$B_{d,s} \to \rho, \omega, K^*, \phi$ decay form factors from light-cone sum rules reexamined

Patricia Ball[1,*] and Roman Zwicky[2,†]

[1]*IPPP, Department of Physics, University of Durham, Durham DH1 3LE, United Kingdom*
[2]*William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*
(Received 8 December 2004; published 26 January 2005)

We present an improved calculation of $B \to$ light vector form factors from light-cone sum rules, including one-loop radiative corrections to twist-2 and twist-3 contributions, and leading order twist-4 corrections. The total theoretical uncertainty of our results at zero momentum transfer is typically 10% and can be improved, at least in part, by reducing the uncertainty of hadronic input parameters. We present our results in a way which details the dependence of the form factors on these parameters and facilitates the incorporation of future updates of their values from, e.g., lattice calculations. We also give simple and easy-to-implement parametrizations of the q^2 dependence of the form factors which are valid in the full kinematical regime of q^2.

DOI: 10.1103/PhysRevD.71.014029 PACS numbers: 13.25.Hw, 12.38.Lg

I. INTRODUCTION

This paper aims to give a new and more precise determination of the decay form factors of $B_{d,u,s}$ mesons into light vector mesons, i.e., ρ, ω, K^*, and ϕ; it is a continuation of of our previous study of B decays into pseudoscalar mesons [1]. The calculation uses the method of QCD sum rules on the light cone, which in the past has been rather successfully applied to various problems in heavy-meson physics cf. Refs. [2][1]; an outline of the method will be given below. Our calculation improves on our previous paper [4] by

(i) including $B \to \omega$ form factors;
(ii) including radiative corrections to 2-particle twist-3 contributions to one-loop accuracy;
(iii) including a new parametrization of the dominant hadronic contributions (twist-2 distribution amplitudes);
(iv) detailing the dependence of form factors on distribution amplitudes;
(v) including a new parametrization of the dependence of the form factors on momentum transfer;
(vi) including a careful analysis of the theoretical uncertainties.

Like in Ref. [1], the motivation for this study is twofold and relates to the overall aim of B physics to provide precision determinations of quark flavor mixing parameters in the standard model. Quark flavor mixing is governed by the unitary CKM matrix which depends on four parameters: three angles and one phase. The constraints from unitarity can be visualized by the so-called unitarity triangles; the one that is relevant for B physics is under intense experimental study. The over-determination of the sides and angles of this triangle from a multitude of processes will answer the question whether there is new physics in flavor-changing processes and where it manifests itself. One of the sides of the unitarity triangles is given by the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$. $|V_{cb}|$ is known to about 2% accuracy from both inclusive and exclusive $b \to c\ell\nu$ transitions [5], whereas the present error on $|V_{ub}|$ is much larger and around 15%. Its reduction requires an improvement of experimental statistics, which is underway at the B factories *BABAR* and Belle, but also and, in particular, an improvement of the theoretical prediction for associated semileptonic spectra and decay rates. This is one motivation for our study of the $B \to \rho$ semileptonic decay form factors A_1, A_2, V, which, in conjunction with alternative calculations, hopefully from lattice, will help to reduce the uncertainty from exclusive semileptonic determinations of $|V_{ub}|$. Secondly, form factors of general $B \to$ light meson transitions also are needed as ingredients in the analysis of nonleptonic two-body B decays, e.g., $B \to \rho\pi$, in the framework of QCD factorization [6]—again with the objective to extract CKM parameters. One issue calling for particular attention in this context is the effect of SU(3) breaking, which enters both the form factors and the K^* and ϕ meson distribution amplitudes figuring in the factorization analysis. We would like to point out that the implementation of SU(3) breaking in the light-cone sum rules approach to form factors is *precisely* the same as in QCD factorization and is encoded in the difference between ρ, ω, K^*, and ϕ distribution amplitudes, so that the use of form factors calculated from light-cone sum rules together with the corresponding meson distribution amplitudes in factorization formulas allows a unified and controlled approach to the assessment of SU(3) breaking effects in nonleptonic B decays.

As we shall detail below, QCD sum rules on the light cone allow the calculation of form factors in a kinematic regime where the final state meson has large energy in the rest system of the decaying B, $E \gg \Lambda_{\rm QCD}$. The physics underlying B decays into light mesons at large momentum transfer can be understood qualitatively in the framework of hard exclusive QCD processes, pioneered by Brodsky

*Patricia.Ball@durham.ac.uk
†zwicky@physics.umn.edu
[1]See also Ref. [3] for reviews.

and Lepage *et al.* [7]. The hard scale in B decays is m_b and one can show that to leading order in $1/m_b$ the decay is described by two different parton configurations: one where all quarks have large momenta and the momentum transfer happens via the exchange of a hard gluon, the so-called hard-gluon exchange, and a second one where one quark is soft and does interact with the other partons only via soft-gluon exchange—the so-called soft or Feynman mechanism. The consistent treatment of both effects in a framework based on factorization, i.e., the clean separation of perturbatively calculable hard contributions from nonperturbative "wave functions," is highly nontrivial and has spurred the development of soft collinear effective theory (SCET), an effective field theory which aims to separate the two relevant large mass scales m_b and $\sqrt{m_b\Lambda_{\rm QCD}}$ in a systematic way [8]. In this approach form factors can indeed be split into a calculable factorizable part which roughly corresponds to the hard-gluon exchange contributions, and a nonfactorizable one, which includes the soft contributions and cannot be calculated within the SCET framework [9,10]. Predictions obtained in this approach then typically aim to eliminate the soft part and take the form of relations between two or more form factors whose difference is expressed in terms of factorizable contributions.

The above discussion highlights the need for a calculational method that allows numerical predictions while treating both hard and soft contributions on the same footing. It is precisely QCD sum rules on the light cone (LCSRs) that accomplish this task. LCSRs can be viewed as an extension of the original method of QCD sum rules devised by Shifman, Vainshtein, and Zakharov [11], which was designed to determine properties of ground state hadrons at zero or low momentum transfer, to the regime of large momentum transfer. QCD sum rules combine the concepts of operator product expansion, dispersive representations of correlation functions, and quark-hadron duality in an ingenious way that allows the calculation of the properties of nonexcited hadron states with a very reasonable theoretical uncertainty. In the context of weak-decay form factors, the basic quantity is the correlation function of the weak current and a current with the quantum numbers of the B meson, evaluated between the vacuum and a light meson. For large (negative) virtualities of these currents, the correlation function is, in coordinate space, dominated by distances close to the light cone and can be discussed in the framework of light-cone expansion. In contrast to the short-distance expansion employed by conventional QCD sum rules à la Shifman, Vainshtein, and Zakharov where nonperturbative effects are encoded in vacuum expectation values of local operators with vacuum quantum numbers, the condensates, LCSRs rely on the factorization of the underlying correlation function into genuinely nonperturbative and universal hadron distribution amplitudes (DAs) ϕ which are convoluted with process-dependent amplitudes T. The latter are the analogues of Wilson coefficients in the short-distance expansion and can be calculated in perturbation theory. The light-cone expansion then reads, schematically:

$$\text{correlation function} \sim \sum_n T^{(n)} \otimes \phi^{(n)}. \tag{1}$$

The sum runs over contributions with increasing twist, labeled by n, which are suppressed by increasing powers of, roughly speaking, the virtualities of the involved currents. The same correlation function can, on the other hand, be written as a dispersion relation, in the virtuality of the current coupling to the B meson. Equating dispersion representation and the light-cone expansion, and separating the B meson contribution from that of higher one- and multi-particle states using quark-hadron duality, one obtains a relation for the form factor describing the decay $B \to$ light meson.

A crucial question is the accuracy of light-cone sum rules. Like with most other methods, there are uncertainties induced by external parameters like quark masses and hadronic parameters and intrinsic uncertainties induced by the approximations inherent in the method. As we shall discuss in Sec. IV, the total theoretical uncertainty for the form factors at $q^2 = 0$ is presently around 10%, including a 7% irreducible systematic uncertainty.

Our paper is organized as follows: in Sec. II we define all relevant quantities, in particular, the meson distribution amplitudes. In Sec. III we outline the calculation. In Sec. IV we derive the sum rules and present numerical results. Section V contains a summary and conclusions. Detailed expressions for distribution amplitudes, a breakdown of the light-cone sum rule results into different contributions, and explicit formulas for the contributions of 3-particle states are given in the Appendices A, B, and C.

II. DEFINITIONS

$B \to V$ transitions, where V stands for the vector mesons ρ, ω, K^*, and ϕ, can manifest themselves as semileptonic decays $B \to V\ell\bar{\nu}_\ell$ or rare flavor-changing neutral current penguin-induced decays $B \to V\gamma$ and $B \to V\ell^+\ell^-$. All these decays are described by a total of seven independent form factors which usually are defined as ($q = p_B - p$)

$$\begin{aligned} c_V\langle V(p)|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle \\ = -ie^*_\mu(m_B+m_V)A_1(q^2) \\ + i(p_B+p)_\mu(e^*q)\frac{A_2(q^2)}{m_B+m_V} \\ + iq_\mu(e^*q)\frac{2m_V}{q^2}[A_3(q^2)-A_0(q^2)] \\ + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\frac{2V(q^2)}{m_B+m_V}, \end{aligned} \tag{2}$$

with

$$A_3(q^2) = \frac{m_B + m_V}{2m_V} A_1(q^2) - \frac{m_B - m_V}{2m_V} A_2(q^2) \quad \text{and} \quad A_0(0) = A_3(0); \tag{3}$$

$$\begin{aligned} c_V\langle V(p)|\bar{q}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle &= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma 2T_1(q^2) + T_2(q^2)\{e_\mu^*(m_B^2 - m_V^2) \\ &\quad - (e^*q)(p_B+p)_\mu\} \\ &\quad + T_3(q^2)(e^*q)\left\{q_\mu - \frac{q^2}{m_B^2 - m_V^2}(p_B+p)_\mu\right\}, \end{aligned} \tag{4}$$

with

$$T_1(0) = T_2(0). \tag{5}$$

A_0 is also the form factor of the pseudoscalar current:

$$\begin{aligned} c_V\langle V|\partial_\mu A^\mu|B\rangle &= c_V(m_b + m_q)\langle V|\bar{q}i\gamma_5 b|B\rangle \\ &= 2m_V(e^*q)A_0(q^2). \end{aligned} \tag{6}$$

$\bar{q}$ in the above formulas stands for $\bar{u}$, $\bar{d}$, and $\bar{s}$ in (2) and (6) and $\bar{d}$, $\bar{s}$ in (4); the actual assignment of different decay channels to underlying $b \to q$ transitions is made explicit in Table I. In our calculation, we assume isospin symmetry throughout, which implies that there are five different sets of form factors: $B_q \to \rho$, $B_q \to \omega$, $B_q \to K^*$, $B_s \to K^*$, and $B_s \to \phi$ (with $q = u, d$). The factor c_V accounts for the flavor content of particles: $c_V = \sqrt{2}$ for ρ^0, ω and $c_V = 1$ otherwise.[2]

The currents in (2) and (4) contain both vector and axial-vector components. V and T_1 correspond to the vector components of the currents, and, as the B meson is a pseudoscalar, to the axial-vector components of the matrix elements. $A_{1,2}$ clearly correspond to the axial-vector component of the $V - A$ current; the term in $A_3 - A_0$ arises as the contraction of (2) with $-iq^\mu$ must agree with (6). As for the penguin current, $T_{2,3}$ correspond to the axial-vector components of the current; there is no analogon to A_0, as the current vanishes upon contraction with q^μ. As we shall see in Sec. IV, for analyzing the dependence of each form factor on q^2, it is best to choose $A_{0,1,2}$ as independent form factors for the A current, and define A_3 by (3), but for the penguin current it will turn out more appropriate to choose a different set of independent form factors: T_1, T_2, and $\tilde{T}_3$ with

[2]To be precise, c_V is $\sqrt{2}$ for ρ^0 in $b \to u$ and for ω, and $-\sqrt{2}$ for ρ^0 in $b \to d$, with the flavor wave functions $\rho^0 \sim (\bar{u}u - \bar{d}d)/\sqrt{2}$ and $\omega \sim (\bar{u}u + \bar{d}d)/\sqrt{2}$. We assume that ϕ is a pure $s\bar{s}$ state.

TABLE I. Allowed decay channels in terms of underlying quark transitions. We assume isospin-symmetry and hence have five different sets of form factors: $B_q \to \rho$, $B_q \to \omega$, $B_q \to K^*$, $B_s \to K^*$, and $B_s \to \phi$ (with $q = u, d$).

	ρ^+	ρ^0, ω	ρ^-	K^{*+}	$K^{*0}(d\bar{s})$	K^{*-}	$\bar{K}^{*0}(s\bar{d})$	ϕ
B_u^-		$b \to u$	$b \to d$			$b \to s$		
$\bar{B}_d$	$b \to u$	$b \to d$					$b \to s$	
$\bar{B}_s$				$b \to u$	$b \to d$			$b \to s$

$$\begin{aligned} c_V\langle V(p)|\bar{q}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle &= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma 2T_1(q^2) + e_\mu^*(m_B^2 - m_V^2)T_2(q^2) \\ &\quad - (p_B+p)_\mu(e^*q)\tilde{T}_3(q^2) + q_\mu(e^*q)T_3(q^2), \end{aligned} \tag{7}$$

and T_3 defined as

$$T_3(q^2) = \frac{m_B^2 - m_V^2}{q^2}[\tilde{T}_3(q^2) - T_2(q^2)]. \tag{8}$$

As the actual calculation is done using an off shell momentum p_B with $p_B^2 \neq m_B^2$, it is crucial to avoid any ambiguity in the interpretation of scalar products like $2pq = p_B^2 - q^2 - p^2 \neq m_B^2 - q^2 - m_V^2$ that occur at intermediate steps of the calculation. This is particularly relevant for the penguin form factors which are defined in terms of a matrix element over the tensor current which is contracted with the physical momentum q^ν. The problem can be avoided by extracting T_i and $\tilde{T}_3$ from sum rules for a matrix element with no contractions:

$$\begin{aligned} \langle V(p)|\bar{q}\sigma_{\mu\nu}\gamma_5 b|B(p_B)\rangle &= A(q^2)\{e_\mu^*(p_B+p)_\nu - (p_B+p)_\mu e_\nu^*\} \\ &\quad - B(q^2)\{e_\mu^* q_\nu - q_\mu e_\nu^*\} \\ &\quad - 2C(q^2)\frac{e^*q}{m_B^2 - m_V^2}\{p_\mu q_\nu - q_\mu p_\nu\}. \end{aligned} \tag{9}$$

A, B, and C are related to T_i and $\tilde{T}_3$ defined in (4) and (7) as

$$T_1 = A, \qquad T_2 = A - \frac{q^2}{m_B^2 - m_V^2}B, \qquad T_3 = B + C, \qquad \tilde{T}_3 = A + \frac{q^2}{m_B^2 - m_V^2}C, \tag{10}$$

which implies

$$T_1(0) = T_2(0) = \tilde{T}_3(0). \tag{11}$$

Relevant for semileptonic decays are, in the limit of vanishing lepton mass, the form factors $A_{1,2}$ and V with q^2, the invariant mass of the lepton-pair, in the range $0 \leq q^2 \leq (m_B - m_V)^2$. $B \to V\gamma$ depends on $T_1(0)$, whereas $B \to V\ell^+\ell^-$ depends on all seven form factors (see Ref. [14] for an explicit formula). The motivation for studying $B \to \rho\ell\bar{\nu}_\ell$ and $B \to \omega\ell\bar{\nu}_\ell$ is to extract information on the CKM matrix element $|V_{ub}|$, whereas the flavor-changing neutral current transitions $B \to (K^*, \rho, \omega)\gamma$ and $B \to (K^*, \rho, \omega)\ell^+\ell^-$ serve to constrain new physics or, in the absence thereof, the ratio $|V_{ts}/V_{td}|$ [15], which would complement the determination of $|V_{ts}/V_{td}|$ from B mixing.

In the LCSR approach the form factors are extracted from the correlation function of the relevant weak current J_W, i.e., either the semileptonic $V - A$ current or the penguin current of (8), and an interpolating field for the B meson, in the presence of the vector meson:

$$\Gamma(q^2, p_B^2) = i \int d^4x e^{iqx} \langle V(p)|TJ_W(x) j_b^\dagger(0)|0\rangle, \qquad (12)$$

with $j_b = m_b \bar{q}' i\gamma_5 b$, $q' \in \{u, d, s\}$. For virtualities

$$m_b^2 - p_B^2 \geq O(\Lambda_{\rm QCD} m_b), \qquad m_b^2 - q^2 \geq O(\Lambda_{\rm QCD} m_b), \qquad (13)$$

the correlation function (12) is dominated by lightlike distances and therefore accessible to an expansion around the light cone. The above conditions can be understood by demanding that the exponential factor in (12) vary only slowly. The light-cone expansion is performed by integrating out the transverse and ''minus'' degrees of freedom and leaving only the longitudinal momenta of the partons as relevant degrees of freedom. The integration over transverse momenta is done up to a cutoff, $\mu_{\rm IR}$, all momenta below which are included in a so-called hadron distribution amplitude (DA) ϕ, whereas larger transverse momenta are calculated in perturbation theory. The correlation function is hence decomposed, or factorized, into perturbative contributions T and nonperturbative contributions ϕ, which both depend on the longitudinal parton momenta and the factorization scale $\mu_{\rm IR}$. If the vector meson is an effective quark-antiquark bound state, as is the case to leading order in the light-cone expansion, one can write the corresponding longitudinal momenta as up and $(1 - u)p$, where p is the momentum of the meson and u a number between 0 and 1. The schematic relation (1) can then be written in more explicit form as

$$\Gamma(q^2, p_B^2) = \sum_n \int_0^1 du T^{(n)}(u, q^2, p_B^2, \mu_{\rm IR}) \phi^{(n)}(u, \mu_{\rm IR}). \qquad (14)$$

As Γ itself is independent of the arbitrary scale $\mu_{\rm IR}$, the scale dependence of $T^{(n)}$ and $\phi^{(n)}$ must cancel each other.[3] If $\phi^{(n)}$ describes the meson in a 2-parton state, it is called a 2-particle DA; if it describes a 3-parton, i.e., quark-antiquark-gluon state, it is called 3-particle DA. In the latter case the integration over u gets replaced by an integration over two independent momentum fractions, say α_1 and α_2.

[3]If there is more than one contribution of a given twist, they will mix under a change of the factorization scale $\mu_{\rm IR}$ and it is only in the sum of all such contributions that the residual $\mu_{\rm IR}$ dependence cancels.

Equation (14) is called a ''collinear'' factorization formula, as the momenta of the partons in the meson are collinear with its momentum. Any such factorization formula requires verification by explicit calculation; we will come back to that issue in the next section.

Let us now define the distribution amplitudes to be used in this paper. All definitions and formulas are well known and can be found in Ref. [16]. In general, the distribution amplitudes we are interested in are related to nonlocal matrix elements of type[4]

$$\langle 0|\bar{q}_2(0)\Gamma[0, x] q_1(x)|V(p)\rangle \quad \text{or}$$
$$\langle 0|\bar{q}_2(0)[0, vx]\Gamma G^a_{\mu\nu}(vx)\lambda^a/2[vx, x] q_1(x)|V(p)\rangle.$$

x is lightlike or close to lightlike and the light-cone expansion is an expansion in x^2; v is a number between 0 and 1; and Γ a combination of Dirac matrices. The expressions $[0, x]$, etc., denote Wilson lines that render the matrix elements, and hence the DAs, gauge invariant. One usually works in the convenient Fock-Schwinger gauge $x^\mu A^a_\mu(x)\lambda^a/2 = 0$, where all Wilson lines are just $\mathbf{1}$; we will suppress them from now on.

