# Order $1 / N^{3}$ corrections to the conformal anomaly of the $(2,0)$ theory in six dimensions 

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#### Abstract

Using Supergravity on $A d S_{7} \times S^{4}$ we calculate the bulk one-loop contribution to the conformal anomaly of the $(2,0)$ theory describing $N$ coincident M5 branes. When this is added to the tree-level result, and an additional subleading order contribution calculated by Tseytlin, it gives an expression for the anomaly that interpolates correctly between the large $N$ theory and the free $(2,0)$ tensor theory corresponding to $N=1$. Thus we can argue that we have identified the exact $N$-dependence of the anomaly, which may have a simple protected form valid away from the large $N$ limit.


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## 1. Introduction

The low energy ( 2,0 ) theory describing $N$ coincident M5 branes is not yet well understood. However, some information about this theory can be obtained via the AdS/CFT correspondence. For example, the elegant calculation of Henningson and Skenderis [12] makes a prediction of the leading order $N$ dependence of the conformal anomaly of this theory.

One might hope that, as in the case of $\mathcal{N}=4 \mathrm{SYM}$, the anomaly has a protected form with simple dependence on $N$. Then provided one could calculate the appropriate sub-leading order corrections, one would have an exact result that is valid beyond the large- $N$ regime.

This is indeed what happens for the R-symmetry anomaly of the $(2,0)$ theory

$$
\begin{equation*}
J_{8}=N J_{8}^{\text {free }}+\left(N^{3}-N\right) p_{2} \tag{1}
\end{equation*}
$$

where $J_{8}^{\text {free }}$ is the anomaly of the free $(N=1)$ theory and $p_{2}$ is the normal bundle of the brane world-volume. As a function of $N$, this interpolates between the $N=1$ theory and the interacting large- $N$ one that can be described by 11 d Supergravity on $A d S_{7} \times S^{4}$.

[^0]The situation is, however, more complicated than the analogous one for $\mathcal{N}=4$ SYM, where a one-loop calculation of the conformal and R-symmetry anomalies gives $N^{2}-1$ copies of the free $N=1$ theory anomalies, and this is protected by supersymmetry from higher loop and stringy corrections. Take, for example, the conformal anomaly. It is given in general by a sum of type-A and type-B anomalies proportional to Euler and Weyl invariants respectively, and in the $\mathcal{N}=4$ SYM theory these are related respectively to three and two point correlators of the stress tensor. Thus known renormalisation theorems for these correlators apply.

In general the conformal anomaly of an even-dimensional theory (up to total derivative terms that are renormalisation scheme dependent) is a sum of type-A and type-B anomalies, where the former is proportional to the Euler density and the latter is a weighted sum of Weyl invariants made out of contractions of the Weyl tensor and its conformally covariant derivatives. In six dimensions there are three such Weyl invariants.

For the $(2,0)$ theory the leading order coefficient of the Euler invariant in the type-A anomaly is related to the four-point correlator of the stress tensor, while the coefficients of Weyl invariants in the type-B anomaly depend on the two and three point correlators. It has been shown [2] that the leading order dependence of the two and three point correlators is given by $4 N^{3}$ times the corresponding correlators of the free tensor multiplet. But there is no reason to expect the four-point correlator to exhibit the same ratio. Indeed, if we look at the leading order result [12] for the $(2,0)$ conformal anomaly, we discover that the type-B anomaly is given by $4 N^{3}$ times that for the free theory, while the type-A anomalies have a different ratio, $16 N^{3} / 7$ [1].

