MASSIVE BLACK HOLES IN GALACTIC HALOS?

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ABSTRACT

We consider the idea that galaxy halos are composed of massive black holes as a possible resolution of two problems: the composition of dark halos, and the heating of stellar disks. We find that in order to account for the amount of disk heating that is observed in our Galaxy and others for which good data exists, the black holes must have masses $M_H \sim 10^6 \, M_\odot$. This heating mechanism makes predictions for the dependence of the velocity dispersion on time, and for the shape of the velocity ellipsoid, that are in good agreement with observations. It also predicts that the stellar distribution function is approximately isothermal at low epicyclic energies, but with a power-law tail at high energies, $N(E)dE \propto E^{-2}dE$, that typically contains about 1% of the stars. The latter provides a possible explanation for the high-velocity A stars found in the solar neighborhood. This mechanism can also account approximately for the observed constancy of the scale height of the stellar disks in spiral galaxies, though this does not result as naturally as the other properties. Among the predictions made are that we expect a significant fraction of normal galaxies to have one or a pair of massive black holes in their nuclei, and also that VLBI measurements will indicate that about 1% of sources are gravitationally lensed at the several milliarcsecond level. We consider the constraints on this picture set by dynamical friction causing black holes to spiral to the Galactic center, by the possible presence of dark matter in dwarf spheroidal galaxies, and by the accretion of interstellar gas by the black holes producing luminous objects in the Galaxy. This last may pose a problem for the scenario.

Subject Headings: black holes — galaxies: internal motions — galaxies: structure — gravitation

I. INTRODUCTION

In this paper, we investigate the possibility of a radical resolution of two important problems in galactic structure and evolution: the unknown composition of the dark halos around galaxies, and the uncertain mechanism for heating stellar disks in galaxies. (1) There is now abundant evidence that most of the mass in spiral galaxies, and possibly in elliptical galaxies also, is in a very low luminosity form, in dark halos that have a more extended distribution than most of the luminosity (Roberts 1975; Ostriker, Peebles, and Yahil 1974; Faber and Gallagher 1979). The observed flatness of the rotation curves at large radii in spiral galaxies shows that the density of these dark halos varies as $\rho_H \propto r^{-2}$ at large distances, and measurements of the flaring of the gas layers in these systems show that the halos are fairly round (Gunn 1980; van der Kruit 1981). However, it is still not known what the dark halos are made of, although the most straightforward possibilities have been ruled out by observations: the dark matter cannot consist of nuclear-burning stars of any mass (Boughn, Saulson, and Seldner 1981; Gilmore and Hewett 1983) or of gas (Peebles 1980; Hegyi and Olive 1983). Some other remaining possibilities are discussed by Hegyi (1984) and Primack and Blumenthal (1984). (2) The observed relation between velocity dispersion and age for stars in the solar neighborhood shows that some mechanism increases the velocity dispersions of stars in galactic disks after they are born (Wielen 1977). However, no mechanism has yet been shown able to account for all of the observed features of the acceleration process: gravitational scattering of stars by massive gas clouds in the galactic disk, as suggested by Spitzer and Schwarzschild (1951, 1953), may possibly be important for accelerating young stars but appears unable to explain the velocity dispersions of old disk stars (Lacey 1984c), while acceleration of stars by transient spiral density waves (Barbanis and Woltjer 1967; Carlberg and Sellwood 1985) has not been demonstrated to produce sufficient heating of the disk in the vertical direction.

As a resolution of both of these problems, we consider the possibility that dark halos may be composed of massive black holes. Then, assuming that the black holes are formed pregalactically, dark halos form by dissipationless collapse: if all parts of the protogalaxy collapse simultaneously, violent relaxation leads to a density profile $\rho_H \propto r^{-3}$ (van Albada 1982; McGlynn 1984), while if mass shells at larger radii collapse at successively later times, a flatter density profile results, with $\rho_H \propto r^{-2}$ (Gunn 1977; Fillmore and Goldreich 1984; Bertschinger 1985), which is in better agreement with the density profiles deduced from rotation curves. Massive black holes will also tend to lose energy to the stars in two-body encounters because of the tendency to equipartition. If we assume that *all* the heating of disk stars results from this effect, we can use it to determine the mass M_H of the holes. (The change in the velocities of spheroid stars by this mechanism will be relatively much less important because of the larger initial velocity dispersion in the latter case.) A rough derivation is as follows (a more detailed calculation is presented in § II): defining V_H to be the typical velocity of the black holes relative to the disk stars, we find for the change in the total velocity dispersion σ of the disk stars after a time Δt

$$\Delta(\sigma^2) \approx \langle(\Delta V)^2\rangle \Delta t \approx \frac{8\pi G^2 \rho_H M_H \ln \Lambda \, \Delta t}{V_H}, \tag{1}$$

for $\sigma \ll V_H$ (Chandrasekhar 1960), where $\ln \Lambda$ is the Coulomb logarithm. Note that for a fixed mass density ρ_H in perturbers,

determined from analysis of the rotation curve, the rate of energy change is proportional to M_H. For the solar neighborhood, we take $\Delta(\sigma^2) \approx \sigma^2$, with $\sigma \approx 80$ km s^{-1} for the oldest disk stars, $\Delta t \approx 10^{10}$ yr, $\rho_H \approx 10^{-2}$ $M_\odot$ pc^{-3}, $V_H \approx 300$ km s^{-1}, and $\ln \Lambda \approx 10$, and find $M_H \approx 10^6$ $M_\odot$. Bahcall, Hut and Tremaine (1985) have set a much smaller upper limit on the masses of the objects that constitute the unseen material in the disk, but their limit does not apply to the dark matter in the halo. We note in advance that a particular result of this type of heating is that while most of the stars will be given a roughly Maxwellian distribution of velocities, there is also a power-law tail due to occasional close encounters with black holes. Of order 1% of stars in the disk would be in the power-law tail if this theory is correct, and these would reach to large distances from the plane.

The plan of the remainder of the paper is as follows: In § II, we derive the evolution of the stellar velocity distribution at a particular point in the disk in more detail, including the effects of the epicyclic motions of the stars and of close encounters between stars and black holes. In § III, we compare the predictions of this mechanism with the observational constraints on disk heating. In § IV, we consider other tests of the idea that galactic halos consist of black holes with this mass. Section V presents our conclusions. Preliminary versions of some of the results of this paper have been presented elsewhere (Lacey 1984a, b).

II. EVOLUTION OF THE VELOCITY DISTRIBUTION OF DISK STARS

In § IIa we calculate the time evolution of the three components of the velocity dispersion of disk stars due to scattering of the stars by halo black holes. In § IIb we consider the shape of the distribution function in more detail, and in particular the high-energy tail which results from close encounters between stars and black holes. Throughout, we assume that the mass of the black holes is much greater than that of the stars, $M_H \gg m_*$, and that the halo velocity dispersion is isotropic and much larger than that of the stars, $\sigma_H \gg \sigma$.

a) Evolution of the Components of the Stellar Velocity Dispersion

The method we use here is the same as that used by Lacey (1984c) in the analogous problem of disk heating by giant molecular clouds: we calculate the evolution of the mean values of the epicyclic energies of the stars and then relate these to the components of the velocity dispersion. First, some definitions: we define (u, v, w) to be the components of velocity of a star relative to the local circular velocity $V_c(r)$:

$$u \equiv \dot{r}, \tag{2a}$$

$$v \equiv r\dot{\theta} - V_c(r), \tag{2b}$$

$$w \equiv \dot{z}, \tag{2c}$$

where (r, θ, z) are cylindrical polar coordinates. The background galactic gravitational field is assumed to be symmetric about $r = 0$ and $z = 0$. The orbits of most disk stars depart only slightly from circular orbits in the plane, and so, in the absence of perturbations, they are described approximately by standard (harmonic) epicyclic theory (e.g., Chandrasekhar 1960): superposed on the circular motion of the guiding center around the galaxy at frequency $\Omega(r) = V_c(r)/r$ is a harmonic oscillation in the plane at frequency $\kappa(r) = 2\Omega(r)[1 + \frac{1}{2} d \ln \Omega/d \ln r]^{1/2}$ and a harmonic oscillation perpendicular to the plane at frequency $\nu(r)$. The corresponding epicyclic energies, which are separately conserved in the absence of encounters, are

$$E_e = \tfrac{1}{2}(u^2 + \beta^2 v^2), \tag{3}$$

$$E_z = \tfrac{1}{2}(w^2 + v^2 z^2), \tag{4}$$

where

$$\beta \equiv 2\Omega/\kappa. \tag{5}$$

(See also equations [A1]–[A4] of the Appendix.) The other constant of the unperturbed motion is the angular momentum J:

$$J = rV_c(r) + rv. \tag{6}$$

We assume that the population of black holes in the halo, with local number density n_H and local mean rotation velocity $\bar{V}_H$, has an isotropic Gaussian distribution of random velocities with one-dimensional dispersion σ_H. Then, for $M_H \gg m_*$, the moments of the velocity change ΔV of a star due to encounters with the black holes, resolved parallel and perpendicular to the velocity of the star relative to the halo population, $V_{\rm rel}$, are, per unit time,

$$\langle \Delta V_\parallel \rangle = -\frac{4\pi G^2 n_H M_H^2 \ln \Lambda}{\sigma_H^2} \gamma G(\gamma), \tag{7a}$$

$$\langle \Delta V_\perp \rangle = 0, \tag{7b}$$

$$\langle (\Delta V_\parallel)^2 \rangle = \frac{4\sqrt{2} \pi G^2 n_H M_H^2 \ln \Lambda}{\sigma_H} G(\gamma), \tag{7c}$$

$$\langle (\Delta V_\perp)^2 \rangle = \frac{4\sqrt{2} \pi G^2 n_H M_H^2 \ln \Lambda}{\sigma_H} H(\gamma), \tag{7d}$$

$$\langle (\Delta V_\parallel)(\Delta V_\perp) \rangle = 0 \tag{7e}$$

(Chandrasekhar 1960), where we have defined

$$\gamma \equiv V_{rel}/\sqrt{2}\,\sigma_H \,, \tag{8}$$

and

$$G(\gamma) \equiv [\mathrm{erf}\,(\gamma) - \gamma\,\mathrm{erf}'\,(\gamma)]/2\gamma^3 \,, \tag{9a}$$

$$H(\gamma) \equiv [(2\gamma^2 - 1)\,\mathrm{erf}\,(\gamma) + \gamma\,\mathrm{erf}'\,(\gamma)]/2\gamma^3 \,, \tag{9b}$$

where erf (γ) is the error function

$$\mathrm{erf}\,(\gamma) \equiv \frac{2}{\sqrt{\pi}} \int_0^\gamma \exp\,(-y^2)dy \,.$$

[Note that the functions $G(\gamma)$ and $H(\gamma)$ defined here differ from those defined by Chandrasekhar.] The Coulomb logarithm has the value

$$\ln \Lambda \approx \ln\,(l_{max}(V_{rel}^2 + 3\sigma_H^2)/GM_H) \,, \tag{10}$$

where the maximum impact parameter $l_{max} \sim r$. In equations (7a)–(7e) we have neglected "nondominant" terms, which are $O(1/\ln \Lambda)$ relative to the leading terms. This is a reasonable approximation, since $\ln \Lambda \approx 10$ for cases of interest. To the same order of approximation, all the higher moments of ΔV vanish. We return later to the effects of these nondominant terms on the velocity distribution.

In terms of the components of the epicyclic velocity, the velocity of the star relative to the black hole population is

$$V_{rel} = [u, v + (V_c - \bar{V}_H), w] \,. \tag{11}$$

Resolving the moments in equations (7a)–(7e) parallel to the (u, v, w) axes, and assuming $u, v, w \ll \sigma_H$, we find

$$\langle \Delta u \rangle \approx 0 \,, \tag{12a}$$

$$\langle \Delta v \rangle \approx \langle \Delta V_\parallel \rangle_0 \,, \tag{12b}$$

$$\langle \Delta w \rangle \approx 0 \,, \tag{12c}$$

$$\langle (\Delta u)^2 \rangle \approx \tfrac{1}{2}\langle (\Delta V_\perp)^2 \rangle_0 \,, \tag{12d}$$

$$\langle (\Delta v)^2 \rangle \approx \langle (\Delta V_\parallel)^2 \rangle_0 \,, \tag{12e}$$

$$\langle (\Delta w)^2 \rangle \approx \tfrac{1}{2}\langle (\Delta V_\perp)^2 \rangle_0 \,, \tag{12f}$$

where the error made is $O(u/\sigma_H)$, and all other moments vanish in the same approximation. The subscript zero means that an expression is to be evaluated at $u = v = w = 0$, i.e., for $V_{rel} = (V_c - \bar{V}_H) \equiv V_0$. Thus, to this order of approximation, the moments of $(\Delta u, \Delta v, \Delta w)$ are independent of the epicyclic motion of the star.

