t\left(i\right) c_t\left(i\right) + Q_t\left(i\right) s_t\left(i\right)\right] di, \quad (2)$$

where $Q_t\left(i\right)$ denotes the nominal price of a share in firm i, $s_t(i)$ denotes the quantity of shares, $\Phi_t\left(i\right) di = D_t(i) s_t(i)$, where $D_t(i)$ is the dividend associated with a unit share, and $\int_0^1 p_t\left(i\right) c_t\left(i\right) di = P_t C_t$ denotes total nominal expenditure on non-durable consumption. We assume that for $t = 0$, $s_{-1}(i) = 1$, for all $i \in [0,1]$. Also, we assume that each household owns an equal share of all the firms. The constraint (2) says that, in each period, income (financial plus labour) must be less than the value of expenditure (on non-durable consumption plus financial investment). The household chooses $c_t(i), n_t(i), s_t(i)$ so as to maximize (1) subject to the constraint (2) and the relevant initial and transversality conditions. Additionally, its optimal allocation across differentiated goods $c_t(i)$ must satisfy:

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t}\right)^{-b}. \quad (3)$$

The aggregate equilibrium nominal magnitudes are determined by a quantity-theory type relation:

$$M_t V_t = \int_0^1 p_t\left(i\right) c_t\left(i\right) di = P_t C_t, \quad (4)$$

where the velocity of circulation, V_t is time-varying. It is given by:

$$V_t = \Omega C_t^{\delta} e^{\rho \pi_t}, \qquad \delta \in [0,1), \rho \in [0,1), \tag{5}$$

where $(1-\delta)$ is the scale elasticity of money demand and ρ is the semi-elasticity with respect to the opportunity cost variable, $\pi_t = \frac{P_t}{P_{t-1}} - 1$, inflation. Different values of parameters δ and ρ capture different degrees of time varying velocity and Ireland's case of a constant velocity is nested as a special case (for $\delta = 0$ and $\rho = 0$), as in (Evans and Nicolae, 2009). For any non-zero positive values of δ or ρ, velocity is time-varying and endogenous to the model.

The agent solves the maximization problem, yielding the following first order conditions:

$$C_t^{-\alpha} = \lambda_t P_t; \tag{6}$$

$$\gamma = \lambda_t W_t; \tag{7}$$

(from (6) and (7))

$$W_t = \gamma P_t C_t^{\alpha}. \tag{8}$$

And for all i

$$Q_t(i) = D_t(i) + \beta(\lambda_{t+1}/\lambda_t) Q_{t+1}(i), \tag{9}$$

where λ_t is an unknown multiplier associated with the budget constraint (2)[2].

2.2. The Corporate Sector

The supply-side of the economy consists of monopolistically competitive firms and there is price rigidity. A continuum of firms, indexed by i over the unit interval, each produces a different, perishable consumption good, indexed by $i \in [0,1]$, where firm i produces good i. The representative household trades shares in each

[2] For simplicity, Ω is here set equal to unity.

firm i, which sell at the nominal price $Q_t(i)$ at the beginning of time t, and pay a nominal dividend $D_t(i)$ at the end of time t.

We assume a simple linear production technology $y_t(i) = l_t(i)$, where $y_t(i)$ and $l_t(i)$ are the output of firm i and the labour used to produce it, respectively. Y_t is aggregate output. Equilibrium returns to shareholders at time t for firm i are given by:

$$D_t(i) = [p_t(i) - W_t(i)] \left(\frac{p_t(i)}{M_t}\right)^{-b} C_t^{1-b(1-\delta)} - I_t(i)W_t(i)k, \tag{10}$$

where

$$I_t(i) = \begin{cases} 1, & \text{if the firm pays the cost of price adjustment } k \text{ at moment } t; \\ 0, & \text{if the firm does not pay the cost } k \text{ at moment } t. \end{cases}$$

Costly price adjustment is central to this model, in which time-dependent and state-dependent strategies are both present. Firms are divided into two categories, such that at time t firms belonging to the first category can freely change their prices, $p_{1,t}(i)$, while firms belonging to the second must sell output at the same price as they set a period before, $p_{2,t}(i) = p_{2,t-1}(i)$, unless they pay the fixed cost $k > 0$, measured in terms of labour. At time $t + 1$, the roles are reversed and the first category of firms keeps prices unchanged, $p_{1,t+1}(i) = p_{1,t}(i)$, unless they are willing to pay the fixed cost k, while the second category of firms can freely set new prices.