In [9-11] we checked the sub-leading order correction to the conformal anomaly of $\mathcal{N}=4$ SYM by a oneloop calculation in $A d S_{5} \times S^{5}$ supergravity. In this Letter we will perform a similar calculation on $A d S_{7} \times S^{4}$ in order to calculate sub-leading order corrections to the conformal anomaly of the large- $N(2,0)$ theory. An attempt to calculate such corrections by considering $R^{4}$ corrections to the supergravity action was made in [3], but our result gives corrections at a different order in $N$ since $R^{4}$ corrections give anomalies of $O(N)$, while one-loop supergravity anomalies are $O(1)$. The different order of these results is explained by the fact that the supergravity loop-counting parameter is $G_{\text {Newton }} \sim 1 / N^{3}$, whereas the string loop-counting parameter is $g_{s}^{2} \sim 1 / N^{2}$.

Summing over contributions from all the Kaluza-Klein towers of supergravity fields gives a contribution to the anomaly which, when properly regularised, is equal to twice the contribution from a free tensor multiplet. Remarkably, the fields that contribute to the regularised sum exactly match the field content of the tensor multiplet; this is similar to what we observed in the $d=4$ case, where the regularised contributions from Kaluza-Klein fields in supergravity correspond to a sum of contributions that exactly match the field content of the $\mathcal{N}=4$ SYM theory [11].

In [3] an $O(N)$ contribution to the type-B anomaly was calculated from $R^{4}$ terms in the string theory effective action, but this contribution was conjectured to be incomplete. (Similar calculations of subleading order anomalies from $R^{4}$ terms were performed for the $\mathcal{N}=4$ SYM case in [5,6].) The contribution calculated in [3] can be seen to be related by supersymmetry to an $O(N)$ term in the chiral anomaly, but we would expect there to be other subleading order corrections [4]. However, any additional corrections due to stringy effects will not contribute at the same order as the supergravity contribution.

If we add our $O(1)$ contribution to the type-B anomaly, we get a result that interpolates correctly between the large- $N$ and $N=1$ cases. Thus our result may give the exact $N$ dependence of the type-B anomaly. The $O(N)$ contribution to the type-A anomaly was not calculated in [3], but our results lead us to a new conjecture for the exact form of the type-A anomaly.

## 2. Leading order anomaly from AdS/CFT

The leading order result of [12] for the conformal anomaly of the large- $N(2,0)$ theory can be written as

$$
\begin{equation*}
\mathcal{A}=-\frac{4 N^{3}}{(4 \pi)^{3} \cdot 288}\left[E_{6}+8\left(12 I_{1}+3 I_{2}-I_{3}\right)+O\left(\nabla_{i} J^{i}\right)\right] \tag{2}
\end{equation*}
$$

where in terms of the 17 invariants

$$
\begin{align*}
\left(A_{1}, A_{2}, \ldots, A_{17}\right)=( & \nabla^{4} R,\left(\nabla_{i} R\right)^{2},\left(\nabla_{i} R_{j k}\right)^{2}, \nabla_{i} R_{j k} \nabla^{j} R^{i k},\left(\nabla_{i} R_{j k l m}\right)^{2}, R \nabla^{2} R, R_{i j} \nabla^{2} R^{i j} \\
& R_{i j} \nabla_{k} \nabla^{j} R^{i k}, R_{i j k l} \nabla^{2} R^{i j k l}, R^{3}, R R_{i j}^{2}, R R_{i j k l}^{2}, R_{i}^{j} R_{j}^{k} R_{k}^{i}, R_{i j} R_{k l} R^{i k j l}, \\
& \left.R_{i j} R^{i k l m} R^{j}{ }_{k l m}, R_{i j}^{k l} R_{k l}^{m n} R_{m n}^{i j}, R_{j k j l} R^{i m j n} R_{m}^{k}{ }_{n}^{l}\right) \tag{3}
\end{align*}
$$

we have

$$
\begin{equation*}
E_{6}=-8 A_{10}+96 A_{11}-24 A_{12}-128 A_{13}-192 A_{14}+192 A_{15}-32 A_{16}+64 A_{17} \tag{4}
\end{equation*}
$$