From equations (12a)–(12f) we derive the mean rates of change of E_e and E_z averaged around an epicyclic orbit, as described in the Appendix:

$$\langle \Delta E_e \rangle_{av} \approx \tfrac{1}{4}\langle (\Delta V_\perp)^2 \rangle_0 + \tfrac{1}{2}\beta^2\langle (\Delta V_\parallel)^2 \rangle_0 \,, \tag{13}$$

$$\langle \Delta E_z \rangle_{av} \approx \tfrac{1}{4}\langle (\Delta V_\perp)^2 \rangle_0 \,, \tag{14}$$

where the fractional error is now only $O(u^2/\sigma_H^2)$ because of the orbit-averaging. We assume that the evolution due to encounters occurs on a time scale long compared to the orbital time scale, so that the stellar distribution function is in a quasi-steady state, with the phase-space density f depending on position and velocity only through the constants of the unperturbed motion (Jeans' theorem), i.e., $f = f(E_e, E_z, J, t)$. We define $\langle E_e \rangle$ and $\langle E_z \rangle$ to be the mean values of the epicyclic energies, averaged over all stars at fixed J. Then, since $\langle \Delta E_e \rangle_{av}$ and $\langle \Delta E_z \rangle_{av}$ are approximately independent of E_e and E_z, we find, combining equations (7), (13), and (14),

$$\frac{d\langle E_e \rangle}{dt} \approx \langle \Delta E_e \rangle_{av}$$

$$\approx \frac{\sqrt{2}\,\pi G^2 \rho_H M_H \ln \Lambda}{\sigma_H}\,[2\beta^2 G(\gamma_0) + H(\gamma_0)] \equiv D_e \,, \tag{15}$$

$$\frac{d\langle E_z \rangle}{dt} \approx \langle \Delta E_z \rangle_{av}$$

$$\approx \frac{\sqrt{2}\,\pi G^2 \rho_H M_H \ln \Lambda}{\sigma_H}\,H(\gamma_0) \equiv D_z \,, \tag{16}$$

where

$$\gamma_0 \equiv V_0/\sqrt{2}\,\sigma_H = (V_c - \bar{V}_H)/\sqrt{2}\,\sigma_H \,, \tag{17}$$

and $\rho_H \equiv n_H M_H$. In the approximation that radial gradients in the stellar distribution function can be neglected, the components of velocity dispersion, averaged over all stars at fixed r, are related to the mean epicyclic energies by

$$\sigma_u^2 \equiv \langle (u - \langle u \rangle)^2 \rangle = \langle E_e \rangle , \tag{18}$$

$$\sigma_v^2 \equiv \langle (v - \langle v \rangle)^2 \rangle = \langle E_e \rangle / \beta^2 , \tag{19}$$

$$\sigma_w^2 \equiv \langle (w - \langle w \rangle)^2 \rangle = \langle E_z \rangle , \tag{20}$$

which follows from equations (A1)–(A4) and the assumption of a quasi-steady state. Radial gradients introduce correction terms of $O(\sigma^2/V_c^2)$, when $\langle E_e \rangle$ and $\langle E_z \rangle$ are treated as functions of r through the zero-order relation $J = rV_c(r)$. Combining equations (18)–(20) and (15)–(16), we find that the components of the velocity dispersion evolve as

$$\sigma_u(t) = (\sigma_{u0}^2 + D_e t)^{1/2} , \tag{21}$$

$$\sigma_v(t) = \sigma_u(t)/\beta , \tag{22}$$

$$\sigma_w(t) = (\sigma_{w0}^2 + D_z t)^{1/2} . \tag{23}$$

Thus the ratio of the tangential to radial velocity dispersions is constant in time

$$\sigma_v/\sigma_u = 1/\beta , \tag{24}$$

which simply results from epicyclic theory, while the ratio of vertical to radial velocity dispersions tends to a constant value for $\sigma^2 \gg \sigma_0^2$:

$$\sigma_w/\sigma_u \rightarrow (D_z/D_e)^{1/2} = [1 + 2\beta^2 G(\gamma_0)/H(\gamma_0)]^{-1/2} . \tag{25}$$

This ratio thus depends on the shape of the rotation curve through β, and on the dimensionless rotation velocity of the halo through γ_0. The total velocity dispersion $\sigma = (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)^{1/2}$ increases with time as

$$\sigma(t) = (\sigma_0^2 + Dt)^{1/2} , \tag{26}$$

where

$$D = \frac{\sqrt{2}\,\pi G^2 \rho_H M_H \ln \Lambda}{\sigma_H} \left[2(\beta^2 + 1)G(\gamma_0) + \left(2 + \frac{1}{\beta^2}\right)H(\gamma_0) \right] . \tag{27}$$

Note that the form derived for $\sigma(t)$, which is our main result, is intuitively obvious, because the stars receive equal but randomly directed increments in velocity in equal time intervals, leading to a random walk in velocity space, for which $\sigma \propto t^{1/2}$. This is quite different from the dependence $\sigma \propto t^{1/4}$ associated with scattering by giant molecular clouds in the disk (Lacey 1984c). Our result that the heating in the vertical direction is comparable to that in the horizontal direction also contrasts with the results for disk heating by transient spiral density waves, for which the vertical heating appears to be much less than the horizontal heating (Carlberg 1984). Results similar to equations (26) and (27) for the evolution of the total velocity dispersion, but not allowing for the epicyclic motions of the stars, have also been derived by Ipser and Semenzato (1985), in a preprint that we received while we were writing this paper. Our results are also related to those of Wielen (1977), who assumes, without deriving this from any specific physical mechanism, that the diffusion coefficients in velocity space are isotropic and independent of epicyclic velocity and derives an equation of the form (26). In our case, the diffusion coefficients *are* velocity-independent, but are *not* isotropic, except in the special case that the halo population is nonrotating relative to the disk population (i.e., $\gamma_0 = 0$), for which our results (24) and (25) for the shape of the velocity ellipsoid reduce to those of Wielen.

Modification of Results for Self-Gravitating Disk.—The previous analysis was for the case that the stellar population moves in a static background potential that is harmonic in z, and it must be modified if the population is self-gravitating in the vertical direction, since disk heating then makes the potential time-dependent. In § IIb we show that the stellar distribution function is approximately isothermal in z, so that in a self-gravitating disk, most of the stars move for most of their orbits in a potential which is approximately harmonic in z, with frequency $\nu(t) \propto \mu_D/\langle E_z \rangle^{1/2}$, where μ_D is the surface density of the disk and $\langle E_z \rangle$ is now an average over *all* the stars in the disk at that radius (see eqs. [48]–[49] and the discussion following them). The frequency ν is a slowly varying function of time through the time dependence of $\langle E_z \rangle$ and possibly of μ_D. For slow changes in the potential, E_e/κ and E_z/ν are adiabatically invariant. The evolution of the epicyclic energy in the plane $\langle E_e \rangle$ is therefore approximately unchanged, but the time dependence of ν leads to an extra contribution to $d\langle E_z \rangle/dt$, $(d\langle E_z \rangle/dt)_{ad} = (\dot\nu/\nu)\langle E_z \rangle$, which must be added to that of equation (16). Therefore equation (16) is replaced by

$$\frac{d\langle E_z \rangle}{dt} \approx \frac{2}{3}\langle \Delta E_z \rangle_{av} + \frac{2}{3}\left(\frac{\dot\mu_D}{\mu_D}\right)\langle E_z \rangle , \tag{28}$$

where the second term allows crudely for the effects of any change in the mass of the disk due to inflow or outflow of matter. If the latter term can be neglected, the ratio of vertical to radial velocity dispersions in the self-gravitating case tends to the value, for $\sigma^2 \gg \sigma_0^2$,

$$\sigma_w/\sigma_u \rightarrow (2D_z/3D_e)^{1/2} = (\tfrac{2}{3})^{1/2}[1 + 2\beta^2 G(\gamma_0)/H(\gamma_0)]^{-1/2} , \tag{29}$$

which is smaller by a factor $(2/3)^{1/2}$ than in the non-self-gravitating case. This result applies to the dominant part of the mass distribution in the disk, while equation (25) applies to a group of test particles with lower velocity dispersion.

b) Shape of the Stellar Distribution Function

Having derived the time-evolution of the components of the stellar velocity dispersion in the previous section, we now consider the shape of the distribution function in more detail. It is convenient to work in terms of the function $N(E_e, E_z, J, t)$, defined such that $N(E_e, E_z, J, t) dE_e dE_z dJ$ is the total number of stars in the range $dE_e dE_z dJ$. $N(E_e, E_z, J, t)$ is related to the phase-space density $f(E_e, E_z, J, t)$ by

$$N(E_e, E_z, J, t) = [(2\pi)^3/\kappa\nu] f(E_e, E_z, J, t) \tag{30}$$

(e.g., Lacey 1984*c*). Henceforth, the *J*-dependence will be suppressed. We also define

$$N_e(E_e, t) \equiv \int_0^\infty N(E_e, E_z, t) dE_z , \tag{31}$$

$$N_z(E_z, t) \equiv \int_0^\infty N(E_e, E_z, t) dE_e . \tag{32}$$

We also generalize our definition of E_z to allow for a potential in z, $\Phi(z)$, which is not harmonic:

$$E_z = \tfrac{1}{2}w^- + \Phi(z) , \tag{33}$$

(cf. eq. [4]). This modification can be important for stars with large *z*-energies. Equation (30) relating N and f remains valid in this case, but with $\nu = \nu(E_z)$. We again assume that the distribution of random velocities of the black holes is isotropic, but in this section we relax the requirement that it be Gaussian. (For an istropic, non-Gaussian velocity distribution, the analysis of § II*a* would proceed in the same way, but with more general expressions [e.g., Henon 1973] replacing eqs. [7*a*], [7*c*] and [7*d*] for $\langle \Delta V_\parallel \rangle$, $\langle (\Delta V_\parallel)^2 \rangle$ and $\langle (\Delta V_\perp)^2 \rangle$ respectively.)

The evolution of the distribution function may be analyzed simply in two limits: For low values of the epicyclic energies, the form of the distribution function results from the cumulative effect on the stars of many distant, weak encounters with black holes, which cause a diffusion in energy space. At high energies, the form of the distribution function results from single close encounters between stars and black holes, which have low probability but cause large changes in the energy of a star. The relative importance of distant and close encounters is determined by the value of ln A (e.g., Henon 1973). We will assume that ln $\Lambda \gg 1$, meaning that the distant encounters dominate the energy input into the stars. We now consider the two limits in turn.

i) *Low Energies*

The evolution in the low-energy, diffusion regime is described approximately by the orbit-averaged Fokker-Planck equation:

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial E_e}(\langle \Delta E_e \rangle_{av} N) - \frac{\partial}{\partial E_z}(\langle \Delta E_z \rangle_{av} N) + \frac{1}{2}\frac{\partial^2}{\partial E_e^2}[\langle (\Delta E_e)^2 \rangle_{av} N] + \frac{\partial^2}{\partial E_e \partial E_z}(\langle \Delta E_e \Delta E_z \rangle_{av} N) + \frac{1}{2}\frac{\partial^2}{\partial E_z^2}[\langle (\Delta E_z)^2 \rangle_{av} N] \tag{34}$$

(e.g., Henon 1973), where we have neglected the *J*-dependence. We assume that for low epicyclic energies, the harmonic approximation is valid for both vertical and horizontal epicyclic oscillations, and that the velocity-space diffusion coefficients are approximately independent of the epicyclic velocity (cf. § II*a*), so that the energy-space diffusion coefficients obey the relations derived in the Appendix. Then the Fokker-Planck equation separates: there are solutions of the form $N(E_e, E_z, t) = N_e(E_e, t) N_z(E_z, t)$, where

$$\frac{\partial N_z}{\partial t} = \langle \Delta E_z \rangle_{av}\left(\frac{\partial N_z}{\partial E_z} + E_z \frac{\partial^2 N_z}{\partial E_z^2}\right) , \tag{35}$$

and similarly for N_e. This has as a solution the isothermal distribution

$$N_z(E_z, t) = \frac{1}{\langle E_z \rangle} \exp\left(-\frac{E_z}{\langle E_z \rangle}\right) , \tag{36}$$

with $\langle E_z \rangle = \langle \Delta E_z \rangle_{av} t + \langle E_z \rangle_0$. We have arbitrarily normalized the integral of N_z over E_z to unity, since equation (35) is linear in N_z. There is an exactly similar solution for $N_e(E_e, t)$.

ii) *High Energies*

The form of the distribution function for large epicyclic energies directly reflects the probability distribution for the energy changes in a single close encounter, since the probability of a star undergoing two such close encounters is very small. Therefore, we start from the expression for the probability per unit time that a star moving at velocity V_{rel} relative to a population of black holes with isotropic distribution function $f_H(V_H)$ has its velocity changed in an encounter by ΔV:

$$R(V_{rel}, \Delta V) = \frac{4\pi G^2 M_H^2}{(\Delta V)^5}\int_0^\infty d(V_H^2) f_H(\sqrt{V_H^2 + a^2}) \tag{37}$$

(Goodman 1983; Ipser and Semenzato 1983), where, for $M_H \gg m_*$,

$$a^2 \equiv \frac{[V_{rel}\cdot\Delta V + \tfrac{1}{2}(\Delta V)^2]^2}{(\Delta V)^2} . \tag{38}$$

Here, $V_{rel} = (u, v + V_0, w)$. Assuming $u, v, w, \Delta u, \Delta v, \Delta w \ll \sigma_H$, $V_0 \sim \sigma_H$, we expand equation (38),

$$a^2 = \frac{(\Delta v)^2 V_0^2}{[(\Delta u)^2 + (\Delta v)^2 + (\Delta w)^2]} + O\left(\frac{v, \Delta v}{V_0}\right). \tag{39}$$

We also define the dimensionless halo distribution function $g_H(x)$ by $f_H(V_H) = (n_H/\sigma_H^3)g_H(V_H^2/\sigma_H^2)$. (A Gaussian distribution function, as assumed in § IIa, corresponds to $g_H(x) = 1/(2\pi)^{3/2} \exp(-x/2)$.) Substituting into equation (37), we obtain

$$R(\Delta V) = \frac{4\pi G^2 n_H M_H^2}{[(\Delta u)^2 + (\Delta v)^2 + (\Delta w)^2]^{5/2}\sigma_H} \int_0^\infty dx \, g_H\left[x + \frac{(\Delta v)^2 V_0^2/\sigma_H^2}{(\Delta u)^2 + (\Delta v)^2 + (\Delta w)^2}\right] + O\left(\frac{v, \Delta v}{\sigma_H}\right), \tag{40}$$

where we have suppressed the dependence of R on V_{rel}.

