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Flow resistance equations for gravel- and boulder-bed streams

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[1] Alternative general forms are considered for equations to predict mean velocity over the full range of relative submergence experienced in gravel- and boulder-bed streams. A partial unification is suggested for some previous semiempirical models and physical concepts. Two new equations are proposed: a nondimensional hydraulic geometry equation with different parameters for deep and shallow flows, and a variable-power resistance equation that is asymptotic to roughness-layer formulations for shallow flows and to the Manning-Strickler approximation of the logarithmic friction law for deep flows. Predictions by existing and new equations using D_{84} as roughness scale are compared to a compilation of measured velocities in natural streams at relative submergences from 0.1 to over 30. The variable-power equation performs as well as the best existing approach, which is a logarithmic law with roughness multiplier. For predicting how a known or assumed discharge is partitioned between depth and velocity, a nondimensional hydraulic geometry approach outperforms equations using relative submergence. Factor-of-two prediction errors occur with all approaches because of sensitivity to operational definitions of depth, velocity, and slope, the inadequacy of using a single grain-size length scale, and the complexity of flow physics in steep shallow streams.

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1. Introduction

[2] In a channel of known slope and shape, the partitioning of discharge between depth, velocity, and width depends on the balance between the downslope component of water weight and the frictional resistance of the bed. Quantifying flow resistance is therefore important in flood estimation, ecological habitat prediction, engineering design, geomorphological regime theory, sediment routing models, and other scientific and practical applications. What is usually needed is a way to predict resistance from readily observable and time-invariant channel properties. In large low-gradient rivers, this can be done tolerably well using sand bed form geometry or gravel grain size. But a recent review concluded that [*Wohl*, 2000, p. 82], "...there is not at present a well-tested, consistently accurate equation for calculating the resistance coefficients of mountain rivers." This reflects differences in the physical sources of resistance in flows with different relative submergence of the bed (d/k, where d is the mean flow depth and k is a representative bed roughness height). At high submergence (say $d/k \sim 10^2$), the main source of resistance is skin friction (form drag on individual particles and viscous friction on their surfaces), plus any large-scale form resistance of dunes, bars, or bends. At low submergence (say $d/k < 10$, and more particularly $d/k \sim 1$), which is characteristic not just of boulder torrents but also small gravel bed streams at low discharge, form drag associated with the turbulent wakes of large roughness elements becomes relatively greater, and there is also spill loss if flow is locally supercritical and wave drag on any elements protruding above the water surface. Several authors have proposed resistance equations specifically for shallow flows [e.g., *Bathurst*, 1978, 1985, 2002; *Jarrett*, 1984; *Rickenmann*, 1991; *Katul et al.*, 2002; *Smart et al.*, 2002], but with the exception of *Smart et al.* [2002], none was intended to work also for deeper flows, and if extrapolated to such conditions, the predicted resistance is usually far too low or high.

[3] This paper examines published empirical relations and conceptual models as a basis for suggesting generalized predictive equations for flow resistance or velocity over a wide range of conditions in streams with beds dominated by gravel, cobbles, or boulders. It does not address sand-bed rivers or the effects of submerged vegetation and large woody debris, but does cover step-pool channels. It is written in the same spirit as the attempts by *Lawrence* [1997] and *Katul et al.* [2002] to establish connections between roughness formulations across traditionally distinct boundary layer types. I propose two new approaches and compare them with existing approaches in terms of ability to predict measured velocities in natural streams.

2. Definition and Calculation of Flow Resistance

[4] Flow resistance is defined and quantified by coefficients in equations which relate cross-sectional average velocity (V) to mean flow depth (d) and gradient ($S = \sin\theta$) on the assumption that frictional retardation of flow is equal and opposite to the downslope component of water weight. The three classic equations are

$$V = C(dS)^{1/2} = (8gdS/f)^{1/2} = d^{2/3}S^{1/2}/n \qquad (1)$$

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where g is the gravitational acceleration and C, f, and n are the Chézy coefficient, Darcy-Weisbach friction factor, and Manning's n, respectively. These are interchangeable via equation (1), but f has two advantages. It is nondimensional and is physically interpretable as a drag coefficient if resistance is equated with gravitational driving force per unit bed area and assumed proportional to the square of velocity:

$$\tau_0 = \rho u_*^2 = \rho g d S = \rho V^2 f/8 \tag{2}$$

where u_* is the shear velocity. Equations (1) and (2) imply that

$$(8/f)^{1/2} = V/u_* \tag{3}$$

and resistance is quantified in this inverse way hereafter. *Bjerklie et al.* [2005] proposed $V \propto d^{2/3}S^{1/3}$ as an alternative to the Manning equation for slope-area discharge estimation and showed that it provided a good fit to a wide range of measurements; the calibrated constant in this equation is effectively a universal resistance coefficient. Other alternatives proposed specifically for steep shallow streams are discussed later.

[5] Accurate determination of f, C, or n at a site involves measuring V, d, and S as precisely as possible then substituting their values in equation (1). If flow is not uniform, S should be the energy slope, incorporating a term $\Delta V^2/2g$ to account for change in velocity head. For narrow channels with rough banks, the mean depth is usually replaced by the hydraulic radius $R = A/P$, where $A = wd$ is the cross-section area, w is the wetted width, and P is the wetted perimeter. Calculated resistance values apply to the discharge at the time and may alter at higher or lower stage, so for predictive purposes, f (or C or n) has to be estimated from measurable and invariant properties of the river channel. Equation (1) can then be used to estimate velocity and discharge or (with $Q = wdV$) to estimate water depth for specified discharge and slope and thus predict water level and mean bed shear stress.

3. Approaches to Estimating Flow Resistance

[6] In deep uniform gravel bed rivers without submerged vegetation, the dominant source of resistance is skin friction. Ideally this should be characterized by statistics of the bed microtopography since the same grain size distribution can offer greater or lesser resistance to flow depending on grain packing [e.g., *Gomez*, 1993]. *Smart et al.* [2002] and *Aberle and Smart* [2003] reported promising results using the standard deviation σ_z of bed elevation in a digital elevation model or dense long profile, but this is seldom measured. Normally all that is available is a grain size distribution, from which k can be equated with or scaled on a representative diameter D. I therefore focus on this approach but discuss later the limitations of D as a measure of roughness. D is variously defined as D_{50} (the median), D_{84} (the size such that 84% is finer), or D_{90}.

[7] There are two standard ways to estimate flow resistance from grain size. The first, attributed to Strickler, relates Manning's n to the 1/6 power of D. This implies

$$(8/f)^{1/2} = a_1(d/D)^{1/6} \tag{4}$$

The constant a_1 is generally quoted as 6.7 if D_{50} is used as roughness scale, or 8.2 using D_{84} or D_{90}, but *Parker* [1991] proposed $(8/f)^{1/2} = 8.1(d/2D_{84})^{1/6}$ which effectively reduces a_1 to 7.3.

[8] The other main approach, attributed to Keulegan, integrates the logarithmic law of the wall throughout the flow depth to obtain

$$(8/f)^{1/2} = (1/\kappa)\ln(d/ez_0) = (1/\kappa)\ln(11d/k) \tag{5}$$

where $\kappa \approx 0.4$ is the von Karman constant, e is the root of natural logarithms, z_0 is the zero-velocity height, and $k = 30z_0$ is the Nikuradse roughness height (originally equated with D_{50}). The constant 11 (=30/e) is for an infinitely wide flow and increases slightly for other cross-section shapes [*Hey*, 1979]; most authors use 12.2 which Keulegan fitted to data from a trapezoidal flume.

