Scattering in THz Imaging

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1. ABSTRACT

A new mathematical method, the *Phase Distribution Model*, is devised for the calculation of attenuation and scattering of THz radiation in random materials. The accuracy of the approximation is tested by comparison with exact calculations and with experimental measurements on textiles and specially constructed phantoms.

Keywords: THz, scattering, attenuation, random materials

1. INTRODUCTION

At Durham, we are carrying out parallel theoretical and experimental work to develop an understanding of THz attenuation and scattering in randomly structured materials such as clothing and powders. On the one hand, scattering has adverse effects: it may produce false signatures in spectra when interference takes place within a scattering structure (e.g. fibres in clothing or granules of powder), or diminish and scramble the return signal from a suspect item secreted below garments. On the other hand, it might be used to advantage to determine the characteristic size, texture and location of an object concealed within a matrix of other material. Scattering effects are particularly relevant in this spectral regime, where the wavelength, and the size and separation of scattering centres, are often commensurable.

A mathematical model is needed to relate THz propagation to the physical properties of the material. Exact solutions of Maxwell's equations are prohibitively difficult for random structures; so empirical approximation methods are needed. This is of especial significance in practical systems for the recognition of hidden illegal substances on a suspect some distance from the observer. Although considerable previous work has been published in this area, it is evident that agreement between theory and experiment is not always good, especially for an assembly of closely-packed scatterers: for example, Pearce and Mittleman [1] have determined the mean free path for scattering through a dense collection of sphere and compared with the predictions of both exact (Mie) scattering and a quasi-crystal approximation; typical differences of one order of magnitude are shown between experiment and theory.

In view of the need to develop a mathematical procedure to predict rapidly the scattering and attenuation properties of an inhomogeneous material, we have developed a *Phase Distribution Model* (PDM). Modelling proceeds by dividing the material into slices perpendicular to the incident beam, with slice thickness chosen to satisfy two criteria:

(a) The thickness should be great enough that there is no average correlation between the scatterer positions in adjacent slices and interference effects between scattered waves for different slices will average out;

(b) The slice should be thin enough to allow simple approximations for transmission to be used.

The first requirement is to produce a satisfactory model for the attenuation of the direct beam. The accuracy of the model can be checked by comparison with exact calculations and with measurements on specifically constructed artefacts with known physical properties. Similar comparisons for a model of scattering then follow. The measurements reported here are undertaken with a standard broadband spectrometer system of the type reported elsewhere [2]. The bandwidth available is approximately 3THz.

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2. PHASE DISTRIBUTION MODEL

Transmission of a plane incident wave of unit amplitude through the slice results in a variation of amplitude $a(\underline{\rho})$ and phase $\varphi(\underline{\rho})$ with co-ordinate $\underline{\rho}$ over the second surface. The forward propagating unscattered wave amplitude is given by

$$F = \int_W a(\rho) e^{i\varphi(\rho)} \frac{d^2\rho}{W},$$

where the integral extends over the wavefront, area W. For the calculation of the unscattered wave, the spatial dependence of the field is not the significant property, and the relevant information can be summarised in a phase distribution function defined by

$$P(\varphi_1) = \int_W a(\rho)\delta(\varphi(\rho) - \varphi_1)\frac{d^2\rho}{W}.$$

Thus the unscattered amplitude is given by

$$F = \int P(\varphi_1) e^{i\varphi} d\varphi,$$

and the transmitted intensity through the slice is $|F|^2$.

The mean free path for transmission through the complete material is then

$$M.F.P. = -\frac{d}{\ln|F|^2},$$

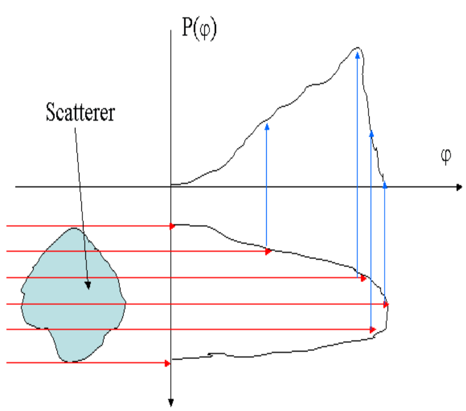

Figure 1 Projection approximation for phase distribution.

where d is the slice thickness.