We now define the auxiliary variables $\Delta\xi = \frac{1}{2}[(\Delta u)^2 + \beta^2(\Delta v)^2]$, $\Delta\eta = \frac{1}{2}(\Delta w)^2$. These have the property that for large energy changes, with $\Delta E_e \gg E_e$, $\Delta E_z \gg E_z$, $\Delta E_e \approx \Delta\xi$, $\Delta E_z \approx \Delta\eta$ (see eqs. [A5] and [A6]). Also, integrating over the effects of all encounters, we have $\langle\Delta E_e\rangle_{av} \approx \langle\Delta\xi\rangle_{av}$, $\langle\Delta E_z\rangle_{av} \approx \langle\Delta\eta\rangle_{av}$, for $u, v, w \ll \sigma_H$ (eqs. [A8] and [A9]). Integrating over all values of $\Delta u, \Delta v$ that lead to the same value of $\Delta\xi$, we find that the probability per unit time for an encounter with $\Delta\xi$, $\Delta\eta$ is

$$K(\Delta\xi, \Delta\eta) \approx \frac{G^2 n_H M_H^2}{(\Delta\xi)^3 \sigma_H} F\left(\frac{\Delta\eta}{\Delta\xi}\right), \qquad (E_e, E_z, \Delta\xi, \Delta\eta) \ll \sigma_H^2, \tag{41}$$

where the error is now only $O\{[(\Delta\xi)^2, v^2]/\sigma_H^2\}$ after averaging over epicyclic phase, and where the dimensionless function $F(y)$ is defined to be

$$F(y) \equiv \frac{\pi}{\beta\sqrt{y}} \int_0^{2\pi} d\phi \frac{1}{(\sin^2\phi + (1/\beta^2)\cos^2\phi + y)^{5/2}} \int_0^\infty dx \, g_H\left[x + \frac{(2\gamma_0^2/\beta^2)\cos^2\phi}{(\sin^2\phi + (1/\beta^2)\cos^2\phi + y)}\right]. \tag{42}$$

Stars in the high-energy tail of the distribution function, with $E_e \gg \langle E_e\rangle$, $E_z \gg \langle E_z\rangle$, have gained essentially all their energy in a single encounter. Therefore, the form of the distribution function at high energies after a time t is, assuming the normalization $\iint dE_e \, dE_z \, N(E_e, E_z, t) = 1$,

$$N(E_e, E_z, t) \approx K(E_e, E_z)t$$

$$\approx \frac{G^2 n_H M_H^2 t}{(E_e)^3 \sigma_H} F\left(\frac{E_z}{E_e}\right), \qquad (\langle E_e\rangle, \langle E_z\rangle) \ll (E_e, E_z) \ll \sigma_H^2. \tag{43}$$

Thus, the high-energy tail of the distribution function has a power-law form, with the horizontal and vertical epicyclic energies E_e and E_z being correlated. In general, it is not possible to perform both integrals in equation (42) for $F(y)$ analytically. Therefore it is more illuminating to consider the projections N_e or N_z of the distribution function obtained by integrating $N(E_e, E_z, t)$ over E_z or E_e (eqs. [31] and [32]). We derive

$$N_e(E_e, t) = \frac{G^2 n_H M_H^2 t}{(E_e)^2 \sigma_H}\left(\int_0^\infty F(y)dy\right), \qquad \langle E_e\rangle \ll E_e \ll \sigma_H^2, \tag{44}$$

$$N_z(E_z, t) = \frac{G^2 n_H M_H^2 t}{(E_z)^2 \sigma_H}\left(\int_0^\infty yF(y)dy\right), \qquad \langle E_z\rangle \ll E_z \ll \sigma_H^2. \tag{45}$$

[N.B. $N(E_e, E_z, t) \neq N_e(E_e, t)N_z(E_z, t)$ in this case.] We can relate the amplitudes of the power-law tails on $N_e(E_e, t)$ and $N_z(E_z, t)$ to the mean energies: from equation (41) we find

$$\langle\Delta\eta\rangle = \int_0^\infty \int_0^\infty \Delta\eta K(\Delta\xi, \Delta\eta)d\Delta\xi d\Delta\eta$$

$$\approx \frac{G^2 n_H M_H^2}{\sigma_H} \ln\left(\frac{\Delta\eta_{max}}{\Delta\eta_{min}}\right) \int_0^\infty yF(y)dy, \tag{46}$$

where we have introduced upper and lower cutoffs $\Delta\eta_{max}$ and $\Delta\eta_{min}$ on the integration over $\Delta\eta$ because equation (41) for $K(\Delta\xi, \Delta\eta)$ is valid only over a certain range. The maximum possible velocity change in an encounter with a typical black hole is $\sim \sigma_H$, thus $\Delta\eta_{max} \sim \sigma_H^2$. The minimum possible velocity change in an encounter is $\sim GM_H/l_{max}\sigma_H$; thus $\Delta\eta_{min} \sim (GM_H/l_{max}\sigma_H)^2$. Therefore $\ln(\Delta\eta_{max}/\Delta\eta_{min}) \approx 2\ln\Lambda$ (see eq. [10]). Now $\langle\Delta E_z\rangle_{av} \approx \langle\Delta\eta\rangle$, and $\langle E_z\rangle = \langle\Delta E_z\rangle_{av}t + \langle E_z\rangle_0$, so combining these results with equations (45) and (46) and assuming that $\langle E_z\rangle \gg \langle E_z\rangle_0$, we find

$$N_z(E_z, t) \approx \frac{1}{2\ln\Lambda}\frac{\langle E_z\rangle}{E_z^2}, \qquad \langle E_z\rangle \ll E_z \ll \sigma_H^2. \tag{47}$$

An exactly similar result holds for $N_e(E_e, t)$. Comparing with equation (36), we find the interesting result that the time evolution of $N_e(E_e, t)$ and $N_z(E_z, t)$ is self-similar (at least in the low-energy and high-energy limits) and is independent of the form of the halo velocity distribution function (if it is isotropic). The fraction of stars in the high-energy tail, when defined in a dimensionless way, depends only on the value of $\ln\Lambda$.

iii) *Phase-Space Density*

To convert from $N(E_e, E_z, t)$ to the phase-space density $f(E_e, E_z, t)$ we need to know the vertical oscilliation frequency $v(E_z)$, which is constant for z-amplitudes smaller than the scale height of the disk but varies with energy for larger amplitudes. As a useful model, we assume that the background density $\rho_D(z)$ producing the potential $\Phi(z)$ is that of a self-gravitating isothermal disk, with $\rho_D(z) = \rho_0 \, \mathrm{sech}^2 \, (z/h_D)$, where the effective half-thickness is $h_D = \sigma_D^2/\pi G \mu_D$, μ_D being the surface density and σ_D the vertical velocity dispersion (see § IIId). This density distribution produces a potential

$$\Phi(z) \approx \tfrac{1}{2}v_0^2 z^2 , \qquad\qquad z \lesssim h_D ,$$
$$\approx \sqrt{2}\,\sigma_D v_0 z - 2 \ln 2 \, \sigma_D^2 , \qquad z \gtrsim h_D , \tag{48}$$

for which the vertical frequency is

$$v(E_z) \approx v_0 , \qquad\qquad E_z \lesssim \sigma_D^2 ,$$
$$\approx v_0 \frac{\pi}{2} \frac{\sigma_D}{E_z^{1/2}} , \qquad E_z \gtrsim \sigma_D^2 , \tag{49}$$

where $v_0 = (4\pi G\rho_0)^{1/2}$. We consider only the distribution function for vertical motions here: defining $f_z(E_z, t)$ analogously to $N_z(E_z, t)$, we have, by equation (30),

$$f_z(E_z, t) = \frac{v(E_z)}{2\pi} N_z(E_z, t) . \tag{50}$$

For the case that $\langle E_z \rangle \leq \sigma_D^2$ but that the power-law tail begins at some energy $E_{zc} > \sigma_D^2$, the phase-space density is approximately (combining eqs. [36], [47], [49], and [50])

$$f_z(E_z, t) \approx \frac{v_0}{2\pi\langle E_z \rangle} \exp\left(-\frac{E_z}{\langle E_z \rangle} \right) , \qquad E_z < E_{zc} ,$$
$$\approx \frac{v_0}{8 \ln \Lambda} \frac{\sigma_D \langle E_z \rangle}{E_z^{5/2}} , \qquad\qquad E_{zc} < E_z \lesssim \sigma_H^2 . \tag{51}$$

The above expression is exact in the low- and high-energy limits but is simply an interpolation in the intermediate range. The quantity E_{zc} is defined in such a way that the approximate expression for $f_z(E_z)$ is continuous:

$$\left(\frac{E_{zc}}{\langle E_z \rangle} \right)^{5/2} \exp\left(-\frac{E_{zc}}{\langle E_z \rangle} \right) \equiv \frac{\pi}{4 \ln \Lambda} \frac{\sigma_D}{\langle E_z \rangle^{1/2}} . \tag{52}$$

If the stellar population of interest is self-gravitating in the vertical direction, then $\sigma_D^2 \approx \langle E_z \rangle$ in the above expressions. The derivation of $N(E_e, E_z, t)$ should then be modified to take account of adiabatic invariance (cf. § IIa), but we ignore this complication here, since it does not change the essential features of the results.

III. OBSERVATIONAL TESTS OF DISK HEATING

We ccnsider four distinct observational tests of the idea that halo black holes account for essentially all the heating of stellar disks in galaxies: (a) the age dependence of the stellar velocity dispersion; (b) the shape of the velocity ellipsoid; (c) constraints on high velocity/large scale height components in disks; (d) the variation of the disk velocity dispersion/scale height with radius. For some of these tests, the best constraints come from detailed data on the solar neighborhood; for others, from observations of the large-scale structure of other galaxies.

a) *Age Dependence of the Velocity Dispersion*

The total velocity dispersion σ of a population of stars is predicted to vary with its age t according to equation (26) (neglecting the effects of self-gravity, which in this case are small), i.e., $\sigma \propto t^{1/2}$ for $\sigma^2 \gg \sigma_0^2$. The relation between velocity dispersion and age in the solar neighborhood has been estimated observationally by Wielen (1977), who derives space velocities from radial velocities, proper motions, and trigonometric parallaxes and assigns mean ages to groups of stars on the basis either of their position on the main sequence or of the statistics of calcium emission line strength, assuming that the star formation rate has been constant over the life of the disk. Wielen's data, which cover the range $t \sim 10^8$–10^{10} yr, are in excellent agreement with the theoretical prediction. The velocity dispersion–age relation has also been investigated observationally by Carlberg *et al.* (1985), using an independent method which involves calculating ages for individual F stars from Strömgren-band photometry and theoretical isochrones. Their results, however, are in disagreement with Wielen's data and with our prediction.

The magnitude of the stellar heating depends on the mass M_H of the black holes, and so cannot be predicted, since M_H is unknown. Instead, we derive a value of M_H on the assumption that this mechanism accounts for all the observed disk heating. The oldest age-group listed by Wielen (1977) has $\sigma \approx 80$ km s^{-1}, while the youngest has $\sigma_0 \approx 10$ km s^{-1}; therefore, assuming that the age of the disk is $t_1 = 15$ Gyr, we find from equation (26) that $D \approx 400$ km^2 s^{-2} Gyr^{-1}. We combine this with equation (27) to estimate M_H. We take $2^{1/2}\sigma_H \approx V_c \approx 220$ km s^{-1}, and in calculating $\ln \Lambda$ we take $l_{\max} \approx r \approx 8$ kpc. The value of $\ln \Lambda$ also depends weakly on M_H; it will turn out (see eq. [54]) that $\ln \Lambda \approx 12$ is appropriate. We also take $\beta = (2)^{1/2}$ (corresponding to a flat rotation

curve) and $\gamma_0 = 1$. (More details of the estimation of β and γ_0 are given in § III*b*, but the result for D is not very sensitive to their values.) We derive

$$\rho_H M_H \approx 2 \times 10^4 \; M_\odot^2 \; \text{pc}^{-3} \; . \tag{53}$$

For a singular isothermal sphere, we would have $\rho_H = V_c^2/4\pi Gr^2 \approx 1.4 \times 10^{-2} \; M_\odot \; \text{pc}^{-3}$, for the parameters quoted above. More detailed estimates of ρ_H can be made from Galactic mass models: the various models of Caldwell and Ostriker (1981); Ostriker and Caldwell (1983); and Bahcall, Schmidt, and Soneira (1983) all give values of ρ_H in the solar neighborhood within a factor of 2 of $\rho_H = 10^{-2} \; M_\odot \; \text{pc}^{-3}$. Adopting the latter value, we find

$$M_H \approx 2 \times 10^6 \; M_\odot \; . \tag{54}$$

If other mechanisms contribute toward heating the disk, then this is actually an upper limit.

b) Shape of the Velocity Ellipsoid

The ratio of velocity dispersions in the plane, σ_v/σ_u, is given by equation (24). It depends only on the assumption of dynamical equilibrium and therefore does not test the heating theory. The ratio of vertical to radial velocity dispersions, σ_w/σ_u, for $\sigma^2 \gg \sigma_0^2$, is given by equation (25) for a non–self-gravitating population that moves in a time-independent potential, and by equation (29) for a population which is self-gravitating in the vertical direction and has constant surface density. The velocity dispersion ratio in the latter case (denoted SG) is smaller by a factor $(\frac{2}{3})^{1/2}$ than in the former (denoted NSG) because of the effect of adiabatic cooling of the vertical motions. However, these results are modified if the disk mass is changed by inflow or outflow of gas. Late infall of matter into the disk, as is invoked in some models of chemical evolution (e.g., Twarog 1980; Lacey and Fall 1985), tends to offset the adiabatic cooling (cf. eq.[28]). For instance, for a constant infall rate and a balancing constant star formation rate, the effects of adiabatic cooling and adiabatic heating on the mean vertical velocity dispersion (averaged over stars of all ages) roughly cancel, so that σ_w/σ_u is approximately the same as in the NSG case. For infall and star formation rates which decay with time, which is probably more realistic, σ_w/σ_u is intermediate between the SG and the NSG cases. Conversely, if there is mass loss from the disk in the form of a galactic wind (e.g., McKee and Ostriker 1977; Wheeler and Ostriker 1985), the adiabatic cooling will be increased, reducing σ_w/σ_u further.

The ratio σ_w/σ_u as given by equation (25) or (29) depends on the parameters $\beta = 2\Omega/\kappa$ and $\gamma_0 = (V_c - \bar{V}_H)/2^{1/2} \; \sigma_H$. The quantity β may be estimated either from measurements of the Galactic rotation curve or from the observed value of the ratio σ_v/σ_u (using eq. [24]). γ_0 obviously cannot be determined directly, since the halo objects are unobserved. If the halo were a self-gravitating, nonrotating, singular isothermal sphere, we would have $\bar{V}_H = 0$, $V_c = 2^{1/2}\sigma_H$, and thus $\gamma_0 = 1$. If the halo rotates, γ_0 is reduced, though the fractional correction $\bar{V}_H/V_c$ is probably small. The value of γ_0 will also be modified due to the gravity of the luminous matter in the Galaxy, etc., which changes V_c and σ_H. The sensitivity of the results to γ_0 is not large, so we assume $\gamma_0 = 1$ in what follows. Figure 1 shows σ_v/σ_u and σ_w/σ_u plotted against β for $\gamma_0 = 1$.