[9] For shallow flows, neither Manning-Strickler nor Keulegan works well in its original form, but several modifications and alternatives have been proposed. *Jarrett* [1984] suggested that n is estimated better from S and R than from D. His best fit equation for n is equivalent to predicting velocity as

$$V = 3.10R^{0.83}S^{0.12} \tag{6}$$

in SI units. High slope acts here as a surrogate for coarse bed material. Three other approaches retain D as a predictor: modified log laws, generalized power laws and nondimensional hydraulic geometry, and roughness-layer models based on a mixing-layer analogy or the concept of form-induced stress. I discuss these in turn, then generalize the nondimensional hydraulic geometry approach and suggest a new empirical approach using a variable-power equation.

3.1. Modified Logarithmic Laws

[10] Setting $k = D_{50}$ in the Keulegan equation gives a lower bound to resistance in gravel bed rivers and nearly always overestimates measured velocity [*Millar*, 1999]. The general trend of measurements is fitted better by setting k to some multiple of D_{84} to allow for small-scale form drag on protruding clasts. Suggested values of k/D_{84} range from 2.2 to 3.5 [*Thompson and Campbell*, 1979; *Bray*, 1979; *Hey*, 1979; *Bathurst*, 1985].

[11] Velocity profiles in shallow flows deviate considerably from logarithmic, often having an inflection near the tops of the roughness elements [*Wiberg and Smith*, 1991], but several authors have shown that logarithmic resistance equations still give adequate predictions. *Thompson and Campbell* [1979] proposed a modified Keulegan equation

$$(8/f)^{1/2} = 2.5(1 - 0.1k/R)\ln(12R/k) \tag{7}$$

with $k = 4.5D_{50}$ (=2.37D_{84} at their field site). The extra term allows for drag on large obstacles which partly block the flow; it becomes significant at $R/k < 1$. *Wiberg and Smith* [1991] calculated the form drag on a distribution of grain sizes protruding above a plane bed and showed that while this generated an inflected velocity profile, the bulk flow properties differed little from equation (5) with $k/D_{84} \approx 3$. *Smart et al.* [2002] and *Aberle and Smart* [2003] found that a log law with k scaled on σ_z gave a good fit to shallow-flow measurements in flume experiments with fixed or armored gravel.

[12] This strand of the literature shows that despite the known inadequacy of the law of the wall to describe velocity profiles in shallow flows, equations based on it work over a wide range of conditions so long as the roughness height is suitably inflated. Logarithmic equations are therefore the standard which others have to beat to become accepted as useful universal resistance laws.

3.2. Generalized Power Laws and Nondimensional Hydraulic Geometry

[13] The Manning-Strickler relation implies a 1/6 power relation between $(8/f)^{1/2}$ and d/k. Keulegan-type logarithmic equations are also closely approximated by a power law over any limited range of d/k, but with different exponents for different ranges: 1/6 for $10 < d/k < 100$, 1/2 for $1 < d/k < 10$, and 1 for $0.5 < d/k < 2$ [*Carson and Kirkby*, 1972, p. 222]. Several authors have proposed the generalized power law

$$(8/f)^{1/2} = a(d/k)^b \tag{8}$$

as a resistance equation, but not surprisingly the best fit exponent depends on the calibration data. *Griffiths* [1981] obtained 0.29 for gravel bed rivers, while *Bathurst* [2002] obtained 0.55 and 0.93 for shallow streams of gradient <0.8% and >0.8%. *Smart et al.* [2002] presented a model for head losses past roughness elements which implies $b = 0.5$ and showed that it fits rough-bed flume data well. Flow measurements in step-pool streams with emergent clasts are best fitted by $b \approx 1$ [*Lee and Ferguson*, 2002; *Comiti et al.*, 2007].

[14] A power law resistance relation is also implicit in the dimensionally consistent hydraulic geometry equation

$$V \propto q^{0.6}(gS)^{0.2}/k^{0.4} \tag{9}$$

(where q is the unit discharge Q/w) which *Rickenmann* [1991] and *Aberle and Smart* [2003] suggested for steep shallow flows on the basis of loose-bed flume experiments. Substituting $q = dV$ yields $V/u_* \propto d/k$, corresponding to $b = 1$ in equation (8). Rickenmann used $k = D_{90}$, whereas Aberle and Smart set $k = \sigma_z$ and found that equation (9) fitted their data even better than a log law. *Comiti et al.* [2007] propose the nondimensional hydraulic geometry equation

$$V_* = cq_*^m \tag{10}$$

for step-pool and cascade reaches. Here $V_* = V/(gD_{84})^{0.5}$, $q_* = q/(gD_{84}^3)^{0.5}$ which by definition is highly correlated with relative submergence, and the best fit to a compilation of step-pool measurements is with $m = 0.66$. This is a generalized form of equation (9) without the slope term. Comiti et al. explored the possible additional influence of slope but found no clear effect.

[15] Whereas equation (10) omits the slope term from equation (9), other authors have proposed resistance laws that are analogous to equation (9) without the roughness height term. *Jarrett*'s [1984] equation for n in terms of R and S implies $V \propto q^{0.45}S^{0.07}$, and *Bjerklie et al.*'s [2005] modified Manning equation is equivalent to $V \propto q^{0.4}S^{0.2}$.

3.3. The Roughness Layer

[16] A third strand in the recent literature considers the sources of resistance in shallow flows and what they imply for spatially averaged velocity profiles and bulk flow resistance. The vertical velocity profile over a rough bed is usually considered to consist of a thin laminar sublayer, a turbulent boundary layer with logarithmic profile, and an outer layer which deviates slightly from logarithmic. In shallow flows, the outer layer is absent and the near-bed profile deviates from logarithmic within a roughness layer that extends above the tops of the roughness elements. For d/k below $\sim$4, the roughness elements affect all levels in the flow, so there is no boundary layer in the conventional sense. *Lawrence* [1997, 2000] proposed a mixing-length model for flow resistance in overland flow over stony soil, and *Katul et al.* [2002] suggested that a shallow stream is analogous to the mixing layer within airflow through vegetation or overbank streamflow on vegetated floodplains. In these situations, the flow penetrates a porous array of roughness elements, with reduced mean velocity because of turbulent eddies generated in and below the shear layer at the top of the canopy or roughness elements. In a complementary approach, *Gimenez-Curto and Corniero Lera* [1996, 2006] and *Nikora et al.* [2001, 2004] noted that double (space as well as time) Reynolds averaging of the Navier-Stokes equations over an irregular bed introduces spatial covariance terms which increase bulk flow resistance. These terms represent form drag on roughness elements and form-induced stress due to vorticity generated by flow separation from roughness elements. *Gimenez-Curto and Corniero Lera* [1996] proposed that form-induced stress dominates the bulk resistance in such situations, in what they termed a "jet regime" as distinct from the rough turbulent regime with a boundary layer above an essentially planar bed.