The distribution function is then amenable to a statistical treatment. The problem is thus reduced to finding suitable approximations for the calculation of P.

For an isolated scattering object occupying a small part, S, of the wavefront, W, the total scattering cross-section is

$$Q = 2S(1 - \int P(\varphi)\cos\varphi d\varphi) \ .$$

This result for the total cross-section can be used to validate approximation methods by comparison with the results of exact calculations, such as Mie theory, for spheres or cylinders.

For a moderate range of refractive indices, a useful method is to project the optical density of the slice on to the second surface [3] and hence calculate the phase change produced [Fig.1]. This can be carried out for any reasonable shapes of scatterers. For the case of a sphere, radius b and refractive index n, the resulting phase change distribution is

$$P(\varphi) = \frac{2\varphi}{\varphi_M^2},$$

for $0 < \varphi \leq \varphi_M$, where φ_M is the maximum phase change along a path through the centre

$$\varphi_M = \frac{4\pi b(n-1)}{\lambda}.$$

3. COMPARISONS WITH THEORY AND EXPERIMENT

The comparison of the resulting calculated cross-section with Mie theory, and a similar comparison for cylindrical scatterers are shown in Fig. 2 and Fig. 3. The agreement is satisfactory over a wide range of wavelengths and diameters for refractive indices not exceeding 2.

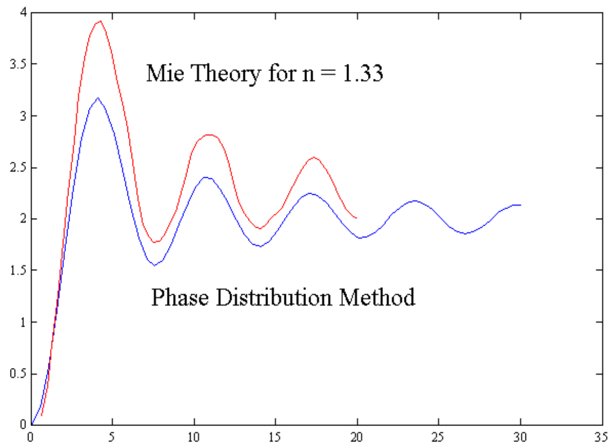

Figure 2. Normalized scattering cross-section for spheres.

Calculations of exact solutions of Maxwell's equations for randomly arranged dielectric cylinders, using Pendry's code [4], are in progress at Durham. At long wavelengths there is good agreement with the phase distribution picture, but at shorter wavelengths the computing is not yet complete. The phase distribution method has also been compared with measurements of transmission through samples of known physical properties.

As noted in section 1, above, Pearce and Mittleman [1] have measured transmission through randomly arranged teflon spheres and have found that theoretical treatments based on the 'quasi-crystalline approximation' or Mie theory for isolated spheres do not give a satisfactory description of the data. Figure 4 shows the comparison of the data with the results from the PDM calculation. The general trend of the data is well described, but the pair distribution function introduces oscillations which are not included.

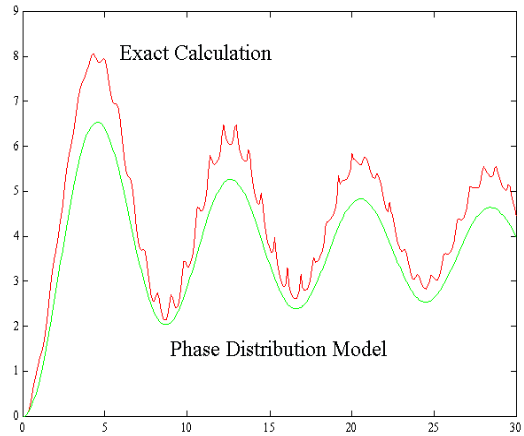

Figure 3. Normalized scattering cross-section for cylinders.

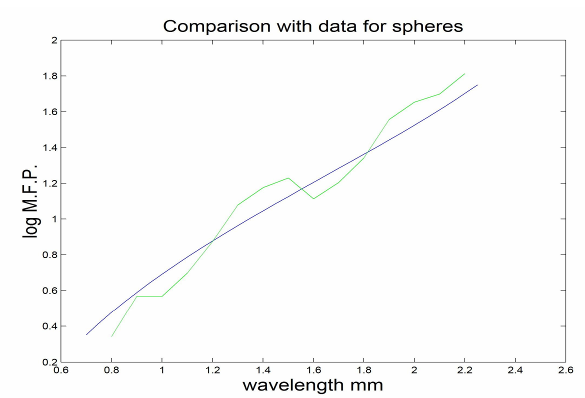

Figure 4. PDM prediction of attenuation in random spheres.

