Brane World Cosmology Without the Z_2 Symmetry

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Abstract

The Friedmann equation for a positive tension brane situated between two bulk spacetimes that posses the *same* 5D cosmological constant, but which does not posses a Z_2 symmetry of the metric itself is derived, and the possible effects of dropping the Z_2 symmetry on the expansion of our Universe are examined; cosmological constraints are discussed. The global solutions for the metric in the infinite extra dimension case are found and comparison with the symmetric case is made. We show that any brane world senario of this type must revert to a Z_2 symmetric form at late times, and hence rule out certain proposed scenarios.

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1 Introduction

Recently there has been considerable interest in the novel suggestion that we live in a Universe that possesses more than four dimensions. The standard model fields are assumed to be confined to a hypersurface (or 3-brane) embedded in this higher dimensional space, in contrast the gravitational fields propagate through the whole of spacetime [1, 2, 3, 4]. In order for this to be a phenomenologically relevant model of our universe, standard 4D gravity must be recoverd on our brane. There are various ways to do this, the most obvious being to assume that the extra dimensions tranverse to our brane are compact. In this case gravity can be recovered on scales larger than the size of the extra dimensions [2]. This is different from earlier proposals since the restrictions on the size of the extra dimensions from particle physics experiments no longer apply, as the standard model fields are confined to the brane. The extra dimensions only have to be smaller than the scale on which gravity experiments have probed, currently of order 1mm. Another way to recover 4D gravity at large distances is to embed a positive tension 3-brane into an AdS_5 bulk [4, 5]. In this scenario 4D gravity is obtained at scales larger than the AdS radius. Randall and Sundrum showed that this could produce sensible gravity even if the extra dimension was not compact.

The cosmology of these extra dimension scenarios has been investigated and the Friedmann equation derived and shown to contain important deviations from the usual 4-dimensional case [6]. Some inflationary models have been investigated [7], as have brane world phase transitions, topological defects and baryogenesis [8]. Most brane world scenarios that involve one extra dimension assume a Z_2 symmetry about our brane, motivated by a model derived from M-Theory proposed by Horava and Witten [9]. However, many recent papers examine models that are not directly derived from M-Theory: for example there has been much interest in the one infinite extra dimension proposal. The motivation for maintaining the Z_2 symmetry is seemingly no longer adequate and it is therefore interesting to analyse a brane world model without this symmetry and to assess its phenomenological implications. There have been some multi-brane scenarios suggested, which involve branes that although lying between two bulk space times with the same cosmological constant, *do not possess a Z_2 symmetry of the metric itself* [10]. This approach generates an altered Friedmann equation as well as giving different bulk solutions. The cosmological solutions have not been analysed and it is therefore interesting to entertain the possibility that we live on such a brane and to determine whether it possesses a sensible cosmology. A different approach has been taken by [11, 12] where they have looked at the effect of having different cosmological constants either side of the brane. Our approach is not equivalent to theirs, as will be discussed in more detail below.

In Section 2 of this paper we utilise the 5D Einstein's equations to derive the Friedmann equation for such a non-Z_2 symmetric 3-brane and we then investigate the associated cosmological consequences of such a model. We show that the Friedmann equation acquires an extra term when there is no Z_2 symmetry, which can give rise to a period of expansion on the brane, and consider constraints on such a term from nucleosynthesis. These constraints apply to any scenario in which we reside on a 3-brane without the Z_2 symmetry, and

demonstrate that scenarios such as [16] are physically unrealistic. In Section 3 we restrict ourselves to the infinite fifth dimensional case and solve Einstein's equations in the bulk in order to generate the global solutions, examining the difference between them and the symmetric solutions and also demonstrating that they remain well-defined. Our conclusions are summarised in Section 4.

2 The Friedmann Equation with no Z_2 Symmetry

In this section we examine a cosmologically realistic positive tension 3-brane in 5 dimensions. We assume that the 5D cosmological constants either side of the brane are identical and then derive the general solution to Einstein's equations without assuming that the metric across the brane is Z_2 symmetric. For the single brane case, this amounts to having non-Z_2 symmetric initial conditions with a symmetric bulk, however many of the results also apply to multi-brane scenarios such as [10].

