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Improved Analysis of $B \to \pi e\nu$ from QCD Sum Rules on the Light-Cone

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Abstract

We present a new calculation of the $B \to \pi$ form-factor f_+, relevant for the measurement of $|V_{ub}|$ from semileptonic $B \to \pi$ transitions, from QCD sum rules on the light-cone. The new element is the calculation of radiative corrections to next-to-leading twist-3 accuracy. We find that these contributions are factorizable at $O(\alpha_s)$, which lends additional support to the method of QCD sum rules on the light-cone. We obtain $f_+(0) = 0.26 \pm 0.06 \pm 0.05$, where the first error accounts for the uncertainty in the input-parameters and the second is a guesstimate of the systematic uncertainty induced by the approximations inherent in the method. We also obtain a simple parametrization of the form-factor which is valid in the entire kinematical range of semileptonic decays and consistent with vector-meson dominance at large momentum-transfer.

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1. The experimental programme of the dedicated B-factories BaBar and Belle will contribute to unravel structure and size of flavour- and CP-violation. The key-observable is the unitarity-triangle of the CKM-matrix whose overdetermination will help to answer the question whether there are additional sources of CP-violation not present in the SM. Overdetermination means independent measurements of sides and angles from different processes. One of the sides is determined by the CKM-matrix element $|V_{ub}|$, whose precise measurement is certainly a challenging task due to the smallness of the corresponding branching ratios. The method of choice is to measure it from semileptonic tree-level decays $b \to ue\nu$ where contamination by new-physics effects is expected to be small. The main complication in this measurement is, as a matter of course, QCD effects whose calculation from first principles is highly challenging. Inclusive semileptonic decays are usually treated in heavy quark expansion and become the more delicate the more accurately experimentally necessary cuts are taken into account (which necessitates the inclusion of other potentially large scales and calls for threshold- and soft-gluon resummation, cf. [1]). The alternative is to study exclusive decays where, owing to the nonrenormalization of vector and axialvector currents, both perturbative and nonperturbative QCD effects are neatly encoded in form-factors, depending on only one variable, the momentum-transfer to the leptons. The simplest such decay-process, involving only a single form-factor[1], is $B \to \pi\ell\nu_\ell$, which hence has received fair attention in the literature. First experimental results are available from CLEO [2]. The most precise calculation of the form-factor will *sans doute* finally come from lattice-simulations; presently, however, the main attention of lattice-practitioners appears to be directed not so much to obtaining phenomenologically relevant results, but rather to controlling lattice artifacts and approximations like discretization errors and the explicit breaking of chiral symmetry with Wilson-fermions, particularly relevant for calculating processes that involve pions; another problem is how to simulate relativistic b-quarks, and how to access the region of phase-space where the pion has large momentum. All these problems are presently under intense debate, and we refer to Ref. [3] for reviews and recent research papers.

Another, technically much simpler, but also less rigorous approach is provided by QCD sum rules on the light-cone (LCSRs) [4, 5]. The key-idea is to consider a correlation-function of the weak current and a current with the quantum-numbers of the B-meson, sandwiched between the vacuum and a pion. For large (negative) virtualities of these currents, the correlation-function is, in coordinate-space, dominated by distances close to the light-cone and can be discussed in the framework of light-cone expansion. In contrast to the short-distance expansion employed by conventional QCD sum rules à la SVZ [6], where nonperturbative effects are encoded in vacuum expectation values of local operators with vacuum quantum numbers, the condensates, LCSRs rely on the factorization of the underlying correlation function into genuinely nonperturbative and universal hadron distribution amplitudes (DAs) ϕ that are convoluted with process-dependent amplitudes T_H, which are the analogues to the Wilson-coefficients in the short-distance expansion and can

[1] This statement is true only as long as lepton-masses are negligible, i.e. for semi-electronic and -muonic decays.

be calculated in perturbation theory, schematically

$$\text{correlation function } \sim \sum_n T_H^{(n)} \otimes \phi^{(n)}. \tag{1}$$

The sum runs over contributions with increasing twist, labelled by n, which are suppressed by increasing powers of, roughly speaking, the virtualities of the involved currents. The same correlation function can, on the other hand, be written as a dispersion-relation, in the virtuality of the current coupling to the B-meson. Equating dispersion-representation and the light-cone expansion, and separating the B-meson contribution from that of higher one- and multi-particle states, one obtains a relation for the form-factor describing $B \to \pi$.