Observations of the Galactic rotation curve (reviewed by Knapp 1983) imply that locally $V_c(r)$ is flat or slowly rising, with $d \ln V_c/d \ln r \approx 0.1 \pm 0.1$, which implies $\beta \approx 1.35 \pm 0.05$. For this value of β, we predict $\sigma_u:\sigma_v = 1:0.74 \pm 0.03$, and $\sigma_u:\sigma_w = 1:0.67 \pm 0.01$ (NSG) or $1:0.55 \pm 0.01$ (SG). The various observational determinations of the shape of the velocity ellipsoid in the solar
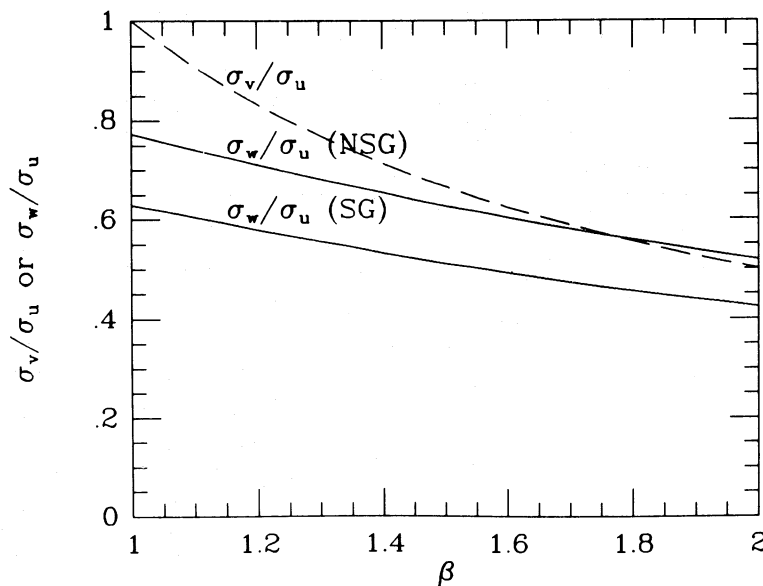

FIG. 1.—Dependence of velocity dispersion ratios on rotation curve shape. Plotted as functions of $\beta = 2\Omega/\kappa$ are σ_v/σ_u (eq. [24]) and σ_w/σ_u, for $\sigma^2 \gg \sigma_0^2$ and $\gamma_0 = 1$, in the self-gravitating (SG, eq. [29]) and non–self-gravitating (NSG, eq. [25]) cases.

neighborhood show significant discrepancies both with each other and with the value of β determined from the rotation curve, which makes a clean comparison with the theory difficult. We choose to compare our predictions with two recent observational determinations: Woolley *et al.* (1977) measured the components of the velocity dispersion from radial velocities for a sample of roughly 900 G and K stars (mainly giants) out to about 500 pc from the Sun. They find $\sigma_u : \sigma_v : \sigma_w = 1 : 0.74 \pm 0.04 : 0.71 \pm 0.06$. Wielen (1974), using space velocities for a sample of roughly 300 McCormick K and M dwarfs within 20 pc of the Sun, finds $\sigma_u : \sigma_v : \sigma_w = 1 : 0.59 \pm 0.05 : 0.51 \pm 0.04$. These two sets of observational ratios disagree with each other at roughly the 2 σ level for the quoted uncertainties. The Woolley *et al.* value of σ_v / σ_u agrees with the estimate based on the rotation curve value of β; the Wielen value of σ_v / σ_u is discrepant, again at the 2 σ level, and corresponds instead to $\beta = 1.70 \pm 0.15$. For the latter value of β, we would predict $\sigma_u : \sigma_w = 1 : 0.58 \pm 0.03$ (NSG) or $1 : 0.47 \pm 0.03$ (SG). These inconsistencies may be caused by observational error or by departures from axisymmetry in the Galaxy, for instance due to a spiral density wave. Both samples of stars have values of the velocity dispersion similar to that of the old disk as a whole; therefore comparison with the prediction for the SG case is probably the more appropriate. (To illustrate the effects of varying γ_0, if we had taken $\gamma_0 = \frac{2}{3}$ instead of $\gamma_0 = 1$, we would have predicted values of σ_w / σ_u smaller by 0.04 (NSG) or 0.03 (SG) for the values of β discussed above.) Considering all the uncertainties, the agreement between prediction and observation for the value of σ_w / σ_u is satisfactory, when compared to the predictions of other mechanisms. For disk heating by molecular clouds, a simplified analytical calculation predicts values of σ_w / σ_u that are somewhat too high (Lacey 1984c), although a numerical simulation gives better agreement with the observed ratio (Villumsen 1985). Heating by transient spiral density waves predicts $\sigma_w / \sigma_u \ll 1$ (Carlberg 1984).

c) Constraints on High Velocity/Large Scale Height Components in Galactic Disks

The high-energy tail on the stellar distribution function that results from disk heating by halo black holes might be detected either as a high-velocity component or as a large scale height component in disks. For $\ln \Lambda = 12$, as derived in the previous section, we find from equations (36) and (47) that the distribution function $N(E)$ changes from exponential to power-law form at $E / \langle E \rangle \approx 7$ and that the fraction of stars in the power-law tail is about 10^{-2}. This fraction is independent of any of the other properties of the disk or halo. There are two observational phenomena with which one might try to identify such a high-energy tail: thick disks, and the high-velocity A stars.

A "thick-disk" is defined to be a stellar component with scale height and flattening intermediate between the ordinary thin disk and the spheroid. Thick disks were first found in S0 galaxies by Burstein (1979), but there has been some controversy regarding their interpretation as a distinct component and their existence in spiral galaxies. Van der Kruit and Searle (1981a, b) find no thick disks in spiral galaxies without bulges but do find analogues of Burstein's thick disks in spiral galaxies with significant bulges. However, their interpretation of the "thick disk" in these cases is that it represents the flattening of the outer parts of the bulge by the gravity of the disk, rather than being a distinct component. There is also some evidence for a thick disk in our own Galaxy, from star counts (Gilmore and Reid 1983; Gilmore 1984) and from the properties of halo giant stars (Ratnatunga 1985). Gilmore claims that this thick disk locally has about 10% of the surface density of the thin disk.

The small fraction of stars predicted to be in a "thick disk" for heating of the disk by halo black holes, about 1%, is consistent with the fact that no thick disks have been detected in bulge-free spiral galaxies, but correspondingly this mechanism cannot explain the thick disks which are claimed to exist in some other spiral galaxies and to contain $\sim 10\%$ of the surface density of the thin disk, because the thick disk fraction depends only on the value of $\ln \Lambda$, which will be practically the same for all disk galaxies. The predicted thick disk has density varying with z approximately as $n(z) \propto z^{-2}$ for large z (from eqs. [48] and [51]).

The high-velocity A stars are a high-velocity stellar population in the solar neighborhood. Rodgers (1971) found that there is a population of main-sequence A stars with normal disk metallicities that extends out to several kiloparsecs from the plane. These stars have ages $\sim 10^9$ yr, and so cannot be a relic from the formation of the Galaxy, unlike thick disks, which might be explained in this way. Stetson (1981a, b) identified the corresponding high-velocity population at $z \approx 0$ and found that they constitute $\sim 10^{-3}$ of the A dwarfs in the solar neighborhood, consistent with the estimate made by Rodgers, Harding, and Sadler (1981) by extrapolating the density at several kiloparsecs back down to the plane. Stetson also found a similar fraction of high-velocity F dwarfs, with metallicities covering the range found in the disk. This suggests that whatever process produces these high-velocity disk stars has been operating over the life of the disk, though not necessarily continuously, rather than being a single event.

From equation (51), we predict for the distribution $f_w(w)$ of vertical velocities at $z = 0$, normalized so that $\int f_w(w) dw = 1$,

$$
\begin{aligned}
f_w(w) &\approx \frac{1}{\sqrt{2\pi}\,\sigma_w} \exp\left(-\frac{w^2}{2\sigma_w^2}\right), && w < w_c, \\
&\approx \frac{\sqrt{\pi}}{\ln \Lambda} \frac{\sigma_D \sigma_w^3}{w^5}, && w_c < w \lesssim \sigma_H,
\end{aligned}
\tag{55}
$$

where $w_c = (2E_{zc})^{1/2}$ and E_{zc} is given by equation (52). Taking $\sigma_w \approx 10$ km s^{-1} for the A stars and $\sigma_D \approx 25$ km s^{-1} for the background disk (Mihalas and Binney 1981), we find $w_c / \sigma_w \approx 3.6$ for $\ln \Lambda = 12$. Stetson defines high-velocity A stars to be those with transverse velocities greater than 80 km s^{-1}, which corresponds roughly to $V/\sigma > 5$, since the total velocity dispersion of A dwarfs is $\sigma \approx 20$ km s^{-1}. A proper estimate of the fraction of high-velocity stars according to Stetson's selection criterion should use the three-dimensional velocity distribution, but we can make a rough estimate by using the one-dimensional distribution (55) to calculate the fraction of stars at $z = 0$ with $w/\sigma_w > 5$. We find a fraction $\sim 3 \times 10^{-4}$, similar to the observed fraction $\sim 10^{-3}$. Thus, disk heating by halo black holes may be able to explain the high-velocity A stars. An analysis similar to the above was performed by Ipser and Semenzato (1985). They estimated a larger fraction of high-velocity stars in the solar neighborhood for $M_H \approx 1 \times 10^6$ $M_\odot$, but their calculation neglects the difference in scale heights of the low- and high-velocity populations.

d) Variation of Disk Scale Height with Radius

The velocity dispersion of the disk is predicted to vary with radius according to equations (26) and (27), where most of the radial dependence is likely to arise from that of the halo density ρ_H. Ideally, one would measure the variation of velocity dispersion with radius in a disk and compare it with what is predicted for halo density models derived from fits to rotation curves. The measurement of $\sigma(r)$ has only been attempted for one spiral galaxy so far (van der Kruit and Freeman 1985), and the results are rather uncertain. The best constraints at present on the radial variation of the velocity dispersion are somewhat less direct, and come from determinations of the radial variation of the disk scale height. Van der Kruit and Searle (1981a, b, 1982) in a series of papers present surface photometry of edge-on spiral galaxies. They find that the surface brightness distributions of these galaxies can be fitted by models in which the volume luminosity density of the old stellar disk varies as $j(r, z) \propto \exp(-\alpha r) \operatorname{sech}^2 (z/h_D)$, with the disk scale height h_D constant with radius. If the disk has constant mass-to-light ratio and is self-gravitating, then a constant scale height implies $\sigma_D^2 \propto \mu_D \propto \exp(-\alpha r)$ (see eqs. [59] and [60]), where σ_D is the vertical velocity dispersion of the disk and μ_D is its surface density. Rather than compare the predicted velocity dispersion for disk heating by black holes with the radial dependence derived in van der Kruit and Searle's model, we prefer to predict the vertical structure and project it to compare directly with the original surface brightness data. We take this approach for two reasons. The first is that the line-of-sight integration through a galaxy will tend to smooth over variations in the scale height and thus make the scale height of the projected disk appear more constant than it really is; it is difficult to assign meaningful error bars to a deprojection to take account of this effect. The second reason is that the vertical gravity of the halo may be important in some parts of the disk, causing the vertical structure to depart from the sech^2 law.

We make the approximation that the z-motions of stars are decoupled from their motions in the plane, and that the old disk population, which dominates the mass in the disk and dominates the light from the disk at large z, is exactly isothermal in the z-direction at each radius, with velocity dispersion $\sigma_D(r)$. Then the z-dependence of the density ρ_D of the old disk is found at each radius by solving the coupled stellar hydrodynamic and Poisson equations (e.g., Bahcall 1984):

$$\frac{\sigma_D^2}{\rho_D} \frac{\partial \rho_D}{\partial z} = -\frac{\partial \Phi}{\partial z},$$ (56)

and

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G (\rho_D + \rho_{H\,\mathrm{eff}}),$$ (57)

where

$$\rho_{H\,\mathrm{eff}} \equiv \rho_H(r) - \frac{1}{4\pi Gr} \frac{\partial}{\partial r} V_c^2(r),$$ (58)

In equation (58), we have neglected the z-dependence of $\rho_{H\,\mathrm{eff}}$ and assumed that the gravity due to the bulge is negligible. The second term on the right-hand side of that equation represents the contribution of distant matter to the vertical gravity and vanishes for a flat rotation curve. The assumption of the decoupling of the vertical and horizontal motions in equation (56) and the neglect of the z-dependence in equation (58) break down for $z \gtrsim r$. The assumption of strict isothermality is also only an approximation, for two reasons: (1) The distribution function has a high-energy tail, as discussed in § IIb and IIIc. However, this should not have a large effect for the values of z at which we compare with observations. (2) There is a range of ages in the old disk population, so that the distribution function should more properly be treated as a sum of Gaussians with different dispersions. However, the range of dispersions in the old disk should not be large, so we neglect this effect.

Equations (56) and (57) have simple analytic solutions for the extreme cases that either the disk self-gravity or the halo gravity dominate locally.

i) Self-gravitating:

$$\rho_D(z) = \frac{\mu_D}{2h_D} \operatorname{sech}^2 \left(\frac{z}{h_D} \right),$$ (59)

where

$$h_D = \frac{\sigma_D^2}{\pi G \mu_D}.$$ (60)

ii) Non–self-gravitating:

$$\rho_D(z) = \frac{\mu_D}{2h_D} \exp \left(-\frac{\pi z^2}{4h_D^2} \right),$$ (61)

where

$$h_D = \frac{\sigma_D}{(8G\rho_{H\,\mathrm{eff}})^{1/2}}.$$ (62)

In the above, h_D is the effective half-thickness, $h_D \equiv \mu_D / 2\rho_D(0)$. For any non-zero $\rho_{H\,\mathrm{eff}}$, the effective halo gravity will dominate that of the disk at large enough z.