Firms are constantly re-evaluating their pricing strategy, weighing the benefits of holding prices fixed against the alternative of changing prices and incurring the fixed penalty. At moment t the firms that can freely change price are able to choose between two strategies, depending on whether the inflation rate is moderate or high. At moderate rates of inflation, they are more likely to keep their prices constant for two periods and hence avoid the cost k (single price strategy). On the other hand, in the case of a high inflation, or in the face of sharp changes in the monetary stance, firms are more likely to choose a new price and pay the cost k (two price strategy). The price-setting decision at time t maximizes the return to shareholders.

The *equilibrium* in the model is given by the market clearance conditions for the three markets present in this model (goods market, labour market and asset market). Clearance in two markets assures clearance in the third. From the market clearance conditions for the goods and labour markets we have:

$$C_t = Y_t = L_t.$$

The clearance condition for the asset market is $s_{t-1}(i) = 1, \forall i \in [0,1]$, in each period.

2.3. The pricing strategies

There are two pricing strategies the firm can follow. Under the *single price strategy,* firm i chooses the price $p_t(i)$ to maximize the expression:

$$\Pi_t(i) = D_t(i) + \beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right) D_{t+1}(i), \tag{11}$$

which follows from (9), and implies that prices are set to maximize market value. Substituting (4) and (8) into (10), and then this into equation (11), yields the price firm i will use for two consecutive time periods:

$$p_t(i) = \frac{b}{b-1}\gamma e^{\rho\pi_t}\frac{M_t^b Y_t^{1-b(1-\delta)} + \beta M_{t+1}^b Y_{t+1}^{1-b(1-\delta)}}{M_t^{b-1}Y_t^{2-b(1-\delta)-\alpha-\delta} + \beta M_{t+1}^{b-1}Y_{t+1}^{2-b(1-\delta)-\alpha-\delta}}.$$

This equation, familiar from the New Keynesian economics literature, shows that the optimal price is a function of current and future anticipated demand and cost conditions; and that, in steady-state, price is a fixed mark-up over marginal costs. As is familiar in models of monopolistic competition, the markup is constant and determined by the elasticity of demand (that is, it is tied down via the preference side of the model): the lower the elasticity, the higher the mark-up.

Under the *two price strategy,* firm i chooses the price $p_t(i)$ to maximize the expression:

$$\Pi_t(i) = D_t(i) \tag{12}$$

and now the optimizing price is:

$$p_t(i) = \frac{b}{b-1}\gamma e^{\rho\pi_t}\frac{M_t}{Y_t^{1-\alpha-\delta}}.$$

Again, prices are a mark-up, but now only current period demand and cost conditions are relevant since only current dividend matters.

2.4. Monetary Policy

The disinflationary policy employed in this paper follows the approach adopted by (Ball 1994), (Ireland 1997) and (Nicolae and Nolan 2006). The monetary policy is designed to bring money growth to zero over some time horizon. Specifically, at period 0, the authorities make a surprise announcement about the path for the money supply, $\{M_t\}_{t=0}^{T}$, such that by time period T inflation will be zero. This announced path for the money supply implies a decrease in the growth rate of the money supply.

Let

$$\theta_t = \frac{M_t}{M_{t-1}}$$

denote the gross rate at which the money supply increases at time t. We adopt a disinflationary process of the following sort:

$$\theta_t = \begin{cases} \theta_{t-1} - \varphi^{T-1}\left(\pi_i - \pi^*\right), & t < T-1 \\ 1, & t \geq T \end{cases}, \qquad \varphi \in (0,1), \tag{13}$$

where π_i is the initial rate of inflation from which the disinflation process starts, π^* is the final (target) inflation to be set here at $\pi^* = 1$.

A horizon of time $T = 1$ implies immediate disinflation, while for $T > 1$ the policymakers engineer a more gradual path towards price stability. To facilitate comparison with the existing literature we employ a linear disinflationary policy following (Ireland 1997, Nicolae and Nolan 2006 and Evans and Nicolae 2009), which we obtain for $\varphi = (1/T)^{\frac{1}{T-1}}$.

2.5. Imperfect Credibility

In this paper imperfect credibility is modelled in the style of Nicolae and Nolan (2006) in which credibility is imperfect, but nevertheless improving over time. The probability mass characterising agents' subjective expectations is shifting through time onto the central bank's announced money supply path, $\{M_s^A\}_{s=0}^{T+J}$, $J \geq 0$. It is assumed that agents perceive of two possible outcomes regarding the path for the money supply: the monetary authority's announced path for the money supply and a more inflationary path for the money supply. There are two choices for the more inflationary path i) agents perceive the authorities as reverting to the previous steady state inflation rate; and ii) agents fear the government will 'run out of steam' such that at time t (for $0 < t < T$) the growth rate of the money stock will be equal to the growth rate between $t-1$ and t.