The DAs are formally ordered by twist, i.e., the difference between spin and dimension of the corresponding operators. In this paper we take into account 2- and 3-particle DAs of twist-2, 3, and 4. The classification scheme of vector meson DAs is more involved than that for pseudoscalars; it has been studied in detail in Ref. [16]. One important point is the distinction between chiral-even and chiral-odd operators, i.e., those with an odd or even number of γ_μ matrices. In the limit of massless quarks the DAs associated with these operators form two completely separate classes that do not mix under a change of $\mu_{\rm IR}$. One more important parameter is the polarization state of the meson, longitudinal ($\parallel$) or transverse ($\perp$), which helps to classify twist-2 and 3 DAs. Up to twist-4 accuracy, we have the following decomposition of chiral-even 2-particle DAs [16]:

$$\begin{aligned}
&\langle 0|\bar{q}_2(0)\gamma_\mu q_1(x)|V(P, \lambda)\rangle \\
&\quad = f_V m_V \Big\{ \frac{e^{(\lambda)} z}{Pz} P_\mu \int_0^1 du\, e^{-iuPz}\Big[\phi_\parallel(u) + \frac{m_V^2 x^2}{16}\mathbb{A}_\parallel(u) \\
&\qquad + O(x^4)\Big] + \Big(e^{(\lambda)}_\mu - P_\mu \frac{e^{(\lambda)} z}{Pz}\Big)\int_0^1 du\, e^{-iuPz}[g_\perp^{(v)}(u) \\
&\qquad + O(x^2)] - \frac{1}{2} z_\mu \frac{e^{(\lambda)} z}{(pz)^2} m_V^2 \int_0^1 du\, e^{-iupz}[g_3(u) \\
&\qquad + \phi_\parallel(u) - 2 g_\perp^{(v)}(u) + O(x^2)]\Big\}, \qquad (15)
\end{aligned}$$

[4]The currents to use for ρ^0 and ω are $(\bar{u}\Gamma u \mp \bar{d}\Gamma d)/\sqrt{2}$, respectively.

$$\langle 0|\bar{q}_2(0)\gamma_\mu\gamma_5 q_1(x)|V(P,\lambda)\rangle = -\frac{1}{4} f_V m_V \epsilon_\mu^{\nu\alpha\beta} e_\nu^{(\lambda)} p_\alpha z_\beta \int_0^1 du\, e^{-iupz}[g_\perp^{(a)}(u) + O(x^2)], \tag{16}$$

and for the chiral-odd ones:

$$\begin{aligned}\langle 0|\bar{q}_2(0)\sigma_{\mu\nu} q_1(x)|V(P,\lambda)\rangle = if_V^T\Big\{&(e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu)\int_0^1 du\, e^{-iuPz}\Big[\phi_\perp(u) + \frac{m_V^2 x^2}{16}\mathbb{A}_\perp(u)\Big]\\ &+ (p_\mu z_\nu - p_\nu z_\mu)\frac{e^{(\lambda)}z}{(pz)^2} m_V^2 \int_0^1 du\, e^{-iupz}\Big[h_\parallel^{(t)}(u) - \frac{1}{2}\phi_\perp(u) - \frac{1}{2}h_3(u) + O(x^2)\Big]\\ &+ \frac{1}{2}(e_\mu^{(\lambda)}z_\nu - e_\nu^{(\lambda)}z_\mu)\frac{m_V^2}{pz}\int_0^1 du\, e^{-iupz}[h_3(u) - \phi_\perp(u) + O(x^2)]\Big\},\end{aligned} \tag{17}$$

$$\langle 0|\bar{q}_2(0)q_1(x)|V(P,\lambda)\rangle = \frac{i}{2} f_V^T (e^{(\lambda)}z) m_V^2 \times \int_0^1 du\, e^{-iupz}[h_\parallel^{(s)}(u) + O(x^2)]. \tag{18}$$

The relevant 3-particle DAs are defined in Appendix B.

Note that we distinguish between lightlike vectors p, z with $p^2 = 0 = z^2$ and the vectors P, x with $P^2 = m_V^2$ and $x^2 \neq 0$; explicit relations between these vectors are given in Appendix B. The DAs are dimensionless functions of u and describe the probability amplitudes to find the vector meson V in a state with minimal number of constituents—quark and antiquark—which carry momentum fractions u (quark) and $1 - u$ (antiquark), respectively. The eight DAs $\phi = \{\phi_{\parallel,\perp}, g_\perp^{(v,a)}, h_\parallel^{(s,t)}, h_3, g_3\}$ are normalized as

$$\int_0^1 du\phi(u) = 1. \tag{19}$$

The nonlocal operators on the left-hand side are renormalized at scale μ, so that the distribution amplitudes depend on μ as well. This dependence can be calculated in perturbative QCD; we will come back to that point below.

The vector and tensor decay constants f_V and f_V^T featuring in Eqs. (15) and (18) are defined as

$$\langle 0|\bar{q}_2(0)\gamma_\mu q_1(0)|V(P,\lambda)\rangle = f_V m_V e_\mu^{(\lambda)}, \tag{20}$$

$$\langle 0|\bar{q}_2(0)\sigma_{\mu\nu} q_1(0)|V(P,\lambda)\rangle = if_V^T(\mu)(e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu). \tag{21}$$

f_V^T depends on the renormalization scale as

$$f_V^T(Q^2) = L^{C_F/\beta_0} f_V^T(\mu^2),$$

with $L = \alpha_s(Q^2)/\alpha_s(\mu^2)$ and $\beta_0 = 11 - 2/3n_f$, n_f being the number of flavors involved.

The DAs as defined above do actually not all correspond to matrix elements of operators with definite twist: $\phi_{\perp,\parallel}$ are of twist-2, $h_\parallel^{(s,t)}$ and $g_\perp^{(v,a)}$ contain a mixture of twist-2 and 3 contributions, and $\mathbb{A}_{\perp,\parallel}$, h_3, and g_3 a mixture of twist-2, 3, and 4 contributions. Rather than as matrix elements of operators with definite twist, the DAs are defined as matrix elements of operators built from fields with a fixed spin projection onto the light cone. For quark fields, the possible spin projections are $s = \pm 1/2$ and the corresponding projection operators $P_+ = 1/(2pz)\not{p}\not{z}$ and $P_- = 1/(2pz)\not{z}\not{p}$. Fields with fixed spin projection have a definite conformal spin, given by $j = 1/2(s +$ canonical mass dimension), and composite operators built from such fields can be expanded in terms of increasing conformal spin.[5] The expansion of the corresponding DAs, suitably dubbed conformal expansion, is one of the primary tools in the analysis of meson DAs, and together with the use of the QCD equations of motion it allows one to parametrize the plethora of 2- and 3-particle DAs in terms of a manageable number of independent hadronic matrix elements. DAs defined as matrix elements of operators with definite twist, on the other hand, do not have a well-defined conformal expansion [18], and this is the reason why we prefer the above definitions. In an admittedly rather sloppy way we will from now on refer to $g_\perp^{(v,a)}, h_\parallel^{(s,t)}$ as twist-3 DAs and to $h_3, g_3, \mathbb{A}_{\perp,\parallel}$ as twist-4 DAs. A more detailed discussion of the relations between the different DAs is given in Appendix B; the upshot is that the eighteen twist-2, 3, and 4 DAs we shall take into account can be paramatrized, to NLO in the conformal expansion, in terms of ten hadronic matrix elements, most of which give only tiny contributions to the LCSRs for form factors.

For the leading twist-2 DAs $\phi_{\parallel,\perp}$ in particular, the conformal expansion goes in terms of Gegenbauer polynomials:

$$\phi(u,\mu^2) = 6u(1-u)\Big[1 + \sum_{n=1}^\infty a_n(\mu^2) C_n^{3/2}(2u-1)\Big]. \tag{22}$$

The first term on the right-hand side $6u(1-u)$ is referred to as asymptotic DA; as the anomalous dimensions of a_n are positive, ϕ approaches the asymptotic DA in the limit $\mu^2 \to \infty$. The usefulness of this expansion manifests itself in the fact that, to leading logarithmic accuracy, the (non-

[5]For a more detailed discussion we refer to the first reference in [16] and to Ref. [17].

perturbative) Gegenbauer moments a_n renormalize multiplicatively with

$$a_n(Q^2) = L^{\gamma_n/(2\beta_0)} a_n(\mu^2), \tag{23}$$

with $L = \alpha_s(Q^2)/\alpha_s(\mu^2)$. The anomalous dimensions $\gamma_n^{\parallel,\perp}$ are given by

$$\gamma_n^{\parallel} = 8C_F\left[\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right], \tag{24}$$

$$\gamma_n^{\perp} = 8C_F[\psi(n+2) + \gamma_E - 1], \tag{25}$$

with $\psi(n+1) = \sum_{k=1}^{n} 1/k - \gamma_E$. As the contributions from different comformal spin do not mix under renormalization, at least to leading logarithmic accuracy, one can construct models for DAs by truncating the expansion at a fixed order. Despite the absence of any ''small parameter'' in that expansion, the truncation is justified inasmuch as one is interested in physical amplitudes rather than the DA itself. If we write

$$\text{amplitude} = \int_0^1 du\ \phi(u)T(u),$$

then, assuming that T is a regular function of u, i.e., with no (end point) singularities, the highly oscillating behavior of the Gegenbauer-polynomials suppresses contributions from higher orders in the conformal expansion. Even for a function T with a mild end point singularity, for instance $T = \ln u$, we find, using the generating function of the Gegenbauer polynomials,

$$\int_0^1 du\ \phi(u)T(u) = -\frac{5}{6}a_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+3)} 3a_n.$$

This result indicates that, assuming the a_n falloff in n, which, as we shall see seen in Sec. IV C is indeed the case, even a truncation after the first few terms should give a reasonable approximation to the full amplitude. A more thorough discussion of the convergence of the conformal expansion for physical amplitudes can be found in Ref. [19]. The major shortcoming of models based on the truncation of the conformal expansion is the fact that the information available on the actual values of the a_n (and, in particular, their analogues in 3-particle DAs) is, to put it mildly, scarce. We therefore use truncated models only for DAs whose contribution to the LCSRs is small as is the case for all 3-particle DAs and the twist-4 DAs; explicit formulas are given in Appendix B. All contributions due to or induced by twist-2 DAs, on the other hand, are treated as described in Sec. IV C.

The major difference between the analysis of LCSRs for $B \to$ vector meson form factors and that of $B \to$ pseudoscalar form factors presented in [1] is probably the identification of a suitable parameter by which to order the relative weight of different contributions to the sum rules. For $B \to$ pseudoscalar form factors, the standard classification in terms of increasing twist proved to be suitable, as the chiral parities of the twist-2 DA and 2-particle twist-3 DAs are different, so that contribution of the latter to the LCSRs is suppressed by a factor $m_\pi^2/(m_u + m_d)/m_b$. In addition, the admixture of twist-2 matrix elements to twist-3 DAs and of twist-2 and three matrix elements to twist-4 DAs is small and moreover vanishes in the chiral limit $m_\pi \to 0$. For vector mesons, the situation is more complex: for instance, both the twist-2 DA $\phi_\perp$ and the twist-3 DA $g_\perp^{(v)}$ contribute at the same order to the form factors A_2 and A_0, in the combination $\phi_\parallel - g_\perp^{(v)}$. Naive twist counting is evidently not very appropriate for classifying the relative size of contributions of different DAs to the form factors. Instead, we decide to classify the relevance of contributions to the LCSRs not by twist but by a parameter $\delta \propto m_V$. The precise definition of δ depends on the kinematics of the process; to leading order in an expansion in $1/m_b$, however, one finds $\delta_{\rm HQL} = m_V/m_b$. The numerical analysis of the LCSRs does indeed display a clear suppression of terms in $O(\delta)$ and higher, which suggests the following classification of 2-particle DAs:

(i) $O(\delta^0)$:$\phi_\perp$;

(ii) $O(\delta^1)$:$\phi_\parallel$, $g_\perp^{(v,a)}$;

(iii) $O(\delta^2)$:$h_\parallel^{(s,t)}$, h_3, $\mathbb{A}_\perp$;

(iv) $O(\delta^3)$:g_3, $\mathbb{A}_\parallel$.

We treat δ as expansion parameter of the light-cone expansion and shall combine it with the perturbative QCD expansion in α_s to obtain a second order expression for the correlation functions (12); terms in δ^3 are dropped.

III. CALCULATION OF THE CORRELATION FUNCTIONS

As we have seen in Sec. II, LCSRs for form factors are extracted from the correlation function of the corresponding weak current with the pseudoscalar current $j_b = m_b\bar{q}' i\gamma_5 b$, evaluated between the vacuum and the vector meson. In this section we describe the calculation of these correlation functions to second order in α_s and δ.

The relevant correlation functions are defined as

$$\begin{aligned} i\int d^4x e^{iqx}\langle V(p)|T(V-A)_\mu(x) j_b^\dagger(0)|0\rangle &= \\ = -i\Gamma_0 e_\mu^* + i\Gamma_+(e^*q)(q+2p)_\mu &+ i\Gamma_-(e^*q)q_\mu \\ + \Gamma_V \epsilon_\mu^{\alpha\beta\gamma} e_\alpha^* q_\beta p_\gamma, \end{aligned} \tag{26}$$

$$\begin{aligned} i\int d^4x e^{iqx}\langle V(p)|T[\bar{q}\sigma_{\mu\nu}\gamma_5 b](x) j_b^\dagger(0)|0\rangle \\ = \mathcal{A}\{e_\mu^*(2p+q)_\nu - e_\nu^*(2p+q)_\mu\} - \mathcal{B}\{e_\mu^* q_\nu \\ - e_\nu^* q_\mu\} - 2C(e^*q)\{p_\mu q_\nu - q_\mu p_\nu\}. \end{aligned} \tag{27}$$

The definitions of $\Gamma^\pm$ and C differ from those used in

Ref. [4] by a factor pq; we shall come back to this point below. In this section we describe the calculation of the contributions of 2-particle DAs to the above correlation functions; those of 3-particle DAs are calculated in Appendix C.

In light-cone expansion and including only contributions from 2-particle Fock states of the mesons, each of the seven invariants $\Gamma_{0,\pm,V}$, $\mathcal{A}, \mathcal{B}, C$ can be written as a convolution integral of type

$$\Gamma^V = \int \frac{dk_+}{2\pi} \phi^V_{ab}(k_+) T_{ba}(k_+, p_B^2, q^2), \tag{28}$$

with a, b being spinor indices. $p^2 = m_V^2$ is set to 0 and k_+ is the longitudinal momentum of the quark in the vector meson V, which is related to the momentum fraction u introduced in Sec. II by $k_+ = up_+$.[6] The above factorization formula implies a complete decoupling of long-distance QCD effects, encoded in the DA ϕ^V, and short-distance effects calculable in perturbation theory, described by T. Factorization also makes it possible to calculate T in a convenient way: if it holds, T must be independent of the specific properties of the external hadron state, and one can calculate Γ^V with a particularly simple state that allows a straightforward extraction of the short-distance amplitudes T.[7] A convenient choice of the external state is a free quark-antiquark pair with longitudinal momenta up and $\bar{u}p$ and spins s and r, respectively, and DA

$$\begin{aligned}\phi^{q_1\bar{q}_2}_{ab}(k_+) &= \int dz_- e^{-ik_+z_-} \langle q_1(up, s)\bar{q}_2(\bar{u}p, r)|(\bar{q}_1)_a(z)[z, 0](q_2)_b(0)|0\rangle \Big|_{z_+=0, z_\perp=0} \\ &= 2\pi \bar{u}^{q_1}_a(up, s) v^{q_2}_b(\bar{u}p, r)\delta(k_+ - up_+),\end{aligned}$$

where $\bar{u}$ and v are the standard fermion spinors. The T amplitudes, to one-loop accuracy, are then given directly by the diagrams shown in Fig. 1 with external on shell quarks with momenta up and $(1-u)p$, respectively. The projection onto a specific Dirac structure is done using the general decomposition

$$\begin{aligned}(\bar{q}_1)_a(q_2)_b = &\frac{1}{4}(\mathbf{1})_{ba}(\bar{q}_1 q_2) - \frac{1}{4}(i\gamma_5)_{ba}(\bar{q}_1 i\gamma_5 q_2) \\ &+ \frac{1}{4}(\gamma_\mu)_{ba}(\bar{q}_1\gamma^\mu q_2) \\ &- \frac{1}{4}(\gamma_\mu\gamma_5)_{ba}(\bar{q}_1\gamma^\mu\gamma_5 q_2) \\ &+ \frac{1}{8}(\sigma_{\mu\nu})_{ba}(\bar{q}_1\sigma^{\mu\nu}q_2).\end{aligned} \tag{29}$$

In order to obtain the convolution integrals for vector mesons, one has to replace the structures $\bar{q}_1\Gamma q_2$ in (29) by the appropriate DAs and include factors of e^*z, pz, and x^2 as given in Eqs. (15)–(18). The translation of explicit terms in z_μ into momentum space is given by

$$z_\mu \to -i\frac{\partial}{\partial(up)_\mu}, \tag{30}$$

as the outgoing q_1 comes with a factor $\exp(iupz)$. Terms in $1/(pz)$ can be treated by partial integration:

$$\frac{1}{pz}\phi(u) \to -i\int_0^u dv\ \phi(v) \equiv -i\Phi(u). \tag{31}$$

There are no surface terms, as for all the relevant structures ϕ, e.g., $\phi_\parallel - g_\perp^{(v)}$, one has $\Phi(0) = 0 = \Phi(1)$. A second, approximate way to deal with factors $(e^*z)/(pz)$ is based on the observation that (e^*z) projects onto the longitudinal polarization state of the vector meson cf. Eq. (B2), and that in the ultrarelativistic limit $E_V \to \infty$ the longitudinal polarization vector is approximately collinear with the meson's momentum:

$$\epsilon^{(0)}_\mu = \frac{1}{m_V}[p_\mu + O(m_V^2)] \quad \longrightarrow \quad \frac{e^*z}{pz} \to \frac{1}{m_V} \quad \text{and} \quad \frac{1}{m_V} \to \frac{e^*q}{pq}.$$

Up to corrections in m_V^2, this procedure yields results identical with those from partial integration—provided that the corresponding DA ϕ is normalized to 0. That is

$$\begin{aligned}-\int_0^1 du\Phi(u)e^*_\kappa\frac{\partial}{\partial(up)_\kappa} &\sim \frac{e^*z}{pz}\int_0^1 du\phi(u) \\ &\sim \frac{e^*q}{pq}\left[\int_0^1 du\phi(u) + O(m_V^2)\right],\end{aligned}$$

where the first relation is valid only if ϕ is normalized to 0, i.e., $\Phi(1) = 0$. This is indeed the case for the mixed-twist structure $\phi_\parallel - g_\perp^{(v)}$ but does not apply if only the pure twist-2 DA $\phi_\parallel$ is included, as done in [4]. In this case, unphysical singularities in $p_B^2 = q^2$ appear in $\Gamma_\pm$ and C and have to be factored out. This explains the appearance of additional factors $1/(pq) = 2/(p_B^2 - q^2)$ in the correlation functions used in [4]. In the calculation presented in

[6]The plus-component of a 4-vector k^μ is defined as $k_+ = (k^0 + k^3)/\sqrt{2}$, the minus component as $k_- = (k^0 - k^3)/\sqrt{2}$.