The particular strength of LCSRs lies in the fact that they allow inclusion not only of hard-gluon exchange contributions, which have been identified, in the seminal papers that opened the study of hard exclusive processes in the framework of perturbative QCD (pQCD) [7], as being dominant in light-meson form factors, but that they also capture the so-called Feynman-mechanism, where the quark created at the weak vertex carries nearly all momentum of the meson in the final-state, while all other quarks are soft. This mechanism is suppressed by two powers of momentum-transfer in processes with light mesons; as shown in [5], this suppression is absent in heavy-to-light transitions[2] and hence any reasonable application of pQCD to B-meson decays should include this mechanism. LCSRs also avoid any reference to a "light-cone wave-function of the B-meson", which is a necessary ingredient in all extensions of the original pQCD method to heavy-meson decays [8, 9], but whose exact definition appears to be problematic [10]. A more detailed discussion of the rationale of LCSRs and of the more technical aspects of the method is beyond the scope of this letter; more information can be found in the literature [11].

LCSRs are available for the $B \to \pi$ form factor f_+ to $O(\alpha_s)$ accuracy for the leading twist-2 contribution and at tree-level for higher-twist (3 and 4) contributions [12, 13, 14]. In this letter we calculate the leading radiative corrections to the twist-3 contributions. The motivation for this calculation is twofold: first, it has been found in [12, 13, 14] that the tree-level twist-3 corrections are chirally enhanced and sizeable, and amount up to 30% of the final result for f_+, which indicates that radiative corrections may be phenomenologically relevant. Second, the existence of the factorization-formula (1) is nontrivial beyond tree-level and, to date, it is only for the twist-2 contribution that factorization has been shown to hold also after inclusion of radiative corrections. In this letter we show that (1), for a certain approximation of the DAs $\phi^{(3)}$ (leading conformal spin), also holds when $O(\alpha_s)$ corrections to the twist-3 contributions are included.

2. Let us now properly define the relevant quantities. The form-factors $f_{+,0}$ are given by $(q = p_B - p)$

$$\langle \pi(p) | \bar{u} \gamma_\mu b | B(p_B) \rangle = f_+(q^2) \left\{ (p_B + p)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} \, q_\mu \right\} + \frac{m_B^2 - m_\pi^2}{q^2} \, f_0(q^2) \, q_\mu; \tag{2}$$

in semileptonic decays the physical range in q^2 is $0 \leq q^2 \leq (m_B - m_\pi)^2$. The starting point

[2] For very large quark masses, though, the Feynman-mechanism is suppressed by Sudakov-logarithms, which are, however, not expected to be effective at the b-quark mass.

for the calculation of the form-factor f_+ in (2) is the correlation function

$$i\int d^4y e^{iqy}\langle\pi(p)|T[\bar{u}\gamma_\mu b](y)[m_b\bar{b}i\gamma_5 d](0)|0\rangle = \Pi_+ 2p_\mu + \dots, \tag{3}$$

where the dots stand for structures not relevant for the calculation of f_+. As mentioned before, for a certain configuration of virtualities, namely $m_b^2 - p_B^2 \geq O(\Lambda_{\rm QCD} m_b)$ and $m_b^2 - q^2 \geq O(\Lambda_{\rm QCD} m_b)$, the integral is dominated by light-like distances and accessible to an expansion around the light-cone:

$$\Pi_+(q^2, p_B^2) = \sum_n \int_0^1 du\, \phi^{(n)}(u;\mu_{\rm IR}) T_H^{(n)}(u; q^2, p_B^2; \mu_{\rm IR}). \tag{4}$$

As in (1), n labels the twist of operators and $\mu_{\rm IR}$ is the (infrared) factorization-scale. The restriction on q^2, $m_b^2 - q^2 \geq O(\Lambda_{\rm QCD} m_b)$, implies that f_+ is not accessible at all momentum-transfers; to be be specific, we restrict ourselves to $0 \leq q^2 \leq 14\,{\rm GeV}^2$. As Π_+ is independent of $\mu_{\rm IR}$, the above formula implies that the scale-dependence of $T_H^{(n)}$ must be canceled by that of the DAs $\phi^{(n)}$.

In (4) we have assumed that Π_+ can be described by collinear factorization, i.e. that the only relevant degrees of freedom are the longitudinal momentum fractions u carried by the partons in the π, and that transverse momenta can be integrated over. Hard infrared (collinear) divergencies occurring in $T_H^{(n)}$ should be absorbable into the DAs, as discussed in detail in Ref. [15]. Collinear factorization is trivial at tree-level, where the b-quark mass acts effectively as regulator, but can, in principle, be violated by radiative corrections, by so-called "soft" divergent terms, which yield divergencies upon integration over u. Such terms break for instance factorization in non-leading twist in the treatment of nonleptonic B-decays à la BBNS [9]; it is thus instructive to see what happens in the simpler case of the correlation function (3), where the convolution involves only one DA instead of up to three in $B \to \pi\pi$. To anticipate the result: we find that factorization also works at one-loop level for twist-3 contributions and that there are no soft divergencies.