and

$$
\begin{align*}
& I_{1}=\frac{19}{800} A_{10}-\frac{57}{160} A_{11}+\frac{3}{40} A_{12}+\frac{7}{16} A_{13}+\frac{9}{8} A_{14}-\frac{3}{4} A_{15}-A_{17}  \tag{5}\\
& I_{2}=\frac{9}{800} A_{10}-\frac{27}{40} A_{11}+\frac{3}{10} A_{12}+\frac{5}{4} A_{13}+\frac{3}{2} A_{14}-3 A_{15}+A_{16}  \tag{6}\\
& I_{3}=-\frac{11}{50} A_{10}+\frac{27}{10} A_{11}-\frac{6}{5} A_{12}-3 A_{13}-4 A_{14}+4 A_{15}+\frac{1}{10} A_{6}-A_{7}+A_{9}+\nabla_{i} \mathcal{J}^{i} \tag{7}
\end{align*}
$$

## 3. Seeley-DeWitt coefficients

The conformal anomaly contributed by a conformal field in six-dimensions is proportional to the Seeley-DeWitt coefficient $b_{6}$ of the associated kinetic operator. The general expression for $b_{6}$ for a six-dimensional operator of the form $-\nabla^{2}-E$ was given in [7] and can be written in the form

$$
\begin{align*}
b_{6}=\frac{1}{(4 \pi)^{3} 7!} \operatorname{tr}[ & 18 A_{1}+17 A_{2}-2 A_{3}-4 A_{4}+9 A_{5}+28 A_{6}-8 A_{7}+24 A_{8}+12 A_{9} \\
& +\frac{35}{9} A_{10}-\frac{14}{3} A_{11}+\frac{14}{3} A_{12}-\frac{208}{9} A_{13}+\frac{64}{3} A_{14}-\frac{16}{3} A_{15}+\frac{44}{9} A_{16}+\frac{80}{9} A_{17} \\
& +14\left(8 V_{1}+2 V_{2}+12 V_{3}-12 V_{4}+6 V_{5}-4 V_{6}+5 V_{7}+6 V_{8}+60 V_{9}+30 V_{10}+60 V_{11}\right. \\
& \left.\left.+30 V_{12}+10 V_{13}+4 V_{14}+12 V_{15}+30 V_{16}+12 V_{17}+5 V_{18}-2 V_{19}+2 V_{20}\right)\right] \tag{8}
\end{align*}
$$

where the invariants $V_{a}$, depending on the connection curvature $F_{i j}$ and the endomorphism $E$, are given by

$$
\begin{align*}
\left(V_{1}, V_{2}, \ldots, V_{20}\right)=( & \nabla_{i} F_{j k} \nabla^{i} F^{j k}, \nabla^{i} F_{j i} \nabla_{k} F^{j k}, F_{i j} \nabla^{2} F^{i j}, F_{i j} F^{j k} F_{k}^{i}, R_{i j k l} F^{i j} F^{k l}, \\
& R_{i j} F^{i k} F^{j}{ }_{k}, R F_{i j} F^{i j}, \nabla^{4} E, E \nabla^{2} E, \nabla_{k} E \nabla^{k} E, E^{3}, E F_{i j}^{2}, R \nabla^{2} E, \\
& \left.R_{i j} \nabla^{i} \nabla^{j} E, \nabla_{i} R \nabla^{i} E, E E R, E \nabla^{2} R, E R^{2}, E R_{i j}^{2}, E R_{i j k l}^{2}\right) \tag{9}
\end{align*}
$$

For a conformally-invariant operator, $b_{6}$ has the general form

$$
\begin{equation*}
b_{6}=a E_{6}+b_{1} I_{1}+b_{2} I_{2}+b_{3} I_{3}+\nabla_{i} J^{i} \tag{10}
\end{equation*}
$$