We must now choose models for the mass distribution in the disk and dark halo. We assume that the surface density of the disk is a truncated exponential in radius, which follows from the observed surface brightness profiles and the assumption of constant mass-to-light ratio (Freeman 1970; van der Kruit and Searle 1981a):

$$\mu_D(r) = \mu_{D0} \exp(-\alpha r), \qquad r < r_{max},$$
$$= 0, \qquad\qquad\qquad r > r_{max}. \tag{63}$$

We assume that the dark halo is spherical, with volume density having the pseudo-isothermal form

$$\rho_H(r) = \frac{\rho_{H0}}{(1 + r^2/r_c^2)} \tag{64}$$

(e.g., Caldwell and Ostriker 1981). This has $\rho_H \propto r^{-2}$ for $r \gg r_c$, which is the form indicated by the flatness of galaxy rotation curves at large distances.

These mass distributions produce a circular velocity in the plane $V_c(r) = [V_{cD}^2(r) + V_{cH}^2(r)]^{1/2}$, with

$$V_{cD}^2(r) = \left(\frac{\pi G \mu_{D0}}{\alpha}\right)(\alpha r)^2 \left[I_0\left(\frac{\alpha r}{2}\right)K_0\left(\frac{\alpha r}{2}\right) - I_1\left(\frac{\alpha r}{2}\right)K_1\left(\frac{\alpha r}{2}\right)\right], \tag{65}$$

where $I_0, I_1, K_0,$ and K_1 are modified Bessel functions (Freeman 1970), and

$$V_{cH}^2(r) = V_\infty^2\left[1 - \frac{\tan^{-1}(r/r_c)}{(r/r_c)}\right], \tag{66}$$

where $V_\infty^2 = 4\pi G \rho_{H0} r_c^2$. The contribution of the disk to the rotation curve has been evaluated in the infinite thin disk approximation. The non-zero disk thickness and finite truncation radius introduce small corrections to equation (65) (e.g., van der Kruit and Searle 1982). These are not important for our calculation, except near the center, where $(1/4\pi Gr)(\partial V_{cD}^2/\partial r)$ diverges logarithmically as $r \to 0$ when evaluated using equation (65). We cut this divergence off at its value at $r = h_D(r = 0)$. Our results for the disk scale height are not sensitive to our approximate treatment of this term because, in practice, where the disk gravity is important, the local contribution dominates.

The vertical velocity dispersion of the disk evolves in time as $d\sigma_D^2/dt = \frac{2}{3}D_z$ when the disk is self-gravitating (ignoring infall), and as $d\sigma_D^2/dt = D_z$ when the halo gravity dominates (eqs. [16] and [28]). We define the critical velocity dispersion σ_{cr}, at which the disk makes the transition from being self-gravitating to being non–self-gravitating, as that for which expressions (60) and (62) for h_D are equal; thus $\sigma_{cr} = (\pi^2 G/8\rho_{H\,eff})^{1/2}\mu_D$. Treating the disk as a single-age population with effective age τ, we then have approximately

$$\sigma_D^2 = \sigma_{D0}^2 + D_z\tau, \qquad\qquad \sigma_{cr}^2 < \sigma_{D0}^2,$$
$$= \tfrac{3}{2}\sigma_{D0}^2 - \tfrac{1}{2}\sigma_{cr}^2 + D_z\tau, \qquad \sigma_{D0}^2 \le \sigma_{cr}^2 < \sigma_{D0}^2 + \tfrac{2}{3}D_z\tau,$$
$$= \sigma_{D0}^2 + \tfrac{2}{3}D_z\tau, \qquad\qquad \sigma_{D0}^2 + \tfrac{2}{3}D_z\tau \le \sigma_{cr}^2. \tag{67}$$

The quantity D_z is given by equation (16), in which M_H and $\ln \Lambda$ are taken to be constant with radius. We assume that $\bar{V}_H = 0$, so that $V_0 = V_c(r)$, and $\sigma_H(r)$ is found by integrating the equation of stellar hydrodynamics for spherical symmetry (e.g., Binney 1982, eq. [18]), for the given halo density profile, where we calculate the disk gravity from $M_D(r)$, the disk mass within a sphere of radius r. The results are not sensitive to the latter approximation.

We can determine the qualitative behavior of the disk scale height as follows: The velocity dispersion depends on radius mainly through the halo density ρ_H (see eq. [16]), thus $\sigma_D^2 \propto \rho_H$ if $\sigma_D^2 \gg \sigma_{D0}^2$. If the disk is locally self-gravitating, which is likely to be a good approximation over much of the disk, then $h_D \propto \sigma_D^2/\mu_D$, and we find

$$h_D \propto \frac{\rho_H}{\mu_D} \propto \frac{\exp(\alpha r)}{(r^2 + r_c^2)}. \tag{68}$$

We see from this that the disk scale height will increase at large radii, where the disk surface density falls off more rapidly than the halo density, and also at small radii, if the halo core radius r_c is small. However, the halo gravity will tend to reduce the flaring of the disk, and the line-of-sight integration will also tend to reduce the apparent scale height variation. We now proceed to include these effects properly.

We numerically integrate equations (56) and (57), to find the vertical structure of the disk at each radius, and then perform line-of-sight integrations through the model, to calculate the projected surface density distribution. We then compare the projected model with the observed surface brightness distribution for an edge-on galaxy. We compare the models with the observational data for two of the van der Kruit and Searle galaxies, NGC 5907 and NGC 4244, chosen because they are essentially bulge-free, which greatly simplifies the comparison, and because they span the range in αr_{max} found in van der Kruit and Searle's sample ($\alpha r_{max} = 3.4$ for NGC 5907 and 5.3 for NGC 4244). We use the sets of J-band luminosity profiles parallel to the z-axis presented in van der Kruit and Searle (1981a, 1982).

We choose the parameters of the models as follows: we adopt van der Kruit and Searle's estimates for the distances to the galaxies, the radial scale-length α^{-1} and truncation radius r_{max}, and the inclination angle i. We calculate the central surface density of the disk μ_{D0} from the inferred face-on central surface brightness I_0 of the *old disk* and an assumed mass-to-light ratio M/L for the old disk; $\mu_{D0} = (M/L)I_0$. We consider values of the mass-to-light ratio for the old disk in the range $M/L = 5$–10, which

corresponds to $M/L \approx 2$–5 for the disk as a whole, since about half the total light comes from the young population (van der Kruit 1983). The galaxies under consideration are observed to have rotation curves that are fairly flat over most of the visible disk, so for given M/L and halo core radius r_c, we calculate the asymptotic rotation velocity V_∞ from $V_{cH}^2(r_m) = V_{rot}^2 - V_{cD\ max}^2$, where V_{rot} is the measured rotation velocity, and $V_{cD\ max} \approx 0.880(\pi G \mu_{D0} \alpha^{-1})^{1/2}$ is the maximum rotation velocity due to the exponential disk alone, which occurs at a radius $r_m \approx 2.15\alpha^{-1}$. The flatness of the rotation curve provides a constraint on r_c. We assume that $\ln \Lambda = 12$ and $\tau = 10$ Gyr in both galaxies but leave M_H as a free parameter (in fact, the results only depend on the product M_H [$\ln \Lambda$]τ) and take $\sigma_{D0} = 4$ km s^{-1} (see Wielen 1977). Thus for each galaxy we attempt to fit, we have three adjustable parameters: M/L, r_c, and M_H. Note that we only try to fit the light distribution at large z, where the old population is expected to dominate; close to the plane, the light distribution is affected by light from the young stellar population and by obscuration by dust.

For NGC 5907, we adopt $\alpha^{-1} = 5.7$ kpc, $r_{max} = 19.3$ kpc, $i = 87°$, $I_0 = 35\ L_\odot$ pc^{-2}, and $V_{rot} = 240$ km s^{-1}, the latter from Casertano (1983). The value of I_0 assumed here differs from that given by van der Kruit and Searle (1982), which is incorrect (van der Kruit 1985). Figure 2 shows the comparison between the observed profiles and a model with $M/L = 10$, $r_c = 5.7$ kpc, and $M_H = 3 \times 10^6\ M_\odot$, which is seen to provide a reasonable fit. An equally good fit is obtained with different values of M/L in the range we consider, provided M_H is also varied to keep the scale height the same, e.g., if M/L is reduced to 5, then M_H must be reduced to $1.5 \times 10^6\ M_\odot$. The quality of the fit is, however, rather more sensitive to the value of r_c: the fit remains reasonable if r_c is doubled to 11.4 kpc, but if r_c is halved to 2.85 kpc then the projected scale height becomes too large near the center, as shown in Figure 2. Thus we require $r_c \approx (1$–2$)\alpha^{-1}$.

A problem with this simple model is posed by the flatness of the observed rotation curve: Casertano (1983) finds that for $5 \lesssim r \lesssim 19$ kpc, the rotation velocity is constant at the value $V_{rot} = 240$ km s^{-1} to within ± 10 km s^{-1}, while the model with $M/L = 10$, $r_c = 5.7$ kpc, and $M_H = 3 \times 10^6\ M_\odot$, which fits the surface photometry, produces a rotation curve which is still rising over this range. The model rotation curve can be made flatter by increasing M/L or by decreasing r_c, but the latter worsens the fit to the surface photometry, as already mentioned. However, the rotation curve near the center is probably affected by a small bulge component, so that consistency may be possible. Such a component seems to be required even when the disk and halo parameters are chosen to optimize the fit to the rotation curve alone (Casertano 1983).

For NGC 4244, we adopt $\alpha^{-1} = 2.6$ kpc, $r_{max} = 13.7$ kpc, $i = 88°$, $I_0 = 37\ L_\odot$ pc^{-2}, and $V_{rot} = 130$ km s^{-1}, the last from Casertano (1985). Figure 3 shows the comparison between the observed profiles and a model with $M/L = 10$, $r_c = 2.6$ kpc, and $M_H = 1 \times 10^6\ M_\odot$. The model is seen to provide a reasonable fit in the inner parts of the galaxy but has too large a scale height in the outer parts. As for the case of NCG 5907, for a reasonable fit in the inner regions we require $r_c \approx (1$–2$)\alpha^{-1}$. The increase in the scale height of the outer parts of the disk is much more dramatic in this case than in the case of NCG 5907 because of the larger value of
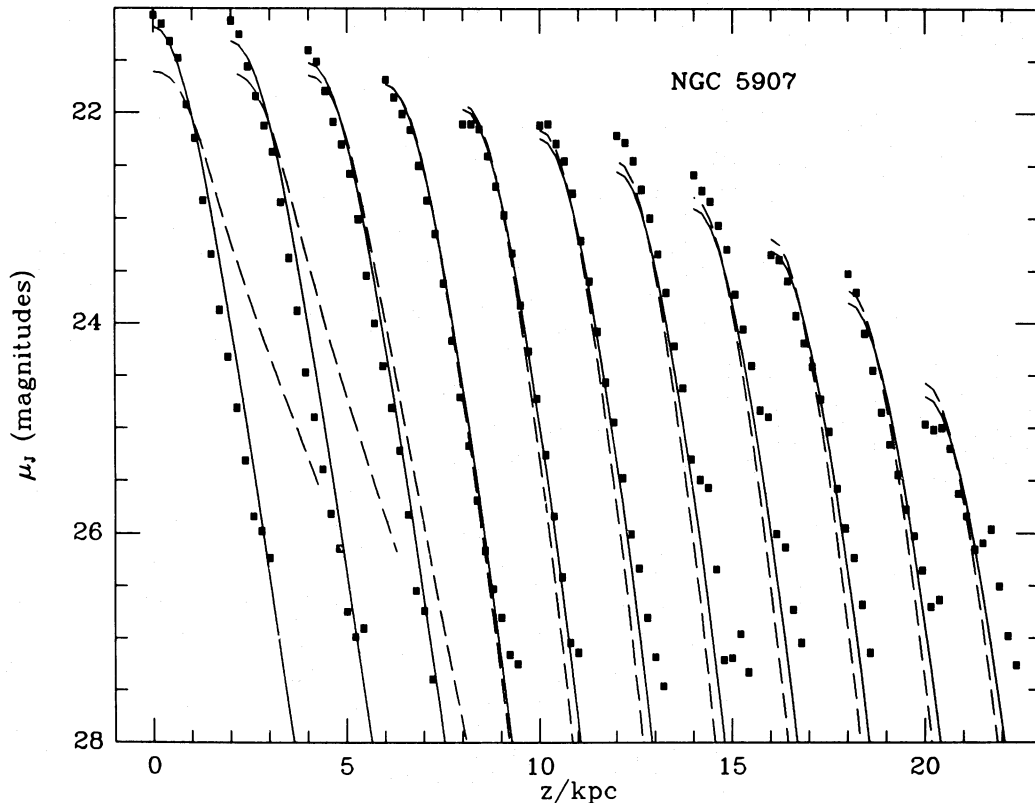

FIG. 2.—Comparison of models with data for surface brightness profiles of the edge-on spiral galaxy NGC 5907. The J-band data are from Van der Kruit and Searle (1982). The surface brightness profiles are taken parallel to the projected z-axis, at equal intervals in radius of $\Delta r = 1.84$ kpc. The models shown have $M/L = 10$, $M_H = 3 \times 10^6\ M_\odot$, and $r_c = 5.7$ kpc (*solid line*) and $r_c = 2.85$ kpc (*dashed line*). (See text for a full description.)
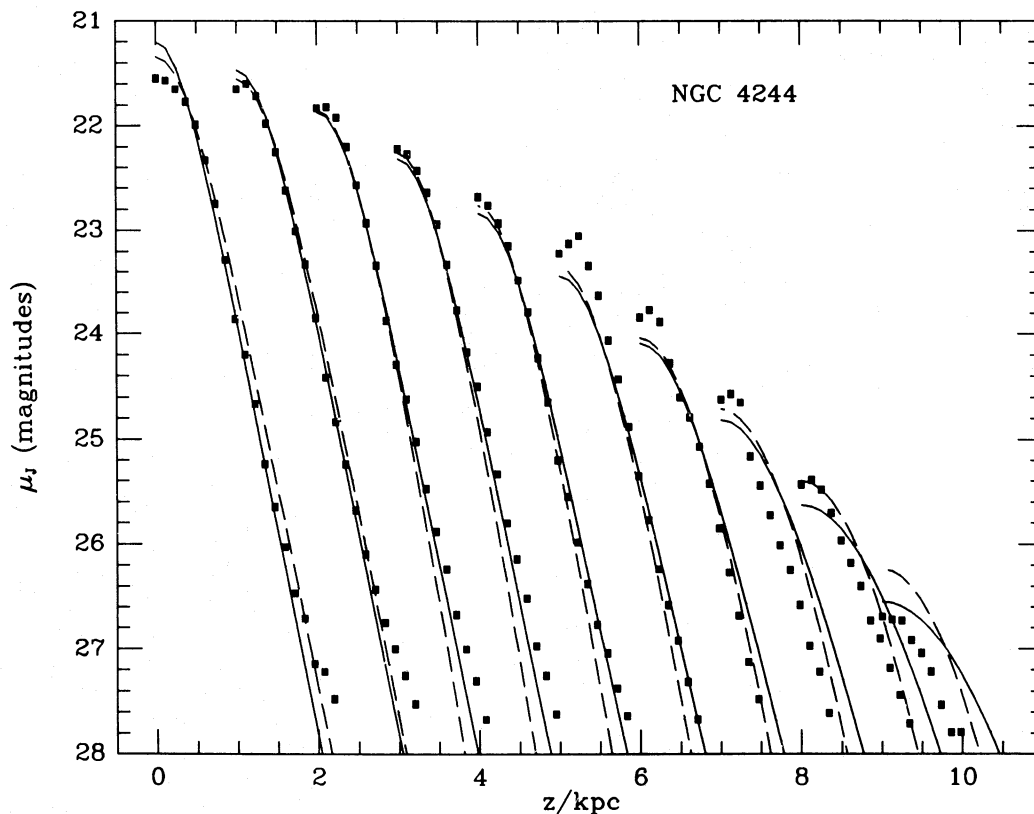
FIG. 3.—Comparison of models with data for surface brightness profiles of the edge-on spiral galaxy NGC 4244. The J-band data are from van der Kruit and Searle (1981a). The surface brightness profiles are taken at intervals in radius of $\Delta r = 1.39$ kpc. The models shown have $M/L = 10$ and $M_H = 1 \times 10^6 \, M_\odot$ (solid line), and $M/L = 5$, $M_H = 0.35 \times 10^6 \, M_\odot$ (dashed line), with $r_c = 3.9$ kpc in both cases. (See text for a full description.)