[7]This is completely analogous to the calculation of Wilson coefficients in a local operator product expansion, which must be independent of the external states and hence are calculated using any convenient state.

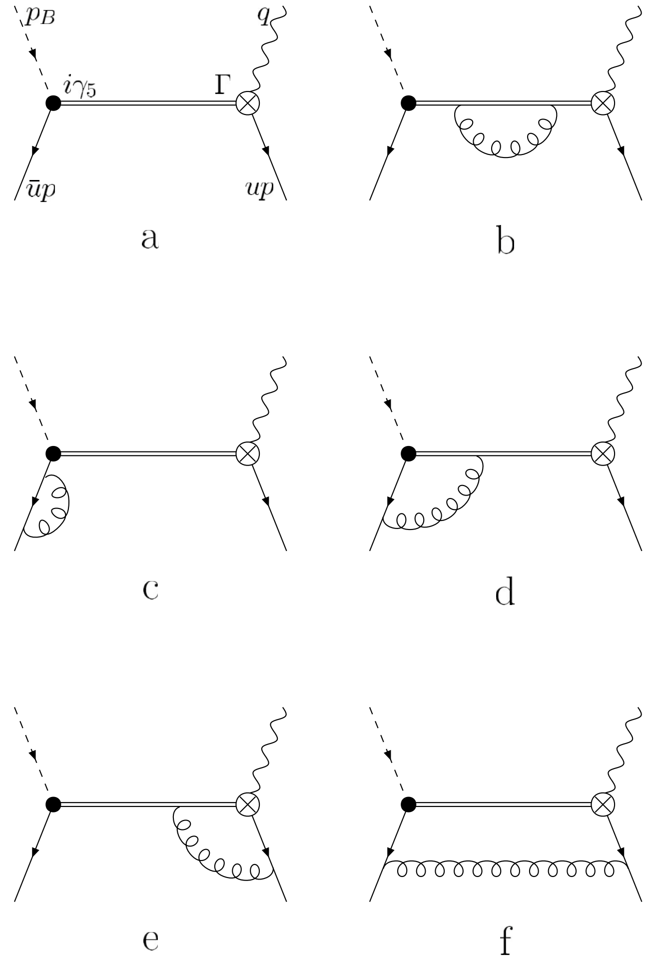

FIG. 1. Diagrams for 2-particle correlation functions. Γ is the weak interaction vertex. The light-quark self-energy diagrams of type c give purely divergent contributions in $1/\epsilon_{\rm IR} - 1/\epsilon_{\rm UV}$.

this paper we use the prescriptions (30) and (31) throughout and hence avoid unphysical singularities in p_B^2. We have checked that indeed the singularity structure of all seven invariants $\Gamma_{0,\pm,V}$, $\mathcal{A}, \mathcal{B}, \mathcal{C}$ is given by a cut on the real axis for $p_B^2 \geq m_b^2$.

The complete correlation function, including 2-particle DAs to $O(\delta^2)$ for the vector current V, axial-vector current A, tensor current T, and scalar current S,[8] can now be written as

$$\Gamma(q^2, p_B^2) = \sum_{C=V,A,T,S} \int_0^1 du \mathfrak{P}^C_{ab}(u;\mu^2) T_{ba}(u, p_B^2, q^2; \mu^2), \tag{32}$$

with

$$\mathfrak{P}^V_{ab} = \frac{1}{4} f_V m_V (\gamma^\alpha)_{ab}\bigg[-P_\alpha e^*_\beta \Phi(u)\frac{\partial}{\partial(up)_\beta} + e^*_\alpha g_\perp^{(v)}(u)\bigg], \tag{33}$$

$$\mathfrak{P}^A_{ab} = -\frac{i}{16} f_V m_V (\gamma^\alpha\gamma_5)_{ab}\epsilon_{\alpha\kappa\lambda\beta} e^{*\kappa} P^\lambda g_\perp^{(a)}(u)\frac{\partial}{\partial(up)_\beta}, \tag{34}$$

$$\mathfrak{P}^T_{ab} = -\frac{i}{4} f_V^T (\sigma^{\alpha\beta})_{ab}\bigg\{e^*_\alpha P_\beta\bigg(\phi_\perp(u) - \frac{1}{16} m_V^2 \mathbb{A}_\perp \frac{\partial^2}{\partial(up)_\kappa \partial(up)^\kappa}\bigg) \tag{35}$$

$$+ m_V^2 P_\alpha \epsilon^*_\gamma I_L(u) \frac{\partial^2}{\partial(up)_\beta \partial(up)_\gamma} - \frac{1}{2} m_V^2 e^*_\alpha H_3(u)\frac{\partial}{\partial(up)_\beta}\bigg\}, \tag{36}$$

$$\mathfrak{P}^S_{ab} = -\frac{1}{8} m_V^2 f_V^T (e^*_\alpha)(\mathbf{1})_{ab} h_\parallel^{(s)}(u)\frac{\partial}{\partial(up)_\beta}, \tag{37}$$

where

$$\Phi(u) = \int_0^v dv[\phi_\parallel(v) - g_\perp^{(v)}(v)],$$
$$I_L(u) = \int_0^u dv \int_0^v dw\bigg[h_\parallel^{(t)}(w) - \frac{1}{2}\phi_\perp(w) - \frac{1}{2} h_3(w)\bigg],$$
$$H_3(u) = \int_0^u dv[h_3(v) - \phi_\perp(v)].$$

All these three functions $F(u)$ fulfill $F(0) = 1 = F(1)$.

Just to give an example, the tree contribution is given by

$$T^{\rm tree}_{ba} = -i[\Gamma(\not{q} + u\not{P} + m_b)\gamma_5]_{ba}/[(q+uP)^2 - m_b^2],$$

with the weak vertex Γ.

In order for factorization to hold, two conditions have to be met:

(a) the long-distance infrared sensitive parts (IR singularities) in T have to cancel against those in the DAs;
(b) the convolution integral $\int du \phi^{\rm ren}(u) T^{\rm ren}(u)$ has to converge. Otherwise factorization is violated by soft end point singularities.

In order to check condition (a), we decompose the bare amplitude into finite and divergent terms as

$$T^{\rm bare}(u) = T^{(0)}(u) + \alpha_s\bigg[T^{(1),{\rm ren}}(u) + \frac{1}{\epsilon} T^{(1),{\rm div}}(u)\bigg].$$

Ultraviolet divergences, which only occur for the penguin current, are easily subtracted using the known renormalization of the corresponding current:

$$T^{\rm bare}(u) \to T^{\rm bare}(u) - \delta Z_{\rm peng} T^{(0)}(u).$$

The remaining divergent terms have to cancel against the divergent parts of the bare DA,

[8] The matrix elements of vector mesons over the pseudoscalar current vanish.

$$\phi^{\mathrm{bare}}(u) = \phi^{\mathrm{ren}}(u) + \alpha_s \frac{1}{\epsilon} \phi^{\mathrm{div}}(u),$$

so that

$$\int_0^1 du[\phi^{\mathrm{ren}}(u) T^{(1),\mathrm{div}}(u) + \phi^{\mathrm{div}}(u) T^{(0)}(u)] = 0.$$

$\phi^{\mathrm{div}}(u)$ is known explicitly for the twist-2 π DA [7] and coincides with that for $\phi_\parallel$ but to the best of our knowledge has not yet been calculated for $\phi_\perp$. Alternatively, one can check the cancellation of divergences order by order in the conformal expansion of the DAs cf. Sec. II and Appendix C, with[9]

$$a_n^{\parallel,\perp,\mathrm{bare}} = a_n^{\parallel,\perp}\left(1 + \frac{\alpha_s}{4\pi} \frac{\gamma_n^{\parallel,\perp}}{2} \frac{1}{\epsilon}\right).$$

We find that all $1/\epsilon$ terms cancel as required.

As for condition (b), we also find that all T are regular at the end points, so that there are no end-point singularities in the convolution.

As an interesting by-product, we also find the following fixed-order evolution equations of the first inverse moment of the DAs:

$$\begin{aligned} \int_0^1 du \frac{\phi_\parallel(u, \mu_2^2)}{u} &= \int_0^1 du \frac{\phi_\parallel(u, \mu_1^2)}{u} \\ &\quad \times \left\{1 + a_s \ln\frac{\mu_2^2}{\mu_1^2}(3 + 2\ln u)\right\}, \\ \int_0^1 du \frac{\phi_\perp(u, \mu_2^2)}{u} &= \int_0^1 du \frac{\phi_\perp(u, \mu_1^2)}{u} \\ &\quad \times \left\{1 + 2a_s \ln\frac{\mu_2^2}{\mu_1^2}\left(2 + \frac{\ln u}{1-u}\right)\right\}. \end{aligned} \tag{38}$$

These equations allow one to calculate the change of that inverse moment directly for a given DA without having to calculate the Gegenbauer moments in an intermediate step. The first of these relations also can be obtained from the known one-loop evolution kernel of the π twist-2 DA [7], whose anomalous dimensions coincide with those of $\phi_\parallel$; the relation for $\phi_\perp$ is new.

As we shall see in the next section, the LCSRs do actually not involve the full correlation functions but only their imaginary parts in p_B^2. As in Ref. [1] we take the imaginary part only after calculating the convolution integral, which results in closed and comparatively simple, albeit lengthy expressions. The distribution amplitudes $\phi_{\perp,\parallel}$, $g_\perp^{(v,a)}$ are given by their respective conformal expansions, which we truncate at a_9. As discussed in Sec. II, the effective expansion parameter of the light-cone expansion is δ, so that the correlation function is expanded in both δ and α_s. We combine both expansions and include terms up to second order, i.e., $O(\delta^{0,1,2}\alpha_s^0)$ and $O(\delta^{0,1}\alpha_s^1)$, but drop $O(\delta^2\alpha_s^1)$.[10] A list of the included terms is given in Table II.

[9] We use dimensional regularization with $D = 4 + 2\epsilon$.
[10] Terms of $O(\delta^0\alpha_s^2)$ are not included, either.

TABLE II. Contributions included in the calculation of the correlation functions (26) and (27). $\delta \propto m_V$ is the effective expansion parameter of the light-cone expansion; we include contributions up to second order in δ and α_s; those marked by $(*)$ are new.

DA		$O(\alpha_s^0)$	$O(\alpha_s)$
twist-2:	$\phi_\perp$	δ^0, δ^2	δ^0
	$\phi_\parallel$	δ	δ
twist-3:	$g_\perp^{(a)}, g_\perp^{(v)}$	δ	$\delta^{(*)}$
	$h_\parallel^{(s)}, h_\parallel^{(t)}$	δ^2	
	$\mathcal{V}, \mathcal{A}$ (3-part. DAs)	δ	
	$\mathcal{T}$ (3-part. DA)	δ^2	
twist-4:	$h_3, \mathbb{A}_\perp$	δ^2	
	chiral-odd 3-part. DAs	δ^2	

Note that we have not calculated the radiative corrections to the contributions from the 3-particle twist-3 DAs $\mathcal{V}, \mathcal{A}$ as they are expected to be very small. This follows in part from the observation that $O(\alpha_s)$ terms in the corresponding twist-3 matrix elements also do show up in the $O(\alpha_s)$ corrections to $g_\perp^{(v,a)}$ and are very small numerically.

Depending on the specific weak vertex and projection onto the DAs, some diagrams contain traces with an odd number of γ_5, which leads to ambiguities when naive dimensional regularization with anticommuting γ_5 is used. We solve this problem by using Larin's prescription for dealing with γ_5 [20] and replace, whenever necessary, $[a_s = C_F\alpha_s/(4\pi)]$

$$\gamma_\mu\gamma_5 \to (1 - 4a_s)\frac{i}{3!}\epsilon_{\mu\nu_1\nu_2\nu_3}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3},$$

$$\gamma_5 \to (1 - 8a_s)\frac{i}{4!}\epsilon_{\nu_1\nu_2\nu_3\nu_4}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4},$$

$$\sigma_{\mu\nu}\gamma_5 \to -(1 - 0a_s)\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}.$$

Note that we use the Bjorken/Drell convention for the ϵ tensor with $\epsilon_{0123} = +1$. For the special case of the axial-vector form factors and the projection onto the DA $g_\perp^{(a)}$, one can implement Larin's prescription by rewriting either the weak vertex or the B vertex. We have checked that we obtain the same result in both cases. One might also think of "Larinizing" the projection operator onto the DA; the corresponding finite renormalization will be u dependent due to the nonlocality of the current and is yet unknown.

IV. NUMERICS

This section is the heartpiece of our paper, in which we derive the sum rules for $B \to V$ form factors and obtain numerical results. The section is organized as follows: in

Sec. IV A we derive the LCSR for one of the seven form factors, V. In Sec. IV B we give values for most of the needed hadronic input parameters and explain how to determine the sum rule specific parameters, i.e., the Borel parameter M^2 and the continuum threshold s_0. We also calculate f_{B_d} and f_{B_s}, which are necessary ingredients in the LCSRs. In Sec. IV C we motivate the need for and introduce models of the twist-2 DAs $\phi_{\perp,\parallel}$. In Sec. IV D we calculate the form factors at $q^2 = 0$ and discuss their uncertainties. In Sec. IV E we present the form factors for central input values of the parameters and provide a simple parametrization valid in the full kinematical regime of q^2. The results for $q^2 = 0$ and central results for arbitrary q^2 are shown in this paper's tables.

A. The sum rules

With explicit expressions for the correlation functions in hand, we are now in a position to derive the LCSRs for the form factors. Let us choose $V(q^2)$ for a B_q transition as example. The corresponding correlation function is Γ_V as defined in Eq. (26). The basic idea is to express Γ_V in two different ways, as dispersion relation of the expression obtained in light-cone expansion on one hand, and as dispersion relation in hadronic contributions on the other hand. Equating both representations one obtains a light-cone sum rule for V. One side of the equation is hence the light-cone expansion result

$$\Gamma_V^{\rm LC}(p_B^2, q^2) = \int_{m_b^2}^{\infty} ds \frac{\rho_V^{\rm LC}(s, q^2)}{s - p_B^2}, \qquad (39)$$

with $\pi\rho_V^{\rm LC}(s, q^2) = {\rm Im}[\Gamma_V^{\rm LC}]$, which has to be compared to the physical correlation function that also features a cut in p_B^2, starting at m_B^2:

$$\Gamma_V^{\rm phys}(p_B^2, q^2) = \int_{m_B^2}^{\infty} ds \frac{\rho_V^{\rm phys}(s, q^2)}{s - p_B^2}; \qquad (40)$$

the spectral density is given by hadronic contributions and reads

$$\rho_V^{\rm phys}(s, q^2) = f_{B_q} m_B^2 \frac{2V(q^2)}{m_B + m_V} \delta(s - m_B^2) + \rho_+^{\text{higher-mass states}}(s, q^2). \qquad (41)$$

Here f_{B_q} is the B_q meson decay constant defined as

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B\rangle = i f_{B_q} p_\mu \quad \text{or} \quad (m_b + m_q)\langle 0|\bar{q}i\gamma_5 b|B\rangle = m_B^2 f_{B_q}. \qquad (42)$$

To obtain a light-cone sum rule for V, one equates the two expressions for Γ_V and uses quark-hadron duality to approximate

$$\rho_V^{\text{higher-mass states}}(s, q^2) \approx \rho_V^{\rm LC}(s, q^2)\Theta(s - s_0), \qquad (43)$$

where s_0, the so-called continuum threshold is a parameter to be determined within the sum rule approach itself. In principle one could now write a sum rule

$$\Gamma_V^{\rm phys}(p_B^2, q^2) = \Gamma_V^{\rm LC}(p_B^2, q^2),$$

and extract V. However, in order to suppress the impact of the approximation (43), one subjects both sides of the equation to a Borel transformation

$$\frac{1}{s - p_B^2} \to \hat{B}\frac{1}{s - p_B^2} = \frac{1}{M^2}\exp(-s/M^2),$$

which ensures that contributions from higher-mass states be sufficiently suppressed and improves the convergence of the operator product expansion. We then obtain

$$e^{-m_B^2/M^2} m_B^2 f_{B_q} \frac{2V(q^2)}{m_B + m_V} = \int_{m_b^2}^{s_0} ds\; e^{-s/M^2}\rho_V^{\rm LC}(s, q^2). \qquad (44)$$

This is the final sum rule for V and explains why, as announced in the previous section, only the imaginary part of the correlation function is needed. Expressions for the other form factors are obtained analogously. The task is now to find sets of parameters M^2 (the Borel parameter) and s_0 (the continuum threshold) such that the resulting form factor does not depend too much on the precise values of these parameters; in addition the continuum contribution, that is the part of the dispersive integral from s_0 to ∞, which has been subtracted from both sides of (44), should not be too large, say less than 30% of the total dispersive integral.

B. Hadronic input parameters

After having derived the LCSRs for the form factors, the next step is to fix the parameters on which they depend. These are the decay constants of the B_q and B_s meson, f_{B_q} and f_{B_s}, the couplings $f_V^{(T)}$ of the vector mesons, introduced in Sec. II, the meson DAs, the quark masses m_b and m_s, α_s and the factorization scale $\mu_{\rm IR}$, and, finally, the sum rule specific parameters M^2 and s_0.

The f_V are known from experiment and are collected in Table III. The f_V^T, on the other hand, are not that easily accessible in experiment and hence have to be determined from theory. For internal consistency, we determine these

TABLE III. Values of the vector meson couplings. f_V is extracted from experiment, f_V^T from QCD sum rules for f_V^T/f_V cf. Ref. [12].

V	ρ	ω	K^*	ϕ
f_V [MeV]	205 ± 9	195 ± 3	217 ± 5	231 ± 4
f_V^T (1 GeV) [MeV]	160 ± 10	145 ± 10	170 ± 10	200 ± 10
f_V^T (2.2 GeV) [MeV]	147 ± 10	133 ± 10	156 ± 10	183 ± 10

parameters from QCD sum rules for the ratio f_V^T/f_V, as explained in Ref. [12]. The results are collected in Table III, too. f_ρ^T had already been determined earlier in Ref. [21]; the result agrees with that in Table III. The ratios f_V^T/f_V also have been determined from lattice [22] and agree with ours within errors. Meson DAs are discussed in the next subsection.

The b quark mass entering our formulas is the one-loop pole mass m_b for which we use $m_b = (4.80 \pm 0.05)$ GeV cf. Table 6 in Ref. [3]. m_s, on the other hand, is the $\overline{\rm MS}$ running mass, $\overline{m}_s(2\ \text{GeV}) = 100$ MeV, which is an average of two recent lattice determinations [23]; the uncertainty in m_s has only a minor impact on our results. As for the strong coupling, we take $\alpha_s(m_Z) = 0.118$ and use NLO evolution to evaluate it at lower scales. All scale-dependent quantities are evaluated at the factoriszation scale $\mu_{\rm IR}$ which separates long- from short-distance physics. The only exception are the form factors T_i, which also depend on an ultraviolet scale $\mu_{\rm UV}$ which is set to m_b. We choose $\mu_{\rm IR} = \sqrt{m_B^2 - m_b^2} = 2.2$ GeV as reference scale; a variation of $\mu_{\rm IR}$ by ± 1 GeV has only small impact on the final results.