The $b_{6}$ coefficients for the fields appearing in the free $(2,0)$ tensor multiplet were calculated in [1]. If we ignore the total derivative terms, and denote the $b_{6}$ coefficients for a conformal scalar, Dirac fermion, and gauge 2-form as $s, f$ and $g_{a_{2}}$, respectively, then we have

$$
\begin{align*}
& s=\frac{1}{(4 \pi)^{3} 7!}\left(-\frac{5}{72} E_{6}-\frac{28}{3} I_{1}+\frac{5}{3} I_{2}+2 I_{3}\right)  \tag{11}\\
& f=\frac{1}{(4 \pi)^{3} 7!}\left(-\frac{191}{72} E_{6}-\frac{896}{3} I_{1}-32 I_{2}+40 I_{3}\right) \tag{12}
\end{align*}
$$

$$
\begin{equation*}
g_{a_{2}}=\frac{1}{(4 \pi)^{3} 7!}\left(-\frac{221}{4} E_{6}-\frac{8008}{3} I_{1}-\frac{2378}{3} I_{2}+180 I_{3}\right) . \tag{13}
\end{equation*}
$$

A free $(2,0)$ tensor multiplet consists of 5 scalars, 2 Weyl fermions and a chiral 2 -form gauge field, and its total conformal anomaly is thus given by

$$
\begin{equation*}
5 s+f+\frac{g_{a_{2}}}{2}=-\frac{1}{(4 \pi)^{3} \cdot 288}\left(\frac{7}{4} E_{6}+8\left(12 I_{1}-4 I_{2}+I_{3}\right)\right) . \tag{14}
\end{equation*}
$$

## 4. One-loop conformal anomalies from AdS/CFT

The one-loop contribution to the conformal anomaly from bulk supergravity fields was found in [8] using Schrödinger functional methods. These are particularly appropriate to the study of the AdS/CFT correspondence, because being Hamiltonian, they allow us to study bulk fields via sources that live near the AdS boundary. The result of [8] can be expressed (for six-dimensional boundaries) as

$$
\begin{equation*}
\delta \mathcal{A}=-\sum \frac{(\Delta-3)}{2} b_{6}, \tag{15}
\end{equation*}
$$

where the sum is taken over all the fields in 11d Supergravity compactified on $A d S_{7} \times S^{4}$, and $\Delta$ is the scaling dimension of the corresponding boundary operator.

To find the coefficient $b_{6}$ appropriate to each bulk field, it is necessary to decompose the seven-dimensional bulk fields into ones appropriate to the six-dimensional boundary. There are some interesting features of this decomposition.

If the boundary is assumed Ricci-flat, then the bulk AdS metric (satisfying the Einstein equations with cosmological constant $-15 / l^{2}$ ) can be written as

$$
\begin{equation*}
d s^{2}=\frac{1}{t^{2}}\left(l^{2} d t^{2}+\sum_{i, j} \hat{g}_{i j} d x^{i} d x^{j}\right), \quad t>0, \tag{16}
\end{equation*}
$$

where $\hat{g}_{i j}$ is proportional to the boundary metric. In this metric, the decomposition into boundary fields exhibits cancellations that ensure that each massive seven-dimensional bulk field contributes to the anomaly via the SeeleyDeWitt coefficient corresponding to an irreducible six-dimensional operator with the same spin. So, for example, the contribution of the massive seven-dimensional vector field is proportional to the $b_{6}$ coefficient for the sixdimensional (gauge-fixed) Maxwell operator. Where there are gauge invariances, there are additional contributions associated with Faddeev-Popov ghosts.

If the boundary is not Ricci-flat, the metric that satisfies Einstein's equations is obtained by multiplying $\hat{g}_{i j}$ in (16) by the factor $\left(1-\hat{R} t^{2} l^{2} / 120\right)^{2}$ where $\hat{R}$ is the Ricci scalar constructed from $\hat{g}_{i j}$. The effect of this on the decomposition into six-dimensional fields is to introduce couplings to $\hat{R}$ that render them conformally covariant. Thus, a seven-dimensional minimally coupled scalar contributes via the $b_{6}$ coefficient for a six-dimensional conformal scalar, and a seven-dimensional gauge field via the $b_{6}$ coefficient of a six-dimensional gauge field.