There are two two-quark twist-3 DAs of the π, ϕ_p and ϕ_σ, which are defined as

$$\langle 0|\bar{u}(x)[x,-x]i\gamma_5 d(-x)|\pi^-(p)\rangle = \mu_\pi^2(\mu_{\rm IR}) \int_0^1 du\, e^{i\xi px}\, \phi_p(u,\mu_{\rm IR}), \tag{5}$$

$$\langle 0|\bar{u}(x)[x,-x]\sigma_{\alpha\beta}\gamma_5 d(-x)|\pi^-(p)\rangle = -\frac{i}{3}\,\mu_\pi^2(\mu_{\rm IR})(p_\alpha x_\beta - p_\beta x_\alpha) \int_0^1 du\, e^{i\xi px}\, \phi_\sigma(u,\mu_{\rm IR}) \tag{6}$$

with $\mu_\pi^2(\mu_{\rm IR}) = f_\pi m_\pi^2/(m_u + m_d)(\mu_{\rm IR})$ and $\xi = 2u - 1$; u is the longitudinal momentum-fraction of the total momentum of the π carried by the (d-)quark. The Wilson-line

$$[x,-x] = {\rm P}\exp[2ig\int_0^1 dt x_\mu A^\mu((2t-1)x)]$$

ensures gauge-invariance of the nonlocal matrix-elements. Exploiting conformal symmetry of massless QCD, which holds to leading-logarithmic accuracy, one can perform a partial-wave expansion of the DAs in terms of increasing conformal spin, which amounts to an expansion in Gegenbauer-polynomials and fixes the functional dependence on u; the amplitudes of the partial-waves are of nonperturbative origin and can be related to local

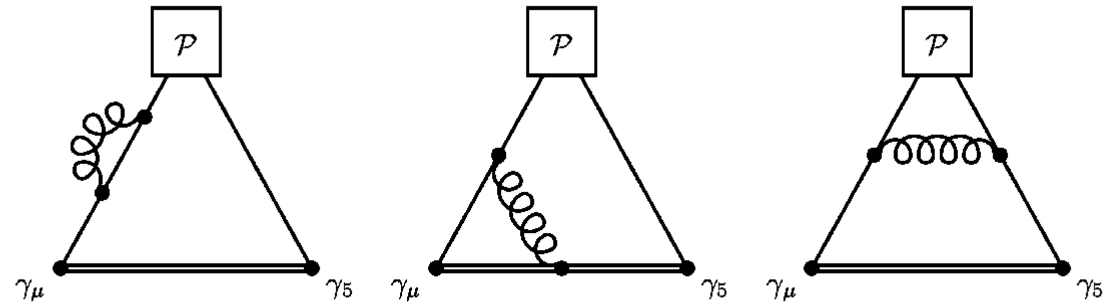

Figure 1: Some of the diagrams contributing to $T_H^{(p,\sigma)}$ in one-loop order. The double line denotes the b-quark propagator, the single lines denote the light u- and d-quark propagators. γ_μ and γ_5 are the weak and the B-vertex, respectively. There are two more self-energy and one more vertex-correction diagrams.

hadronic matrix-elements by virtue of the QCD equations of motion as discussed in detail in [16, 17]. It turns out, in particular, that the two DAs ϕ_σ and ϕ_p are not independent, but mix with each other and the twist-3 three-particle DA $\mathcal{T}$ that parametrizes the matrix-element $\langle 0|\bar{u}(x)[x,vx]\sigma_{\mu\nu}\gamma_5 gG_{\alpha\beta}(vx)[vx,-x]d(-x)|\pi^-(p)\rangle$. Thus, for consistency, when calculating radiative corrections to Π_+ to twist-3 accuracy, one has to include all three DAs,

$$\Pi_+^{(3)} \sim \phi_p \otimes T_H^{(p)} + \phi_\sigma \otimes T_H^{(\sigma)} + \mathcal{T} \otimes T_H^{(\mathcal{T})},$$

and it is only in the sum of these three terms that hard infrared divergencies and the scale-dependence are expected to cancel. The analysis of [16] has shown, however, that the two-quark DAs are very well approximated by the lowest partial-wave, i.e. the one with smallest conformal spin, and that mixing with $\mathcal{T}$ sets in only at higher conformal spin. In calculating radiative corrections to $\Pi_+^{(3)}$ we thus restrict ourselves to leading conformal spin, i.e. the so-called asymptotic DAs, and use [16]

$$\phi_\sigma(u) = 6u(1-u), \quad \phi_p(u) = 1, \quad \mathcal{T} = 0. \tag{7}$$