As it is assumed that the authorities stick to the announced path of disinflation, these alternate expectations are:

$$E_{t+j-1}M_{t+j} = \sigma_{t+j}\theta_{-1}M_{t+j-1}^A + (1-\sigma_{t+j})M_{t+j}^A;$$

$$E_{t+j-1}M_{t+j} = \sigma_{t+j}\theta_{t-1}M_{t+j-1}^A + (1-\sigma_{t+j})M_{t+j}^A,$$

where $\{\sigma_s\}_{s=0}^{T+J}$ is given by two plausible characterizations of the transition between imperfect credibility and perfect foresight. These are given by:

$$\sigma_t = \begin{cases} (-1)^{\delta}\,\tau(N^2-(t-\eta N)^2)^{\frac{1}{2}} + \eta\sigma_0, & t < N-1 \\ 0, & t \geq N \end{cases} \qquad (14)$$

where N captures the time it takes for agents to believe completely the central bank's announcements (we assumed in this paper $N \leq T$) , σ_0 is a measure of the initial level of credibility, and $\tau = N/\sigma_0$. For $\eta = 0$ we have what is labeled as 'concave' learning[3] and $\eta = 1$ 'convex' learning[4].

For the purposes of comparison we adopt $\sigma_0 = 1$ which means that agents start the learning process from total lack of credibility and $N = 6$ which means that after three years the agents take the central bank announcment at face value (implying that agents finally believe completely the announcement when, and only when, price stability is actually achieved)[5]. Furthermore, all the analysis in this paper is conducted assuming 'concave' learning.

Introducing uncertainty into our framework results in some computational complexity which requires a more complex nonlinear solution method to solve this model.

2.6. Model Calibration

This section presents the calibration of the model. Again, to facilitate comparison with the existing literature, we employ parameter values drawn from the wider literature. For ease of reference, Table 1 sets out the parameter values used in the calibration. We allow the newly introduced parameters δ and ρ to take a number

[3] That is, σ plots as a concave function of time on the $x-y$ plane, where the $x-$axis measures time. This captures the intuitive idea that agents may be reluctant to update their priors initially. However, as time goes by and the central bank sticks to its announced money supply targets, they increasingly come to believe the announced target path. We shall refer to this case as *concave (expectations) updating.*

[4] This reflects a population, although happy to accept that the monetary authority dislikes the current relatively high rate of inflation, nevertheless worries that as the slope of the short-run Phillips curve flattens the monetary authority may be tempted to renege. The importance of the exploitability of the Phillips curve has been emphasized by Ball, Mankiw and Romer (1988) and is a crucial factor in high inflation equilibria in games of the Barro and Gordon (1983) sort. We refer to this as *convex (expectations) updating.*

[5] If σ_t takes a longer time to reach zero, output obviously also takes a longer time to reach its new steady state level.

of different values in order to explore the effect of time varying velocity on output (Ireland's case ($\delta = 0$ and $\rho = 0$) is a special case of the work carried out here).

Parameter	**Value**	**Description**
α	0.1	intertemporal elasticity of substitution (value as in Ball, Mankiw and Romer, 1988)
b	6	price elasticity of demand (value as in Rotemberg and Woodford, 1992)
k	0.1075	cost of price adjustment (value as in Ireland, 1997)
β	0.97	discount factor; each interval of time corresponds to 6 months (value as in Ball and Mankiw, 1994)
γ	1	degree of disutility from work (value as in Nicolae and Nolan, 2006)
Parameters	*capturing*	*the degree of time varying velocity*
δ	$[0, 1)$	$(1 - \delta)$ is the scale elasticity of money demand
ρ	$[0, 1)$	opportunity cost semi-elasticity of money demand

Table 1. *Parameter values used in the model calibration.*

In the following section, we present benchmark results from the existing literature. These describe the behaviour of output during immediate and gradual disinflations starting from low initial inflation rates, where velocity is assumed constant and there is perfect credibility. The subsequent section analyses the output response to gradual disinflation in the full model when velocity is allowed to vary.and there is imperfect credibility.

3. Benchmark Results

This section presents results familiar from the literature for the specific case where velocity is assumed constant and there is perfect credibility (Ireland, 1997). Figure 3.1 shows two key results: (i) that immediate ($T = 1$) disinflation from a low (3%) inflation rate brings about a significant early output loss (some 1.47% in the first period and 1.67% in the second period) before reaching its new steady-state level; and (ii) that gradual ($T = 6$) disinflation from a low (3%) initial inflation rate brings about a much smaller early fall in output (now 0.2% in the first period) which is then followed by a substantive (compensatory) output boom before the new steady-state is reached[6]. Ireland's policy conclusion that small inflations should be disinflated gradually is clearly evident.