Now this necessarily spoils some of the cancellations that we observed in the Ricci-flat case. For example, by decomposing a seven-dimensional massive vector into transverse and longitudinal parts, we can show that the $b_{6}$ coefficient for it differs from that of the gauge field by a conformal scalar contribution. In the Ricci-flat case this cancelled the contribution from the Faddeev-Popov ghosts, but since the latter are minimally coupled, the cancellation is now incomplete. However, this is exactly what is needed to make the overall sum of $b_{6}$ coefficients a sum of $b_{6}$ coefficients of conformal operators.

In Table 1 we display the values of $\Delta-3$ for the Kaluza-Klein spectrum. These are related to the bulk masses, which were first given in [13]. The supermultiplets are labelled by an integer $p \geqslant 2$ and form representations

Table 1
The $(a, b)$ representation of $U S p(4)$ has dimension $(a+1)(b+1)(a+b+2)(2 b+a+3) / 6$

| Field | $S U(4)$ rep $^{\mathrm{n}}$ | $U S p(4)$ rep $^{\mathrm{n}}$ | $\Delta-3$ |  |
| :--- | :---: | :---: | :---: | :---: |
| $\phi^{(1)}$ | $(0,0,0)$ | $(0, p)$ | $2 p-3$, | $p \geqslant 2$ |
| $\psi^{(1)}$ | $(1,0,0)$ | $(1, p-1)$ | $2 p-5 / 2, \quad p \geqslant 2$ |  |
| $A_{\mu \nu \rho}^{(1)}$ | $(2,0,0)$ | $(0, p-1)$ | $2 p-2$, | $p \geqslant 2$ |
| $A_{\mu}^{(1)}$ | $(0,1,0)$ | $(2, p-2)$ | $2 p-2$, | $p \geqslant 2$ |
| $\psi_{\mu}^{(1)}$ | $(1,1,0)$ | $(1, p-2)$ | $2 p-3 / 2$, | $p \geqslant 2$ |
| $h_{\mu \nu}$ | $(0,2,0)$ | $(0, p-2)$ | $2 p-1$, | $p \geqslant 2$ |
| $\psi^{(2)}$ | $(0,0,1)$ | $(3, p-3)$ | $2 p-3 / 2$, | $p \geqslant 3$ |
| $A_{\mu \nu}$ | $(1,0,1)$ | $(2, p-3)$ | $2 p-1$, | $p \geqslant 3$ |
| $\psi_{\mu}^{(2)}$ | $(0,1,1)$ | $(1, p-3)$ | $2 p-1 / 2$, | $p \geqslant 3$ |
| $A_{\mu \nu \rho}^{(2)}$ | $(0,0,2)$ | $(0, p-3)$ | $2 p$, | $p \geqslant 3$ |
| $\phi^{(2)}$ | $(0,0,0)$ | $(4, p-4)$ | $2 p-1$, | $p \geqslant 4$ |
| $\psi^{(3)}$ | $(1,0,0)$ | $(3, p-4)$ | $2 p-1 / 2$, | $p \geqslant 4$ |
| $A_{\mu}^{(2)}$ | $(0,1,0)$ | $(2, p-4)$ | $2 p$, | $p \geqslant 4$ |
| $\psi^{(4)}$ | $(0,0,1)$ | $(0, p-4)$ | $2 p+1 / 2$, | $p \geqslant 4$ |
| $\phi^{(3)}$ | $(0,0,0)$ | $2 p+1$, | $p \geqslant 4$ |  |

of $U S p(4)$. The $p_{6}$ coefficients of the fields can be calculated using the formula (8), but we will not give them all explicitly, since the only ones we will need in the final result are the ones involved in the free $(2,0)$ tensor multiplet.