[17] Although these authors started from different heuristic models, they mostly reached the same conclusion about bulk flow resistance in a roughness layer. For partial inundation ($d/k < 1$), *Lawrence* [1997] and *Nikora et al.* [2001] proposed that resistance consists mainly of form drag on roughness elements so that $f \propto d$ and V/u_* increases as the flow becomes even shallower. *Lawrence* [2000] subsequently found experimentally that f is near constant for $d < k$ and is much higher than predicted by form-drag models, perhaps because of additional resistance from free surface deformation and wake interference. *Nikora et al.* [2004] noted that several alternative velocity profiles can be justified heuristically. For d/k from 1 to $\sim$4, *Lawrence* [1997, 2000], *Gimenez-Curto and Corniero Lera* [1996, 2006], and *Nikora et al.* [2001] all assumed a mixing length that scales with k. This implies a linear resistance relation

$$(8/f)^{1/2} \propto d/k \tag{11}$$

The papers of *Rickenmann* [1991], *Lawrence* [1997], *Nikora et al.* [2001], *Aberle and Smart* [2003], and *Gimenez-Curto and Corniero* [2006] imply a proportionality constant of $\sim$1 to $\sim$4 in equation (11) depending on the assumptions made about obstacle shape, how k is defined in terms of measurable bed properties, and whether the height of the mixing layer is equated with k or some multiple of it.

[18] *Katul et al.* [2002] suggested that a mean velocity profile with an inflection at the top of a mixing layer can be represented by a hyperbolic tangent function. If the thickness of the mixing layer is equated with k, integration of the profile function gives the flow resistance as

$$(8/f)^{1/2} \propto \{1 + (k/d)\ln[0.65\cosh(d/k - 1)]\} \quad (12)$$

This approach assumes that the length scale of vorticity in the mixing layer is of the same order as the flow depth and obstacle height, but equation (12) was considered applicable throughout the range $0.2 < d/k < 7$. At $d/k \approx 1$, it is almost as steep as equation (11), but the resistance is asymptotically constant if the equation is extrapolated beyond its intended range to very low or very high d/k.

4. Synthesis and New Resistance Equations

4.1. Generalized Nondimensional Hydraulic Geometry

[19] The approaches of *Aberle and Smart* [2003] and *Comiti et al.* [2007] can be integrated with each other and with much previous literature on flow resistance by starting from the general power law equation (8) and setting $k = D$:

$$(8/f)^{0.5} = V/u_* = a(d/D)^b \quad (13)$$

Writing $d = q/V$ and rearranging yields the nondimensional hydraulic geometry relation

$$V_* = a^{1-m}S^{(1-m)/2}q_*^m \quad (14)$$

in which V_* and q_* are defined as in (10) and the exponents b and m are related by $m = (2b + 1)/(2b + 3)$ and $b = (3m - 1)/2(1 - m)$. This predicts velocity for known slope, grain size, and unit discharge; depth follows immediately and shear stress can then be estimated from depth $\times$ slope. This equivalence between power law resistance and nondimensional hydraulic geometry doubtlessly has been worked out by previous researchers, but I cannot find it in print. *Aberle and Smart* [2003] proposed an equation for V as a product of powers of g, S, q, and D and noted that for dimensional homogeneity the exponents of g and D are determined by that of q; equation (14) suggests the slope exponent is also constrained. The particular equations proposed by *Rickenmann* [1991] and *Aberle and Smart* [2003] involve $q^{0.6}$ which is consistent with $m = 0.6$ in equation (14) and $b = 1$ in the power law resistance equation (13); that is, they are equivalent to the roughness-layer resistance relation (11).

[20] Evidently several different heuristic and empirical analyses of shallow flows all converge on $(8/f)^{1/2} \propto d/D$, suggesting that this can be adopted as a default model for shallow flows without having to justify any particular interpretation of the dominant physical processes. This roughness-layer (RL) relation can be regarded as one end-member of a range of possible power law resistance functions for different conditions. The other end-member, for relatively deep rivers with lower slopes, is the Manning-Strickler (MS) friction law [equation (4), or $b = 1/6$ in equation (13)] which corresponds in equation (14) to $m = 0.4$ and a slope exponent of 0.3. Thus as attention moves from steep shallow streams to deeper rivers, the exponent of discharge in equation (14) decreases and that of slope increases.

[21] The neatest way to synthesize the MS and RL end-members is through a variable-power resistance equation, which I develop below, but a first approximation is simply to use MS for "deep" flows and RL for "shallow" flows, with "deep" and "shallow" defined by cutpoints of d/D or q_*. This MS/RL approach has two variants: predict f and V/u_* from d/D using equation (13) with $b = 1/6$ for deep flows but 1 for shallow flows, or predict V directly from q, S, and D using equation (14) with $m = 0.4$ for deep flows but 0.6 for shallow flows. It would be possible to use a threefold division with $b = 1/6$, 1/2, 1 or $m = 0.4$, 0.5, 0.6. Using a simple two-way cut, the prediction equations using relative submergence are

$$(8/f)^{1/2} = V/u_* = a_1(d/D)^{1/6} \text{ (deep flows)} \quad (15a)$$

$$= a_2 d/D \text{ (shallow flows)} \quad (15b)$$

where the previously cited literature suggests $a_1 \approx 7-8$ and $a_2 \approx 1-4$. In the second variant, velocity is predicted using

$$V_* = a_1^{0.6}q_*^{0.4}S^{0.3} \text{ (deep flows)} \quad (16a)$$

$$= a_2^{0.4}q_*^{0.6}S^{0.2} \text{ (shallow flows)} \quad (16b)$$

with V_*, q_*, a_1, and a_2 as previously defined. In testing this MS/RL approach below, I use $D = D_{84}$, R rather than d, and define "shallow" flows in equation (15) as $R/D_{84} < 4$ based on the previously cited literature. The equivalent cutpoint for equation (16) is taken as $q_* = 2$ based on the strong (since partly spurious) correlation between q_* and R/D_{84} in the data set used below.

4.2. A Variable-Power Equation

[22] The alternative synthesis is a function that is asymptotic to the MS and RL equations as d/D becomes very large or very small, respectively. This approach does not appear to have been considered previously. It treats the Darcy-Weisbach friction factor f as a sum of two components, as done when combining skin friction and bed form effects [e.g., *Yalin*, 1992, pp. 13–14]. The Manning-Strickler equation (4) implies

$$f/8 = (D/d)^{1/3}/a_1^2 \quad (17)$$

and the roughness-layer relation (11) implies

$$f/8 = (D/d)^2/a_2^2 \quad (18)$$

Both components are present in coarse-bedded streams but in varying proportions. Adding them gives

$$\begin{aligned} f/8 &= (D/d)^2/a_2^2 + (D/d)^{1/3}/a_1^2 \\ &= (D/d)^2[a_1^2 + a_2^2(d/D)^{5/3}]/a_1^2a_2^2 \end{aligned} \quad (19)$$

and therefore

$$(8/f)^{1/2} = a_1a_2(d/D)/[a_1^2 + a_2^2(d/D)^{5/3}]^{1/2} \quad (20)$$

Table 1. Field Data Sets Used to Evaluate Performance of Alternative Flow Resistance Equations[a]

Source	Field Area	Channel Morphology	Number of Sites	Number and Type of Measurements
Charlton et al. [1978]	UK	Not stated	23	23 (b, g)
Bathurst [1978]	UK	Run	3	9 (c)
Jarrett [1984]	US (CO)	Not stated	19	66 (g)
Bathurst [1985]	UK	Riffle or run	16	41 (c)
Thorne and Zevenbergen [1985]	US (CO)	Run	2	12 (g)
Hicks and Mason [1991]	New Zealand	Pool-riffle or run	14	94 (g)
Lee and Ferguson [2002]	UK	Step-pool	6	70 (s)
MacFarlane and Wohl [2003]	US (WA)	Step-pool	17	17 (s)
Comiti et al. [in press]	Italy	Mainly step-pool	10	44 (g, s)

[a]Notes on measurement methods: (b) data refer to bankfull discharge; (c) discharge by current metering; (g) discharge from gauging-station rating curve; (s) velocity from salt wave.