4. Output Effects of Immediate and Gradual Disinflation with Time Varying Velocity and Imperfect Credibility

Evans and Nicolae 2009 have shown that when time varying velocity is endogenized in Ireland's model, the early output loss will be larger and the output boom may disappear raising questions about the net impact on output over time. In the next section we explore this question more fully, but first we present some output paths for a specific gradual disinflation from a low initial inflation rate for 3 cases: i) $\delta = 0$, $\rho = 0$, $\sigma_0 = 0$ (Ireland's model); ii) $\delta = 0.01$, $\rho = 0.05$, $\sigma_0 = 0$ (model allowing for time varying velocity); and iii) $\delta = 0.01$, $\rho = 0.05$, $\sigma_0 = 1$ (allowing

[6] Such disinflationary booms are typically understood as follows. Under perfect credibility, agents respond in advance of the change in policy by lowering their prices, knowing that inflation is going to be lower in the future. Because agents set prices for two periods, and because inflation will be lower in the future, they set lower prices today, inducing a boom (Ball 1994).

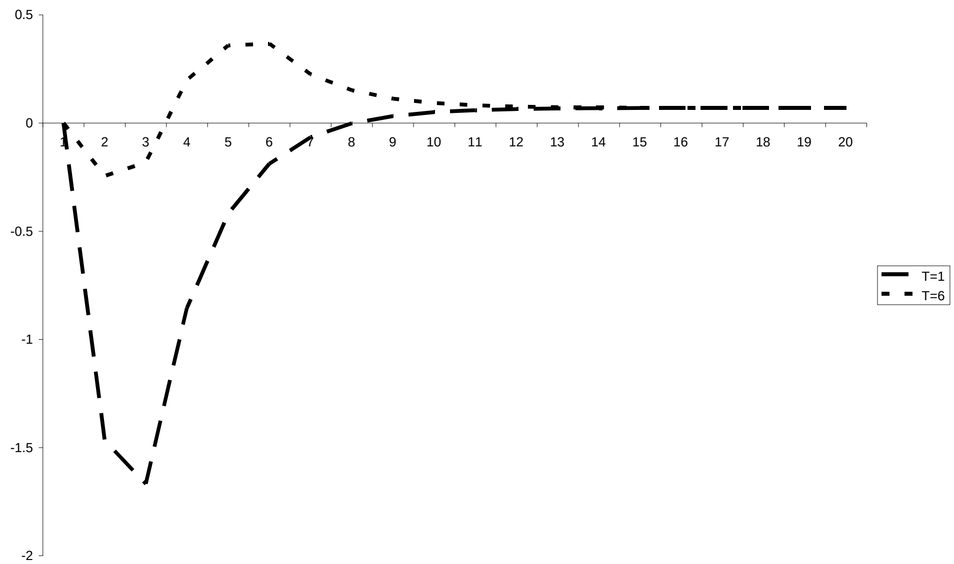

Figure 3.1: **Benchmark Result (Ireland, 1997):** *Output effect of an immediate and gradual disinflation of a 'small' (3%) initial annual inflation rate* ($\delta = 0$, $\rho = 0$, $\sigma_0 = 0$).

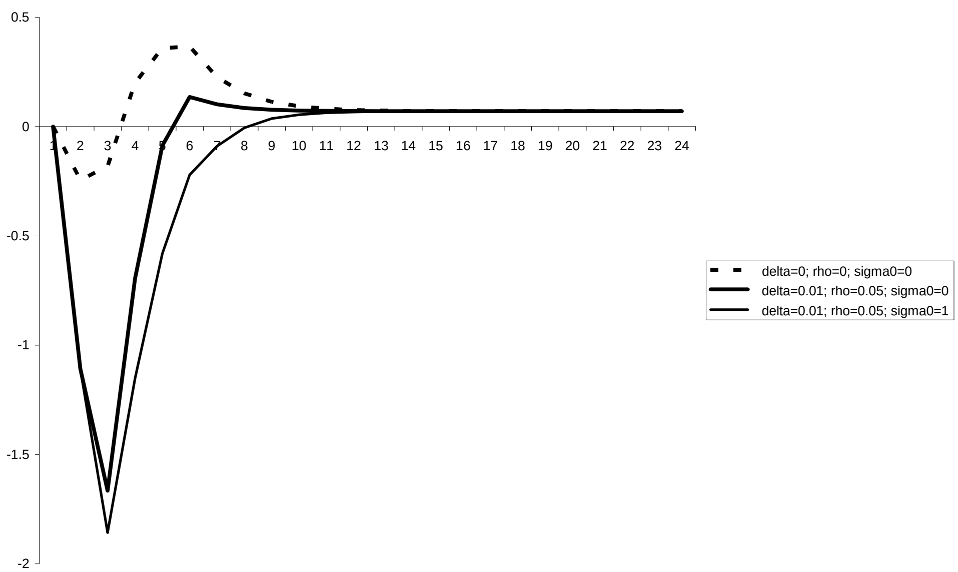

Figure 4.1: **Imperfect Credibility Result:** *Output effect of a gradual* ($T = 6$) *disinflation from a 'small' (3%) initial annual inflation rate.*

for time varying velocity and imperfect credibility). Our emphasis is on gradual disinflation from low initial inflation rates.