The remaining parameters are $f_{B_{q,s}}$, M^2, and s_0. $f_{B_{q,s}}$ has been determined from both lattice and QCD sum rule calculations. The state of the art of the former are unquenched NRQCD simulations with 2 + 1 light flavors, yielding $f_{B_s} = (260 \pm 30)$ MeV [24], which is slightly larger than the 2003 recommendation $f_{B_s} = (240 \pm 35)$ MeV [25]. For f_{B_d}, it is difficult to find any recent numbers, the consensus being that more calculations at smaller quark masses are needed in order to bring the extrapolation to physical $m_{u,d}$ under sufficient control [24]. As for QCD sum rules, both f_{B_d} and f_{B_s} have been determined to $O(\alpha_s^2)$ accuracy: $f_{B_d} = (208 \pm 20)$ MeV and $f_{B_s} = (224 \pm 21)$ MeV [26], in agreement with lattice determinations. The impact of $O(\alpha_s^2)$ corrections on $f_{B_{q,s}}$ is nonnegligible. As the diagrams responsible for these corrections, for instance B vertex corrections, are precisely the same that will enter LCSRs at $O(\alpha_s^2)$, we proceed from the assumption that these corrections will tend to cancel in the ratio (correlation function)/f_B. We hence evaluate $f_{B_{d,s}}$ from a QCD sum rule to $O(\alpha_s)$ accuracy, which reads [27]:[11]

$$
\begin{aligned}
f_{B_q}^2 m_B^2 e^{-m_B^2/M^2} &= \int_{m_b^2}^{s_0} ds\ \rho^{\rm pert}(s) e^{-s/M^2} + C_{\bar q q}\langle \bar q q\rangle \\
&\quad + C_{\bar q G q}\langle \bar q \sigma g G q\rangle \\
&\equiv \int_{m_b^2}^{s_0} ds\ \rho^{\rm tot}(s) e^{-s/M^2}. \qquad (45)
\end{aligned}
$$

[11] The contribution of the gluon condensate is not sizable and we therefore neglect it.

Here $\langle \bar q q\rangle$ and $\langle \bar q \sigma g G q\rangle$ are the quark and mixed condensate, respectively, for which we use the following numerical values at $\mu = 1$ GeV:

$$
\begin{aligned}
\langle \bar q q\rangle &= -(0.24 \pm 0.01)^3\ \text{GeV}^3 \quad \text{and} \\
\langle \bar q \sigma g G q\rangle &= 0.8\ \text{GeV}^2 \langle \bar q q\rangle. \qquad (46)
\end{aligned}
$$

The C are perturbative Wilson coefficients multiplying the condensates. $C_{\bar q q}$ is known to $O(\alpha_s)$ accuracy [26,28], $C_{\bar q G q}$ at tree-level.

The criteria for choosing M^2 and s_0 in the above sum rule are very similar to those to be used for the LCSRs. Ideally, if the correlation function were known exactly, the sum rule would be independent of M^2. In practice it is not, but "good" sum rules, plotted as function of M^2, still exhibit a flat extremum. We hence require the existence of such an extremum in M^2 and evaluate the sum rule precisely at that point. This eliminates M^2 as independent parameter and leaves us with s_0. As already mentioned after Eq. (44), the purpose of the Borel transformation is to enhance the contribution of the ground state to the physical spectral function with respect to that of higher states. We hence require that continuum contribution, that is the integral over $\rho^{\rm tot}(s)$ for $s > s_0$, must not be too large. To be specific, we require

$$
\left[\int_{s_0}^{\infty} ds\ \rho^{\rm tot}(s) e^{-s/M^2}\right] \Bigg/ \left[\int_{m_b^2}^{\infty} ds\ \rho^{\rm tot}(s) e^{-s/M^2}\right] < 30\%.
$$

This puts a lower bound on s_0. The larger s_0, the smaller M^2, the position of the minimum, and the larger nonperturbative contributions to (45). As the condensates are meant to yield small nonperturbative corrections, but blow up at small M^2, requiring the nonperturbative corrections to be not too large puts an upper bound on s_0. For $f_{B_{q,s}}$, we require the highest term in the condensate expansion, the mixed condensate, to contribute less than 10% to the correlation function. For LCSRs, which rely on an expansion in higher twist rather than higher condensates, we correspondingly require the contribution of higher twists to the LCSR not to exceed 10%. One more requirement on the s_0 is that they not stray away too much from "reasonable" values: s_0 is to separate the ground state from higher-mass contributions, and hence should be below the next known clear resonance in that channel. Assuming an

TABLE IV. Parameter sets for f_{B_q} and f_{B_s} to $O(\alpha_s)$ accuracy. f_{B_q} and f_{B_s} are given in MeV, s_0, and M^2 in GeV^2. Note that the values of $f_{B_{q,s}}$ given in the table are *not* to be interpreted as meaningful determinations of these quantities cf. text.

	m_b	s_0	M^2	f_{B_q}	s_0	M^2	f_{B_s}
set one	4.85	33.8	3.8	148	34.9	4.2	169
set two	4.80	34.2	4.1	161	35.4	4.4	183
set three	4.75	34.6	4.4	174	35.9	4.6	197

TABLE V. Parameter sets for $B_q \to \rho$ for V, A_0, A_1, A_2, T_1, and T_3. As $T_1(0) = T_2(0)$ the corresponding parameters are equal. s_0 and M^2 in GeV^2.

	m_b	s_0^V	c_c^V	$s_0^{A_0}$	$c_c^{A_0}$	$s_0^{A_1}$	$c_c^{A_1}$	$s_0^{A_2}$	$c_c^{A_2}$	$s_0^{T_1}$	$c_c^{T_1}$	$s_0^{T_3}$	$c_c^{T_3}$
set one	4.85	35.2	1.7	33.0	1.7	33.7	1.7	34.1	1.7	34.8	1.7	34.7	1.7
set two	4.80	35.8	2.1	33.6	1.6	34.2	1.8	34.7	1.8	35.3	1.9	35.2	1.8
set three	4.75	36.4	2.1	34.2	1.6	34.7	1.9	35.3	1.9	35.8	2.1	35.7	1.9

excitation energy of 0.4 to 0.8 GeV, we thus expect the s_0 to lie in the interval 32 to 37 GeV^2.

Applying the above criteria to (45), we obtain the sets of (s_0, M^2) collected in Table IV, together with the resulting $f_{B_{q,s}}$. We would like to stress that these values are not to be interpreted as new independent determinations of $f_{B_{q,s}}$, but are intermediate results to be used in the evaluation of the LCSRs.

We proceed to determine the continuum thresholds and Borel parameters for the LCSRs, using the same criteria as above. In order to keep the complexity of the calculation at a manageable level, for each form factor the corresponding set is determined only once, at $q^2 = 0$. To avoid confusion between parameters entering (45) and those entering the LCSRs, let us call the latter ones $M^2_{\rm LC}$ and s_0^F where F is the form factor. For larger q^2, these parameters are expected to change slightly. Part of this effect can be taken into account in the following way: the tree-level LCSR to twist-2 accuracy reads, basically,

$$\int_{u_0}^1 du \frac{\phi(u)}{u} e^{-[m_b^2-(1-u)q^2]/(uM^2_{\rm LC})} \quad \text{with} \quad u_0 = \frac{m_b^2 - q^2}{s_0 - q^2},$$

which implies that the expansion parameter is $uM^2_{\rm LC}$ rather than $M^2_{\rm LC}$. We hence rescale the Borel parameter as

$$M^2_{\rm LC} \to M^2_{\rm LC}/\langle u\rangle(q^2),$$

with the average value of u, $\langle u\rangle(q^2)$, given by

$$\langle u\rangle(q^2) \equiv \left(\int_{u_0}^1 du\; u \frac{\phi(u)}{u} e^{-[m_b^2-(1-u)q^2]/(uM^2_{\rm LC})}\right) \Big/ \left(\int_{u_0}^1 du \frac{\phi(u)}{u} e^{-[m_b^2-(1-u)q^2]/(uM^2_{\rm LC})}\right),$$

with, approximately, $\langle u\rangle(0\ {\rm GeV}^2) = 0.86$ and $\langle u\rangle \times (14\ {\rm GeV}^2) = 0.77$. The optimum Borel parameter hence becomes larger with increasing q^2, which agrees with what one finds when $M^2_{\rm LC}$ is determined without rescaling. Parametrizsing the relation between the Borel parameters of local and light-cone correlation functions as

$$M^2_{\rm LC} \equiv c_c M^2/\langle u\rangle, \tag{47}$$

we obtain, for $B_q \to \rho$, the values collected in Table V. The sets for other transitions are similar.

C. Models for distribution amplitudes

As mentioned in Sec. II and detailed in Appendix B, the DAs entering the LCSRs can be modeled by a truncated conformal expansion. It turns out that the dominant contributions to the sum rules come from the twist-2 DAs $\phi_{\perp,\parallel}$, which to NLO in the conformal expansion are described by the lowest three Gegenbauer moments: $a_0^{\perp,\parallel} \equiv 1$, which follows from the normalization of the DAs $a_1^{\perp,\parallel}$, which is nonzero only for K^*, and $a_2^{\perp,\parallel}$. In Ref. [4], it was these three parameters that were used to define the models for $\phi_{\perp,\parallel}$; all terms $a_{n\geq 3}$ were dropped.

The numerical values of $a_{1,2}$ (and higher moments) are largely unknown. a_1 has been determined from QCD sum rules in [12,29,30]. Averaging over the determinations, we choose

$$a_1^{\parallel}(K^*, 1\ {\rm GeV}) = 0.10 \pm 0.07 = a_1^{\perp}(K^*, 1\ {\rm GeV}), \tag{48}$$

as our preferred values. Note that positive a_1 refer to a K^* containing an s quark—for a $\bar{K}^*$ with an $\bar{s}$ quark, a_1 changes sign.

Predictions for $a_2^{\perp,\parallel}$ also come from QCD sum rules [12,16,21,29] and read

$$\begin{aligned} a_2^{\parallel}(\rho, 1\ {\rm GeV}) &= 0.18 \pm 0.10, \\ a_2^{\parallel}(K^*, 1\ {\rm GeV}) &= 0.09 \pm 0.05, \\ a_2^{\parallel}(\phi, 1\ {\rm GeV}) &= 0 \pm 0.1, \\ a_2^{\perp}(\rho, 1\ {\rm GeV}) &= 0.2 \pm 0.1, \\ a_2^{\perp}(K^*, 1\ {\rm GeV}) &= 0.13 \pm 0.08, \\ a_2^{\perp}(\phi, 1\ {\rm GeV}) &= 0 \pm 0.1. \end{aligned} \tag{49}$$

All these determinations have to be taken *cum grano salis*, as the sum rules do not exhibit a clear Borel window and also become increasingly unreliable for larger n.[12]

But even assuming $a_{1,2}$ were known to sufficient accuracy—under what conditions is a truncation of ϕ after a_2 is justified? We have seen in Sec II that after the convolution with a smooth short-distance function T the contributions of higher a_n fall off sharply. So the actual question is not so much how the truncated expansion compares to the full convolution integral, but rather how the neglected

[12]This is due to the different power behavior of perturbative and nonperturbative terms in n cf. Ref. [21].

terms compare to other terms, for instance originating from 3-particle DAs, which are included in the LCSR. For instance, assuming $a_i \geq 0.05$ it is necessary to include $a_2^{\parallel}$ and $a_{2,4,6,8}^{\perp}$ in order to match the size of the contributions from quark-quark-gluon matrix elements, and even for $a_i \geq 0.01$ one still needs $a_2^{\parallel}$ and $a_{2,4}^{\perp}$. If, on the other hand, one consistently neglects terms that contribute less than 1% to the form factor, one can drop nearly all contributions from quark-quark-gluon matrix elements, unless their values as given in Appendix B are grossly underestimated. If $a_i \geq 0.05$, one then has to keep $a_{2,4,6}^{\perp}$, but can drop all $a_{n>0}^{\parallel}$. The upshot is that, in view of the lack of information on $a_n^{\perp,\parallel}$, it is a good idea to devise models for $\phi_{\perp,\parallel}$ with a small number of parameters, possibly tied to experimental observables, and a well-defined "tail" of higher-order Gegenbauer moments. This task is undertaken in Ref. [19].

Following Ref. [19], we introduce two-parameter models for (the symmetric part of) $\phi_{\perp,\parallel}$ which are defined by the fall-off behavior of the Gegenbauer moments a_n in n and the value of the integral

$$\Delta = \int_0^1 du \frac{\phi(u)}{3u} \equiv 1 + \sum_{n=1}^{\infty} a_{2n}; \tag{50}$$

$\Delta = 1$ for the asymptotic DA. In particular we require Δ to be finite, which implies that the a_n must fall off sufficiently fast. The choice of Δ as characteristic parameter of ϕ relies on the fact that it is directly related to an experimental observable, at least for π and η, namely, the $\pi(\eta)\gamma\gamma^*$ transition form factor, for which experimental constraints exist from CLEO [31]. We assume that the vector meson DAs are not fundamentally different and take the range of $\Delta(\pi)$ extracted from CLEO as the likely range for $\Delta(\rho)$. The second parameter characterizing our models is the fall-off behavior of the Gegenbauer moments in n, which we assume to be powerlike. We then can define a model DA $\tilde{\phi}_a^+$ in terms of its Gegenbauer moments

$$a_n = \frac{1}{(n/2+1)^a}: \tag{51}$$

using the generating function of the Gegenbauer-polynomials,

$$f(\xi, t) = \frac{1}{(1-2\xi t + t^2)^{3/2}} = \sum_{n=0}^{\infty} C_n^{3/2}(\xi) t^n,$$

this model can be summed to all orders in the Gegenbauer expansion:

$$\tilde{\phi}_a^+(u) = \frac{3u\bar{u}}{\Gamma(a)} \int_0^1 dt(-\ln t)^{a-1}[f(2u-1, \sqrt{t}) + f(2u-1, -\sqrt{t})]. \tag{52}$$

The corresponding value of Δ is $\Delta_a^+ = \zeta(a)$. In order to obtain models for arbitrary values of Δ, we split off the asymptotic DA and write

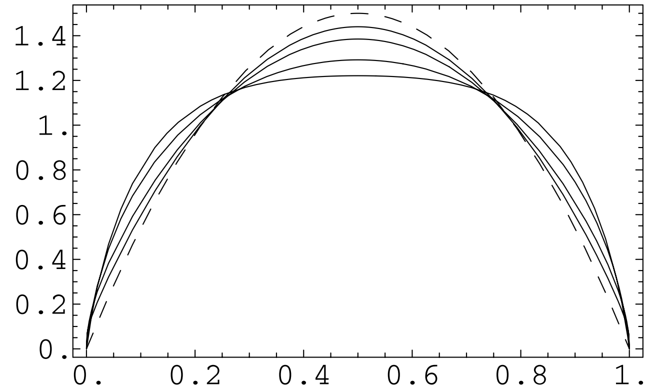

FIG. 2. Examples for model DAs ϕ_a^+ as functions of u, for $\Delta = 1.2$ and $a = 1.5, 2, 3, 4$ (solid curves), as compared to the asymptotic DA (dashed curve). For $a \to 1$, ϕ_a^+ approaches the asymptotic DA.

$$\phi_a^+(\Delta) = 6u\bar{u} + \frac{\Delta - 1}{\Delta_a^+ - 1}[\tilde{\phi}_a^+(u) - 6u\bar{u}]. \tag{53}$$

Evidently one recovers the asymptotic DA for $\Delta = 1$ and the truncated conformal expansion with $a_2 = \Delta - 1$ and $a_{n\geq 4} = 0$ for $a \to \infty$. The above formula is only valid for $a > 1$, as otherwise Δ_a^+ diverges, or, equivalently, ϕ_a^+ does not vanish at the endpoints $u = 0, 1$. In Fig. 2 we plot several examples of ϕ_a^+ for a fixed value of Δ.

Our preferred values for Δ, a, and the corresponding values of $a_{2,4}^{\perp,\parallel}(1\text{ GeV})$ are collected in Table VI. We choose $a = 3 \pm 1$ in order to obtain nonnegligible effects from higher-order a_n. The choice of $\Delta(\rho)$ is motivated by the fact that all available calculations indicate $a_2 > 0$, hence $\Delta > 1$. We then fix the maximum Δ in such a way that it yields $a_2 < 0.2$, which, given the fact that the sum rule results (49) are likely to overshoot the true value of a_2, appears as to be the likely maximum value. We then obtain $\Delta(\rho) = 1.15 \pm 0.10$, with a rather conservative error. We choose the same values for ω. For K^* and ϕ, we take into account that the values of a_2 appear to have the tendency to decrease, which was noticed already in Ref. [29]. Assuming that the decrease is 20% from ρ to K^*, and another 20% from K^* to ϕ, we arrive at the numbers quoted in Table VI.

TABLE VI. Values for $\Delta(1\text{ GeV}) \equiv \Delta^{\perp,\parallel}(1\text{ GeV})$, a, and the corresponding values of $a_{2,4}^{\perp,\parallel}(1\text{ GeV})$.

	Δ	a	$a_2^{\perp,\parallel}(1\text{ GeV})$	$a_4^{\perp,\parallel}(1\text{ GeV})$
ρ, ω	1.15 ± 0.10	3 ± 1	$0.09^{+0.10}_{-0.07}$	0.03 ± 0.02
K^*	1.12 ± 0.10	3 ± 1	$0.07^{+0.09}_{-0.07}$	0.02 ± 0.01
ϕ	1.10 ± 0.10	3 ± 1	$0.06^{+0.09}_{-0.07}$	$0.02^{+0.01}_{-0.02}$

Evidently, DAs defined in dependence of Δ also require a specification of the scale at which they are valid. As we presume that $\Delta(\mu)$ will be measured, if at all, at the low scale $\mu = 1$ GeV, we choose this as the reference scale. Δ at higher scales can be obtained from Eq. (50), using the leading-order renormalization group-improved expressions for $a_n(\mu)$, or, if μ is not too different from 1 GeV, from the unimproved expression Eq. (38).

Models for the asymmetric part of the DA, relevant for K^*, can be constructed in a similar way as

$$\tilde{\psi}_b^+(u) = \frac{3u\bar{u}}{\Gamma(b)} \int_0^1 dt(-\ln t)^{b-1}[f(2u-1, \sqrt{t}) - f(2u-1, -\sqrt{t})]. \quad (54)$$

One relevant parameter is b, and as the second one we choose a_1. Models for the asymmetric part of ϕ with arbitrary a_1 can then be defined as

$$\phi_b^{\mathrm{asym},+} = a_1(3/2)^b \tilde{\psi}_b^+(u). \quad (55)$$

Examples for such models are shown in Fig. 3.