This plots as a smooth curve, asymptotic to a 1/6 power relation at $d/D \gg 1$ (when the a_1^2 term in the denominator becomes negligible) but to a linear relation at $d/D < 1$ (when the second term in the denominator becomes negligible; it is the smaller term even at $d/D = 1$ since $a_1 > a_2$). This variable-power equation (VPE hereafter) provides a single resistance equation applicable to both shallow and deep flows over coarse river beds. It quantifies the concept that the dominant sources of resistance alter as flow becomes shallower, and avoids forcing a logarithmic resistance law on circumstances in which velocity profiles are far from logarithmic. A possible physical rationale is discussed below.

5. Comparison With Field Data

[23] The potential value of the two-part MS/RL equation in its variants (15) and (16), and the variable-power equation (20), can be assessed by testing how well they and existing equations predict measured mean velocity. Velocity, rather than f or $(8/f)^{1/2}$, is chosen as the variable to be predicted because f itself is not normally of interest when resistance equations are used for scientific or practical applications.

5.1. Selection of Data and Range of Conditions

[24] Data on velocity and the variables needed to predict it were compiled from the sources listed in Table 1. Criteria for inclusion of a data set were that (1) it refers to near-straight reaches of natural streams with gravel/cobble or cobble/boulder beds and no bedrock or woody debris; (2) field methods are clearly described or could be ascertained by asking the authors and appear robust; (3) depth and velocity are averaged over several cross sections; (4) pebble-count estimates of D_{84} are listed; (5) hydraulic radius is either listed or calculable as $A/(w + d)$ which is a close approximation for trapezoidal sections with bank angles of $\sim 45°$; and (6) either energy slopes are listed, or it is clear that flow is sufficiently close to uniform that listed water surface slopes are a close approximation to energy slopes. The use of D_{84} as roughness height, and hydraulic radius rather than mean depth, reflects dominant practice and therefore data availability. The data include at-a-site changes in velocity as well as between-site comparisons. A versatile resistance formulation should be able to predict both, although *Bathurst* [1985, 2002] found a tendency for V/u_* to increase faster with R/D at sites than between sites.

[25] The first six data sets in Table 1 were obtained by surveying several cross sections in a straight reach to obtain mean width and depth, surveying slope over a substantial distance (usually >20 widths), measuring Q by current meter or from the rating curve for an adjacent gauging station, and subsequently deriving V and f. *Jarrett* [1984] and *Hicks and Mason* [1991] listed true energy slopes allowing for nonuniform flow, *Bathurst* [1978, 1985] stated that his listed water surface slopes are within 5% of the energy slope, and the other authors stated that their reaches were nearly uniform. The three most recent data sets are for small step-pool streams in which Q was measured by gauging structure or salt dilution, reach-average velocity was measured by salt-wave traveltime, S was surveyed over the same distance, and a reach-average depth was obtained as Q/wV or by averaging measurements at several sections. This procedure does not yield separate velocity measurements at the start and end of the reach from which to compute $\Delta V^2/g$, but this correction term cannot be significant in step-pool channels with high slope and low or moderate velocity. A few data points are omitted: bedrock reaches in *MacFarlane and Wohl*'s [2003] data set, measurements which Jarrett and *Bathurst* [1985] flagged as overbank flows through dense vegetation, two of Jarrett's sites for which D_{84} is not available, and those of Hicks and Mason's sites for which the quoted uncertainty in Q exceeds 10%.

[26] The complete data set (N = 376) spans slopes from 0.0007 to 0.21, D_{84} from 0.05 to 0.8 m, relative submergence R/D_{84} from 0.1 to 26 with one value of 87, and q^* from 0.002 to >100. The median Froude number in the entire data set is only 0.37, but this does not exclude the possibility of local hydraulic jumps, and five measurements in flood conditions have a reach-average Froude number in excess of 0.8.

5.2. Predictive Performance of Alternative Equations: Visual Assessment

[27] Figure 1 shows the correlation between $(8/f)^{1/2}$ and relative submergence for the data as a whole. Figure 1a is a log-log plot which differentiates more clearly in low-submergence, high-resistance conditions. The scatter in percentage terms is perceptibly wider at this end of the plot. Figure 1b is the traditional semilog plot on which the log law plots as a straight line. This hides the large relative, but small absolute, differences in predictions at very low submergence, and instead emphasizes differences at higher submergence. The q^* version of the MS/RL method cannot be shown in these plots, and nor can the equations of *Jarrett* [1984] and *Bjerklie et al.* [2005], but curves for almost all other equations are displayed in Figure 1a and/or 1b to allow comparison between equations and with the data. The equations of *Rickenmann* [1991], *Smart et al.* [2002], and

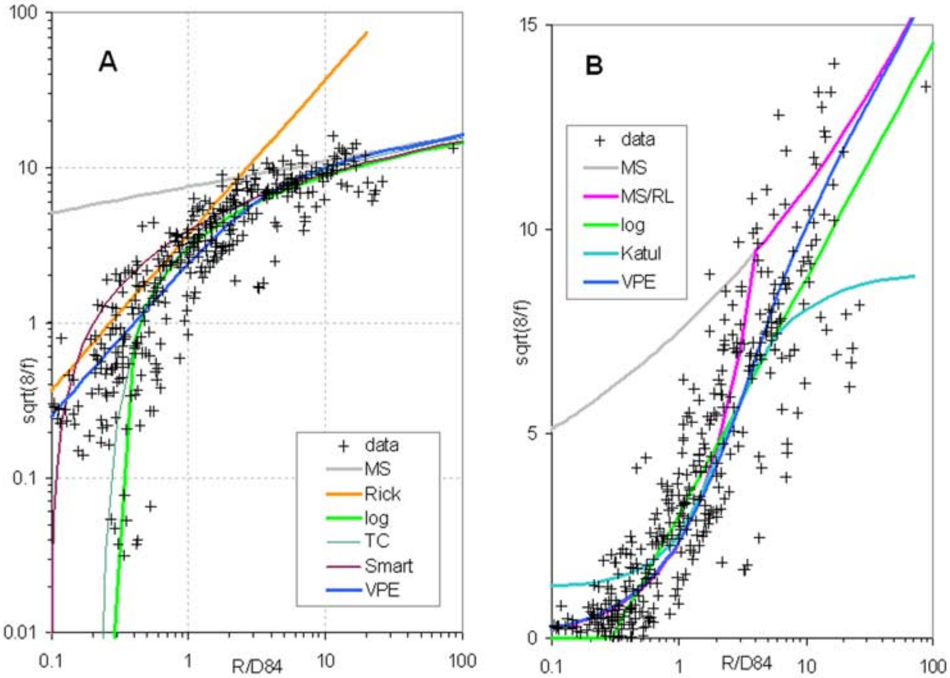

Figure 1. (a) Log-log and (b) semilog plots of $(8/f)^{1/2}$ (= V/u_*) against relative submergence R/D_{84} using all 376 data points from the sources listed in Table 1. Curves show alternative resistance equations, without calibration to these data.