Figure 4.1 shows the output effect of a gradual ($T = 6$) disinflation from a low (3%) initial annual inflation rate when we have perfect credibility and constant velocity and with time varying velocity with both perfect ($\sigma_0 = 0$) and imperfect credibility ($\sigma_0 = 1$). Different values for the parameters δ and ρ capture different degrees of time varying velocity, but here they have been set to reflect empirical estimates of the relevant opportunity cost and income elasticities of the demand for money; $\rho = 0.05$ (as in Ball 2001) while δ takes the minimum value for the range of values $[0.01, 0.03]$ reported in a survey of recent empirical money demand studies (Sriram 2001). The minimum value has been chosen here because the early output loss is an increasing function of δ and the arguments we are about to develop become even stronger for higher values of δ. Figure 4.1 shows that relative to Ireland's early output loss and compensating boom, the impact of having endogenized time varying velocity is to induce much larger early output losses and a much moderated boom. The impact of having also allowed for imperfect credibility is to further deepen the early output loss and leave no hint of an output boom. It is clear from the output path from the full model that the disinflation i) involves sacrificing output for a considerable time - in excess of 3 years; and ii) brings about a higher steady state value of output which could only compensate for the early output loss after a substantial period of time has elapsed. Indeed, as can be seen from Figure 4.2, in which we superimpose the output path resulting from an immediate disinflation in Ireland's model our full model yields early output losses broadly similar to the impact of an immediate disinflation in Ireland's original model. Of course, in Ireland's case these large losses could be avoided by opting for a gradual disinflation. However, there is no similar option here - thereby raising a key question about whether gradual disinflation is beneficial.

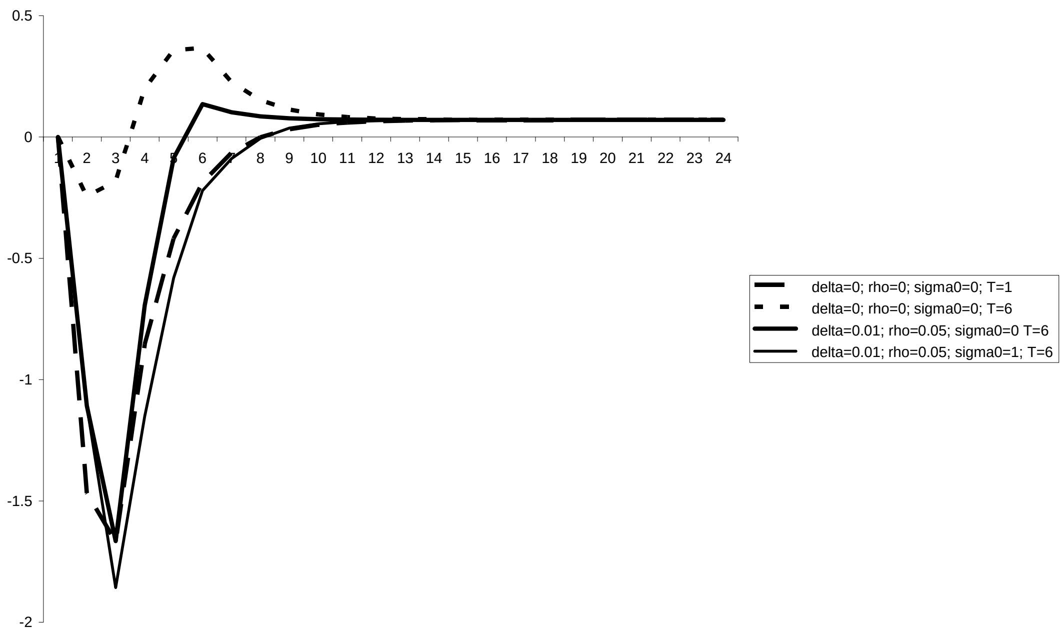

Figure 4.2: **Imperfect Credibility - Comparison Result:** *Output effect of a gradual disinflation from a 'small' (3%) initial annual inflation rate compared with that of immediate disinflation of Ireland's benchmark model .*

To explore this issue further, we construct a crude measure of the overall impact on output by projecting forward over a 30 year time horizon and calculating the net output gain. Tables 2 and 3 sets out the value of the area between the 'output path' and the x axis for a range of δ values for two cases of perfect and imperfect credibility respectively[7]. The area below the axis gives the output loss, and that above the axis gives the output gain. The absolute size of the overall impact is noted in the final column and defined to be the net output gain. We can

[7] δ has been allowed to vary but $\rho = 0$ is maintained. For values of $\rho \in (0, 1)$ the calculated output loss is yet higher.

see that for sufficiently high values of δ the overall impact on output is negative. (If we were to calculate present values, overall net losses would arise at even lower levels of δ). Also, we can see that imperfect credibility serves to ;lower the value of δ at which the overall impact on putput is negative.