$\phi_b^{\mathrm{asym},+}$ also contributes to the value of Δ:

$$\Delta^{\mathrm{asym},+} = \int_0^1 du \frac{\phi_b^{\mathrm{asym},+}(u)}{3u} = -a_1(3/2)^b \zeta(b, 3/2),$$

where $\zeta(b, s) = \sum_{k=0}^{\infty} 1/(k+s)^b$ is the Hurwitz ζ function. Our models for the K^* DA are hence characterized by four parameters: Δ, a of the symmetric part, and a_1, b of the asymmetric part. The total value of Δ is given by

$$\Delta^{\mathrm{total},+} = \Delta + \Delta^{\mathrm{asym},+}.$$

In the actual calculation we choose $a = b$.

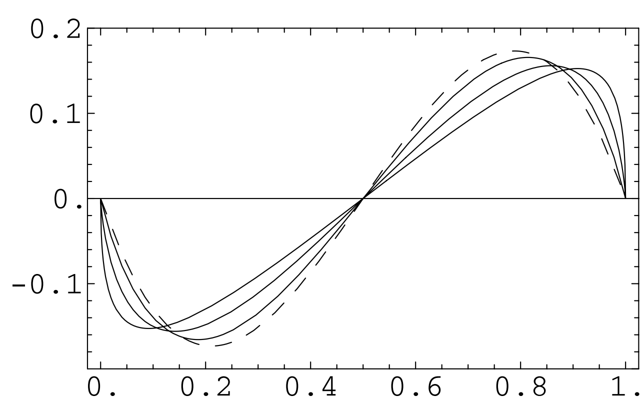

FIG. 3. Models for the asymmetric contributions to the twist-2 DA for $a_1 = 0.1$. Solid curves: $\phi_b^{\mathrm{asym},+}$ as function of u for $b \in \{2, 3, 5\}$; dashed curve: $\phi_\infty^{\mathrm{asym},+}$.

D. Results for $q^2 = 0$

Let us first analyze the form factors for $q^2 = 0$. Using the input parameters given in Tables V and VI, we obtain the results collected in Table VII.

For the discussion of theoretical uncertainties, we distinguish between uncertainties that can be reduced by future more accurate determinations of the corresponding hadronic parameters and others that are either systematic uncertainties, inherent to the method of LCSRs, or parameter uncertainties not likely to be reduced in the near future. The latter comprise the dependence of the form factors on the LCSR parameters s_0, M^2, μ_{IR} and, via f_B, the quark and mixed condensate. Our results also depend, very mildly, on m_s and, more importantly, on the meson DAs which are described by the 2-parameter model (53). All these parameters induce a theoretical error of the form factors which we determine by varying

(i) the threshold s_0 by ± 1.0 GeV^2;
(ii) the Borel parameter M^2 in Eq. (47) by ± 1.5 GeV^2;
(iii) the infrared factorization scale $\mu_{\mathrm{IR}} = \sqrt{m_B^2 - m_b^2}$ by ± 1 GeV;
(iv) the quark condensate and the mixed condensate as indicated in Eq. (46);
(v) the first inverse moment of the twist-2 DAs Δ by ± 0.1;
(vi) the power behavior of the Gegenbauer moments a by ± 1;
(vii) the strange quark mass m_s by $\pm 20\%$.

The largest deviation of the form factor from its central value, in this 7-parameter space, is dubbed Δ_{7p} and amounts to typically 7% to 11%. In Fig. 4 we show the dependence of selected form factors on Δ and a. The uncertainty in these parameters is the most important single source of error of the form factors and amounts to half of the total error.

The form factors also depend, rather mildly, on m_b: varying m_b by ± 0.05 GeV around the central value 4.8 GeV, and using s_0 and M^2 as given in Tables IV and V, we obtain the error Δ_{m_b} which ranges from 1% to 5%.

One more source of uncertainty of the form factors is due to f_V and f_V^T, the vector and tensor coupling of the vector mesons. This is easily understood by splitting the generic form factor F into two terms proportional to f^T and $f^L \equiv f$:

$$F = f^L F^L + f^T F^T. \quad (56)$$

As argued in Sec. II the first term is of order $\delta \propto m_V$, the second of order one and indeed, for most form factors, is the dominant contribution. The present errors of f^T, as collected in Table III, are nonnegligible. f^T is accessible to lattice calculations and first results have been reported in Ref. [22], which indicates that a reduction of the error of f^T seems feasible. In order to allow the adjustment of our

TABLE VII. Form factors at $q^2 = 0$ for parameter set 2 of Tables IV and V, i.e., $m_b = 4.8$ GeV. The form factors are defined in Eqs. (2) and (4). The penguin form factors T_i are evaluated at the UV scale $\mu = m_b$. Δ_{m_b} is the variation of the result with m_b, i.e., the maximum deviation between the results obtained for sets 1, 2, and 3. Δ_{7p} is the maximum deviation found by scanning the 7-parameter space discussed in the text. Δ_L and Δ_T are the uncertainties induced by the vector and tensor couplings in Table III. The total error $\Delta_{\rm tot}$ is obtained by adding $\Delta_{(m_b,7p,L,T)}$ in quadrature. Form factors involving K^* carry one more uncertainty Δ_{a_1} induced by the Gegenbauer moment a_1, with $\delta_{a_1} = [a_1(K^*, 1\text{ GeV}) - 0.1]$.

	$F(0)$	Δ_{m_b}	Δ_{7p}	Δ_L	Δ_T	$\Delta_{\rm tot}$	Δ_{a_1}
$V^{B_q\to\rho}$	0.323	0.007	0.025	0.005	0.013	0.029	
$A_0^{B_q\to\rho}$	0.303	0.004	0.026	0.009	0.006	0.028	
$A_1^{B_q\to\rho}$	0.242	0.007	0.020	0.004	0.010	0.024	
$A_2^{B_q\to\rho}$	0.221	0.008	0.018	0.002	0.011	0.023	
$T_1^{B_q\to\rho}$	0.267	0.004	0.018	0.004	0.010	0.021	
$T_3^{B_q\to\rho}$	0.176	0.001	0.013	0.001	0.009	0.016	
$V^{B_s\to\bar{K}^*}$	0.311	0.006	0.021	0.003	0.013	0.026	$-0.43\delta_{a_1}$
$A_0^{B_s\to\bar{K}^*}$	0.363	0.003	0.032	0.006	0.009	0.034	$-0.37\delta_{a_1}$
$A_1^{B_s\to\bar{K}^*}$	0.233	0.007	0.019	0.002	0.010	0.023	$-0.32\delta_{a_1}$
$A_2^{B_s\to\bar{K}^*}$	0.181	0.008	0.021	0.001	0.010	0.025	$-0.30\delta_{a_1}$
$T_1^{B_s\to\bar{K}^*}$	0.260	0.005	0.021	0.003	0.010	0.024	$-0.33\delta_{a_1}$
$T_3^{B_s\to\bar{K}^*}$	0.136	0.003	0.013	0.000	0.008	0.016	$-0.17\delta_{a_1}$
$V^{B_q\to K^*}$	0.411	0.008	0.029	0.003	0.013	0.033	$0.44\delta_{a_1}$
$A_0^{B_q\to K^*}$	0.374	0.009	0.031	0.005	0.008	0.034	$0.39\delta_{a_1}$
$A_1^{B_q\to K^*}$	0.292	0.009	0.025	0.002	0.009	0.028	$0.33\delta_{a_1}$
$A_2^{B_q\to K^*}$	0.259	0.009	0.023	0.001	0.010	0.027	$0.31\delta_{a_1}$
$T_1^{B_q\to K^*}$	0.333	0.005	0.026	0.003	0.010	0.028	$0.34\delta_{a_1}$
$T_3^{B_q\to K^*}$	0.202	0.002	0.016	0.001	0.008	0.018	$0.18\delta_{a_1}$
$V^{B_q\to\omega}$	0.293	0.006	0.025	0.002	0.013	0.029	
$A_0^{B_q\to\omega}$	0.281	0.012	0.027	0.003	0.006	0.030	
$A_1^{B_q\to\omega}$	0.219	0.008	0.021	0.001	0.010	0.025	
$A_2^{B_q\to\omega}$	0.198	0.007	0.018	0.001	0.011	0.022	
$T_1^{B_q\to\omega}$	0.242	0.003	0.019	0.002	0.010	0.022	
$T_3^{B_q\to\omega}$	0.155	0.000	0.012	0.000	0.009	0.015	
$V^{B_s\to\phi}$	0.434	0.004	0.032	0.003	0.014	0.035	
$A_0^{B_s\to\phi}$	0.474	0.002	0.031	0.005	0.019	0.037	
$A_1^{B_s\to\phi}$	0.311	0.007	0.027	0.002	0.009	0.029	
$A_2^{B_s\to\phi}$	0.234	0.011	0.024	0.001	0.009	0.028	
$T_1^{B_s\to\phi}$	0.349	0.004	0.031	0.002	0.010	0.033	
$T_3^{B_s\to\phi}$	0.175	0.003	0.016	0.000	0.007	0.018	

form factors to new results for f^T, we give explicit results for F^L and F^T in Appendix A. The uncertainties $\Delta_{T,L}$ of the form factors due to the present values of $f^{T,L}$ are included in Table VII. Δ_T is typically of order 4%, Δ_L is much smaller.

For transitions involving the K^*, an additional uncertainty is induced by the first Gegenbauer moment a_1, and is given by Δ_{a_1} in Table VII, where the quantity δ_{a_1} is defined as $[a_1(K^*, 1\text{ GeV}) - 0.1]$. Note that $a_1(K^*)$ refers to a $s\bar{q}$ bound state and hence $a_1(\bar{K}^*) = -a_1(K^*)$, which explains the negative sign of the corresponding entries in Table VII. Again we aim to make our results adjustable to any future improvement in the determination of a_1 and give explicit results for the corresponding contributions in Appendix A.

Some important features of the results collected in Table VII are

(i) the form factors for $B_q \to K^*$ transition are about 20% larger than those for $B_q \to \rho$. The reason for this is twofold: on the one hand, the K^* vector and tensor couplings are larger than those of the ρ. On the other hand, the SU(3) breaking of the twist-2 DAs, parametrized by the first Gegenbauer moment a_1, gives a positive contribution to the form factors;

(ii) the form factors for $B_s \to K^*$ have a tendency to be smaller than those for $B_q \to \rho$. The reason for this is a negative contribution of a_1 and the fact that f_{B_s} is larger than f_{B_q}. On the other hand, the optimum s_0 are also larger than for $B_q \to \rho$, which partially compensates the first two effects;

(iii) the $B_q \to \omega$ form factors are slightly smaller than those for $B_q \to \rho$. This is a consequence of the fact that the ω vector and tensor couplings are smaller than those of the ρ;

(iv) the total theoretical error is dominated by that of the twist-2 DAs and the sum rule parameters s_0 and M^2. The former can, in principle, be reduced by future calculations, the second is systematic and irreducible.

The typical total uncertainty of each form factor is 10%, ranging between 8% and 13%. Any significant reduction of the error requires more accurate information on the twist-2 DAs. The minimum irreducible theoretical uncertainty is set by the systematic uncertainty of the LCSR approach and encoded in the dependence of the results on s_0 and M^2; it amounts to about 6% to 7%.

Let us also compare our results to those obtained in Ref. [4] by the same method, but with less sophistication. The main difference between our present and our previous analysis is the inclusion of radiative corrections to 2-particle twist-3 contributions, the description of the twist-2 DAs by models including all-order effects in the conformal expansion, the more accurate determination of the sum rule parameters s_0 and M^2, and the much more detailed error analysis. The most striking difference between the actual results affects the $B_s \to K^*$ transition, whose form factors were predicted, in [4], to be between 10% and 30% smaller than those in Table VII. The reason for this discrepancy is mainly the more accurate determination of M^2 and s_0 we employ in the present analysis—in Ref. [4] all

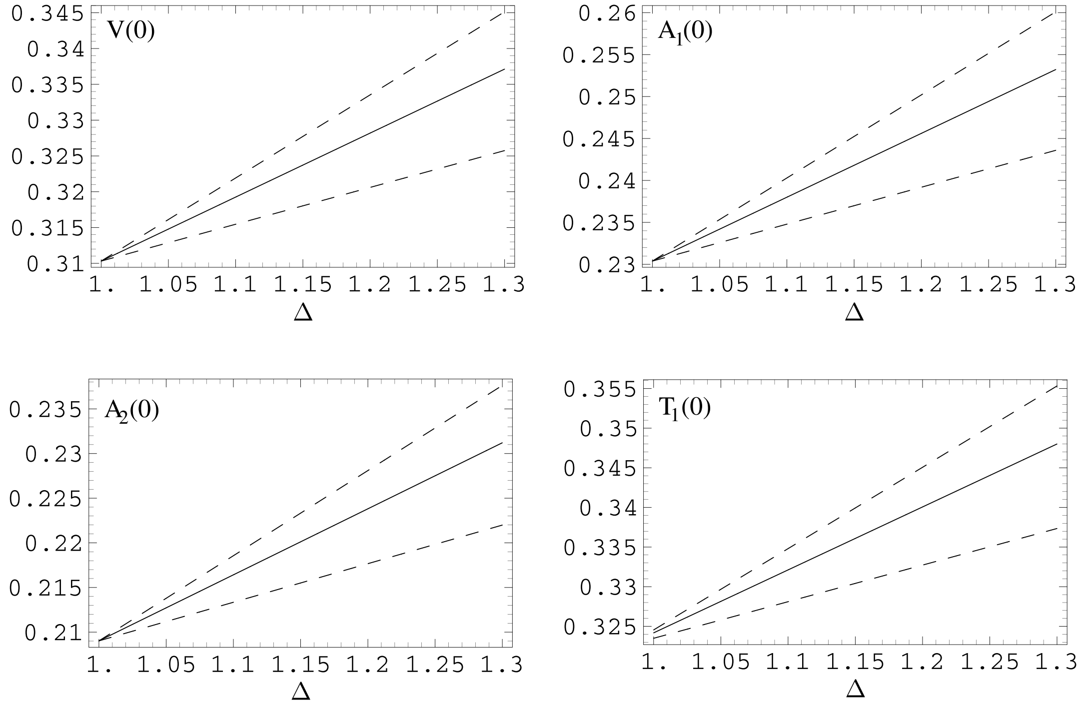

FIG. 4. The form factors $V(0)$, $A_1(0)$, $A_2(0)$ for $B_q \to \rho$ and $T_1(0)$ for $B_q \to K^*$ as functions of Δ, the main parameter of the twist-2 DAs. Solid lines: central values of input parameters. Dashed lines: variation of the form factors with a change of a, the second parameter of the DAs, by ± 1. Allowed values of Δ: cf. Table VI.

form factors were determined for the *same* values of M^2 and s_0. All other form factors quoted in [4] agree, within $\pm 15\%$, with those of Table VII, which is within the theoretical uncertainty stated in [4]. The only exception is $T_3(0)$, which deviates by between 15% and 45% from the numbers obtained in [4]. The reason for this discrepancy lies in the (correct) treatment of factors $1/(pz)$ in our present paper cf. Sec. III.

E. Results for $q^2 \neq 0$, fits and extrapolations

In this subsection we discuss the q^2 dependence of the form factors. The results of the LCSR calculation are plotted in Figs. 5 and 6. They can be parametrized in terms of simple formulas with two or three parameters, which are valid in the full kinematical regime in q^2. The corresponding parameters are collected in Table VIII.

As mentioned in Sec. II, the LCSR method is valid for large energies of the final state vector meson $E_V \gg \Lambda_{\rm QCD}$, which implies, via the relation $q^2 = m_B^2 - 2m_B E_V$, a restriction to not too large q^2. We include values in the regime

$$0 \leq q^2 \leq q^2_{\rm LCSR,max} = 14\ {\rm GeV}^2, \tag{57}$$

which has to be compared with the maximum physical $q^2_{\rm phys,max} = (m_B - m_V)^2$ of 20.3 GeV^2 for $B_q \to (\rho, \omega)$, 19.2 GeV^2 for $B_q \to K^*$, 20.0 GeV^2 for $B_s \to \bar{K}^*$, and 18.2 GeV^2 for $B_s \to \phi$. The main aim of this subsection is to provide fits for the LCSR results, which are valid in the full physical regime of q^2. We will comment below on the dependence of the fit results on the actual value used for $q^2_{\rm LCSR,max}$.

We closely follow the procedure we used in our previous paper on $B \to$ pseudoscalar form factors, Ref. [1]. Generically, barring the occurrence of anomalous thresholds, any form factor $F(q^2)$ has singularities (poles and cuts) for positive real q^2, starting at the position of the lightest resonance coupling to the relevant current and hence can be written as a dispersion integral in q^2. Splitting off the lowest-lying resonance with mass m_R, one has

$$F(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \int_{t_0}^{\infty} ds \frac{\rho(s)}{s - q^2}, \tag{58}$$

where t_0 is the threshold for multi-particle contributions, which can be above or below m_R^2. Keeping only the first term and neglecting the integral altogether one obtains the vector meson dominance (VMD) approximation.[13] Even though this approximation is expected to work very well close to the pole, it certainly will not work far away from it, e.g., at $q^2 = 0$. For $B \to \pi$ transitions it was argued

[13]This notion comes from the analysis of electromagnetic form factors, where the first resonance is the ρ. In weak decays, however, the lowest resonance is, in general, not a vector meson, so that the notion VMD is, strictly speaking, obsolete.

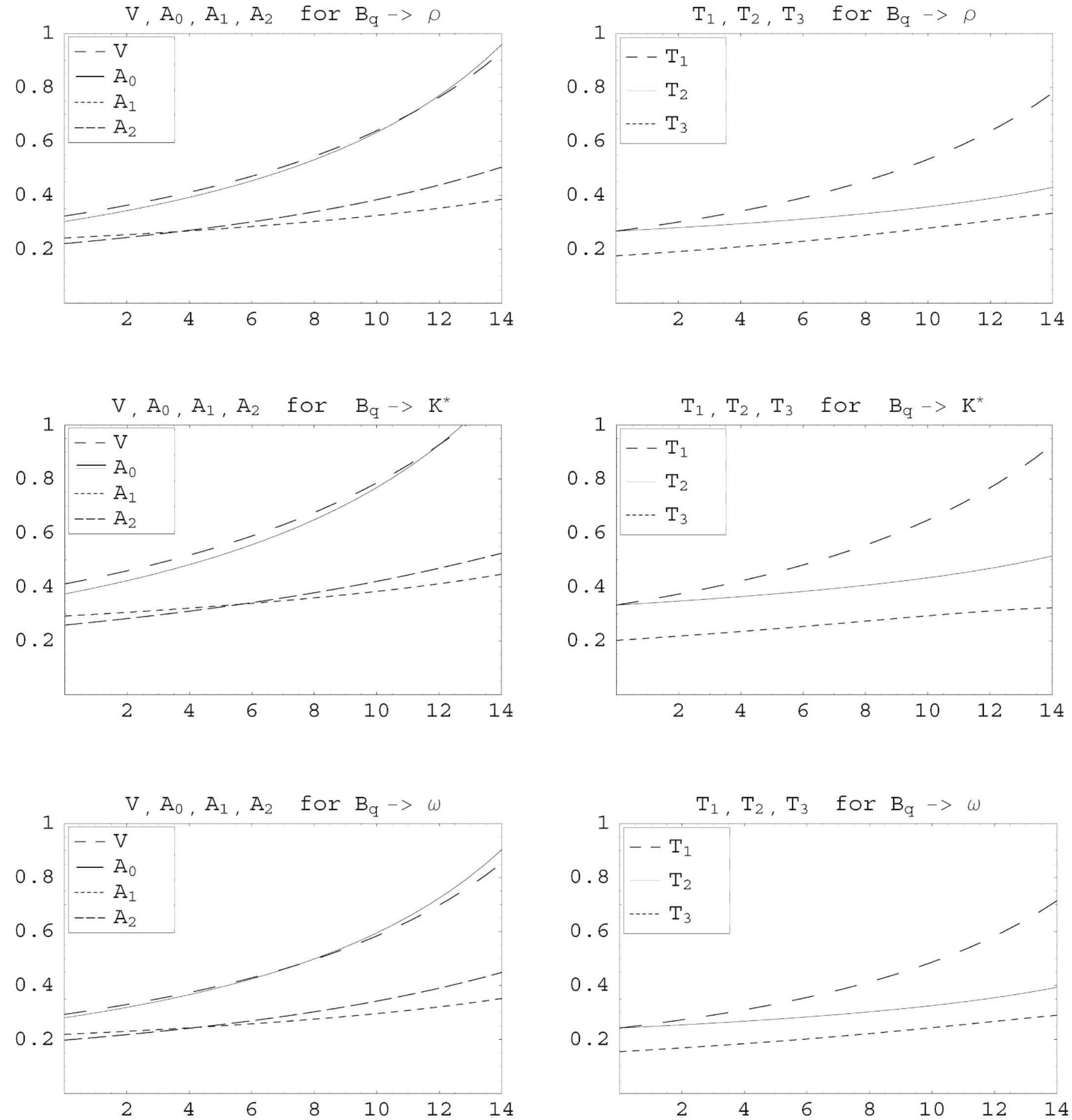

FIG. 5. Form factors for B_q decays as functions of q^2, for central values of input parameters.

in Ref. [32] that the integral can be modeled by a second pole at larger q^2, which is unrelated to any physical resonance:

$$F(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{\rm fit}^2}, \qquad (59)$$

with the three independent parameters $r_{1,2}$ and $m_{\rm fit}$.