Katul et al. [2002] are plotted using their authors' recommended coefficients. The logarithmic equation is plotted using a constant of 12.2 and k/D_{84} = 3.5, following *Hey* [1979]. The Manning-Strickler, MS/RL, and variable-power equations are plotted using a_1 = 7.5 which is in the middle of the range of literature values and a_2 = 2.36 which gives smooth matching of the two parts of the MS/RL relation at the cutpoint R/D_{84} = 4. Improvements in fit after calibration to the present data are discussed below.

[28] It can be seen from Figure 1 that the 1/6 power Manning-Strickler equation overestimates velocity in all conditions, but with recalibration, it could evidently describe quite well the trend for $R/D_{84} > 4$. *Rickenmann*'s [1991] linear equation parallels the data in the partial-submergence conditions for which it was intended, but its flume-calibrated constant overestimates velocity in this field data set. The RL part of MS/RL is parallel to Rickenmann's equation in Figure 1a but offset from it because of the different coefficient value, as shown by the lower part of the VPE curve. *Bathurst*'s [2002] equations are omitted for clarity since they plot close to Rickenmann at lower values of R/D_{84} and close to Manning-Strickler at higher submergence; this confirms Bathurst's suggestion that they define a lower limit for resistance (an upper envelope in Figure 1a). The flume-calibrated logarithmic equation of *Smart et al.* [2002] also plots close to the upper edge of the shallow-flow data, but nearer the middle of the scatter at higher submergence. *Katul et al.*'s [2002] equation fits the general trend well in the range for which it was intended, but not when extrapolated to deep or very shallow flows.

[29] The only plotted equations that follow the trend of the data over most or all of the range are the VPE and the Hey and Thompson-Campbell logarithmic equations. The latter are more sharply curved in Figure 1a, predicting lower resistance than the VPE at intermediate submergence but very high resistance at low submergence where the general trend is more nearly tracked by the VPE. The Hey and Thompson-Campbell relations differ little except at very low R/D_{84} where the latter declines slightly less steeply. All logarithmic equations make nonphysical predictions once R/D_{84} is so low that the logarithm becomes negative.

[30] In the semilogarithmic plot of Figure 1b, the VPE has an inflection, rather like *Katul et al.*'s [2002] relation but with asymptotes that are sloping not horizontal. Its visual fit is good despite using uncalibrated values of the two parameters. The MS/RL relation converges on the VPE at very low or high submergence; it fits shallow flows well but is near the upper edge of the scatter at intermediate submergence.

[31] The nondimensional hydraulic geometry variant of the MS/RL predictor [equation (16)] can only be illustrated in a plot of V_* against q_* (Figure 2). This shows the expected curvature of the data cloud with a perceptibly flatter trend at higher q_*. Predictions of V_* depend on slope as well as q_*, so Figure 2 shows prediction envelopes calculated using the lowest and highest slopes in each half of the data set: 0.0007 and 0.039 for $q_* > 2$, 0.0015 and 0.21 for $q_* < 2$. Despite the use of uncalibrated values of a_1 and a_2, most of the data points do fall within the envelopes, the outliers are on both sides suggesting no major bias, and the precise choice of cutpoint does not appear to be critical.

5.3. Predictive Performance of Alternative Equations: Statistical Assessment

[32] The plots of resistance against submergence in Figure 1 show the differences between equations that can be plotted in that way, but predictive performance is more logically considered for velocity as the prediction target. This

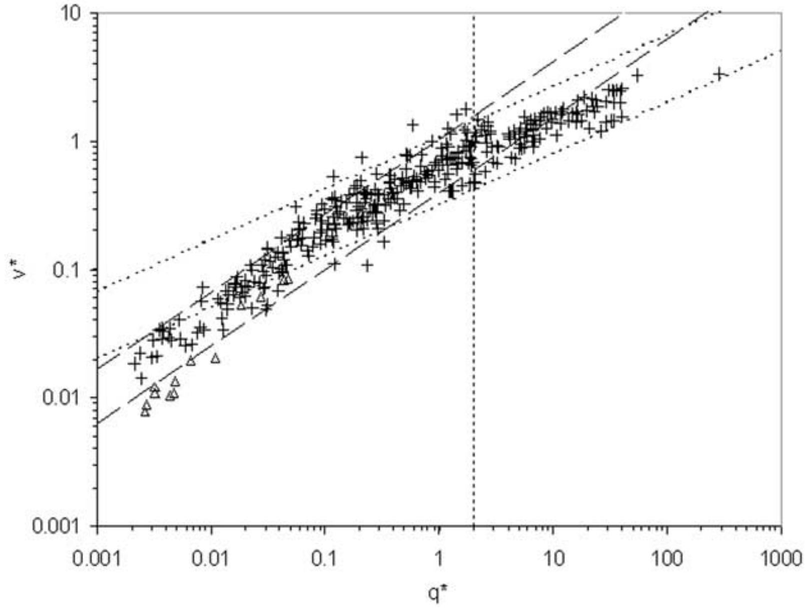

Figure 2. Nondimensional hydraulic geometry plot of $V_* = V/(gD_{84})^{1/2}$ against $q_* = q/(gD_{84}^3)^{1/2}$ for the same data as in Figures 1a and 1b. Parallel broken lines show the envelopes of predictions by equation (16) using the Manning-Strickler (MS) equation for $q_* > 2$ and the roughness-layer (RL) equation for $q_* < 2$. The envelopes correspond to the highest and lowest slope values in each half of the data.

also allows comparison with the MS/RL q_* approach plotted in Figure 2 and the equations of *Jarrett* [1984] and *Bjerklie et al.* [2005] which do not use D.

[33] Predictive performance can be assessed statistically in many ways. The metrics used here are root mean square (RMS) error $s_e = [\sum(V_p - V_m)^2/N]^{1/2}$ (where V_p and V_m denote predicted and measured velocity) and RMS logarithmic error $s_{log} = [\sum(\log V_p - \log V_m)^2/N]^{1/2}$. The former assesses typical error in m s^{-1} and therefore emphasizes errors in predicting faster (usually deeper) flows, whereas s_{log} assesses relative error and therefore gives greater weight to overprediction or underprediction of slow (usually shallow) flows. I also consider the number of prediction errors that exceed a factor of 2 ($V_p/V_m > 2$ or < 0.5).

[34] Table 2 compares the predictive performance of selected equations which, according to Figure 1 or the literature, work over a wide range of conditions and describe the central trend of field data rather than a minimum-resistance envelope. The equations of *Rickenmann* [1991], *Bathurst* [2002], *Smart et al.* [2002], and *Katul et al.* [2002] are therefore not considered. The tabulated statistics are for published values of each equation's parameters without calibration to the present data. *Hey*'s [1979] coefficient values are used for the logarithmic equation, except that negative predictions by this and the Thompson-Campbell equation at very low R/D_{84} are replaced by a small positive number.