δ	**Loss**	**Gain**	**Net Output Gain**
0	(0.42)	4.97	4.55
0.001	(0.65)	4.82	4.17
0.005	(1.72)	4.38	2.66
0.01	(3.22)	4.00	0.77
0.02	(6.60)	3.61	-2.99
0.03	(10.22)	3.49	-6.73
0.05	(17.56)	3.40	-14.15

Table 2. *Overall impact on real output of a gradual disinflation from a 3% initial annual inflation rate under perfect credibility for different values of the velocity parameter* δ (with $\rho = 0, \sigma_0 = 0$).

δ	**Loss**	**Gain**	**Net Output Gain**
0	(0.98)	3.63	2.64
0.001	(1.33)	3.60	2.26
0.005	(2.76)	3.54	0.77
0.01	(4.58)	3.49	-1.08
0.02	(8.23)	3.44	-4.79
0.03	(11.89)	3.41	-8.48
0.05	(19.17)	3.36	-15.81

Table 3. *Overall impact on real output of a gradual disinflation from a 3% initial annual inflation rate under imperfect credibility for different values of the velocity parameter* δ (with $\rho = 0$, $\sigma_0 = 1$).

In the light of these results, Ireland's (1997) conclusion that small inflations are best ended gradually may need to be qualified: it seems that even disinflating a low inflation gradually may be undesirable since the net 'overall impact' on the real economy may be negative. This shift in the potential policy conclusion is attributable to the introduction of time varying velocity and imperfect credibility

The intuition of the impact of having endogenized time varying velocity is that a disinflation reduces both inflation and consumption in the short run and it is also associated with a decline in velocity (from (5)). However, this decline in velocity means equivalently, that for given levels of prices and consumption, nominal money demand would have to rise. This increase in money demand exacerbates the excess demand created by the disinflationary reduction in the money supply. Since prices are slow to adjust, the recessionary output effect is magnified, making time varying velocity costly on output.

With imperfect credibility, agents only gradually come to realize that the price-level is to grow at a zero rate—a realization that is all the more tardy because of the gradualness of the disinflationary process itself. This tardiness results in more of the necessary adjustment being borne by output losses than prices. Under imperfect credibility, the initial contraction in output is more severe for any initial inflation rate than under perfect credibility.

The analysis in this section raises important question for policymakers faced with low inflation. When considering the choice between disinflating gradually and immediately, Ireland advocated a preference for gradual disinflation since immediate disinflation generates unambiguously bigger output losses under the assumptions of perfect credibility and constant velocity. Using the full model developed here (which relaxes both of these strong assumption simultaneously) we have found that, of itself, this scale of bigger will ensues from gradual disinflation. This shifts our focus to the policy question of alternative rates of gradual disinflations. Put more succinctly, we turn to the question of an optimal speed of disinflation.

5. Optimal Speed of Disinflation

As (Ireland 1997) recognises, the optimal speed of disinflation is important information for policymakers who must decide on the time horizon over which they bring about price stability. In this paper we calculate the optimal speed of disinflation in our time varying velocity model[8]. To do this, we use our extended non-linear solution method to calculate the level of utility associated with different speeds of disinflation for a range of initial inflation rates from 1% to 25%. The optimal speed of disinflation is given by the length of time for which utility is maximized[9].[11]

As we have seen so far, a gradual disinflation in a model with price stickiness and constant velocity results in output rising above the new steady-state for some time after the initial fall[10].[12] However, from higher initial inflation rates, the contraction in output in the early periods of the disinflation is more pronounced, increasingly offsetting the utility gain from any subsequent boom pushing the optimal speed of disinflation up. Since the time varying velocity effect on output makes the initial contraction yet more severe and makes the disinflationary boom almost disappear, utility is much lower than in the case where velocity is constant ($\delta = 0$, $\rho = 0$). Since more gradual disinflation boosts utility, the time needed for the disinflation would need to be longer (i.e. the optimal speed of disinflation slower).

Indeed, it turns out that a more gradual period of disinflation is optimal for the case in which $\delta > 0$ and/or $\rho > 0$. This can be seen in Figure 5.1 where we illustrate

[8] At this stage, we limit our attention to the case of perfect credibility, $\sigma_0 = 0$. Work is currently in progress on the case of imperfect credbility, $\sigma_0 = 1$.