3. Some of the diagrams contributing to $T_H^{(p,\sigma)}$ to one-loop order are shown in Fig. 1. The light quarks are massless and have momenta up and $\bar{u}p \equiv (1-u)p$, respectively. They are projected onto the desired DA by closing the trace with an appropriate projection operator $\mathcal{P}$, which is just $-\mu_\pi^2 i\gamma_5/4$ for ϕ_p and involves a derivative in p for ϕ_σ. The calculation is performed in dimensional regularization for both ultraviolet and infrared divergencies. Carefully distinguishing between the two types of divergent terms, we find that the ultraviolet divergencies cancel upon renormalization of the bare b-quark mass in the tree-level expression, as they should. The infrared divergent terms, on the other hand, do not cancel between $\mu_\pi^{2,\text{bare}}\phi_{p(\sigma)}$ and $T_H^{(p(\sigma),\text{bare})}$ separately, but only in the sum of both contributions. The renormalized $T_H^{(p,\sigma)}(u)$ are regular at the endpoints, i.e. for $u \to 0$, $u \to 1$, which entails the absence of soft divergent terms.

As discussed below, the LCSR for f_+ involves the continuum-subtracted Borel-transform $\hat{B}_{\text{sub}}T_H$ of T_H. We calculate it by splitting T_H into two terms, $T_H = T_H^{\text{pole}} + T_H^{\text{dis}}$, where T_H^{dis} can be written as a dispersion-relation in p_B^2 and its Borel-transform is obtained by

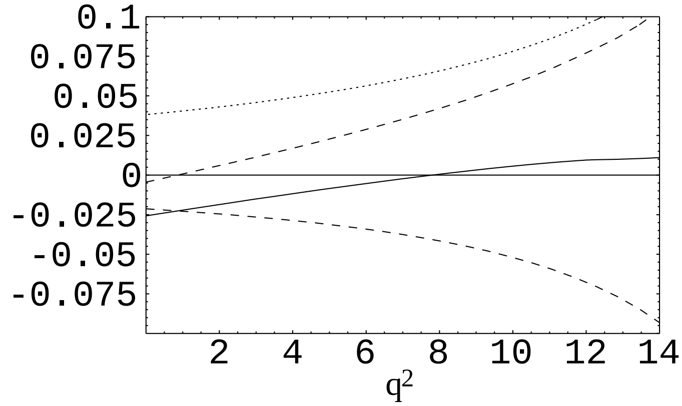

Figure 2: Radiative corrections to twist-2 and 3 contributions, in units of $\alpha_s/(3\pi)$, for representative input-parameters, as functions of q^2. Dashed lines: $T_H^{(p)} \otimes \phi_p$ and $T_H^{(\sigma)} \otimes \phi_\sigma$, respectively. Solid line: sum of dashed lines. Dotted line: $T_H^{(2)} \otimes \phi^{(2)}$. The total twist-3 correction (solid line) is much smaller than the twist-2 correction (dotted line).

applying Eq. (9). $T_H^{\rm pole}$ has a (single or double) pole in $s = m_b^2 - up_B^2 - \bar{u}q^2 \to 0$; the Borel-transforms of these terms are a bit more involved, master-formulae are given in the appendix. The final expressions for $\hat{B}_{\rm sub}T_H^{(p,\sigma)}$ are too bulky to be presented here. A compact version of the tree-level expression can be found in [13]. In Fig. 2 we compare the relative size of radiative corrections to twist-2 and 3 contributions. Whereas the absolute value of the two twist-3 corrections is, separately, of roughly the same size as that of the twist-2 correction, the only relevant quantity, the sum of both twist-3 corrections, is much smaller than the twist-2 contribution, which indicates a good convergence of the light-cone expansion also at $O(\alpha_s)$.

The result of this calculation is the light-cone expansion of Π_+, $\Pi_+^{\rm LC}$, and its continuum-subtracted Borel-transform, $\hat{B}_{\rm sub}\Pi_+^{\rm LC}$.

4. Let us now derive the LCSR for f_+. The correlation function Π_+, calculated for unphysical p_B^2, can be written as dispersion-relation over its physical cut. Singling out the contribution of the B-meson, one has

$$\Pi_+ = f_+(q^2)\,\frac{m_B^2 f_B}{m_B^2 - p_B^2} + \text{higher poles and cuts}, \tag{8}$$

where f_B is the leptonic decay constant of the B-meson, $f_B m_B^2 = m_b\langle B|\bar{b}i\gamma_5 d|0\rangle$. In the framework of LCSRs one does not use (8) as it stands, but performs a Borel-transformation,