[35] The best fit equation in terms of s_e is Jarrett's, closely followed by Hey's and the new VPE and q* relations. However, Jarrett's equation has fairly high s_{log} and many factor-of-2 overpredictions; inspection shows that it predicts high velocities well but overestimates all 71 measured velocities below 0.2 m s^{-1}. The modified Manning equation of *Bjerklie et al.* [2005] has the same bias, but more so, and the highest s_e. In terms of percentage error, the new q* equation is by far the best, then VPE and MS/RL, with the logarithmic equations performing rather poorly.

[36] Table 3 shows the improvement in predictive ability when selected equations are calibrated to minimize either s_e or s_{log}. The equations using R and S are not shown because they continue to have higher s_{log} than others and systematic bias at low velocities. The Thompson-Campbell equation is also omitted since after calibration, its blockage coefficient is almost zero and the equation converges on the simple

Table 2. Ability of Alternative Resistance Equations to Predict Measured Velocity in Combined Data Set Summarized in Table 1 (N = 376), Without Calibration to These Data

Equation	RMS Error in V, m s^{-1}	RMS Error in Log V	Number of Very High/Low Predictions
Jarrett [1984]; equation (6) here	0.41	0.35	84/2
Bjerklie et al. [2005]	0.62	0.48	118/7
Logarithmic[a]	0.43	0.53	31/84
Thompson and Campbell [1979][a]	0.46	0.44	45/57
VPE [equation (20)]	0.43	0.29	41/26
MS/RL using R/D_{84} [equation (15)]	0.50	0.30	46/21
MS/RL using q* [equation (16)]	0.43	0.17	14/16

[a]Predictions below 0.01 m s^{-1} replaced by that value.

Table 3. Ability of Selected Resistance Equations to Predict Measured Velocity in Combined Data Set Summarized in Table 1, After Calibration to These Data

Equation	Calibrated Parameter Values	RMS Error in V, m s^{-1}	RMS Error in Log V	Number of Very High/ Low Predictions
Logarithmic[a]	m = 4.5, V_{min} = 0.25	0.39	0.29	60/20
	m = 3.8, V_{min} = 0.14	0.40	0.27	41/37
	m = 4, V_{min} = 0.2	0.40	0.28	53/22
VPE [equation (20)]	a_1, a_2 = 6.1, 2.4	0.40	0.29	40/28
	a_1, a_2 = 7.3, 2.3	0.42	0.29	39/33
	a_1, a_2 = 6.5, 2.5	0.40	0.29	42/24
MS/RL using R/D_{84} [equation (15)]	a_1, a_2 = 5.3, 2.1	0.40	0.29	38/38
	a_1, a_2 = 5.5, 2.2	0.40	0.29	38/31
	a_1, a_2 = 5.5, 2.2	0.40	0.29	38/31
MS/RL using q_* [equation (16)]	a_1, a_2 = 4.8, 2.5	0.34	0.16	12/17
	a_1, a_2 = 5.7, 2.6	0.35	0.16	12/15
	a_1, a_2 = 5.5, 2.5	0.35	0.16	12/17

[a]Predictions below V_{min} replaced by that value.
Underlined RMS errors are those minimized by the parameter values shown. Third row for each equation gives suggested round-number parameter set giving near-optimal fit to both V and log V.

logarithmic equation. The percentage error of the latter is much improved by introducing a minimum velocity prediction of ~0.2 m s^{-1} and increasing k/D_{84} slightly to ~4. The fits of the three new equations are improved by reducing a_1 to 5–6 (MS/RL and q*) or 6–7 (VPE). After these changes, the q* approach emerges as by far the best predictor whichever criterion is used. Of the approaches using relative submergence, the logarithmic equation has the lowest RMS errors, but VPE and MS/RL are only marginally inferior and the VPE gives slightly fewer factor-of-2 errors than the others.

[37] The equation which does best overall in this statistical comparison is thus the q_* version of the MS/RL approach, followed by two relative-submergence equations: logarithmic with a positive lower limit to predicted velocity and an inflated k/D_{84} on the lines of *Bray* [1979] and *Hey* [1979], and the new variable-power equation. The patterns of prediction error for these equations are illustrated in Figure 3 using round-number parameter values that are near-optimal for both s_e and s_{log} (Table 2). Each plot has a dense cloud of data points along the 1:1 line, showing good agreement of each equation with much of the data, but with some points well off the line. The scatter is visibly less for the q_* equation than for the equations using R/D_{84}, in accordance with the values of s_{log} in Table 3. Computing s_e and s_{log} separately for the RL and MS domains in Figure 3c reveals that s_e is much higher for the MS domain, in which velocities tend to be higher, but s_{log} is slightly higher in the RL domain. Most of the biggest prediction errors in all three plots relate to a single site indicated by a distinctive symbol in Figure 3. This is site GB of *Lee and Ferguson* [2002], which is a very steep (S = 0.18) and coarse (D_{84} = 0.78 m, D_{max} = 2.7 m) step-pool reach in which the nine lowest discharges were a mere trickle between boulders. Just why all equations fail on this site is unclear, but omitting it improves the error statistics of every equation, particularly the VPE which now has the same s_e as the logarithmic equation and an appreciably lower s_{log}. This can be understood by inspecting Figure 1a in which the nine low-flow measurements at site GB form the isolated cluster of points with $(8/f)^{1/2} < 0.1$. The downturn of the logarithmic

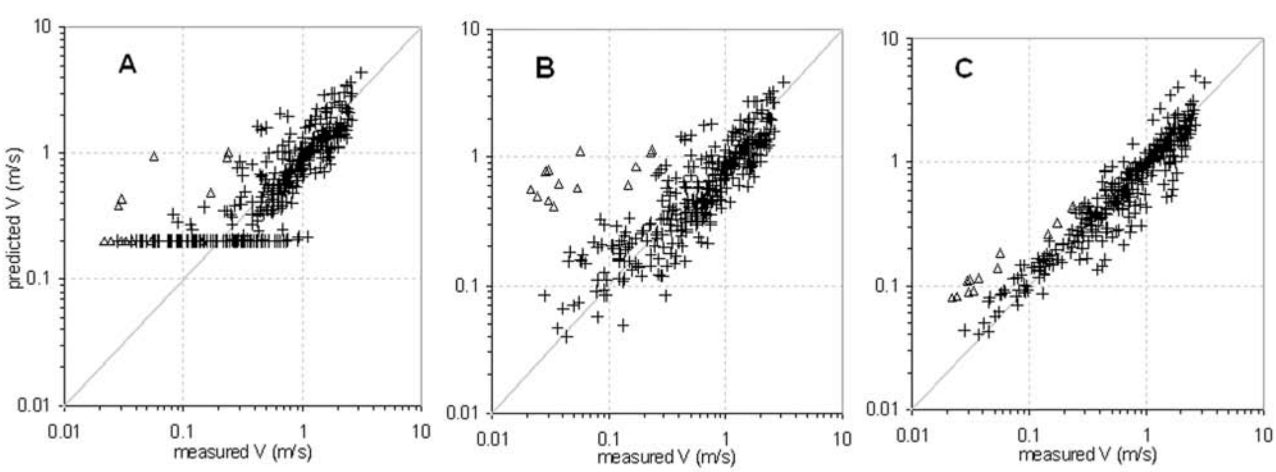

Figure 3. Patterns of velocity prediction error for different resistance equations: (a) logarithmic with k/D_{84} = 4 and low or negative predictions increased to 0.2 m s^{-1}; (b) proposed variable-power equation using a_1 = 6.5, a_2 = 2.5; (c) q_* version of proposed MS/RL approach using a_1 = 5.5, a_2 = 2.5. The Δ symbol is used for data points from site GB of *Lee and Ferguson* [2002].

curve at low R/D_{84} takes it close to these points but systematically away from the main trend of the data.