[9] The calculation was conducted for lengths of the disinflation period ranging from immediate $T = 1$ to more gradual $T = 45$, where T is the length of the period of disinflation measured in half yearly intervals.

[10] To calculate the optimal speed of disinflation, we maximize utility which derives from labour as well as consumption. In this framework, consumption follows the same path as output.

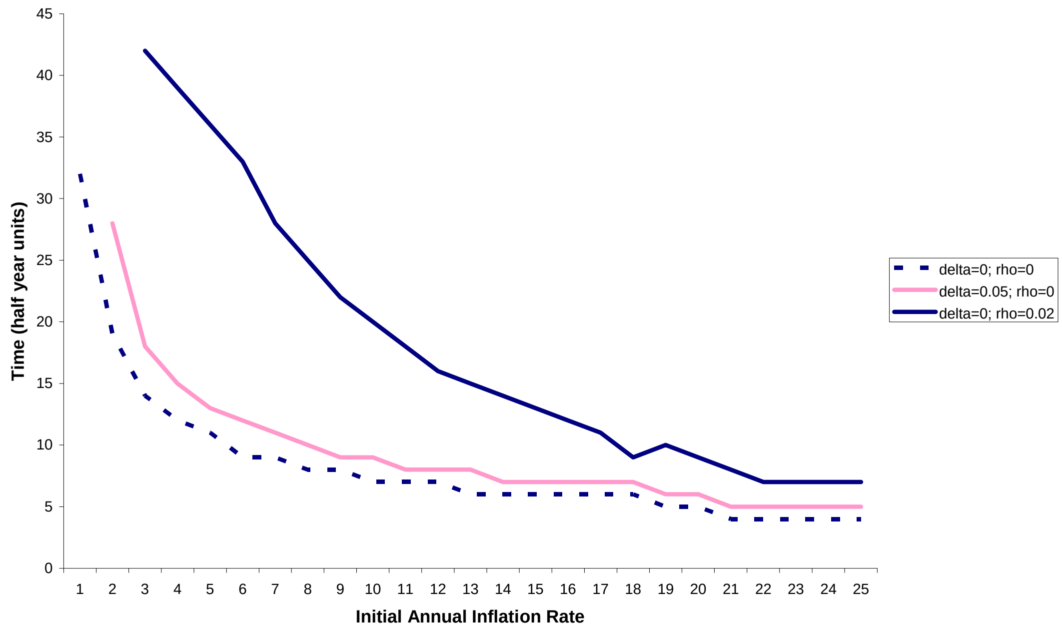

Figure 5.1: **Optimal Speed of Disinflation:** *For any given values of δ and ρ, higher initial inflation rates need to be deflated over shorter time horizons. Higher values of δ and/or ρ shift the relationship such that deflation periods need to be longer,* $(\sigma_0 = 0)$.

the cases in which $\delta = 0.05$, $\rho = 0$ and $\delta = 0$, $\rho = 0.02$ alongside the 'benchmark case' ($\delta = 0$, $\rho = 0$). Across the range of initial inflation rates, the optimal speed of disinflation is decreased. For example, for an initial inflation rate of around 10%, approximately a year longer is required for the disinflationary time period when $\delta = 0.05$, $\rho = 0$, and approximately 6 years is required for the deflationary time period when $\delta = 0$, $\rho = 0.02$. For initial inflation rates greater than the upper teens, the disinflationary period would need to be approximately 6 months longer when $\delta = 0.05$, $\rho = 0$ and approximately 18 months longer when $\delta = 0$, $\rho = 0.02$. It is evident that allowing for time varying velocity through δ (the scale elasticity of money demand parameter) affects the optimal speed of disinflation by less than allowing for time varying velocity through ρ (the opportunity cost parameter). This is because, as inflation is brought down, the demand for money rises, moderating the downward pressure on money demand from the induced output change.

When velocity is time varying, utility maximizing policymakers cannot necessarily rely on a disinflationary boom to compensate the (greater) early losses in output. To avoid the early extra cost imposed by time varying velocity, a more gradual disinflation makes the contractions in activity in the early period of the disinflation less sharp.

6. Discussion and Conclusions

This paper examines the case for making the transition from low inflations to price stability. In crude terms, if the benefits of price stability exceed the cost of transition, disinflationary policy actions may be justified. The analysis is carried out with a DSGE model with sticky prices, time varying velocity and imperfect credibility. The nonlinear solution method reveals that early output losses may be more pronounced and more prolonged than previously suggested in the literature and there may be insufficient compensation from a subsequent higher steady state to justify taking any disinflationary policy action in some cases.