$$\hat{B}\,\frac{1}{t - p_B^2} = \frac{1}{M^2}\exp(-t/M^2), \tag{9}$$

with the Borel-parameter M^2; this transformation enhances the ground-state B-meson contribution to the dispersion-representation of Π_+ and suppresses contributions of higher twist to the light-cone expansion of Π_+. The next step is to invoke quark-hadron duality to approximate the contributions of hadrons other than the ground-state B-meson by the imaginary part of the light-cone expansion of Π_+, so that

$$\hat{B}\Pi_+^{\rm LC} = \frac{1}{M^2}\, m_B^2 f_B\, f_+(q^2)\, e^{-m_B^2/M^2} + \frac{1}{M^2}\,\frac{1}{\pi}\int_{s_0}^\infty dt\, {\rm Im}\Pi_+^{\rm LC}(t)\, \exp(-t/M^2) \quad (10)$$

$$\text{and} \quad \hat{B}_{\rm sub}\Pi_+^{\rm LC} = \frac{1}{M^2}\, m_B^2 f_B\, f_+(q^2)\, e^{-m_B^2/M^2}. \quad (11)$$

Eq. (11) is the LCSR for f_+. s_0 is the so-called continuum threshold, which separates the ground-state from the continuum contribution. At tree-level, the continuum-subtraction in (11) introduces a lower limit of integration, $u \geq (m_b^2 - q^2)/(s_0 - q^2) \equiv u_0$, in (4), which behaves as $1 - \Lambda_{\rm QCD}/m_b$ for large m_b and thus corresponds to the dynamical configuration of the Feynman-mechanism, as it cuts off low momenta of the u-quark created at the weak vertex. At $O(\alpha_s)$, there are also contributions with no cut in the integration over u, which thus correspond to hard-gluon exchange contributions. Numerically, these terms turn out to be very small, $\sim O(1\%)$ of the total result for f_+. As with standard QCD sum rules, the use of quark-hadron duality above s_0 and the choice of s_0 itself introduce a certain model-dependence (or systematic error) in the final result for the form-factor, which is difficult to estimate. In this letter we opt for being rather conservative and add a 20% systematic error to the final result for f_+. Another hadronic parameter showing up in (11), which actually allows one to fix the value of s_0, is f_B. f_B can in principle be measured from the decay $B \to \ell\bar{\nu}_\ell$, which, due to the expected smallness of its branching ratio, BR$\sim O(10^{-6})$, has, up to now, escaped experimental detection. f_B is one of the best-studied observables in lattice-simulations with heavy quarks; the current world-average from unquenched calculations with two dynamical quarks is $f_B = (200 \pm 30)\,$MeV [18]. It can also be calculated from QCD sum rules: the most recent determinations [19] include $O(\alpha_s^2)$ corrections and find $(206 \pm 20)\,$MeV and $(197 \pm 23)\,$MeV, respectively. For consistency, we do not use these results, but replace f_B in (11) by its QCD sum rule to $O(\alpha_s)$ accuracy, including dependence on s_0 and M^2. For the b-quark mass, we use an average over recent determinations of the $\overline{\rm MS}$ mass, $\overline{m}_{b,\overline{\rm MS}}(\overline{m}_b) = (4.22 \pm 0.08)\,$GeV [20, 21], which corresponds to the one-loop pole-mass $m_{b,\rm 1L-pole} = (4.60 \pm 0.09)\,$GeV; this is different from the value $m_b = 4.8\,$GeV we used in our previous paper, the first reference in [13]. With these values we find $f_B = (192 \pm 22)\,$GeV (the error only includes variation with m_b and M^2, at optimized s_0), in very good agreement with both lattice and QCD sum rules to $O(\alpha_s^2)$ accuracy. For $m_b = (4.51, 4.60, 4.69)\,$GeV the optimized s_0 are $(34.5, 34.0, 33.5)\,$GeV^2, and the relevant range in M^2 is $M^2 \approx$(4.5–8) GeV^2.

The infrared factorization-scale is set to $\mu_{\rm IR}^2 = m_B^2 - m_b^2$ [22]; the dependence of f_+ on $\mu_{\rm IR}$ is very small, as all numerically sizeable contributions are now available in next-to-leading order in QCD, which ensures good cancellation of the scale-dependence. For the π-DAs we use for the most part the same expressions as in the first reference of [13], except for the new $O(\alpha_s)$ corrections to $T_H^{(p,\sigma)}$, where we use the DAs given in (7), and for the twist-2 DAs, where new analyses of the available experimental data on the $\gamma^*\gamma\pi$ and the π electromagnetic form-factor indicate that $\phi^{(2)}$ is closer to its asymptotic form than assumed previously and well approximated by [23]

$$\phi^{(2)}(u, \mu_{\rm IR}) = 6u(1-u)\left\{1 + a_2(\mu_{\rm IR})\left(\frac{15}{2}\,(2u-1)^2 - \frac{3}{2}\right)\right\} \quad (12)$$