If we denote the values of $b_{6}$ for the fields $\phi, \psi, A_{\mu}, A_{\mu \nu}, A_{\mu \nu \rho}, \psi_{\mu}, h_{\mu \nu}$ by $s, f, v, a_{2}, a_{3}, r$, and $g$, respectively, then the contribution from a generic ( $p \geqslant 4$ ) multiplet is

$$
\begin{align*}
\left(\sum(\Delta-3) b_{6}\right)_{p \geqslant 4}= & \left(-13 s+2 v-a_{2}-4 f\right)+\left(65 s+54 f-14 a_{3}+6 v+2 r-g+21 a_{2}\right) \frac{p}{6} \\
& +\left(-37 s-6 f+22 a_{3}+18 v+14 r+5 g-9 a_{2}\right) \frac{p^{2}}{6} \\
& +\left(-28 s-48 f-8 a_{3}-24 v-16 r-4 g-12 a_{2}\right) \frac{p^{3}}{3} \\
& +\left(14 s+24 f+4 a_{3}+12 v+8 r+2 g+6 a_{2}\right) \frac{p^{4}}{3} \tag{17}
\end{align*}
$$

whilst for the $p=3$ multiplet it is

$$
\begin{equation*}
\left(\sum(\Delta-3) b_{6}\right)_{p=3}=90 s+230 f+140 v+94 r+2 g+50 a_{2}+62 a_{3} \tag{18}
\end{equation*}
$$

The $p=2$ multiplet contains gauge fields requiring the introduction of Faddeev-Popov ghosts. These are detailed in Table 2, and the total contribution of the $p=2$ multiplet is

$$
\begin{equation*}
\left(\sum(\Delta-3) b_{6}\right)_{p=2}=-16 s+10 f+16 v+10 r+3 g+10 a_{3} . \tag{19}
\end{equation*}
$$

Note that if we substitute the values of the $b_{6}$ coefficients, the contributions from each supermultiplet are nonzero, even in the Ricci-flat case (this is unlike the $d=4$ case). To deal with the sum over multiplets labelled by $p$, we will use the regularisation introduced in [11]. The divergent sum is evaluated by weighting the contribution of each supermultiplet by $z^{p}$. The sum can be performed for $|z|<1$ and we take the result to be a regularisation of the weighted sum for all values of $z$. Multiplying this by $1 /(z-1)$ and integrating around the pole at $z=1$ gives a regularisation of the original divergent sum.

Table 2
Decomposition of gauge fields for the massless multiplet

| Original field | Gauge-fixed fields | $\Delta-3$ |
| :---: | :---: | :---: |
| $A_{\mu}$ | $A_{i}$ | 2 |
|  | $A_{0}$ | 3 |
| $\psi_{\mu}$ | $b_{\mathrm{FP}}, c_{\mathrm{FP}}$ | 3 |
|  | $\psi_{i}^{\mathrm{irr}}$ | $5 / 2$ |
|  | $\gamma^{i} \psi_{i}$ | $7 / 2$ |
| $h_{\mu \nu}$ | $\psi_{0}$ | $7 / 2$ |
|  | $\lambda_{\mathrm{FP}}, \rho_{\mathrm{FP}}$ | $7 / 2$ |
| $\sigma_{\mathrm{GF}}$ | $7 / 2$ |  |
|  | $h_{i j}^{\mathrm{irr}}$ | 3 |
| $h_{0 i}$ | 4 |  |
|  | $h_{00}, h_{\mu}^{\mu}$ | $B_{0}^{\mathrm{FP}}, C_{0}^{\mathrm{FP}}$ |
| $B_{i}^{\mathrm{FP}}, C_{i}^{\mathrm{FP}}$ | $\sqrt{18}$ |  |

This yields

$$
\begin{equation*}
\sum(\Delta-3) b_{6}=26 s+4 f-4 v+a_{2} . \tag{20}
\end{equation*}
$$