References

[1] Abel, Andrew. (1997) "Comment on 'The Costs and Benefits of Going from Low Inflation to Price Stability'." In *Reducing Inflation: Motivation and Strategy*, edited by Christina D. Romer and David H. Romer, pp.156-66. Chicago: University of Chicago Press for NBER.

[2] Almeida, Heito and Marco Bonomo. (2002) "Optimal State-Dependent Rules, Credibility and Inflation Inertia." *Journal of Monetary Economics*, 49:7, 1317-36.

[3] Bailey, Martin J. (1956) "The Welfare Cost of Inflationary Finance." *Journal of Political Economy*, 64:April, 93-110.

[4] Ball, Laurence. (1994) "Credible Disinflation with Staggered Price-Settings." *American Economic Review*, 84:1, 282-89.

[5] Ball, Laurence. (2001) "Another Look at Long-Run Money Demand." *Journal of Monetary Economics*, 47:1, 31-44.

[6] Ball, Laurence, N. Gregory Mankiw, and David Romer. (1988) "The New Keynesian Economics and the Output-Inflation Trade-Off." *Brookings Papers on Economic Activity*, 1:1, 1-65.

[7] Ball, Laurence, and N. Gregory Mankiw. (1994) "Asymmetric Price Adjustment and Economic Fluctuations." *Economic Journal*, 104:423, 247-61.

[8] Barro, Robert J. and David B. Gordon. (1983) "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics*, 12:11, 101-21.

[9] Bonomo, Marco and Carlos Carvalho. (2009) "Imperfectly-Credible Disinflation Under Endogenous Time-Dependent Pricing." *Working Paper, Graduate School of Economics, Getulio Vargus Foundation – FGV, Brazil.*

[10] Chada, Jagjit S., Andrew G. Haldane, and Norbert G.J. Janssen. (1998), "Shoe-leather Costs Reconsidered." *Economic Journal*, 108, 363-382.

[11] Cooley, Thomas F. and Gary D. Hansen. (1991) "The Welfare Costs of Moderate Inflations." *Journal of Money, Credit and Banking*, 23:3(2), 483-503.

[12] Dibooglu, Sel and Turalay Kenc. (2009) "Welfare Cost of Inflation in a Stochastic Balanced Growth Model." E*conomic Modelling*, 26, 650-58.

[13] Dotsey, Michael and Peter J. Ireland. (1996) "The Welfare Cost of Inflation in General Equilibrium." *Journal of Monetary Economics*, 37:1, 29-47.

[14] Evans, Lynne and Anamaria Nicolae. (2009) "The Output Effect of a Transition to Price Stability When Velocity is Time-varying." *Journal of Money, Credit and Banking*, forthcoming

[15] Feldstein, Martin. (1997) "The Costs and Benefits of Going from Low Inflation to Price Stability." In *Reducing Inflation: Motivation and Strategy*, edited by Christina D. Romer and David H. Romer, pp.123-56. Chicago: University of Chicago Press for NBER.

[16] Friedman, Milton. (1969) *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine Publishing Company.

[17] Ireland, Peter N. (1997) "Stopping Inflations, Big and Small." *Journal of Money, Credit and Banking*, 29:4, 759-75.

[18] Ireland, Peter J. (2009) "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review*, 99:3, 1040-52.

[19] Khan, Aubik, Robert G. King, and Alexander L. Wolman. (2003) "Optimal Monetary Policy." *Review of Economic Studies*, 70:4, 825-60.

[20] King, Robert G, and Alexander L. Wolman. (1999) "What Should the Monetary Authority Do When Prices Are Sticky?" In *Monetary Policy Rules*, edited by John Taylor, pp. 349-404. Chicago: University of Chicago Press for NBER.

[21] Lucas, Robert E. (2000) "Inflation and Welfare." *Econometrica*, 68:2, 247-274.

[22] Nicolae, Anamaria, and Charles Nolan. (2006) "The Impact of Imperfect Credibility in a Transition to Price Stability." *Journal of Money, Credit and Banking*, 38:1, 47-66.

[23] Rotemberg, Julio J, and Michael Woodford. (1992) "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity." *Journal of Political Economy*, 100:6, 1153-207.

[24] Serletis, Apostolos and Kazem Yavari. (2004) "The Welfare Cost of Inflation in Canada and the United States." *Economics Letters*, 84, 199-204.

[25] Shiller, Robert J. (1997) "Why Do People Dislike Inflation?" In *Reducing Inflation: Motivation and Strategy*, edited by Christina D. Romer and David H. Romer, pp.13-69. ChicGO: University of Chicago Press for NBER.

[26] Sriram, Subramanian S. (2001) "A Survey of Recent Empirical Money Demand Studies." *IMF Staff Papers*, 47:3, 334-65.