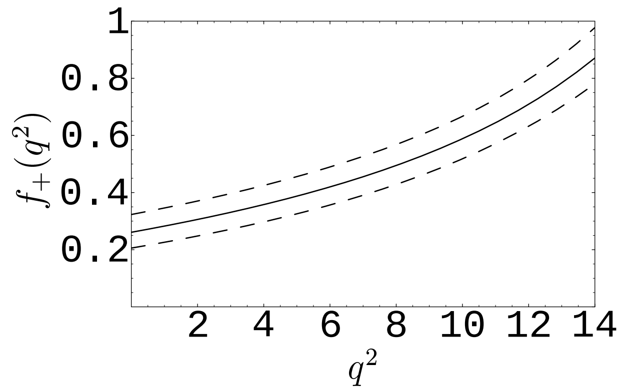

Figure 3: $f_+(q^2)$ as function of momentum-transfer q^2 from the LCSR (11). Solid line: LCSR for central values of input-parameters and $M^2 = 6\,\mathrm{GeV}^2$. Dashed lines: dependence of f_+ on variation of input-parameters as specified in the text and $5\,\mathrm{GeV}^2 \leq M^2 \leq 8\,\mathrm{GeV}^2$.

with $a_2(1\,\mathrm{GeV}) = 0.1 \pm 0.1$. The two-particle twist-3 DAs ϕ_p and ϕ_σ are proportional to $\mu_\pi^2 = m_\pi^2 f_\pi/(m_u + m_d)$, which in the chiral limit equals $-2\langle\bar{q}q\rangle/f_\pi$. Using the "standard value" of the quark-condensate, $\langle\bar{q}q\rangle(1\,\mathrm{GeV}) = (-0.24\,\mathrm{GeV})^3$, one has $\mu_\pi^2(\mu_{\mathrm{IR}}^2) = 0.25\,\mathrm{GeV}^2$. With the central value for the flavour-averaged light-quark mass, $\overline{m}_{ud}(2\,\mathrm{GeV}) = 4.5\,\mathrm{MeV}$, from lattice [21], one has $\mu_\pi^2(\mu_{\mathrm{IR}}^2) = 0.30\,\mathrm{GeV}^2$, with a quoted uncertainty of 20%. In our analysis we use the average $\mu_\pi^2(\mu_{\mathrm{IR}}^2) = (0.27 \pm 0.07)\,\mathrm{GeV}^2$.

With these parameters, we obtain the results shown in Fig. 3. The form-factor can be accurately fitted by

$$f_+(q^2) = \frac{f_+(0)}{1 - a\,(q^2/m_B^2) + b\,(q^2/m_B^2)^2}\,, \tag{13}$$

with $f_+(0)$, a and b given in Tab. 1, for different values of m_b, s_0 and M^2. The above parametrization reproduces the actual values calculated from the LCSR, for $q^2 \leq 14\,\mathrm{GeV}^2$, to within 2% accuracy. At $q^2 = 0$ we find $f_+(0) = 0.26 \pm 0.06$, when all input-parameters are varied within the ranges specified above. This has to be compared to 0.31 ± 0.05 from our previous analysis [13]. At fixed $M^2 = 6\,\mathrm{GeV}^2$, and the central value for m_b, $m_b = 4.6\,\mathrm{GeV}$, we obtain, in this letter, $f_+(0) = 0.261$. Without the new radiative corrections, the result becomes 0.294. Using in addition the parametrization of $\phi^{(2)}$ employed in [13], this becomes 0.321. And switching to $m_b = 4.8\,\mathrm{GeV}$ and $s_0 = 33.5\,\mathrm{GeV}^2$, one obtains 0.308, i.e. the central value for $f_+(0)$ quoted in [13]. The tree-level result for the central values of input-parameters is 0.247, i.e. the total effect of radiative corrections is below 10%.

It is instructive to compare the parametrization (13), obtained at not too large q^2, $q^2 \leq 14\,\mathrm{GeV}^2$, with the vector-meson pole-dominance approximation valid at large $q^2 \approx (m_B - m_\pi)^2 = 26.4\,\mathrm{GeV}^2$: here f_+ is dominated by the B*-pole, located at $q^2 = m_{B^*}^2 = 28.4\,\mathrm{GeV}^2$, and can be expressed as

$$f_+(q^2) = \frac{c}{1 - \frac{q^2}{m_{B^*}^2}}\,. \tag{14}$$

The residue of the pole, c, can be related to physical couplings as $c = f_{B^*} g_{BB^*\pi}/(2m_{B^*})$, where f_{B^*} is the leptonic decay constant of the B* and $g_{BB^*\pi}$ is the coupling of the B* to