αr_{max} in this case (5.3 for NGC 4244, as against 3.4 for NGC 5907). The amount of flaring is not sensitive to r_c, but it is reduced if M/L is reduced to $M/L = 5$, with $M_H = 0.35 \times 10^6 \, M_\odot$, as also shown in Figure 3. The quality of the fits to the luminosity profiles in the outer parts could also be improved if we relaxed the assumptions in the models that the disk age and mass-to-light ratio are constant with radius, for instance allowing τ to decline by a factor of 2 from the center to the edge, as in models of galactic evolution similar to those of Gunn (1982), or allowing M/L to increase by a factor of 2.

In conclusion, disk heating by halo black holes will account for the observed disk thickness of other spiral galaxies as well as of our own, if the black holes have masses $M_H \approx 2 \times 10^6 \, M_\odot$. However, the computed radial variation of the stellar scale height is larger than observed unless there is some fine-tuning of parameters. To fit the scale height in the inner regions, one must choose the halo core radius to be in the range $\alpha r_c \approx 1$–2, for which there is no obvious physical reason, although it appears to be consistent with the observed form of rotation curves. For galaxies with large αr_{max}, to avoid too much flaring of the stellar disk in the outer parts, one must in addition choose a value for the mass-to-light ratio of the old disk that is on the low side ($M/L = 5$ rather than 10), or assume that the outer parts of the disk form after the inner parts.

IV. OTHER TESTS

In this section, we consider some other tests of our disk heating hypothesis that do not involve disk heating directly. These are (a) the effects of two-body encounters (between black holes or between stars and black holes) on the distribution of black holes in the Galaxy, (b) effects of dark matter in dwarf spheroidal galaxies, (c) radiation by black holes accreting from the interstellar medium, and (d) the gravitational lensing of cosmologically distant sources by black holes.

a) Evolution of the Black Hole Distribution Due to Two-Body Encounters

Consider first the effects of encounters between the black holes themselves. A measure of the two-body relaxation time for a population of objects with mass m, density ρ, and one-dimensional velocity dispersion σ is the "reference" relaxation time (Spitzer and Hart 1971):

$$t_{\rm rel} = 0.340 \, \frac{\sigma^3}{G^2 \rho m \ln \Lambda} \, . \tag{69}$$

Assuming that ρ_H is given by equation (64) and $\sigma_H = V_\infty/2^{1/2}$, with $V_\infty = 220$ km s^{-1}, $M_H = 2 \times 10^6 \, M_\odot$ and $\ln \Lambda = 12$, we find that the central two-body relaxation time of the halo is $t_{HH}(0) = 3.2 \times 10^9$ yr $(r_c/{\rm kpc})^2$. The halo core radius r_c is rather weakly constrained by observations: Galactic mass models (Caldwell and Ostriker 1981; Ostriker and Caldwell 1983) suggest $r_c \approx 2$–8 kpc,

while the requirement of a constant disk scale height (see § IIId) requires $r_c \approx (1-2)\alpha^{-1}$, which for our Galaxy corresponds to $r_c \approx 4$–8 kpc (e.g., Bahcall and Soneira 1980). Thus $t_{HH}(0)$ probably exceeds the age of the Galaxy, and encounters between black holes should not have led to significant evolution.

More important for the evolution of the black hole distribution will be encounters between black holes and stars. Two-body encounters cause the black holes to lose kinetic energy to the stars, as the two populations try to reach energy equipartition; if stars and black holes initially have the same velocity dispersion, the time scale t_{H*} for this process is roughly given by equation (69), with $\rho = \rho_*$ and $m = M_H$ (Spitzer 1969); thus $t_{H*} \sim (\rho_H/\rho_*)t_{HH}$. In the inner regions of the Galaxy, the density of stars (in the spheroid) probably exceeds that of the halo, so that the effects of star–black hole encounters dominate those of black hole–black hole encounters. As a black hole loses orbital energy to the background of stars through two-body encounters, its orbit decays in radius. We now consider this process of dynamical friction in more detail. It has previously been discussed by Carr (1978).

Within the central few kiloparsecs of the Galaxy, the mass density in stars is dominated by the spheroid. We take for the density $\rho_s(r)$ of the inner spheroid a model suggested by observations of the stellar infrared luminosity distribution at small radii ($r \lesssim 50$ pc) and of the rotation curve at larger radii ($r \gtrsim 50$ pc) (Sanders and Lowinger 1972; Oort 1977; Sanders 1979). These observations indicate that $\rho_s \propto r^{-1.8}$ at small radii. At larger radii, the density must fall off more steeply; observations of our own and other galaxies suggest $\rho_s \propto r^{-3}$. On the basis of rotation curve measurements, Sanders (1979) suggests that the change of power law slope occurs at $r \approx 800$ pc (see also Matsumoto *et al.* 1982). Based on all this evidence, we adopt as our idealized model for the inner spheroid

$$\rho_s(r) = \rho_1\left(\frac{r_1}{r}\right)^{1.8}, \qquad r \le r_1 ,$$
$$= \rho_1\left(\frac{r_1}{r}\right)^{3}, \qquad r > r_1 , \tag{70}$$

with $\rho_1 = 1.8 \; M_\odot \; \mathrm{pc}^{-3}$, $r_1 = 800$ pc. We have also replaced the observed density distribution, which is flattened at small radii, by the spherical distribution which contains the same mass.

To calculate the rate at which the orbits of the black holes decay due to dynamical friction, we make the idealization that their orbits are all circular. Then the rate of change of the radius of the orbit is given by a straightforward extension of the analysis of Tremaine, Ostriker, and Spitzer 1975):

$$\frac{dr}{dt} = -\frac{4\pi G^2 M_H \ln \Lambda \; B(V_c/\sqrt{2}\,\sigma_s)}{(1 + d \ln V_c/d \ln r)} \frac{r\rho_s(r)}{V_c^3(r)}, \tag{71}$$

where $B(x) = \mathrm{erf}\,(x) - x\,\mathrm{erf}'\,(x)$ and $\sigma_s(r)$ is the one-dimensional velocity dispersion of the spheroid stars, assumed isotropic. Assuming that the spheroid described by equation (70) is self-gravitating, it is straightforward to derive expressions for $V_c(r)$ and $\sigma_s(r)$, using the equation of stellar hydrodynamics in the latter case. For $r > r_1$, we find $V_c \propto r^{-1/2}$ approximately, with $B[V_c/2^{1/2}\sigma_s]$, $1 + d \ln V_c/d \ln r$, and $\ln \Lambda$ slowly varying, so that $dr/dt \propto r^{-1/2}$, and the time for a black hole to spiral into the center is $t_{df} \approx \frac{2}{3}r/|dr/dt|$. Evaluating the various slowly varying factors, we find that over the lifetime of the Galaxy, $t_1 \approx 1.5 \times 10^{10}$ yr, all the black holes with initial orbital radii $r \le r_{df} \approx 2$ kpc will have spiraled into the center, assuming $M_H = 2 \times 10^6 \; M_\odot$. Clearly, this estimate of r_{df} is rather rough: the density distribution $\rho_s(r)$ is uncertain, as is M_H, and the assumption of circular orbits for the black holes cannot be correct in detail. White (1982) finds that for objects with an initially isotropic velocity distribution moving through an isothermal background (with $\rho \propto r^{-2}$), the mean value of r_{df} differs only slightly from the value derived for circular orbits. However, the case of interest here has a different background density distribution, and it is possible that the orbits of the black holes are very radially elongated because of the gravitational influence of the spheroid, so that the true value of r_{df} might differ considerably from the estimate above.

If $r_{df} < r_c$, then the mass in black holes that is accreted by the center over the lifetime of the Galaxy, assuming the same halo density law as previously, is $M_{\rm acc} \approx 2 \times 10^9 \; M_\odot (r_{df}/2 \; \mathrm{kpc})^3 (r_c/4 \; \mathrm{kpc})^{-2}$. In fact, this is probably an underestimate, because the halo density near the center is likely to be increased by the gravitational attraction of the spheroid. Note the sensitivity of this estimate to the uncertain parameters r_{df} and r_c. The observational constraints on the mass distribution close to the Galactic center are fairly stringent: Serabyn and Lacy (1985) estimate that the mass in the form of a compact object at the center is $M_c \approx 3$–$4 \times 10^6 \; M_\odot$, similar to the mass of the individual black holes derived in this paper, but small compared to the total mass that would have been accreted.

However, gravitational interaction between the black holes once they reach the center is likely to cause ejection of the holes by the slingshot effect (Saslaw, Valtonen, and Aarseth 1974; Begelman, Blandford and Rees 1980), in which a three-body system disrupts with the ejection of a single body and a tight binary, although it is not clear how far out from the center the black holes will be ejected. In this case, the number of black holes at the center for most of the time would be 0, 1, or 2, assuming that the three-body disruption time is short compared to the time between successive black holes falling into the center. A problem remains, however: the orbital energy that the black holes lose by dynamical friction is deposited in the stellar background, so that in sinking to the center, a black hole tends to evaporate stars from the cusped distribution around the center. In the final stages of its decaying orbit, the orbital velocity of the black hole will be comparable to that of the stars it interacts with, and it only interacts with stars out to about the radius of its orbit, so it will evaporate a mass in stars $\sim M_H$ from the central region. Thus if mass $M_{\rm acc}$ in black holes is accreted. even if most of it is subsequently ejected, the $\rho_s \propto r^{-1.8}$ cusp will be destroyed out to a radius $a_c \sim 0.4$ pc $(M_{\rm acc}/10^6 \; M_\odot)^{0.83}$, for the density distribution of equation (70). For $r_{df} = 2$ kpc and $r_c = 4$ kpc, we find $a_c \sim 200$ pc . Two-body encounters between the stars tend to recreate a cusp around a central compact object (e.g., Bahcall and Wolf 1976), but this only occurs on the relaxation time scale for the stars, which is long: $t_{\rm rel} \approx 3 \times 10^9$ yr $(r/\mathrm{pc})^{2.1}$ for the original density distribution for $r \ll r_1$, using

equation (69). Thus it seems unlikely that the stellar cusp would be repopulated by stellar dynamical processes, once it had been destroyed. Even with fairly optimistic assumptions, (say) $r_{df} = 1$ kpc, $r_c = 8$ kpc, giving $M_{acc} \approx 6 \times 10^7 \, M_\odot$, we find $a_c \sim 10$ pc. In contrast, the observations show that the light from the stellar population varies as $\rho_s \propto r^{-1.8}$ down to $r < 0.1$ pc (Allen, Hyland, and Jones 1983). A possible resolution of this problem is to repopulate the cusp with stars formed from the large amounts of gas that collect at the center. Interestingly, measurements of ionized gas velocities close to the center indicate that the *mass* density in stars flattens off compared to the *light* within $r \lesssim 10$ pc (Crawford *et al.* 1985), which supports this picture, since a young stellar population would have a lower mass-to-light ratio. The presence of supergiant stars close to the center (Becklin *et al.* 1978) also suggests that there has been star formation relatively recently.

b) Dynamics of Dwarf Spheroidal Galaxies

There are observational indications that at least some dwarf spheroidal galaxies may contain large amounts of dark matter (Aaronson 1983; Faber and Lin 1983), although this is still controversial. For dark matter in the form of massive black holes, we must consider what the effects on a dwarf galaxy will be. Interactions with black holes cause the total energy E of a star moving at velocity V through such a galaxy on the average to increase at a rate

$$\langle \Delta E \rangle = V \langle \Delta V_\parallel \rangle + \tfrac{1}{2} \langle (\Delta V_\parallel)^2 \rangle + \tfrac{1}{2} \langle (\Delta V_\perp)^2 \rangle$$

$$= \frac{4\sqrt{2\pi}\, G^2 \rho_H M_H \ln \Lambda}{\sigma_H} \exp\left(-\frac{V^2}{2\sigma_H^2}\right) \tag{72}$$

(using eq. [7]). For the Draco dwarf galaxy, Aaronson (1983) measures the stars as having a one-dimensional velocity dispersion $\sigma_* = 6.5$ km s^{-1} within the luminous core of radius $r_* = 140$ pc. If the dark matter dominates the mass within this radius, and is assumed to have core radius $r_c \geq r_*$, this implies it has a central density $\rho_{H0} \approx 9\sigma_*^2/4\pi G r_c^2 \approx 0.36 \, M_\odot$ pc^{-3}. For stars with orbits confined to the core, $\langle E \rangle = 3\sigma_*^2$; integrating equation (72), we find that heating over a time $t \approx 10^{10}$ yr will cause the stellar velocity dispersion to exceed the observed value unless $M_H/\sigma_H \lesssim 2 \times 10^2 \, M_\odot$ (km s^{-1})$^{-1}$. A similar bound can be derived for the case that the dark matter is distributed according to equation (64) with $r_c < r_*$. If we assume $\sigma_H \approx \sigma_* \approx 6.5$ km s^{-1}, then we derive the limit $M_H \lesssim 10^3 \, M_\odot$, while if instead we assume $M_H \sim 10^6 \, M_\odot$, then we require $\sigma_H \gtrsim 10^4$ km s^{-1}, or $r_c \gtrsim 100$ kpc. The latter is clearly unacceptable. A way around this problem of $10^6 \, M_\odot$ black holes producing too much heating is to suppose that the black holes form a compact cluster, with density falling off more rapidly than $\rho_H \propto r^{-2}$ outside the core, and core radius small compared to the present stellar core. Then, if the stars initially had a similar distribution, interactions would have ejected them on almost radial orbits from the black hole cluster. The resulting halo of *stars* would continue to be heated and to expand as long as the stellar orbits passed through the cluster core. However, tidal interactions with neighboring large galaxies would deflect the stars onto less radial orbits, and thus terminate the stellar heating, when the characteristic radius of the stellar halo became comparable to, but somewhat less than, the tidal radius.