The dominant poles at $q^2 = m_R^2$ correspond to resonances with quantum numbers $J^P = 1^-$ for V and T_1, 0^- for A_0, and 1^+ for $A_{1,2,3}$ and $T_{2,3}$, $\tilde{T}_3$. As discussed in Sec. II, not all these form factors are independent, and the question arises which ones to fit to the above equation—or any similar formula—and which ones to define in terms of the others. As Eq. (59) contains two explicit poles, we decide the above question in favor of the form factors with the steepest increase in q^2, which means that the independent form factors are V, $A_{0,1,2}$, and $T_{1,2}$, $\tilde{T}_3$, whereas T_3 and A_3 are the dependent ones, defined as in Eqs. (3) and (8).

The values of the resonance masses m_R in (59) are known from experiment for 0^- and 1^- in the B_q channel and 0^- in the B_s channel; the other masses are obtained using heavy quark symmetry relations [13], the numerical values are collected in Table IX.

We shall use fits to Eq. (59) for the form factors V, A_0, and T_1, where the lowest pole m_R^2 lies well below the multi-particle threshold $(m_{B_{q,s}} + m_{\pi,K})^2$.

If, on the other hand, the lowest physical pole lies sufficiently close to the multi-particle threshold t_0 or even above it, then it may be impossible to "resolve" the poles from a low-q^2 "perspective." In this case it is more

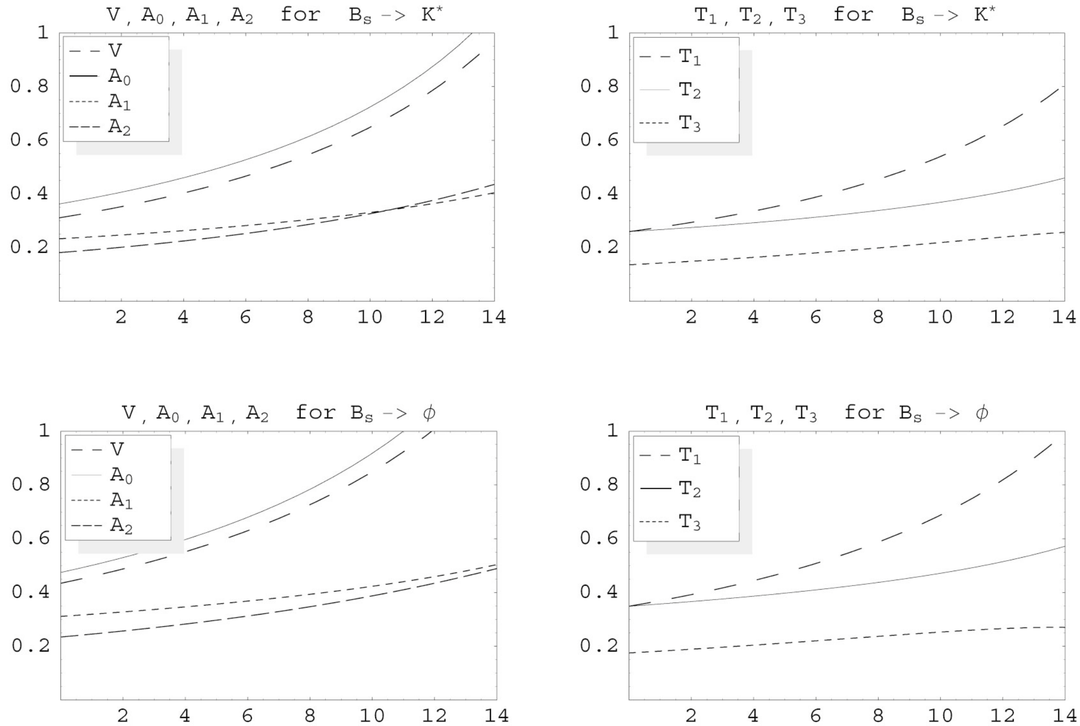

FIG. 6. Form factors for B_s decays as functions of q^2, for central values of input parameters.

appropriate to expand the form factor to second order around the pole, yielding

$$F(q^2) = \frac{r_1}{1 - q^2/m_{\rm fit}^2} + \frac{r_2}{(1 - q^2/m_{\rm fit}^2)^2}, \qquad (60)$$

with the three parameters $r_{1,2}$ and $m_{\rm fit}$. This is the fit formula we shall use for the axial-vector form factors, in particular A_2 and $\tilde{T}_3$. For A_1 and T_2, on the other hand, the residue of the double pole in $m_{\rm fit}$ turns out to be extremely small, so that it can be dropped and one is back to the VMD formula

$$F(q^2) = \frac{r_2}{1 - q^2/m_{\rm fit}^2}, \qquad (61)$$

albeit with an effective pole mass $m_{\rm fit}$ unrelated to any resonance.

The fits of the LCSR results to the above formulas are collected in Table VIII; they differ from the LCSR results obtained for $q^2 \le 14$ GeV^2, by no more than 2.5%. In Table VIII we indicate the "quality" of the fit by δ, which is the maximum deviation of the fit relative to the mean value of the form factor in percent and defined as

$$\delta = 100 \frac{\sum_t |f(t) - f^{\rm fit}(t)|}{\sum_t |f(t)|}, \qquad (62)$$

where the sum runs over $t \in \{0, 0.5, 1, \ldots, 14\}$.

We also have tried fits to the two pole ansatz (59) without fixing one of the masses. In this case the lowest pole is fitted to lie below the actual resonance pole, by up to 1.5 GeV^2. Given the fact that LCSRs are valid for small q^2 far away from the pole, one cannot expect them to resolve its position with perfect accuracy. Nonetheless we take it as an indication for the consistency of our approach that the double pole formula with unrestricted pole positions gives results that agree qualitatively with those from the restricted fits. We also have checked the dependence of the fits on the maximum value of $q^2_{\rm max,LCSR}$ up to which LCSR results are included into the fit. It turns out that the fits are very robust against lowering $q^2_{\rm max,LCSR}$; lowering it from 14 GeV^2 to 7 GeV^2 changes the fitted values at 20 GeV^2 by at most 8%, T_2 being the odd one sticking out. In Fig. 7 we show the effects of a change of $q^2_{\rm max,LCSR}$ on $T_1^{B\to\rho}$ and $A_1^{B\to\rho}$.

Let us now turn to a consistency check of our fits. One can express the residues of V, T_1, and A_0 for $B \to \rho$ in terms of decay constants and strong couplings as follows:

$$r_1^V = \frac{m_B + m_V}{2m_B} f_{B^*} g_{BB^*\rho}, \qquad r_1^{T_1} = \frac{f_{B^*}^T}{2} g_{BB^*\rho},$$
$$r_1^{A_0} = \frac{f_B}{2m_V} g_{BB\rho}, \qquad (63)$$

where $f_{B^*}^T$ is the tensor coupling of the B^* meson defined in the same way as light vector tensor couplings, Eq. (21). f_B has been discussed in Sec. IV B; its value is about 200 MeV

TABLE VIII. Fits for the form factors valid for general q^2. Columns 2–4 give the results of Table VII for $q^2 = 0$, including the errors $\Delta_{\rm tot}$ and Δ_{a_1}. The remaining columns give the fit parameters. Note that we fit the form factor $\tilde{T}_3$, defined in Eq. (10), instead of T_3. The fit formulas to use are given in the last column, the masses m_R are given in Table IX. The penultimate column gives the fit error δ as defined in Eq. (62).

	$F(0)$	$\Delta_{\rm tot}$	Δ_{a_1}	r_1	m_R^2	r_2	$m_{\rm fit}^2$	δ	fit eq.
$V^{B_q\to\rho}$	0.323	0.030		1.045	$m_{1^-}^2$	-0.721	38.34	0.1	(59)
$A_0^{B_q\to\rho}$	0.303	0.029		1.527	$m_{0^-}^2$	-1.220	33.36	0.1	(59)
$A_1^{B_q\to\rho}$	0.242	0.023				0.240	37.51	1.0	(61)
$A_2^{B_q\to\rho}$	0.221	0.023		0.009		0.212	40.82	0.1	(60)
$T_1^{B_q\to\rho}$	0.267	0.023		0.897	$m_{1^-}^2$	-0.629	38.04	0.1	(59)
$T_2^{B_q\to\rho}$	0.267	0.023				0.267	38.59	2.3	(61)
$\tilde{T}_3^{B_q\to\rho}$	0.267	0.023		0.022		0.246	40.88	0.1	(60)
$V^{B_s\to\bar{K}^*}$	0.311	0.026	$-0.43\delta_{a_1}$	2.351	$m_{1^-}^2$	-2.039	33.10	0.1	(59)
$A_0^{B_s\to\bar{K}^*}$	0.360	0.034	$-0.37\delta_{a_1}$	2.813	$m_{0^-}^2$	-2.509	31.58	0.1	(59)
$A_1^{B_s\to\bar{K}^*}$	0.233	0.022	$-0.32\delta_{a_1}$			0.231	32.94	0.8	(61)
$A_2^{B_s\to\bar{K}^*}$	0.181	0.025	$-0.30\delta_{a_1}$	-0.011		0.192	40.14	0.1	(60)
$T_1^{B_s\to\bar{K}^*}$	0.260	0.024	$-0.33\delta_{a_1}$	2.047	$m_{1^-}^2$	-1.787	32.83	0.1	(59)
$T_2^{B_s\to\bar{K}^*}$	0.260	0.024	$-0.33\delta_{a_1}$			0.260	33.01	1.9	(61)
$\tilde{T}_3^{B_s\to\bar{K}^*}$	0.260	0.024	$-0.33\delta_{a_1}$	0.043		0.217	39.38	0.1	(60)
$V^{B_q\to K^*}$	0.411	0.033	$0.44\delta_{a_1}$	0.923	$m_{1^-}^2$	-0.511	49.40	0.0	(61)
$A_0^{B_q\to K^*}$	0.374	0.033	$0.39\delta_{a_1}$	1.364	$m_{0^-}^2$	-0.990	36.78	0.1	(61)
$A_1^{B_q\to K^*}$	0.292	0.028	$0.33\delta_{a_1}$			0.290	40.38	1.0	(63)
$A_2^{B_q\to K^*}$	0.259	0.027	$0.31\delta_{a_1}$	-0.084		0.342	52.00	0.2	(62)
$T_1^{B_q\to K^*}$	0.333	0.028	$0.34\delta_{a_1}$	0.823	$m_{1^-}^2$	-0.491	46.31	0.0	(61)
$T_2^{B_q\to K^*}$	0.333	0.028	$0.34\delta_{a_1}$			0.333	41.41	2.5	(63)
$\tilde{T}_3^{B_q\to K^*}$	0.333	0.028	$0.34\delta_{a_1}$	-0.036		0.368	48.10	0.1	(62)
$V^{B_q\to\omega}$	0.293	0.029		1.006	$m_{1^-}^2$	-0.713	37.45	0.1	(59)
$A_0^{B_q\to\omega}$	0.281	0.030		1.321	$m_{0^-}^2$	-1.040	34.47	0.1	(59)
$A_1^{B_q\to\omega}$	0.219	0.024				-0.217	37.01	1.1	(61)
$A_2^{B_q\to\omega}$	0.198	0.023		0.006		0.192	41.24	0.1	(60)
$T_1^{B_q\to\omega}$	0.242	0.021		0.865	$m_{1^-}^2$	-0.622	37.19	0.1	(59)
$T_2^{B_q\to\omega}$	0.242	0.021				0.242	37.95	2.1	(61)
$\tilde{T}_3^{B_q\to\omega}$	0.242	0.021		0.023		0.220	40.87	0.1	(60)
$V^{B_s\to\phi}$	0.434	0.035		1.484	$m_{1^-}^2$	-1.049	39.52	0.1	(59)
$A_0^{B_s\to\phi}$	0.474	0.033		3.310	$m_{0^-}^2$	-2.835	31.57	0.1	(59)
$A_1^{B_s\to\phi}$	0.311	0.030				0.308	36.54	1.0	(61)
$A_2^{B_s\to\phi}$	0.234	0.028		-0.054		0.288	48.94	0.2	(60)
$T_1^{B_s\to\phi}$	0.349	0.033		1.303	$m_{1^-}^2$	-0.954	38.28	0.1	(59)
$T_2^{B_s\to\phi}$	0.349	0.033				0.349	37.21	2.4	(61)
$\tilde{T}_3^{B_s\to\phi}$	0.349	0.033		0.027		0.321	45.56	0.1	(60)

and we expect f_{B^*} and $f_{B^*}^T$ to be of about the same size. The values of the strong couplings $g_{BB\rho}$ and $g_{BB^*\rho}$ are more controversial as discussed below. As a first check, consider the g-independent ratio

$$\alpha \equiv \frac{r_1^V}{r_1^{T_1}} = \frac{m_B + m_V}{m_{B^*}} \frac{f_{B^*}}{f_{B^*}^T} \sim 1.14. \tag{64}$$

The fitted values of r_1 are collected in Table X and yield

TABLE IX. B meson masses in units GeV, taken from Ref. [13].

	0^-	0^+	1^-	1^+
B_q	5.28	5.63	5.32	5.68
B_s	5.37	5.72	5.42	5.77

$\alpha_{\rm fit} = 1.16$—very close to (64). For r_1 fitted using parameter sets 1 and 3 we find $\alpha = 1.16$ and 1.17, respectively. Any further check of the fitted r_1 requires information on the couplings $g_{BB^{(*)}\rho}$, which have been calculated from both LCSRs [33] and within the constituent quark meson (CQM) model [34]—with significantly different results. The situation resembles that for $g_{DD^*\pi}$, where LCSR determinations are typically by a factor two smaller than lattice and CQM calculations [35]. For this coupling there actually exists an experimental measurement by CLEO [36], which agrees with the lattice and CQM determinations, but disagrees with LCSRs. For the corresponding B coupling $g_{BB^*\pi}$ there is no experimental measurement, as the decay $B^* \to B\pi$ is forbidden by phase space, but one can use heavy quark scaling to obtain $g_{BB^*\pi}$ from the measured $g_{DD^*\pi}$ and compare it with the corresponding theoretical predictions. It turns out that again lattice and CQM calculations are favored, whereas the LCSR calculation gives a too small result, which can be understood following the discussion in Ref. [37]. The recent LCSR determination [33] has up-to-date input parameters and they get from a tree-level analysis

$$g_{BB\rho} = 5.37, \qquad g_{BB^*\rho} = 5.70\ {\rm GeV}^{-1}. \tag{65}$$

For pseudoscalar mesons, NLO calculations have consistently yielded smaller values than tree-level determinations cf. Ref. [38], which, if true also for the ρ, would widen the gap between the results from different methods even further. The CQM-model predictions are [34]

$$g_{BB\rho} = -\sqrt{2}\beta\frac{m_\rho}{f_\pi} = 7.2 \quad \text{with} \quad \beta = -0.86, \tag{66}$$

$$g_{BB^*\rho} = \sqrt{8}\lambda\sqrt{\frac{m_{B^*}}{m_B}}\frac{m_\rho}{f_\pi} = 10.0\ {\rm GeV}^{-1} \quad \text{with} \quad \lambda = 0.6\ {\rm GeV}^{-1}. \tag{67}$$

It is hard for us to judge on the validity of this approach, but as far as we understand the model is further based on empirical success. In Table X we compare the residues for the $B \to \rho$ transition as obtained from our fits, Table IV, to their values given in Eq. (63), using the couplings (65)–(67). For V and T_1 with a 1^- pole the CQM residues are about 10% larger and the LCSR about 40% lower than the fitted values. As discussed above, the LCSR results are expected to fall short of the real values, so this is an excellent confirmation of our results. The A_0 form factor shows some discrepancy which may indicate that either the estimate of the $g_{BB\rho}$ coupling is too low or that the second pole in the fit $m_{\rm fit}^2 \approx 33\ {\rm GeV}^2$ is too close to the resonance pole to allow a clean determination of its residue. Taken altogether, however, the agreement of our fitted results to that of independent calculations is an excellent confirmation of our results.

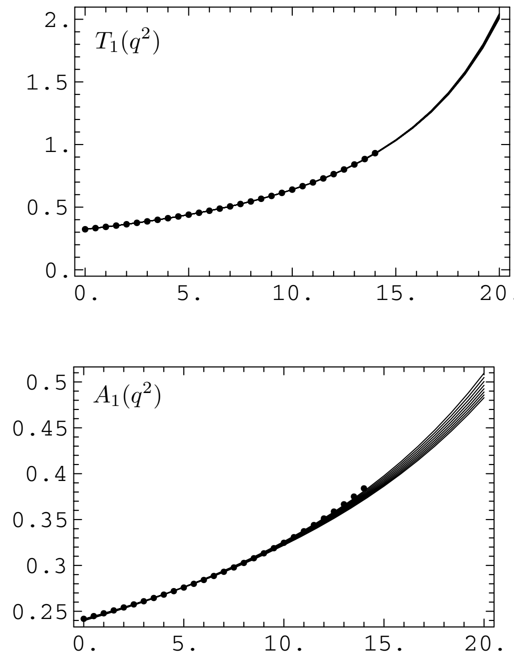

FIG. 7. Comparison of the consistency of fits of $T_1^{B_q\to\rho}$ and $A_1^{B_q\to\rho}$ obtained for different values of $q^2_{\rm LCSR,max}$. Dots: LCSR results for $q^2 \leq 14\ {\rm GeV}^2$. Lines: fits according to Eqs. (59) for T_1 and (61) for A_1, for $q^2_{\rm LCSR,max}$ between 7 and 14 GeV^2. The maximum discrepancy between the fit results at $q^2 = 20\ {\rm GeV}^2$ is 2% for T_1 and 5% for A_1.