m_b[GeV]	4.69	4.60	4.51
s_0[GeV^2]	33.5	34.0	34.5
$a_2(1\,\mathrm{GeV})$	0	0.1	0.2
$\mu_\pi^2(\mu_{\mathrm{IR}}^2)$[GeV^2]	0.34	0.27	0.20
M^2[GeV^2]	8	6	5
$f_+(0)$	0.206	0.261	0.323
a	2.34	2.03	1.76
b	1.77	1.29	0.87
q_0^2[GeV^2]	14.3	15.7	18.5
c_{fit}	0.384	0.439	0.523
$c_{\mathrm{LCSR}} = \dfrac{f_{B^*}g_{BB^*\pi}}{2m_{B^*}}$	0.396	0.414	0.430

Table 1: Fit-parameters of Eqs. (13) and (14), including all variations of input-parameters and M^2.

the Bπ-pair. c can be calculated from LCSRs itself: it is known up to twist-4 at tree-level; $O(\alpha_s)$ radiative corrections are known for the twist-2 contribution [22, 24].

We can now try to match the parametrizations (13) and (14). To this end, we treat c as fit-parameter and require that the transition between both parametrizations be smooth, i.e. that at q_0^2 to be fitted, the values of both $f_+(q_0^2)$ and its derivative are equal for both parametrizations. The resulting values of q_0^2 and c_{fit} are tabulated in Tab. 1, and the corresponding form-factors, obtained from plotting (13) for $q^2 \leq q_0^2$ and (14) for $q^2 \geq q_0^2$, are shown in Fig. 4. The last row in Tab. 1 gives the values of c_{LCSR} obtained directly from the LCSR calculated in [22, 24]. The agreement between the direct and the fitted values is remarkably good, in view of the fact that the LCSR for c is less accurate than ours for f_+, as it does not include $O(\alpha_s)$ corrections at twist-3. Also the values of q_0^2 are well within expectation: sufficiently below the pole on the one hand, but not too small on the other hand. Motivated by these results, we suggest a new parametrization of f_+ in terms of 5 parameters: Eq. (13) for $q^2 \leq q_0^2$ and Eq. (14) for $q^2 \geq q_0^2$, with the set of parameters given in Tab. 1 which comprises the full dependence of $f_+(q^2)$ on input-parameters and the Borel-parameter.

5. In this letter we have investigated the light-cone sum rule for the form-factor $f_+^{B\to\pi}$, including the calculation of $O(\alpha_s)$ radiative corrections to the next-to-leading twist-3 contribution. The calculation has demonstrated the validity of the factorization formula (4) and the absence of soft divergent terms to the considered accuracy, i.e. twist-3 to $O(\alpha_s)$ with the DAs in the approximation of leading conformal spin, Eq. (7). As already found in [14], the Feynman-mechanism is the dominant contribution to f_+ and hard perturbative corrections are numerically small.

In view of the systematic uncertainties inherent in LCSRs, further refinement of the LCSR for $f_+^{B\to\pi}$ by including even higher twist contributions or more perturbative corrections is not likely to increase the overall accuracy. Improvement of the central value of the

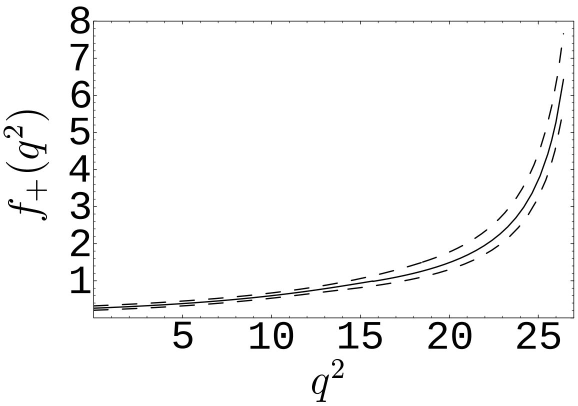

Figure 4: $f_+(q^2)$ as function of q^2 in the entire physical range in $B \to \pi e\nu$, from Eqs. (13) and (14), including all variations of input-parameters and M^2. Solid line obtained from 3rd column in Tab. 1, lower dashed curve from 2nd column and upper dashed curve from 4th column.

result may, however, come from a reduced uncertainty of input-parameters: whereas the dependence of $f_+(q^2)$ on m_b is rather small, a reduction of uncertainty in the non-leading conformal spin contributions to $\phi^{(2)}$ would be useful. As for the approximation of leading conformal spin DAs in twist-3, Eq. (7), at tree-level it yields about 90% of the actual result, which strengthens confidence that this approximation works similarly well also at $O(\alpha_s)$. The change in central values of $f_+(q^2)$ as compared to our previous results, Refs. [13, 14], is partially due to the $O(\alpha_s)$ corrections to twist-3 we have calculated in this letter and partially due to updated input-parameters.