A further effect that must be taken into account is interactions between the black holes themselves: if the number of black holes N exceeds 2, then these interactions will cause ejection of the black holes from the galaxy on a time scale $t_{evap} \approx 300 t_{rh}$, where t_{rh} is the mean two-body relaxation time of the black holes within the half-mass radius (Spitzer 1975). If the ratio of dark to luminous matter in the galaxy is initially 10:1, then the simultaneous ejection of the last three black holes by gravitational slingshot will unbind and disrupt the galaxy for initial values of N in the range $2 < N < 30$. For Draco, if we assume $M_H = 2 \times 10^6 \, M_\odot$, we calculate $t_{evap} \approx 10^{10}$ yr if the black holes have the same distribution as the stars, and $t_{evap} \ll 10^{10}$ yr if they have a much smaller mean radius, as was postulated above. However, for this value of M_H, Aaronson's mass estimate for the galaxy gives $N = 2$, so that disruption will not in fact occur. Galaxies which were initially somewhat more massive than Draco would have been disrupted by now, if the black holes were in a dense cluster, because they would have had $2 < N < 30$ and $t_{evap} < 10^{10}$ yr. Galaxies with $N > 30$ (i.e., total stellar masses $M_* > 6 \times 10^6 \, M_\odot$) would avoid disruption. In those with $t_{evap} < 10^{10}$ yr, all the black holes would have been ejected, leaving a purely stellar system.

To summarize our argument: the dark matter in a dwarf spheroidal may be in the form of massive ($M_H \approx 2 \times 10^6 \, M_\odot$) black holes, but these must be in a dense cluster, to avoid too much stellar heating. The stellar system will then be heated only until its radius is comparable to the tidal radius. This is consistent with the ratios of core to tidal radius measured in local group spheroidals (see Hodge 1971) and provides an explanation of why the dwarf spheroidals have much lower stellar densities than the other branch of low-luminosity ellipticals (cf. Wirth and Gallagher 1984). Evaporation of the black holes should cause disruption of systems with *initial* masses somewhat greater than that of Draco ($M \approx 4 \times 10^6 \, M_\odot$), and thus systems with *present* total masses in that range should be mainly stellar and have normal mass-to-light ratios. We also predict that in the systems with large M/L, the stellar velocities should increase inward because of the central concentration of the black holes.

c) Radiation by Accreting Black Holes

A black hole moving through the disk or halo of a galaxy will accrete gas from the surrounding medium, generating radiation which makes the black hole potentially visible. A typical halo black hole moves supersonically relative to gas in the disk or halo (though only marginally so in the latter case). In this case (e.g., Hunt 1971), a bow shock forms at a distance $\sim r_{acc} = 2GM_H/V_H^2$ from the black hole, at which the gas is slowed down, relative to the black hole, and heated to temperature $T_{sh} \approx 3m_H V_H^2/32k$ (where m_H is the mass of a hydrogen atom and k is the Boltzmann constant). Gas within a distance r_{acc} of the axis is captured by the black hole, leading to an accretion rate

$$\dot{M} = \frac{4\pi G^2 M_H^2 n m_H}{V_H^3} , \tag{73}$$

where n is the local density of hydrogen atoms. In this section, we will evaluate numerical values of accretion rates, luminosities, etc., for the fixed values $M_H = 2 \times 10^6 \, M_\odot$, $V_H = 300$ km s^{-1} but will display the dependence on n, since we shall be interested in densities covering the range $n \sim 10^{-4}$ to 10^3 cm^{-3}. Thus we find $r_{\rm acc} = 0.2$ pc, $T_{\rm sh} \approx 1 \times 10^6$ K, and $\dot{M} = 5.5 \times 10^{19}$ g s^{-1} $(n/$ cm$^{-3})$. If the accreted matter is converted to radiation with an efficiency ϵ, the accretion luminosity of the black hole is

$$L = \epsilon \dot{M} c^2 = 5.0 \times 10^{40} \text{ ergs s}^{-1} \, \epsilon \, (n/\text{cm}^{-3}) \tag{74}$$

for the same parameters. The efficiency ϵ and the spectral distribution of the radiation depend on uncertain details of the accretion process, so that it is not possible to give definite answers. An extensive discussion of this problem has been given by Ipser and Price (1977), and much of our discussion will be patterned on theirs.

At radii somewhat smaller than $r_{\rm acc}$, the accretion flow onto the black hole will be roughly spherical. However, two qualitatively different cases must be distinguished for the form of the gas flow at radii comparable to the Schwarzschild radius of the black hole, $r_S = 2GM_H/c^2$, where most of the energy is released. If the accreted gas has very low angular momentum, the flow will remain roughly spherical, while if the angular momentum is larger, the gas will settle into an accretion disk at small radii. To form an accretion disk with outer radius r_d, the accreted gas must have specific angular momentum $J_d = (GM_H r_d)^{1/2}$, which may occur if there is either a velocity gradient or a density gradient in the ambient medium from which accretion is occurring. If there is a velocity difference Δv across the radius $r_{\rm acc}$ of the accretion front, the accreted gas has angular momentum $J_{\rm acc} \approx \Delta v r_{\rm acc}$, while if there is a density difference Δn across $r_{\rm acc}$, then $J_{\rm acc} \approx (\Delta n/n) r_{\rm acc} V_H$. Taking the dividing line between disklike and spherical accretion as corresponding to $r_d = 20 r_S$ (cf. Ipser and Price), we find that formation of an accretion disk requires either a velocity gradient exceeding 5 km s^{-1} pc^{-1} or a relative density gradient exceeding 0.02 pc^{-1} on the scale $r_{\rm acc}$. While the former is rather large compared to typical values in the interstellar medium, the latter condition seems likely to be met in any dense cloud in the disk, for which typical radii are $a \sim 10$ pc.

For spherical accretion, the efficiency ϵ is rather uncertain because of uncertainties in the treatment of magnetic fields, turbulence, etc., but might be rather low. For compressional heating and bremsstrahlung cooling only, $\epsilon \approx 1 \times 10^{-7} (n/\text{cm}^{-3})$, with most of the radiation being in the form of hard X-rays or gamma-rays with $E \approx 10$ MeV (Shapiro 1973a). If the energy of the magnetic field embedded in the accreted gas reaches rough equipartition with the kinetic energy of the flow, then cooling by synchrotron emission is important, and most of the radiation is in the infrared. Shapiro (1973b) included only compressional heating of the gas and found $\epsilon \approx 4 \times 10^{-3} (n/\text{cm}^{-3})$ for $n \lesssim 1$ cm^{-3}, with emission at a typical wavelength $\lambda \approx 600 \, \mu\text{m}(n/\text{cm}^{-3})^{-1/2}$. Ipser and Price (1982) included additional (and uncertain) heating of the gas by magnetic fields and turbulence and found slightly larger efficiencies: $\epsilon \approx 5 \times 10^{-2} (n/\text{cm}^{-3})$ for $n \lesssim 0.2$ cm^{-3} and $\epsilon \approx 1 \times 10^{-2}$ for $n \gtrsim 0.2$ cm^{-3}, with emission at a typical wavelength $\lambda \approx 10 \, \mu\text{m}$ $(n/\text{cm}^{-3})^{-1/2}$ for $n \lesssim 0.2$ cm^{-3}.

For disk accretion, the efficiency is fairly high in the standard thin disk models: $\epsilon \approx 0.06$ for a nonrotating black hole, and $\epsilon \approx 0.4$ for a maximally rotating black hole. For the nonrotating case, the inner edge of the accretion disk is at $r = 3r_S$, and the flux emitted per unit area from either side of the accretion disk in the steady state is, ignoring relativistic corrections,

$$F(r) = \frac{3GM_H \dot{M}}{8\pi r^3} Q(r) \, , \tag{75}$$

where $Q(r) \equiv 1 - (3r_S/r)^{1/2}$ (e.g., Novikov and Thorne 1973). If the disk is optically thick and the opacity is dominated by absorption rather than electron scattering, then each element of the disk radiates as a blackbody with surface temperature $T_s(r) = [F(r)/\sigma]^{1/4}$, where σ is the Stefan-Boltzmann constant. According to equation (75), half the luminosity of the accretion disk is emitted within a radius $r_h = 12 r_S$; therefore we may characterize the emitted spectrum for the whole disk by the surface temperature at r_h:

$$T_s(r_h) = 1.4 \times 10^4 \text{ K } (n/\text{cm}^{-3})^{1/4} \, . \tag{76}$$

Thus most of the radiation will typically be in the visible range. A detailed discussion of optically thick accretion disks is given by Novikov and Thorne, and using their results one may show that the assumption of optical thickness is self-consistent for the accretion rates of interest here. For accretion rates corresponding to $n \gtrsim 1–10^3$ cm^{-3}, electron scattering opacity may become important at small radii in the disk, but the effect of this on the emitted spectrum is only marginal, and equation (76) for the radiation temperature should still be approximately valid (see Novikov and Thorne).

The optically thick accretion disk models are not the only possible equilibrium solutions, however. Optically thin solutions are also possible, in which the disk radiates at a much higher temperature (Payne and Eardley 1977): assuming bremsstrahlung cooling with cooling function $\Lambda(T) \approx 2 \times 10^{-27}$ ergs cm^3 s$^{-1}(T/K)^{1/2}$ for $T \gtrsim 10^7$ K, the temperature at r_h for an "α-disk" is

$$T_s(r_h) \approx 5 \times 10^8 \text{ K } (\alpha/0.1)^{-1} \, (n/\text{cm}^{-3})^{1/2} \tag{77}$$

for $10^{-4}(\alpha/0.1)^2 \lesssim (n/\text{cm}^{-3}) \lesssim 10^2$, where the viscosity parameter α is thought to lie in the range $10^{-2} \lesssim \alpha \lesssim 1$. In this case, most of the radiation is in the form of X-rays. Payne and Eardley argue that an optically thin rather than optically thick accretion disk will be formed if the accreted gas is preheated to a sufficiently high temperature. Now in the region of roughly spherical inflow onto the black hole ($r_d \lesssim r \lesssim r_{\rm acc}$), adiabatic compression of the gas results in a temperature variation $T(r) \sim 10^{12}$ K (r_S/r), so that it is likely that sufficient preheating will occur. However, optically thin disks are thermally unstable (Pringle 1976), so that their existence is in some doubt (but see Payne and Eardley).

We must also consider possible feedback effects of the emitted radiation on the accretion of gas, which may act to limit the accretion rate below that given by equation (73). The first of these is radiation pressure on the inflowing gas, which prevents the

luminosity due to accretion from exceeding the Eddington value, $L_{Edd} = 4\pi G M_H c m_H/\sigma$, where σ is the photon cross section per hydrogen atom. If the only source of opacity were electron scattering, we would have $L_{Edd,es} = 2.5 \times 10^{44}$ ergs s^{-1}. However, the radiation is also absorbed by dust: for photons above the Lyman limit, the cross section per hydrogen atom is $\sigma_d \approx 2.0 \times 10^{-21}$ cm^2 (Spitzer 1978), and thus $L_{Edd,d} \approx 8.2 \times 10^{40}$ ergs s^{-1} for ionizing radiation. There are other possible feedback effects, but these appear to be less important than radiation pressure on dust. If the emission is in the X-ray band, then heating of the gas in the spherical inflow region by the radiation makes steady inflow impossible above a certain critical luminosity (Ostriker *et al.* 1976); this would be relevant for an optically thin accretion disk, but, using equations (74) and (77), one finds that the critical luminosity somewhat exceeds $L_{Edd,d}$. The radiation from the black hole would also heat the interstellar medium ahead of the accretion shock, causing it to expand and thus reducing the density of the medium into which the black hole is moving. However, the pocket of hot gas ahead of the black hole can only expand at roughly its own sound speed, so the accretion rate would only be significantly reduced if the sound speed exceeded the velocity of the black hole, which would require heating the gas to $T \gtrsim 5 \times 10^6$ K. The ultraviolet radiation from an optically thick accretion disk would ionize and heat the gas to only $T \sim 10^4$ K. The X-rays from an optically thin accretion disk might heat the gas to a higher temperature (Buff and McCray 1974), but still not by a large enough factor to significantly affect the accretion rate, because an element of gas at distance r ahead of the black hole is only heated for a time $\sim r/V_H$ before encountering the bow shock.