V. SUMMARY AND CONCLUSIONS

In this paper we present a thorough and careful examination of the predictions of QCD sum rules on the light cone for the form factors of B_q and B_s transitions to ρ, ω, K^*, and ϕ. Our main results for zero momentum transfer $q^2 = 0$ are collected in Table VII, those for general q^2 in Table VIII.

The present analysis is a sequel of our work on $B \to$ pseudoscalar form factors, Ref. [1], and an extension of the

TABLE X. Residues of the lowest-lying pole for $V^{B_q\to\rho}$, $T_1^{B_q\to\rho}$, and $A_0^{B_q\to\rho}$ obtained from our fits as compared to Eq. (65) with input values from LCSR and CQM determinations.

	our fit	LCSR [33]	CQM [34]
r_1^V	1.05	0.65	1.14
$r_1^{T_1}$	0.90	0.57	1.00
$r_1^{A_0}$	1.53	0.70	0.94

previous work of one of us on $B \to$ vector form factors, Ref. [4]. It improves upon the latter by

(i) including predictions for all form factors of $B_{q,s}$ transitions to $O(\alpha_s)$ accuracy for twist-2 and 3 2-particle contributions;
(ii) including a more sophisticated method for fixing sum rule specific parameters cf. Sec. IV B;
(iii) implementing recently developed new models for the dominant nonperturbative vector meson contributions, the twist-2 vector meson distribution amplitudes cf. Ref. [19];
(iv) allowing the possibility to implement future updates of some hadronic parameters in a straightforward way cf. Appendix A;
(v) giving a careful assessment of uncertainties at zero momentum transfer cf. Sec. IV D;
(vi) including a parametrization of the q^2 dependence of form factors valid in the full physical regime of momentum transfer cf. Sec. IV E;
(vii) giving a variety of consistency checks for the robustness of the q^2 fits and their numerical results.

The accuracy of our results is limited, on the one hand, by the uncertainty of hadronic input parameters and, on the other hand, by the systematic uncertainty induced by the fact that QCD sum rules on the light cone are an approximative method. The uncertainty due to the variation of only the sum rule specific parameters is about 7%, which cannot be reduced any further and hence sets the minimum theoretical uncertainty that can be achieved within this method. An equally large theoretical uncertainty is induced by hadronic parameters and can, in principle, be improved upon. We quote, in particular, the tensor couplings f_V^T of vector mesons, which presently come with the rather large error quoted in Table III. Improvement should be possible by dedicated lattice calculations, a first example of which is Ref. [22]. Another relevant hadronic parameter is Δ, the first inverse moment of the twist-2 vector meson distribution amplitudes, as defined in Sec. IV C. We have inferred a likely range for this parameter for ρ and ω mesons from the known experimental constraints on $\Delta(\pi)$, and further determined a range for $\Delta(K^*)$ and $\Delta(\phi)$ from the observed decrease, within QCD sum rule calculations, of the second Gegenbauer moment a_2 with increasing meson mass. Comparing the theoretical errors collected in Table VII with the global theoretical uncertainty $\sim$15% quoted in our previous publication [4], we have achieved a reduction to about 10%. This is partially due to a reduction of the uncertainties of the hadronic input parameters, in particular m_b, and partially due to a refinement of the assessment of sum rule specific uncertainties as discussed in Sec. IV B. Any future reduction of the total uncertainty will depend on more accurate determinations of Δ, which are absolutely essential not only for light-cone sum rule calculations but also for exploiting the full potential of QCD factorization formulas for nonleptonic exclusive B decays [6]. We take this occasion to urge lattice practitioners to take up the challenge and develop new and ingenious methods to tackle this problem—or just give us an accurate value of a_2, which would already be a big step forward.

The prospects for future direct determinations of $B \to V$ form factors from lattice calculations do appear a bit clouded. On the one hand, there are two recent studies, by the SPQcdR and UKQCD collaborations, Ref. [39], using an improved Wilson action and the quenched approximation. The b quarks are fully relativistic and have typical masses of about 2–3 GeV, so they need to be extrapolated to the physical b quark mass. On the other hand, we conclude from [40] that an unquenched calculation in NRQCD is not really on the menu, which, as far as we understand, is due to an improvement in the treatment of light-quark masses on the lattice, causing the ρ and other vector mesons to become instable particles without a pronounced plateau in the falloff of the correlation function and essentially prevents a precise determination of their properties from lattice. We do not pretend to be sufficient experts in LQCD to be able to meaningfully comment on these issues but remain hopeful that the situation will be clarified in due course.

We have calculated all form factors for $0 \le q^2 \le 14\ \text{GeV}^2$; the upper bound on q^2 is due to the limitations of the light-cone expansion which requires the final state meson to have energies $E \gg \Lambda_{\text{QCD}}$: for $q^2_{\max} = 14\ \text{GeV}^2$ the meson energy is $E = 1.3$ GeV. In order to facilitate the use of our results we have given, in Sec. IV E, Eqs. (59)–(61), simple parametrizations that include the main features of the analytical properties of the form factors and are valid in the full physical regime $0 \le q^2 \le (m_B - m_V)^2$. The corresponding results for our preferred set of input parameters are given in Table VIII. We have checked that the fit results are fairly insensitive to the maximum value of q^2 included and that reducing the latter to, e.g., 7 GeV^2 changes the extrapolated values of the form factors at $q^2 = 20\ \text{GeV}^2$ by typically only 1% to 2%, and by 8% at most (for T_2).

In Sec. I we mentioned factorization formulas for form factors derived in SCET, Ref. [8–10], which, in particular, imply certain (heavy quark) symmetry relations. Since the objective of this paper was to provide numerical results, ready for use in phenomenological applications, we did not discuss the question whether and to what extent our results fulfill these relations, nor the size of symmetry-breaking corrections. A previous study of the corresponding effect in $B \to$ pseudoscalar decays has indicated that such corrections are likely to be nonnegligible [41]. We plan to come back to these points in a future publication.

ACKNOWLEDGMENTS

R. Z. is supported by the Swiss National Science Foundation and would like to thank Antonio Polosa for correspondence on Ref. [34].

APPENDIX A: FORM FACTORS FOR DIFFERENT $f_V^{(T)}$ AND $a_1(K^*)$

The form factors can be written as sum of two contributions which are proportional to the vector meson's vector coupling $f_V \equiv f^L$ and the tensor coupling $f_V^T \equiv f^T$, respectively. The uncertainties of these parameters as tabled in Table III are nonnegligible but amenable to future improvement by, e.g., lattice calculations cf. Ref. [22]. The same applies to $a_1(K^*)$ which also comes with a considerable uncertainty cf. Eq. (48). In order to allow the adjustment of our results to improved determinations of these parameters, we write the generic form factor F as

$$F = \hat{f}^L(F^L + \hat{a}_1^L F^{L,a_1}) + \hat{f}^T(F^T + \hat{a}_1^T F^{T,a_1}), \qquad \text{(A1)}$$

where the hatted quantities are normalized to the central values used in our calculation, i.e., the couplings of Table III and $a_1^T \equiv a_1^\perp$, $a_1^L \equiv a_1^\parallel$ as given in (48). For instance $\hat{f}_\rho^L = f_\rho/(205\ \text{MeV})$. Note that $\hat{a}_1(K^*) \equiv \hat{a}_1(\bar{K}^*)$ and that hatted quantities are trivially invariant under LO scaling. F^L and F^T are collected for $q^2 = 0$ in Table XI and F^{a_1} in Table XII. To give an example, for $A_0^{B_s\to\bar{K}^*}$ we obtain

$$A_0^{B_s\to\bar{K}^*}(0) = \frac{f_{K^*}}{217\ \text{MeV}}\left(0.2469 - \frac{a_1^{K^*}(1\ \text{GeV})}{0.1}0.0398\right) + \frac{f_{K^*}^T(1\ \text{GeV})}{170\ \text{MeV}}\left(0.1532 + \frac{a_1^{K^*}(1\ \text{GeV})}{0.1}0.0024\right),$$

which, choosing the central values of the couplings and a_1, yields 0.3627, in agreement with Table VII.

TABLE XI. Contributions in f^L and f^T to the form factors at $q^2 = 0$. The numbers correspond to the central values of parameter set 2, i.e. $m_b = 4.8$ GeV. $T_2(0)$ follows from $T_1(0) = T_2(0)$.

$F(0)$	F^L	F^T	$F(0)$	F^L	F^T
$V^{B_q\to\rho}$	0.1092	0.2139	$A_0^{B_q\to\rho}$	0.2036	0.0990
$A_1^{B_q\to\rho}$	0.0867	0.1552	$A_2^{B_q\to\rho}$	0.0467	0.1743
$T_1^{B_q\to\rho}$	0.1034	0.1641	$T_3^{B_q\to\rho}$	0.0303	0.1455
$V^{B_s\to\bar{K}^*}$	0.1275	0.2289	$A_0^{B_s\to\bar{K}^*}$	0.2469	0.1532
$A_1^{B_s\to\bar{K}^*}$	0.1022	0.1641	$A_2^{B_s\to\bar{K}^*}$	0.0445	0.1684
$T_1^{B_s\to\bar{K}^*}$	0.1215	0.1737	$T_3^{B_s\to\bar{K}^*}$	0.0211	0.1339
$V^{B_q\to K^*}$	0.1415	0.2234	$A_0^{B_q\to K^*}$	0.2071	0.1269
$A_1^{B_q\to K^*}$	0.1034	0.1545	$A_2^{B_q\to K^*}$	0.0614	0.1658
$T_1^{B_q\to K^*}$	0.1301	0.1665	$T_3^{B_q\to K^*}$	0.0436	0.1386
$V^{B_q\to\omega}$	0.1048	0.1884	$A_0^{B_q\to\omega}$	0.1967	0.0838
$A_1^{B_q\to\omega}$	0.0834	0.1357	$A_2^{B_q\to\omega}$	0.0443	0.1536
$T_1^{B_q\to\omega}$	0.0984	0.1440	$T_3^{B_q\to\omega}$	0.0288	0.1264
$V^{B_s\to\phi}$	0.1594	0.2748	$A_0^{B_s\to\phi}$	0.2647	0.2098
$A_1^{B_s\to\phi}$	0.1226	0.1884	$A_2^{B_s\to\phi}$	0.0558	0.1784
$T_1^{B_s\to\phi}$	0.1469	0.2019	$T_3^{B_s\to\phi}$	0.0316	0.1433

TABLE XII. Contributions of a_1 to the form factors at $q^2 = 0$. Parameters like in Table XI.

$F(0)$	F^{L,a_1}	F^{T,a_1}	$F(0)$	F^{L,a_1}	F^{T,a_1}
$V^{B_s\to\bar{K}^*}$	−0.0057	−0.0396	$A_0^{B_s\to\bar{K}^*}$	−0.0398	0.0024
$A_1^{B_s\to\bar{K}^*}$	−0.0057	−0.0276	$A_2^{B_s\to\bar{K}^*}$	0.0079	−0.0394
$T_1^{B_s\to\bar{K}^*}$	−0.0056	−0.0297	$T_3^{B_s\to\bar{K}^*}$	0.0104	−0.0293
$V^{B_q\to K^*}$	0.0060	0.0403	$A_0^{B_q\to K^*}$	0.0403	−0.0001
$A_1^{B_q\to K^*}$	0.0059	0.0281	$A_2^{B_q\to K^*}$	−0.0080	0.0395
$T_1^{B_q\to K^*}$	0.0059	0.0303	$T_3^{B_q\to K^*}$	−0.0103	0.0299

APPENDIX B: DISTRIBUTION AMPLITUDES

In this appendix we collect explicit expressions for some of the twist-3 and 4 DAs that enter the LCSRs. These expressions are well known and have been taken from Ref. [16]. The twist-2 DAs have already been discussed in Sec. IV C. We also motivate and justify the use of models for DAs based on a truncated conformal expansion.

Before defining the DAs, we introduce the lightlike vectors in which they are expressed. We denote the meson momentum by P_μ (with $P^2 = m_V^2$) and the separation between fields in a nonlocal operator by x_μ (with x^2 close to 0) and introduce lightlike vectors p and z such that

$$p_\mu = P_\mu - \frac{1}{2}z_\mu\frac{m_V^2}{pz}, \qquad z_\mu = x_\mu - P_\mu\left[xP - \sqrt{(xP)^2 - x^2m_V^2}\right]/m_V^2. \qquad \text{(B1)}$$

The meson polarization vector $e_\mu^{(\lambda)}$ is decomposed into projections onto the two lightlike vectors and the orthogonal plane as

$$e_\mu^{(\lambda)} = \frac{(e^{(\lambda)}z)}{pz}\left(p_\mu - \frac{m_V^2}{2pz}z_\mu\right) + e_{\perp\mu}^{(\lambda)}. \qquad \text{(B2)}$$

We also need the projector onto the directions orthogonal to p and z:

$$g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{1}{pz}(p_\mu z_\nu + p_\nu z_\mu). \qquad \text{(B3)}$$

The dual gluon field strength tensor is defined as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$. We use the standard Bjorken-Drell convention [42] for the metric tensor and the Dirac matrices; in particular $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$

is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$. This convention differs in sign by the one of Itzykson/Zuber [43] used in some programming packages, e.g., FEYNCALC. We use a sign convention for the strong coupling g where the covariant derivative is defined as $D_\mu = \partial_\mu - igA_\mu$ and hence the Feynman rule for qqg vertices is $+ig\gamma_\mu$.

Let us also clarify the treatment of SU(3)-breaking effects in DAs. SU(3) breaking occurs in three different ways:

(i) the contribution of odd Gegenbauer-moments $a_{1,3,\ldots}$ to the DAs of the K^*;
(ii) a difference in the values of the couplings $f_V^{(T)}$, the even Gegenbauer moments $a_2^\rho \neq a_2^{K^*}$ and 3-particle matrix elements;
(iii) the modification of relations between DAs by terms in $m_{q_1} \pm m_{q_2}$.

We will take into account the first effect wherever it occurs, except for terms in $O(\delta^2)$, the reason being that the structure of δ^2 terms is very involved and there are yet unknown contributions in $m_V^2 a_1^\perp$ induced by 3-particle twist-4 DAs. The second effect is taken into account for the decay constants and parametrized by the dependence of the form factors on the parameters Δ, as discussed in Sec. IV C; we do not include SU(3) breaking for the 3-particle matrix elements as information on these effects is virtually nonexistant. The third effect is taken into account at $O(\delta\alpha_s^0, \delta\alpha_s)$, i.e., for the chiral-even DAs $g_\perp^{(a,v)}$. It does not occur at $O(\delta^0)$ and the corresponding terms are unknown at $O(\delta^2)$.

The 2-particle DAs have been defined in Eqs. (15)–(18). Up to twist-4 and $O(\delta^2)$, there are seven chiral-odd 3-particle DAs which can be defined as [16]

$$\begin{aligned}
&\langle 0|\bar q_2(z)\sigma_{\alpha\beta} g G_{\mu\nu}(vz) q_1(-z)|V(p,\lambda)\rangle \\
&= f_V^T m_V^2 \frac{e^{(\lambda)}z}{2(pz)}[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}]\mathcal{T}(v,pz) + f_V^T m_V^2 [p_\alpha e^{(\lambda)}_{\perp\mu} g^\perp_{\beta\nu} - p_\beta e^{(\lambda)}_{\perp\mu} g^\perp_{\alpha\nu} \\
&\quad - p_\alpha e^{(\lambda)}_{\perp\nu} g^\perp_{\beta\mu} + p_\beta e^{(\lambda)}_{\perp\nu} g^\perp_{\alpha\mu}] T_1^{(4)}(v,pz) + f_V^T m_V^2 [p_\mu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\nu} - p_\mu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\nu} - p_\nu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\mu} + p_\nu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\mu}] T_2^{(4)}(v,pz) \\
&\quad + \frac{f_V^T m_V^2}{pz}[p_\alpha p_\mu e^{(\lambda)}_{\perp\beta} z_\nu - p_\beta p_\mu e^{(\lambda)}_{\perp\alpha} z_\nu - p_\alpha p_\nu e^{(\lambda)}_{\perp\beta} z_\mu + p_\beta p_\nu e^{(\lambda)}_{\perp\alpha} z_\mu] T_3^{(4)}(v,pz) \\
&\quad + \frac{f_V^T m_V^2}{pz}[p_\alpha p_\mu e^{(\lambda)}_{\perp\nu} z_\beta - p_\beta p_\mu e^{(\lambda)}_{\perp\nu} z_\alpha - p_\alpha p_\nu e^{(\lambda)}_{\perp\mu} z_\beta + p_\beta p_\nu e^{(\lambda)}_{\perp\mu} z_\alpha] T_4^{(4)}(v,pz),
\end{aligned} \tag{B4}$$

$$\begin{aligned}
&\langle 0|\bar q_2(z) g G_{\mu\nu}(vz) q_1(-z)|V(p,\lambda)\rangle \\
&\quad = i f_V^T m_V^2 [e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu] S(v,pz), \\
&\langle 0|\bar q_2(z) i g \tilde G_{\mu\nu}(vz)\gamma_5 q_1(-z)|V(p,\lambda)\rangle \\
&\quad = i f_V^T m_V^2 [e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu]\tilde S(v,pz).
\end{aligned} \tag{B5}$$

Of these seven amplitudes, $\mathcal{T}$ is of twist-3 and the other six of twist-4; higher twist terms are suppressed. In the above equations, we use

$$\mathcal{T}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_2 - \alpha_1 + v\alpha_3)}\mathcal{T}(\underline{\alpha}), \tag{B6}$$

etc., and $\underline{\alpha}$ is the set of three momentum fractions $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\}$. The integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \int_0^1 d\alpha_3 \delta\Big(1 - \sum \alpha_i\Big). \tag{B7}$$

As for chiral-even DAs, to order $O(\delta^2)$ only the twist-3 DAs contribute, which we define as

$$\begin{aligned}
&\langle 0|\bar q_2(z) g \tilde G_{\mu\nu}(vz)\gamma_\alpha\gamma_5 q_1(-z)|V(p,\lambda)\rangle \\
&\quad = f_V m_V p_\alpha [p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{A}(v,pz) + O(m_V^3),
\end{aligned} \tag{B8}$$

$$\begin{aligned}
&\langle 0|\bar q_2(z) g G_{\mu\nu}(vz) i\gamma_\alpha q_1(-z)|V(p)\rangle \\
&\quad = f_V m_V p_\alpha [p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{V}(v,pz) + O(m_V^3).
\end{aligned} \tag{B9}$$

At first glance, the sheer number of different DAs, two of twist-2, seven of twist-3, and nine of twist-4, seems to preclude any predictivity of the LCSRs. Appearances are deceiving, though: not all these DAs are independent of each other, and one can disentangle their mutual interdependencies using the *QCD equations of motion*, which results in integral relations between different DAs, e.g., the chiral-odd DAs $\phi_\parallel$, $g_\perp^{(a,v)}$, g_3, etc. We shall see examples of such relations below. The other important organizing principle for DAs is *conformal expansion*, i.e., a partial wave expansion of DAs in terms of contributions of increasing conformal spin. Conformal expansion relies on the fact that massless QCD displays conformal symmetry[14] which allows one to organize the DAs in terms of irreducible representations of the corresponding symmetry group $SL(2, \mathbb{R})$. The coefficients of these different partial waves renormalize multiplicatively to LO in QCD, but mix at NLO, the reason being that the symmetry is anomalous.