The LCSR is valid only for large energies of the π, i.e. not too large values of q^2. In this letter we have fixed, somewhat arbitrarily, the maximum allowed value of q^2 at $q^2_{\rm max}$ at $14\,\mathrm{GeV}^2$ and have parametrized the form-factor by Eq. (13). On the other hand, for q^2 close to the kinematical maximum allowed in semileptonic decays, $q^2 = 26.4\,\mathrm{GeV}^2$, the form-factor is dominated by the close-by B*-pole and can be parametrized by Eq. (14). The residue of that pole can also be calculated from LCSRs. We have tried to match both parametrizations, Eqs. (13) and (14), by requiring smoothness at the matching-point $q^2 = q_0^2$, which is a parameter of the fit itself. The resulting values of q_0^2 are within expectations, and the fitted values of the residue agree well with the direct calculation from LCSRs, as demonstrated in Tab. 1. This result lends additional confidence to our final parametrization of the form-factor, i.e. the combination of Eqs. (13) and (14), with a total of 5 parameters, which is valid in the complete range of kinematically allowed q^2 in $B \to \pi e\nu$, $0 \leq q^2 \leq (m_B - m_\pi)^2 = 26.4\,\mathrm{GeV}^2$.

Appendix

In this appendix we collect formulas for non-standard Borel-transformations. Generally, the Borel-transform $\hat{B}f(P^2)$ of a function $f(P^2)$ of the Euclidean momentum P is defined as

$$\hat{B}f(P^2) = \lim_{\substack{P^2\to\infty,\, N\to\infty \\ P^2/N=M^2 \text{ fixed}}} \frac{1}{N!}\,(-P^2)^{N+1}\,\frac{d^{N+1}}{(dP^2)^{N+1}}\,f(P^2).$$

By $\hat{B}_{\rm sub}f(P^2)$ we denote the Borel-transform including continuum-subtraction above the threshold s_0, i.e. if f has the dispersion-representation (here p is Minkowskian)

$$f(p^2) = \int\limits_{m^2}^{\infty} dt\,\frac{\rho(t)}{t-p^2},$$

we define

$$\hat{B}_{\rm sub}f(p^2) = \frac{1}{M^2}\int\limits_{m^2}^{s_0} dt\,\rho(t)\,e^{-t/M^2}. \tag{A.1}$$

We need in particular the following transforms ($s = m^2 - up^2 - \bar{u}q^2$):

$$\hat{B}\,\frac{1}{(t-p^2)^\alpha\,s^\beta} = \frac{1}{\Gamma(\alpha+\beta)}\,\frac{1}{u^\beta(M^2)^{\alpha+\beta}}\,e^{-t/M^2}\,{}_1F_1\left(\beta,\alpha+\beta,-\frac{m^2-ut-\bar{u}q^2}{uM^2}\right),$$

from which the Borel-transforms of expressions with additional logarithms are obtained as derivatives, e.g. $\hat{B}s^{-\beta}\ln s = -\frac{d}{d\beta}\,\hat{B}\,s^{-\beta}$. Including continuum subtraction, we find

$$\hat{B}_{\rm sub}\,\frac{1}{s^\beta} = \frac{e^{-\frac{m^2-\bar{u}q^2}{uM^2}}}{(uM^2)^\beta}\,\frac{1}{\Gamma(\beta)}\left(1-\frac{\Gamma(1-\beta,\frac{us_0+\bar{u}q^2-m^2}{uM^2})}{\Gamma(1-\beta)}\right)\Theta(u-u_0)$$

with $u_0 = (m^2-q^2)/(s_0-q^2)$. For integer β, the second terms becomes a sum over $\delta(u-u_0)$ and its derivatives. We also give the spectral function ρ of the general expression $(m^2-p^2)^{-\alpha}s^{-\beta}$, from which the Borel-transform $\hat{B}_{\rm sub}$ including continuum subtraction can be obtained using (A.1):

$$\begin{aligned}\rho(t) \,&=\, \Theta(\tilde{m}^2-t)\Theta(t-m^2)\,\frac{\sin\alpha\pi}{\pi}\,\frac{1}{(t-m^2)^\alpha\,u^\beta(\tilde{m}^2-t)^\beta} \\ &\quad + \Theta(t-\tilde{m}^2)\,\frac{\sin(\alpha+\beta)\pi}{\pi}\,\frac{1}{(t-m^2)^\alpha\,u^\beta(t-\tilde{m}^2)^\beta}\end{aligned}$$

with $\tilde{m}^2 = (m^2-\bar{u}q^2)/u$.

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