6. Discussion

[38] A cautious conclusion to this paper would be that little has changed since *Wohl* [2000] made her pessimistic assessment of equations for calculating flow resistance in mountain rivers. Figure 1 shows that some resistance equations track field data well within a limited range of submergence but are not universal, and others approximately define the minimum-resistance envelope of the data, but few follow the central trend in a more or less unbiased way over the full range of submergence. Moreover, the scatter in the data plots is very wide, so even if a resistance equation is approximately unbiased, it will be subject to considerable predictive uncertainty. In this study, all submergence-based equations get the measured velocity wrong by more than a factor of 2 in at least 15% of cases, although the q^*-based approach reduces this below 8% for reasons which will be discussed later. Big relative errors are most frequent in conditions of partial submergence ($R/D_{84} < 1$) which is common during low flow in steep streams. Prediction errors for faster-flowing rivers, or small streams at times of flood, are smaller in relative terms but still frustratingly large in absolute terms.

[39] Accurate prediction of mean velocity in gravel- and boulder-bed streams is difficult for two kinds of reason: practical difficulties in measuring the bed and flow properties used to calibrate or apply resistance equations, and theoretical limitations on how well it is possible to predict reach-average velocity using very simple equations. Practical problems can in principle be overcome, but the theoretical issues imply that predictions can never be perfect. I consider the practical problems first, then the theoretical limitations of any simple approach, and finally suggest implications for practice and future research.

6.1. Susceptibility to Measurement Error

[40] Uncertainties in input data lead to uncertainties in predictions no matter how good the resistance equation used. The large scatter in Figure 1 may partly reflect measurement errors in the nondimensional variables on one or both axes. Calculation of f and d/D requires values of d, D, S, and V and each is subject to error. As noted above, the slope used to calculate f is ideally the energy slope; using the water-surface slope or mean bed slope can introduce bias if there is substantial flow nonuniformity [*Jarrett*, 1984; *Hicks and Mason*, 1991]. This source of uncertainty is potentially most serious for fast low-gradient flows, i.e., high d/D. The shortage of data points at submergences much above 10 in Figure 1 is because few data sets are explicit about uniformity or velocity head.

[41] Significant measurement errors in variables other than slope are most likely at low d/D, which is where Figure 1 shows greatest scatter. The definitions of flow depth and other geometric variables are obvious in rigid-bed flumes but become increasingly fuzzy in steep natural channels. The absolute precision of cross-section surveys decreases with increasing bed grain size (or rather σ_z), and coarser beds are often associated with shallower flow, so that the relative precision of an estimate of d or R becomes even lower. Estimating V and Q from current-meter measurements is also increasingly unreliable the lower the relative submergence. Most recent studies of step-pool channels have avoided this problem by estimating reach-average mean velocity from salt-wave traveltime between two sites, and some have estimated reach-average depth as $d = Q/wV$ with w averaged over numerous cross sections, but *Lee and Ferguson* [2002] showed by error propagation analysis that $(8/f)^{1/2}$ in their step-pool reaches still had an uncertainty of ±17%. Grain size measurement is also more difficult for boulder beds: The random-pacing method of pebble counting becomes impossible, the grid method becomes harder, and grains are too heavy to lift for measurement. In step-pool systems, there is also the question of whether to use a reach-average grain size or one for the steps which contribute most of the flow resistance. The uncertainty of estimates of D_{84} in such streams is at least ±10% and probably ±20%. If one adds an uncertainty of at least 10% in d or R, horizontal plotting positions in Figure 1 could be out by half a log cycle.

[42] Robustness to measurement error is probably a factor in the superior predictive performance of the MS/RL approach using q^* [equations (16a) and (16b)] compared to the same approach using R/D_{84} [equations (15a) and (15b)]. In Figure 3c, using q_*, any error in D_{84}, Q, or V affects both observed and predicted velocity so that the data point moves almost parallel to the 1:1 line. But when predictions are made using R/D_{84}, error in any one input variable affects either predicted or measured velocity, not both, so tends to increase the scatter.

6.2. Theoretical Limitations

[43] Even with uniform flow and accurate measurements, the predictive ability of simple flow resistance equations is limited by two key assumptions which can be challenged on theoretical grounds. One is that total resistance to flow can be parameterized by a small-scale property of the bed, such as D_{84}; the other is that, for a given bed roughness, there is a unique relation between mean depth and mean velocity.

[44] The assumption that the roughness height in the log law can be equated with a grain size percentile goes back to Nikuradse's classic experiments with roughened pipes. It is accepted as valid for well-sorted sediment without bed forms, but few gravel- or boulder-bed rivers are like this. Grain size distributions are typically wide, so individual large clasts can protrude into the flow and groups of them can form clusters, polygons, or steps. Skin resistance is then supplemented by small-scale form drag and possibly also spill losses, so that the log law and its 1/6 power approximation overpredict velocity unless k/D_{84} is increased in the former and a reduced in the latter. Calculations of the combined form drag on clasts of different size protruding above a plane bed [*Wiberg and Smith*, 1991] give a similar k/D_{84} ratio to the curve fitting of *Bray* [1979], *Hey* [1979], and subsequent workers, but as pointed out by *Smart et al.* [2002], k should really be scaled on statistics of bed microtopography rather than grain size if drag is the main source of resistance. Microtopography is considered explicitly by authors proposing versions of what I have termed the roughness-layer equation. *Lawrence* [2000], *Nikora et al.*

[2001], and *Gimenez-Curto and Corniero* [2006] all assumed that characteristic obstacle height must scale with characteristic grain size, but the precise scaling depends on what is assumed about obstacle shape which tends to be far more variable in river beds than in the better-studied case of plant stems. It is now possible to measure the microtopography of river beds to high resolution by digital photogrammetry and of exposed parts to even higher resolution by laser scanning, but it remains unclear whether a single statistic like σ_z can adequately characterize roughness. Furthermore, in streams with step-pool or bar-pool-riffle morphology, there are two distinct scales of topographic variability. A grain-size-based roughness length will tend to underestimate total flow resistance in such situations, the best choice of topographic roughness length is unclear, and a way to use two length scales may need to be devised. Large woody debris adds further resistance which is difficult to separate from other sources [*Wilcox et al.*, 2006].