As mentioned above, the plethora of vector meson DAs is not mutually independent, but related by the QCD equations of motion. These relations are discussed at length in Ref. [16], whose formulas we adapt to the present case. The chiral-even twist-3 DAs are of order δ, so we keep the full dependence on terms induced by $\phi_\parallel$, but use conformal expansion for the admixture of 3-particle DAs:

$$(1-\delta_+)g_\perp^{(a)} = \bar{u}\int_0^u dv\frac{\Psi(v)}{\bar{v}} + u\int_u^1 dv\frac{\Psi(v)}{v} + 10\zeta_3\left(1-\frac{3}{16}\omega_3^A+\frac{9}{16}\omega_3^V\right)\{5(2u-1)^2 - 1\}, \tag{B10}$$

$$g_\perp^{(v)} = \frac{1}{4}\left[\int_0^u dv\frac{\Psi(v)}{\bar{v}} + \int_u^1 dv\frac{\Psi(v)}{v}\right] + 5\zeta_3\{3(2u-1)^2 - 1\} + \frac{15}{64}\zeta_3(3\omega_3^V-\omega_3^A)[3-30(2u-1)^2 + 35(2u-1)^4], \tag{B11}$$

with $\Psi(u) = 2\phi_\parallel(u) + \delta_+(2u-1)\phi'_\perp(u) + \delta_-\phi'_\perp(u)$, $\delta_\pm = (f_V^T/f_V)(m_{q_2}\pm m_{q_1})/m_V$. The dimensionless coupling ζ_3 is defined by the (local) matrix element

$$\begin{aligned}\langle 0|\bar{q}_2 g\tilde{G}_{\mu\nu}\gamma_\alpha\gamma_5 q_1|V(P,\lambda)\rangle &= f_V m_V\zeta_3\left[e_\mu^{(\lambda)}\left(P_\alpha P_\nu - \frac{1}{3}m_V^2 g_{\alpha\nu}\right) - e_\nu^{(\lambda)}\left(P_\alpha P_\mu - \frac{1}{3}m_V^2 g_{\alpha\mu}\right)\right] \\ &\quad + \frac{1}{3}f_V m_V^3\zeta_4\left[e_\mu^{(\lambda)}g_{\alpha\nu} - e_\nu^{(\lambda)}g_{\alpha\mu}\right],\end{aligned} \tag{B12}$$

where ζ_4 is a matrix element of twist-4. $\omega_3^{A,V,T}$ are matrix elements of quark-quark-gluon operators involving derivatives and defined in the second reference of [16].

The chiral-odd twist-3 DAs, on the other hand, are $O(\delta^2)$, so we model them in conformal expansion truncated after the first non-leading-order:

$$h_\parallel^{(s)}(u) = 6u\bar{u}\left[1+\left(\frac{1}{4}a_2^\perp + \frac{5}{8}\zeta_3\omega_3^T\right)(5(2u-1)^2-1)\right], \tag{B13}$$

[14]See Ref. [17] for a review on the use of conformal symmetry in QCD.

$$h_\parallel^{(t)}(u) = 3(2u-1)^2 + \frac{3}{2}a_2^\perp(2u-1)^2[5(2u-1)^2-3] + \frac{15}{16}\zeta_3\omega_3^T[3-30(2u-1)^2+35(2u-1)^4]. \tag{B14}$$

As mentioned above, we drop contributions in the odd Gegenbauer moment $a_1^\perp$, as not all $m_V^2 a_1^\perp$ terms are known.

As for the 3-particle twist-3 DAs, we have, quoting from Ref. [16]:

$$\mathcal{V}(\underline{\alpha}) = 540\zeta_3\omega_3^V(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3^2, \tag{B15}$$

$$\mathcal{A}(\underline{\alpha}) = 360\zeta_3\alpha_1\alpha_2\alpha_3^2\left[1+\omega_3^A\frac{1}{2}(7\alpha_3-3)\right], \tag{B16}$$

$$\mathcal{T}(\underline{\alpha}) = 540\zeta_3\omega_3^T(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3^2. \tag{B17}$$

These expressions are valid to NLO in the conformal expansion.

The chiral-even 2-particle DAs of twist-4, g_3 and $\mathbb{A}_\parallel$ in Eq. (15), are $O(\delta^3)$, so we drop them. For the chiral-odd twist-4 DAs h_3 and $\mathbb{A}_\perp$ we use NLO conformal expansion (with $a_1^\perp \to 0$):

$$h_3(u) = 1 + \left\{\frac{3}{7}a_2^\perp - 1 - 10(\zeta_4^T + \tilde{\zeta}_4^T)\right\}C_2^{1/2}(2u-1) + \left\{-\frac{3}{7}a_2^\perp - \frac{15}{8}\zeta_3\omega_3^T\right\}C_4^{1/2}(2u-1), \tag{B18}$$

$$\begin{aligned}\mathbb{A}_\perp(u) &= 30u^2\bar{u}^2\left\{\frac{2}{5}\left(1+\frac{2}{7}a_2^\perp + \frac{10}{3}\zeta_4^T - \frac{20}{3}\tilde{\zeta}_4^T\right) + \left(\frac{3}{35}a_2^\perp + \frac{1}{40}\zeta_3\omega_3^T\right)C_2^{5/2}(2u-1)\right\} \\ &\quad - \left(\frac{18}{11}a_2^\perp - \frac{3}{2}\zeta_3\omega_3^T + \frac{126}{55}\langle\langle Q^{(1)}\rangle\rangle + \frac{70}{11}\langle\langle Q^{(3)}\rangle\rangle\right)[u\bar{u}(2+13u\bar{u}) + 2u^3(10-15u \\ &\quad + 6u^2)\ln u + 2\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln\bar{u}].\end{aligned} \tag{B19}$$

The formulas for chiral-odd 3-particle DAs of twist-4 are rather lengthy and we refrain from reproducing them here. They can be found in the second reference of [16].

The numerical values of 3-particle matrix elements are given in Tables XIII and XIV for the scales 1 GeV and

TABLE XIII. 3-particle parameters of chiral-even distribution amplitudes.

μ	ζ_3	ω_3^A	ω_3^V
1 GeV	0.032 ± 0.010	-2.1 ± 1.0	3.8 ± 1.8
2.2 GeV	0.018 ± 0.006	-1.7 ± 0.9	3.6 ± 1.7

TABLE XIV. 3-particle parameters of chiral-odd distribution amplitudes. Terms in $a_2^\perp$ are treated as described in Sec. IV C.

μ	ω_3^T	ζ_4^T	$\tilde{\zeta}_4^T$	$\langle\langle Q^{(1)}\rangle\rangle$	$\langle\langle Q^{(3)}\rangle\rangle$
1 GeV	7.0 ± 7.0	0.10 ± 0.05	-0.10 ± 0.05	-0.15 ± 0.15	0
2.2 GeV	7.2 ± 7.2	0.06 ± 0.03	-0.06 ± 0.03	-0.07 ± 0.07	0

$\sqrt{m_B^2 - m_b^2} = 2.2$ GeV. The corresponding one-loop anomalous dimensions are also given in [16]. The numerical values for the decays constants $f_V^{(T)}$ are collected in Table III.

APPENDIX C: 3-PARTICLE CONTRIBUTIONS TO THE LCSRS

In this paper we include contributions of 3-particle DAs to the correlation function (12) at tree level. This appendix contains explicit formulas for these contributions.

The 3-particle DAs of twist-3 have been defined in Appendix B; the definitions for twist-4 DAs can be found in Ref. [16]. Their contributions to the correlation functions are most easily calculated in the external field method proposed in Ref. [44]. The light-cone b quark propagator in an external field reads, in the Fock-Schwinger gauge $x_\mu A^\mu(x) = 0$:

$$\langle 0|Tb(x)\bar{b}(0)|0\rangle_A = iS_b^{(0)}(x) + iS_b^{(2)}(x, 0), \tag{C1}$$

with

$$S_b^{(2)}(x, 0) = -\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \int_0^1 dv \frac{1}{2}[\bar{v}S_2(k, m)\sigma_{\mu\nu}G^{\mu\nu}(vx) + v\sigma_{\mu\nu}G^{\mu\nu}(vx)S_2(k, m)], \tag{C2}$$

where $S_n(k, m) = (\not{k} + m)/\Delta^n(k)$ with $\Delta(k) \equiv 1/(k^2 - m^2)$.[15] This expression is equivalent to Eq. 2.25 in Ref. [4]. The decomposition (29) selects the chiral-odd DAs (B4) and (B5), and the chiral-even DAs (B8) and (B9). Terms in $e^*_\alpha x_\beta/px$ are treated by partial integration; we have checked that all boundary terms vanish. Upon partial integration, we hence have

$$\frac{e^*_\alpha x_\beta}{px}\int_\alpha e^{ix\cdot(k-l)} f(\alpha_1, \alpha_3) S_2(k, m) \to \int_\alpha e^{ix\cdot(k-l)} f(\tilde{\alpha}_1, \alpha_3)[4S_3(k, m)e^*_\alpha k_\beta - e^*_\alpha\gamma_\beta\Delta(k)^2],$$

with $l = q + (\alpha_1 + v\alpha_3)p$, $f(\tilde{x}, y) = \int^x da f(a, y)$, and $\int_\alpha = \int_0^1 d\alpha_3 \int_0^{1-\alpha_3} d\alpha_1$.

The contribution of 3-particle DAs to the correlation function (12) then reads:

$$\begin{aligned}
&\frac{i}{4} f_V m_V \int_0^1 dv \int D\alpha \Delta(l)^2 (pq)[\mathcal{V}(\alpha) + \mathcal{A}(\alpha)] 2v \mathrm{tr}[\Gamma \not{e}^* \not{p}\gamma_5] + O(m_V^3)\\
&\quad + \frac{i}{4} f_V^T m_V^2 \Big(\int_0^1 dv \int D\alpha \Delta(l)^2 S(\alpha)(\bar{v}\mathrm{tr}[\Gamma(\not{q} + m)\not{e}^*\not{p}\gamma_5] + v\mathrm{tr}[\Gamma\not{e}^*\not{p}(\not{q} + m)\gamma_5])\\
&\quad - \int_0^1 dv \int D\alpha \Delta(l)^2 \tilde{S}(\alpha)(\bar{v}\mathrm{tr}[\Gamma(\not{q} + m)\not{e}^*\not{p}\gamma_5] + v\mathrm{tr}[\Gamma\not{e}^*\not{p}(-\not{q} + m)\gamma_5])\\
&\quad + \int_0^1 dv \int D\alpha \Delta(l)^2 \bar{v}\mathrm{tr}[\Gamma(\not{q} + m)\not{e}^*\not{p}\gamma_5] T_3^{(4)}(\alpha_1, \alpha_3) + \int_0^1 dv \int D\alpha \Delta(l)^3 4v(pq)\mathrm{tr}[\Gamma(-\not{q} + m)\not{e}^*\not{p}\gamma_5] T_3^{(4)}(\tilde{\alpha}_1, \alpha_3)\\
&\quad - \int_0^1 dv \int D\alpha \Delta(l)^2 \bar{v}\mathrm{tr}[\Gamma(\not{q} + m)\not{e}^*\not{p}\gamma_5] T_4^{(4)}(\alpha_1, \alpha_3)\\
&\quad - \int_0^1 dv \int D\alpha \Delta(l)^3 4v(pq)\mathrm{tr}[\Gamma\not{e}^*\not{p}(-\not{q} + m)\gamma_5] T_4^{(4)}(\tilde{\alpha}_1, \alpha_3)\\
&\quad + \int_0^1 dv \int D\alpha [(16\bar{v}\{p(e^*q)\} + 4v\{e^*pq\} - 4vm\{e^*p\})(pq)\Delta(l)^3 T_1^{(4)}(\tilde{\alpha}_1, \alpha_3)
\end{aligned}$$

[15] Note that $S^{(2)}(x, 0) \neq S^{(2)}(0, -x)$ as the Fock-Schwinger gauge breaks translational invariance.

$$+ ((\bar{v} - v)\{qe^* p\} - 4v\{e^*(pq)\} + v\{e^* pq\} + m(\bar{v} + 2v)\{e^* p\})\Delta(l)^2 T_1^{(4)}(\alpha_1, \alpha_3)] + \int_0^1 dv \int D\alpha\Big[(-16\bar{v}\{p(e^* q)\}$$
$$- 4v\{qe^* p\} - 4vm\{e^* p\})(pq)\Delta(l)^3 T_2^{(4)}(\tilde{\alpha}_1, \alpha_3) + ((\bar{v} + v)\{qe^* p\} - 4v\{e^*(pq)\}$$
$$+ v\{e^* pq\} + m(\bar{v} + 2v)\{e^* p\})\Delta(l)^2 T_2^{(4)}(\alpha_1, \alpha_3)\Big]\Big).$$

In the above formula, we use $\{abc\} = \mathrm{tr}[\Gamma \not{a} \not{b} \not{c} \gamma_5]$ and $\{a(bc)\} = b \cdot c\ \mathrm{tr}[\Gamma \not{a} \gamma_5]$.

[1] P. Ball and R. Zwicky, hep-ph/0406232 [Phys. Rev. D to be published)].

[2] V. M. Belyaev, A. Khodjamirian, and R. Rückl, Z. Phys. C **60**, 349 (1993); P. Ball and V. M. Braun, Phys. Rev. D **55**, 5561 (1997); A. Khodjamirian *et al.*, Phys. Lett. B **410**, 275 (1997); E. Bagan, P. Ball, and V. M. Braun, Phys. Lett. B **417**, 154 (1998); P. Ball, J. High Energy Phys. 09 (1998) 005; A. Khodjamirian *et al.*, Phys. Rev. D **62**, 114002 (2000); P. Ball and R. Zwicky, J. High Energy Phys. 10 (2001) 019.

[3] P. Colangelo and A. Khodjamirian, hep-ph/0010175; A. Khodjamirian, AIP Conf. Proc. **602**, 194 (2001).

[4] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).

[5] P. Ball *et al.*, hep-ph/0003238; *Proceedings of the Second Workshop on the CKM Unitarity Triangle, Durham, England, 2003*, edited by P. Ball *et al.*, eConf C0304052.

[6] M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).

[7] V. L. Chernyak and A. R. Zhitnitsky, JETP Lett. **25**, 510 (1977) [Pis'ma Zh. Eksp. Teor. Fiz. **25**, 544 (1977)]; Sov. J. Nucl. Phys. **31**, 544 (1980) [Yad. Fiz. **31**, 1053 (1980)]; A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B**, 245 (1980); Theor. Math. Phys. (Engl. Transl.) **42**, 97 (1980) [Teoreticheskaya i Matematicheskaya Fizika **42**, 147 (1980)]; G. P. Lepage and S. J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. D **22**, 2157 (1980); V. L. Chernyak, A. R. Zhitnitsky, and V. G. Serbo, JETP Lett. **26**, 594 (1977); [Pis'ma Zh. Eksp. Teor. Fiz. **26**, 760 (1977)]; Sov. J. Nucl. Phys. **31**, 552 (1980) [Yad. Fiz. **31**, 1069 (1980)].

[8] C. W. Bauer *et al.*, Phys. Rev. D **63**, 114020 (2001).

[9] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **67**, 071502 (2003).

[10] R. J. Hill, hep-ph/0411073.

[11] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979); **147**, 448 (1979).

[12] P. Ball and M. Boglione, Phys. Rev. D **68**, 094006 (2003).

[13] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).

[14] A. Ali *et al.*, Phys. Rev. D **61**, 074024 (2000).

[15] A. Ali, E. Lunghi, and A. Y. Parkhomenko, Phys. Lett. B **595**, 323 (2004).

[16] P. Ball *et al.*, Nucl. Phys. **B529**, 323 (1998); P. Ball and V. M. Braun, Nucl. Phys. **B543**, 201 (1999).

[17] V. M. Braun, G. P. Korchemsky, and D. Müller, Prog. Part. Nucl. Phys. **51**, 311 (2003).

[18] P. Ball and M. Lazar, Phys. Lett. B **515**, 131 (2001).

[19] P. Ball, A. Talbot, (to be published).

[20] S. A. Larin, Phys. Lett. B **303**, 113 (1993).

[21] P. Ball and V. M. Braun, Phys. Rev. D **54**, 2182 (1996).

[22] D. Becirevic *et al.*, J. High Energy Phys. 05 (2003) 007.

[23] HPQCD Collaboration, C. Aubin *et al.* Phys. Rev. D **70**, 031504 (2004);QCDSF Collaboration, M. Göckeler *et al.*, hep-ph/0409312.

[24] M. Wingate *et al.*, Phys. Rev. Lett. **92**, 162 001 (2004).

[25] A. S. Kronfeld, Nucl. Phys. B, Proc. Suppl. **129**, 46 (2004).

[26] A. A. Penin and M. Steinhauser, Phys. Rev. D **65**, 054006 (2002); M. Jamin and B. O. Lange, Phys. Rev. D **65**, 056005 (2002).

[27] See, for instance, T. M. Aliev and V. L. Eletsky, Sov. J. Nucl. Phys. **38**, 936 (1983) [Yad. Fiz. **38**, 1537 (1983)]; E. Bagan *et al.*, Phys. Lett. B **278**, 457 (1992).

[28] P. Ball, Ph.D. thesis, Heidelberg 1992 (unpublished).

[29] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[30] V. M. Braun and A. Lenz, Phys. Rev. D **70**, 074020 (2004).

[31] CLEO Collaboration, J. Gronberg *et al.*, Phys. Rev. D **57**, 33 (1998).

[32] D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478**, 417 (2000).

[33] Z. H. Li *et al.*, Phys. Rev. D **65**, 076005 (2002).

[34] A. Deandrea *et al.*, Phys. Rev. D **59**, 074012 (1999).

[35] A. Abada *et al.*, Phys. Rev. D **66**, 074504 (2002).

[36] CLEO Collaboration, S. Ahmed *et al.*, Phys. Rev. Lett. **87**, 251 801 (2001); CLEO Collaboration, A. Anastassov *et al.*, Phys. Rev. D **65**, 032003 (2002).

[37] D. Becirevic *et al.*, J. High Energy Phys. 01 (2003) 009.

[38] V. M. Belyaev *et al.*, Phys. Rev. D **51**, 6177 (1995); A. Khodjamirian *et al.*, Phys. Lett. B **457**, 245 (1999).

[39] SPQcdR Collaboration, A. Abada *et al.*, Nucl. Phys. B, Proc. Suppl. **119**, 625 (2003); UKQCD Collaboration, K. C. Bowler *et al.*, J. High Energy Phys. 05 (2004) 035.

[40] P. McKenzie, Proceedings of V_{ux} Workshop at SLAC, Boston, 2003; C. H. Davies, Proceedings of United Kingdom *BABAR* Meeting, Durham, 2004.

[41] P. Ball, hep-ph/0308249.

[42] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[43] C. Itzykson and J. B. Zuber, *Quantum Field Theory*, (McGraw-Hill, New York, 1980).

[44] V. A. Novikov *et al.*, Fortschr. Phys.. **32**, 585 (1985).