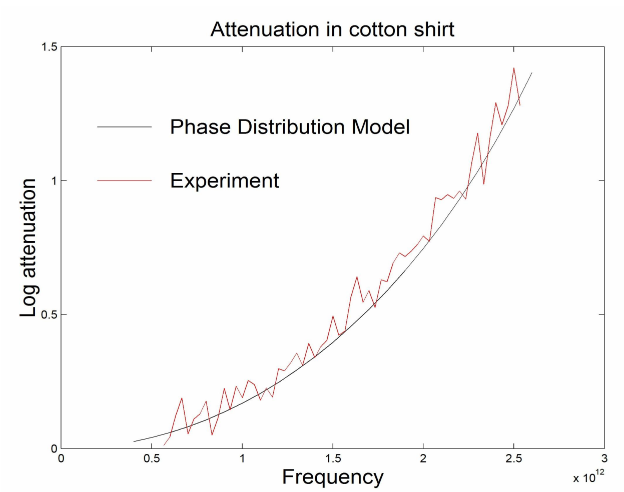

Figure 5: PDM fit with measured shirt parameters.

Transmission measurements through clothing have been carried out by other workers [5] and also at Durham. The data for a variety of shirt fabrics have been successfully modelled using the projected optical density to calculate the phase changes. The yarns are treated as uniform cylinders with an effective refractive index, and diameters of order 0.2 mm. The fibres composing the yarn are typically 20 micron in diameter, too small to be resolved at THz frequencies. The parameters of the model are the diameters and spacing of the yarns, which can be measured directly, and the effective refractive index, treated as an empirical parameter. Figure 5 shows that the model is capable of describing a particular shirt transmission: fits to data taken by other workers [5] using the PDM indicates that this approach can also satisfactorily describe the attenuation of a range of different fabrics such as polyester, terylene and cotton.

Phantom test pieces in the form of randomly arranged parallel dielectric cylinders are under construction at Durham for investigation of attenuation and scattering. All the relevant physical properties of these phantoms, including the THz refractive index of the material, will then be available for a rigorous comparison of theory and experiment.

4. PHASE DISTRIBUTION MODEL FOR SCATTERING

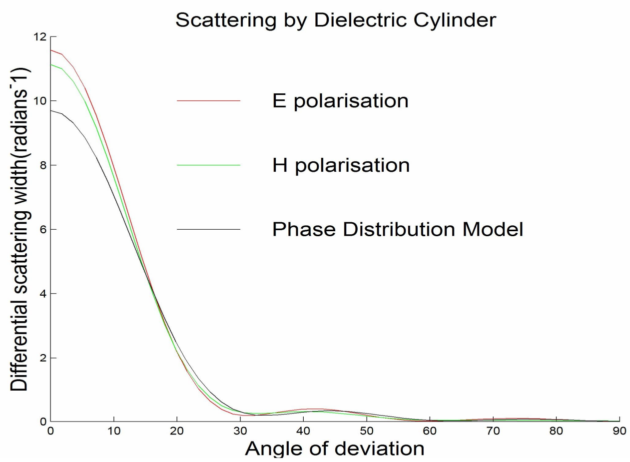

Figure 6. PDM with oblique projection.

Calculation of scattering in a randomly structured material is dependent on knowledge of the attenuation. If the mean free path is smaller than the sample thickness, multiple scattering and diffusion is expected. For a long mean free path, first-order scattering throughout the volume will predominate. The projected optical density method used for calculation of the unscattered amplitude is being adapted for the calculation of scattering by a slice containing scatterers. As before, the projected density can be calculated for any reasonably shaped scatterers, and comparison can be made with exact calculations of differential scattering cross-sections for spheres or cylinders in order to develop the method. The oblique direction of projection is taken to be intermediate between incident and scattering directions. Figure 7 shows the differential cross-section for a dielectric cylinder with refractive index 1.2 for E and H polarisations, together with the oblique projection approximation.

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