We now consider the properties of black holes accreting from various environments in the Galaxy. We assume that the interstellar medium consists of the following components, in order of increasing density (McKee and Ostriker 1977; Sanders, Scoville, and Solomon 1985): a gaseous halo with radius $r \approx 10$ kpc and density $n \approx 10^{-4}$ cm^{-3}; a hot interstellar medium in a disk with $r \approx 10$ kpc, half-thickness $h \approx 2$ kpc, and density $n \approx 4 \times 10^{-3}$ cm^{-3}; and three cooler components in the form of clouds partially filling a disk with $r \approx 10$ kpc and $h \approx 120$ pc: the warm H I, with filling factor $f \approx 0.2$ and $n \approx 0.3$ cm^{-3}, the cold H I, with $f \approx 0.02$ and $n \approx 40$ cm^{-3}, and the molecular clouds, with $f \approx 0.003$ and $n \approx 600$ cm^{-3}. We shall assume $n_H = 5$ kpc^{-3} for the number density of black holes. We consider black holes accreting in each of these environments in turn.

i) *Halo gas.*—There would be $N \approx 2 \times 10^4$ such sources in the Galaxy, with the nearest at $d \approx 2$ kpc. These sources are unlikely to form accretion disks because the density gradients are too low. Using Ipser and Price's (1982) analysis of spherical accretion, we estimate a luminosity of $L \approx 3 \times 10^{31}$ ergs s^{-1} at wavelengths around $\lambda \approx 1000$ μm. Such sources would be difficult to detect.

ii) *Hot interstellar medium.*—There would be $N \approx 6 \times 10^3$ such sources with the closest at $d \approx 0.6$ kpc. Most of them would probably not form accretion disks, though a small fraction ($\sim 10\%$) in the more inhomogeneous gas close to the plane might. For spherical accretion, we find a luminosity $L \approx 4 \times 10^{34}$ ergs s^{-1} at $\lambda \approx 200$ μm. Such sources would probably not have been noticed. For an optically thick accretion disk, we would have $L \approx 1 \times 10^{37}$ ergs s^{-1}; with an effective temperature $T_{eff} \approx 3600$ K. In color and luminosity, this corresponds roughly to an M0 supergiant star. For an optically thin accretion disk, we would have an X-ray source with $L \approx 10^{37}$ ergs s^{-1} and typical photon energy $kT \approx 3$ keV. There are only ~ 30 X-ray sources in the Galaxy with $L_x \gtrsim 10^{37}$ ergs s^{-1} in this energy range (Clark *et al.* 1978), so even a small number of such black hole sources would violate observational constraints.

iii) *Warm H I.*— There would be $N \approx 80$ of these sources, with the closest at $d \approx 2$ kpc. Radiation would almost certainly be from an accretion disk, so that $L \approx 9 \times 10^{38}$ ergs s^{-1}. If the accretion disk were optically thick, the radiation would have an effective temperature $T_{eff} \approx 11,000$ K. This color and luminosity corresponds roughly to a B8 supergiant star. Assuming the spectrum to be roughly blackbody, the flux of ionizing photons would be $S \approx 1 \times 10^{46}$ s^{-1}, which would produce a very small H II region. If the accretion disk were optically thin, it would produce X-rays with typical energy $kT \approx 30$ keV. The observations rule out any X-ray source this luminous existing in the Galaxy.

iv) *Cold H I.*—There would be $N \approx 8$ such sources, with the nearest at $d \approx 6$ kpc. For disk accretion, the luminosity would be $L \approx 1.2 \times 10^{41}$ ergs s^{-1}. which slightly exceeds the Eddington luminosity for radiation pressure on dust. For an optically thick accretion disk, $T_{eff} \approx 36,000$ K. This corresponds in color roughly to an O8 star, but the luminosity is much greater. Again assuming a blackbody spectrum, the ionizing flux would be $S \approx 1 \times 10^{51}$ s^{-1}. In a clumpy medium with clump density $n = 40$ cm^{-3} and filling factor $f = 0.02$, this would ionize a region of radius $r \approx 100$ pc. Thus, the accreting black hole would produce a giant H II region, somewhat brighter than Carina ($S \approx 4 \times 10^{50}$ s^{-1}), but not as bright as 30 Doradus ($S \approx 1 \times 10^{52}$ s^{-1}). Kennicutt (1984) concludes from the observations that several giant H II regions with $S \sim 10^{51}$ s^{-1} may exist in the Galaxy, and even brighter H II regions exist in some other galaxies. A difficulty with this interpretation, however, is that the observed giant H II regions correlate strongly with spiral arms. If the accretion disk were optically thin, again it would produce a much brighter X-ray source than is consistant with observations.

v) *Molecular Clouds.*—There would be $N \approx 1$ such source in the Galaxy, at a distance $d \approx 10$ kpc. If there were no feedback effects, this would have a luminosity $L \approx 1.8 \times 10^{42}$ ergs s^{-1} for disk accretion. However, the effects of radiation pressure on dust should limit the luminosity to $L_{Edd,d} \approx 8 \times 10^{40}$ ergs s^{-1}, similar to the value for a black hole in a cold H I cloud. In contrast to that case however, the H II region would be confined to the interior of the cloud, and most of the radiation would be absorbed directly by dust and reradiated in the infrared, so that the black hole would appear as a very luminous infrared source.

d) Gravitational Lensing by Black Holes

Massive black holes in the halos of galaxies will cause gravitational lensing of cosmologically distant sources. If the black holes comprise a fraction $\Omega_H = 0.1$ of the cosmological critical density, then the probability that an object at redshift $z = 1$ will be significantly lensed (with total amplification greater than 1.3) is approximately $1-2 \times 10^{-2}$ for $0.1 \leq \Omega_{tot} \leq 1$, if there is no amplification bias, and the typical splitting of the images is approximately 7×10^{-3} arcsec, for $M_H = 2 \times 10^6$ $M_\odot$ (Press and Gunn 1973; Turner, Ostriker, and Gott 1984). The observational bias of investigating brighter (hence amplified) sources could increase this fraction by an order of magnitude. This effect is potentially detectable by means of VLBI, though the number of compact sources studied so far is only about 100 (e.g., Pearson and Readhead 1984), which may not be enough to separate out the possible effects of

lensing from those of intrinsic complications in source geometry. In multiple image systems where a quasar is observed to be lensed by a galaxy, minilensing by black holes in the galaxy halo will also cause fluctuations in the intensities of the images around the values calculated assuming a smooth distribution of matter (e.g., Gott 1981; Young 1981). Scaling the results of Gott and of Young to $M_H = 2 \times 10^6\ M_\odot$, we find that massive black holes will lens the optical continuum and line emitting regions of the quasar together, thus leaving their relative intensities unchanged, and cause variations in the intensities on a time scale of $\sim 10^5$ yr, much too long to be observationally interesting. Minilensing splits the images by roughly 10^{-2} arcsec. For Q0967+561, the only system for which a satisfactory lensing model exists at present (Young *et al.* 1981), the inputs into the model are sufficiently uncertain that additional effects on the image intensities due to minilensing by randomly situated black holes probably cannot be excluded. In conclusion, gravitational lensing by galactic halos composed of massive black holes appears to provide a promising test for the picture presented in this paper.

V. CONCLUSIONS

We have shown that if the dark halos around galaxies are composed of massive black holes, then this can provide an explanation of the heating of the stellar disk in our Galaxy and in other galaxies. Gravitational scattering of disk stars by black holes automatically results in the stellar velocity dispersion increasing with age as $\sigma \propto t^{1/2}$ for large t, in good agreement with what is observed in the solar neighborhood, and also predicts axial ratios for the velocity ellipsoid in agreement with observations. The magnitude of the velocity dispersion predicted for old disk stars agrees with that observed in the solar neighborhood if the mass of the black holes is taken to be $M_H \approx 2 \times 10^6\ M_\odot$, with the same value satisfactory for other galaxies also.

Heating by halo black holes results in a distribution function for the stars which is approximately isothermal at low epicyclic energies, in the Fokker-Planck regime in which the evolution is due to many weak encounters, but with a power-law tail at high energies produced by the relatively rare close encounters. The number of stars per unit epicyclic energy in the high energy tail varies as $N(E) \propto E^{-2}$. The evolution of the distribution function is self-similar in the low- and high-energy regimes, so that the fraction of stars in the high-energy tail is independent of the age of the stellar population, and in fact only depends on the value of the Coulomb logarithm $\ln \Lambda$ for scattering of stars by black holes, independent of any of the other properties of the disk or halo. The Coulomb logarithm is expected to have the value $\ln \Lambda \approx 10$, for which we predict the fraction of disk stars in the high-velocity tail to be about 1%, almost independent of the properties of the galaxy or the radius within the galaxy. This fraction is too small to account for the "thick disks" already found in some other galaxies, but is of about the right magnitude to explain the high-velocity main-sequence A and F stars observed in the solar neighborhood.

The heating mechanism can also account for the approximate constancy of the disk scale height with radius that is observed in other spiral galaxies, though this property does not arise as naturally as the others discussed above: the core radius r_c of the halo must be chosen in the range $r_c \approx (1-2)\alpha^{-1}$, where α^{-1} is the exponential scale length of the disk. There is also a problem that the flaring of the outer parts of the disk is predicted to be larger than is observed in disks with large αr_{max}, where r_{max} is the outer truncation radius of the disk, unless the outer parts of spirals form later than the inner parts or have larger mass-to-light ratios.

We do not propose any specific scenario for the formation of these black holes. However, it is interesting that the mass derived for the individual black holes is close to the baryon Jeans mass at recombination, which may be the preferred mass scale for the first bound objects which form in the universe (Peebles and Dicke 1968; Carr and Rees 1984). The subsequent evolution of such an object is uncertain, but the baryons in it might form a single supermassive star, which then collapses to a black hole (Carr 1978). Alternatively, if the mass of the universe is dominated by weakly interacting elementary particles, density fluctuations will collapse directly to black holes if the random motions of the particles are small enough (Shapiro and Teukolsky 1985).

We also predict the effect of the halo black holes on the central region of the Galaxy. A rough calculation of the effect of dynamical friction on the spatial distribution of the black holes suggests that of order 100–1000 will have spiraled into the Galactic center over the life of the Galaxy, but that at any given time there will probably never be more than two in the center due to the gravitational slingshot interaction between single and binary black holes. Thus we naturally account for the compact object postulated to be at the center of the Galaxy by several observers.

We have calculated the effects of gravitational lensing by the black holes and find that they are probably consistent with observational limits. Approximately 1% of sources at redshift $z \approx 1$ will be significantly lensed, with typical image splittings of several milliarcseconds.

We have also calculated the effect that such black holes would have on dwarf spheroidal galaxies, and find that heating of the stars by the black holes might provide an explanation for the low stellar densities of these systems.

There is one potentially serious problem, that radiation by black holes as they accrete gas from the interstellar medium should make some of them very luminous. In particular, there should be ~ 10 black holes in the Galaxy accreting from dense gas clouds, and if the standard thin accretion disk theory is correct, these will each radiate $L \approx 1 \times 10^{41}$ ergs s^{-1} and emit enough ionizing radiation to produce a giant H II region comparable in luminosity to those seen in some other spiral galaxies. Whether some of the observed giant H II regions might in fact be produced by this mechanism is problematical.

In conclusion, although massive black holes in halos can explain disk heating and account for condensed objects in galactic nuclei, the lack of clear, direct evidence for nearby accreting massive objects places the scenario in some doubt.

APPENDIX

In this Appendix we derive expressions for the orbit-averaged diffusion coefficients for E_e and E_z in terms of the velocity space diffusion coefficients. We assume that $\langle \Delta u \rangle$, $\langle \Delta v \rangle$, $\langle \Delta w \rangle$, $\langle (\Delta u)^2 \rangle$, $\langle (\Delta v)^2 \rangle$ and $\langle (\Delta w)^2 \rangle$ are constant around an orbit, with all the other moments vanishing, and that the unperturbed orbits of the stars are described by harmonic epicyclic theory, for which

$$u = (2E_e)^{1/2} \sin (\kappa t + \delta) , \tag{A1}$$

$$v = [(2E_e)^{1/2}/\beta] \cos (\kappa t + \delta) , \tag{A2}$$

$$z = [(2E_z)^{1/2}/\nu] \sin (\nu t + \epsilon) , \tag{A3}$$

$$w = (2E_z)^{1/2} \cos (\nu t + \epsilon) , \tag{A4}$$

where E_e, E_z, and β are defined by equations (3)–(5). Then

$$\Delta E_e = u\Delta u + \tfrac{1}{2}(\Delta u)^2 + \beta^2[v\Delta v + \tfrac{1}{2}(\Delta v)^2] , \tag{A5}$$

$$\Delta E_z = w\Delta w + \tfrac{1}{2}(\Delta w)^2 , \tag{A6}$$

assuming that the change in position of a star during an encounter is negligible. The expected instantaneous rate of change of E_e is therefore

$$\langle \Delta E_e \rangle = u\langle \Delta u \rangle + \tfrac{1}{2}\langle (\Delta u)^2 \rangle + \beta^2([v\langle \Delta v \rangle + \tfrac{1}{2}\langle (\Delta v)^2 \rangle]) . \tag{A7}$$

Orbit-averaging, using the assumptions about the constancy of the moments and the result (from [A1] and [A2]) $\langle u \rangle_{av} = \langle v \rangle_{av} = 0$, we derive

$$\langle \Delta E_e \rangle_{av} = \tfrac{1}{2}[\langle (\Delta u)^2 \rangle + \beta^2 \langle (\Delta v)^2 \rangle] . \tag{A8}$$

Similarly,

$$\langle \Delta E_z \rangle_{av} = \tfrac{1}{2}\langle (\Delta w)^2 \rangle . \tag{A9}$$

Squaring equation (A5), and assuming that mixed moments of $(\Delta u, \Delta v, \Delta w)$ and moments higher than the second vanish, we derive

$$\langle (\Delta E_e)^2 \rangle = u^2\langle (\Delta u)^2 \rangle + \beta^4 v^2 \langle (\Delta v)^2 \rangle . \tag{A10}$$

Orbit-averaging, using the result $\langle u^2 \rangle_{av} = \beta^2 \langle v^2 \rangle_{av} = E_e$, we obtain

$$\langle (\Delta E_e)^2 \rangle_{av} = 2E_e \langle \Delta E_e \rangle_{av} . \tag{A11}$$

Similarly,

$$\langle (\Delta E_e)(\Delta E_z) \rangle_{av} = 0 , \tag{A12}$$

$$\langle (\Delta E_z)^2 \rangle_{av} = 2E_z \langle \Delta E_z \rangle_{av} . \tag{A13}$$

The results for $\langle (\Delta E_e)^2 \rangle_{av}$ and $\langle (\Delta E_z)^2 \rangle_{av}$ clearly depend on the assumption that the epicyclic oscillations are harmonic in time, but the results for the other moments are valid for any periodic oscillations.

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