[45] The other problem, about nonuniqueness of the depth-velocity relation, is more subtle and applies whether bed roughness is characterized by a grain size or a topographic statistic. It can be understood by considering the three-dimensional spatial variability of velocity within the control volume for which a grand mean velocity is to be predicted. The first issue is that any universal relation between local depth d and local vertically averaged velocity V is likely to be nonlinear, so substituting the mean of a spatially variable depth into it will give a biased estimate of the spatially averaged mean velocity. More fundamentally, it is unlikely that any single relation does exist between d and V if there is a combination of skin and drag resistance (I am indebted to a reviewer for this insight and suggestions about how to develop it). In a Reynolds-averaged formulation of uniform flow in a bed-parallel slice at height z, the gravity driving force per unit mass of water is balanced by a combination of turbulent shear and drag on any obstacles present at that height:

$$gS = \frac{\partial \overline{u'w'}}{\partial z} + 0.5 C_D A |U| U \tag{21}$$

where U is the time-averaged velocity, u' is the instantaneous deviation from U, w' is the instantaneous vertical velocity, the overbar denotes time averaging, A is the frontal area of obstacles per unit mass of water, and C_D is a drag coefficient. This is a simplified version of equation (1) in the study by *Nikora et al.* [2004] and is equivalent to equation (2) in *Poggi et al.*'s [2004] paper on turbulence in intermediate-density canopy flow. The turbulent shear term is most easily related to the mean flow by Prandtl's hypothesis

$$\overline{u'w'} = -l^2 \left| \frac{\partial U}{\partial z} \right| \frac{\partial U}{\partial z} \tag{22}$$

where l is an eddy mixing length. If the turbulence term in equation (21) is small compared to the drag term, the value of U for a given slope depends only on obstacle characteristics, but when turbulent shear is significant, the solution for U from equations (21) and (22) depends on l as well as A and C_D. Mixing length varies with height and not necessarily in a universal way. *Poggi et al.* [2004] proposed that flow above a canopy approximates a conventional boundary layer with l proportional to height above the canopy, flow within the canopy is dominated by Karman streets with eddy length independent of height, and flow near the canopy top is essentially a Kelvin-Helmholtz shear layer with l controlled by U and its gradient. For a given roughness height, any one of these scalings implies a particular time-average vertical velocity profile which when integrated gives a particular relation between d and V. Thus $l = \kappa z$ at all heights implies $\mathrm{d}V/\mathrm{d}d \propto d^{-1/2}$, $V \propto d^{1/2}$, and a constant value of $(8/f)^{1/2}$, whereas l = constant implies $\mathrm{d}V/\mathrm{d}d \propto d^{1/2}$, $V \propto d^{3/2}$, and $(8/f)^{1/2} \propto d$ which is the roughness-layer equation. The 1/6 power approximation of the log law falls between these extremes with $V \propto d^{2/3}$ and the log law itself is close to this. A 1/2 power law [*Smart et al.*, 2002] would give a linear velocity profile, and there are endless other possibilities. For shallow flows in which there are two or more mixing length regimes, as suggested by *Nikora et al.* [2004] and *Poggi et al.* [2004], the vertical velocity profile will be more complicated than is implied by any power law with a fixed exponent. This suggests that there is some physical basis for the variable-power equation proposed in this paper, in which the power used to relate V to d alters according to d/k. Another possible way to deal with this situation is to treat V as a weighted average of the mean velocities in each of two layers, computed using different relations (Canovaro and Solari, poster presented at 6th International Workshop on Gravel-Bed Rivers, September 2005).

6.3. Conclusions

[46] The foregoing discussion has suggested both practical and theoretical reasons why simple flow resistance equations do not give precise predictions of mean velocity. There is a clear need for further research directed toward identifying effective topographic indices of resistance, and for detailed flow and turbulence measurements in streams or self-formed laboratory channels to help elucidate the physics of shallow flows over irregular beds. Yet there will surely continue to be a role for simple flow resistance equations that can be used for generic calculations or practical applications, so it is relevant to ask which equations are most reliable and versatile for predicting resistance and velocity in relatively shallow streams. Some clear and moderately optimistic conclusions can be reached about this.

[47] The longstanding and widely used Keulegan approach to flow resistance and velocity prediction, based on integrating the logarithmic law of the wall, predicts velocity at least as well as existing alternative equations even in conditions where a full-depth logarithmic velocity profile is unrealistic. This empirical adequacy is achieved by setting the roughness height k to a substantial multiple of D_{84} and replacing near-zero or negative predictions at very low submergence by a small positive value. The best fit value of k/D_{84} for the present data compilation is $\sim$4, just a little higher than *Hey*'s [1979] value of 3.5.

[48] Two new approaches developed in this paper also predict velocity fairly well at any relative submergence. Each assumes that in very shallow flows $(8/f)^{1/2}$ (or equivalently V/u^*) increases linearly with relative submergence in what I term the roughness-layer relation [RL; equation (11)], whereas in deep flows, an approximately logarithmic rela-

tion holds which can be described by the 1/6 power Manning-Strickler relation [MS; equation (4)]. The variable-power equation [VPE; equation (20)] with RL and MS as asymptotes fits the data compilation as well as any existing resistance law. It is not derived from a rigorous physical analysis, but nothing about it is incompatible with known physics. There are precedents for the concept of additive sources of resistance; the low-submergence asymptote shares the heuristic basis of the mixing-length and jet-regime approaches; the other asymptote is a close approximation of the log law for the deep flows in which it is physically plausible; and a gradual shift from one to the other is what would be expected from likely vertical changes in turbulent eddy scales in flows which have a boundary layer above a roughness layer. The variable-power equation may therefore be a useful tool for anyone wanting to predict velocity by a single equation over a wide range of conditions. The best fit to the present data compilation was obtained by reducing the MS coefficient a_1 by about 20% from its traditional value, although predictions using the latter are at least as good as those by other equations with default coefficients. A reduction in a_1 has the same effect as an increase in k/D_{84} in the generalized log law, and the need for both of these adjustments probably has a common cause in form drag on protruding clasts in poorly sorted riverbeds. The optimum value of the RL coefficient a_2 is within the range suggested by *Nikora et al.* [2001] and close to what *Gimenez-Curto and Corniero* [2006] suggested.

[49] The other new approach is based on a general nondimensional hydraulic geometry relation that includes many previous resistance laws as special cases. The simplest way to apply it is to use either of two specific versions, equivalent to the MS and RL relations, according to the value of the nondimensional unit discharge $q_* = q/(gD^3)^{0.5}$. This MS/RL approach using q^* has by far the lowest velocity prediction error of any of the methods tested. It cannot be used to estimate discharge for a given depth (for example, bankfull), but it can be used to partition a known or assumed discharge between depth and velocity when slope and grain size are known. This situation often arises in ecological and geomorphological applications. The way in which width changes with discharge must be known, but that is no more restrictive than the need to know how depth alters with discharge if using d/D to predict V in the same type of situation, and there is the advantage that width can usually be measured more precisely and easily than depth. One reason why q^* is a better predictor than d/D is probably its lesser sensitivity to measurement error, as already discussed, but there is another. In gravel bed rivers with significant macro-scale form resistance associated with bars or bends, and in step-pool boulder torrents with significant spill resistance, V is lower and d higher than would be expected based on skin resistance only. The increased depth means that a resistance equation using d/D predicts high, not low, velocity unless k_s/D_{84} is set to a very high multiple [e.g., *Millar*, 1999]. But in the hydraulic geometry approach, using unit discharge $q = dV$ as predictor, the effects of large-scale resistance on depth and velocity cancel out and there is no overall bias.

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