As discussed earlier, the $b_{6}$ coefficients of massive fields depend on the decomposition from seven to six dimensions. For the massive vector, we have

$$
\begin{equation*}
2 v=2 v_{0}+3 s-3 s_{0}, \tag{21}
\end{equation*}
$$

where $v_{0}, s_{0}$ are the coefficients for the gauge-fixed six-dimensional Maxwell operator and minimally coupled scalar, respectively [14]. The $b_{6}$ coefficients for all other massive fields are what we would expect for the appropriate spin, for example, the contribution for the massive graviton corresponds to the heat-kernel coefficient for the transverse traceless part of a six-dimensional spin-2 operator, and the contribution for a two-index antisymmetric tensor is the heat-kernel coefficient for an irreducible six dimensional operator of the same spin.

The final expression for the one-loop contribution to the conformal anomaly is

$$
\begin{equation*}
\delta \mathcal{A}=-\sum(\Delta-3) \frac{b_{6}}{2}=-2\left(5 s+f+g_{a_{2}}\right) . \tag{22}
\end{equation*}
$$

## 5. Discussion

If we express the result (22) in terms of the Euler and Weyl invariants we get

$$
\begin{equation*}
\delta \mathcal{A}=\frac{1}{(4 \pi)^{3} \cdot 288}\left(\frac{7}{2} E_{6}+16\left(12 I_{1}+3 I_{2}-I_{3}\right)\right) \tag{23}
\end{equation*}
$$

which is to be added to the leading order result

$$
\begin{equation*}
\mathcal{A}=-\frac{4 N^{3}}{(4 \pi)^{3} \cdot 288}\left[E_{6}+8\left(12 I_{1}+3 I_{2}-I_{3}\right)+O\left(\nabla_{i} J^{i}\right)\right] \tag{24}
\end{equation*}
$$

In [3] an additional subleading order contribution to the anomaly was identified. Since the topology of the boundary was assumed to be trivial, implying the vanishing of the Euler density, only the contribution to the
coefficients of the Weyl invariants was determined. This is given by

$$
\begin{equation*}
\delta \mathcal{A}=\frac{N}{(4 \pi)^{3} \cdot 288}\left(8\left(12 I_{1}+3 I_{2}-I_{3}\right)\right) \tag{25}
\end{equation*}
$$

We can speculate that there is a similar contribution proportional to the Euler density with an undetermined coefficient $\alpha$ :

$$
\begin{equation*}
\delta \mathcal{A}=\frac{N}{(4 \pi)^{3} \cdot 288} \alpha E_{6} \tag{26}
\end{equation*}
$$

Adding all the contributions together gives

$$
\begin{equation*}
\mathcal{A}=-\frac{1}{(4 \pi)^{3} \cdot 288}\left[\left(4 N^{3}-\alpha N-\frac{7}{2}\right) E_{6}+\left(4 N^{3}-N-2\right) \cdot 8\left(12 I_{1}+3 I_{2}-I_{3}\right)+O\left(\nabla_{i} J^{i}\right)\right] . \tag{27}
\end{equation*}
$$

Putting $N=1$, we observe that the coefficient of the Weyl invariants coincides with the result (14) for the free $(2,0)$ tensor multiplet. If $\alpha=-5 / 4$, the coefficient of the Euler density would coincide as well. Thus we conjecture that there is an $O(N)$ contribution to the conformal anomaly corresponding to (26) with $\alpha=-5 / 4$, and that the exact $N$-dependence of the conformal anomaly is thus

$$
\begin{equation*}
\mathcal{A}=-\frac{1}{(4 \pi)^{3} \cdot 288}\left[\left(4 N^{3}+\frac{5}{4} N-\frac{7}{2}\right) E_{6}+\left(4 N^{3}-N-2\right) \cdot 8\left(12 I_{1}+3 I_{2}-I_{3}\right)+O\left(\nabla_{i} J^{i}\right)\right] \tag{28}
\end{equation*}
$$

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