Since we are interested in cosmological solutions, we take a metric of the form:

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{1}$$

where y is the coordinate of the fifth dimension and we adopt a brane-based approach where the brane is the hypersurface defined by $y = 0$. Here γ_{ij} is a maximally symmetric 3-dimensional metric with $k = -1, 0, 1$ parameterising the spatial curvature. We assume immediately that $b^2(t,y)$ is not a function of time and therefore y can be scaled so that $b(y) = 1$. The metric is found by solving the 5D Einstein's equations,

$$G_{AB} = \kappa^2 T_{AB}, \tag{2}$$

where we define $\kappa^2 = 1/\widetilde{M}_5^3$ where $\widetilde{M}_5$ is the fundamental (reduced) 5D Planck Mass. The stress-energy-momentum tensor can be written as,

$$T^A{}_B = T^A{}_B|_{brane} + T^A{}_B|_{bulk}. \tag{3}$$

Again with cosmology in mind, we assume a homogeneous and isotropic geometry in the brane and this makes it possible to write the first term as:

$$T^A{}_B|_{brane} = \delta(y)\text{diag}(-\rho_b, p_b, p_b, p_b, 0). \tag{4}$$

The second term, which describes the same negative bulk cosmological constant ρ_B either side of the brane, is of the form,

$$T^A{}_B|_{bulk} = \text{diag}(-\rho_B, -\rho_B, -\rho_B, -\rho_B, -\rho_B). \tag{5}$$

Note that here we have taken an alternative approach from [11, 12] who assume different cosmological constants either side of the brane. Our lack of Z_2 symmetry enters when we consider the solution of the metric itself. As is shown in [13], for the setup defined above, any set of functions $a(t,y)$ and $n(t,y)$ satisfying,

$$\left(\frac{\dot{a}}{na}\right)^2 = \frac{1}{6}\kappa^2\rho_B + \left(\frac{a'}{a}\right)^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}, \tag{6}$$

together with $G_{05} = 0$, will be solutions to all the Einstein's equations locally in the bulk. The last term in (6) comes from the electric part of the 5D Weyl tensor (where $\mathcal{C}$ is some constant) and its existence and possible problematic effects will be discussed below. Now $G_{05} = 0$ is satisfied if

$$n(t, y) = \frac{\dot{a}(t, y)}{\dot{a}(t, 0)}. \tag{7}$$

Here, we have normalised $n(t, y)$ so that $n(t, 0) = 1$. In order to obtain the Friedmann Equation on the brane, we evaluate (6) at $y = 0$. This is easily done except for the $(a'/a)^2$ term. To evaluate this we need the junction conditions,

$$\frac{[a']}{a_0} = -\frac{\kappa^2}{3}\rho_b, \tag{8}$$

$$\frac{[n']}{n_0} = \frac{\kappa^2}{3}(3p_b + 2\rho_b), \tag{9}$$

where $[Q] = Q(0^+) - Q(0^-)$ and $Q(0) = Q_0$. Now instead of assuming the Z_2 symmetry $y \leftrightarrow -y$ on the metric itself which would give $a'(0^+) = -a'(0^-)$ and therefore $[a'] = 2a'(0^+)$, we write,

$$a'(0^+) = -a'(0^-) + d(t). \tag{10}$$

where $d(t)$ is some function of time only and has yet to be determined. $d(t)$ represents the asymmetry of the metric across the brane and the fact that it is not identically zero for all time is what is referred to as the brane being 'non-Z_2 symmetric' in the rest of this paper. Now in order to determine $d(t)$ we use (8) which gives,

$$a'(0^+) = -\frac{\kappa^2}{6}\rho_b a_0 + \frac{d(t)}{2}, \tag{11}$$

$$a'(0^-) = \frac{\kappa^2}{6}\rho_b a_0 + \frac{d(t)}{2}. \tag{12}$$

Assuming that $a'^2(0) = (a'^2(0^+) + a'^2(0^-))/2$ results in,

$$\frac{a_0'^2}{a_0^2} = \frac{\kappa^4}{36}\rho_b^2 + \frac{d^2(t)}{4a_0^2}. \tag{13}$$

To find an expression for $d(t)$ we take the jump of the (5,5) component of Einstein's equations as is done in [6],

$$\frac{\bar{a}'}{a_0}p_b = \frac{1}{3}\rho_b\frac{\bar{n}'}{n_0}, \tag{14}$$

where $\bar{Q} = (Q(0^+) + Q(0^-))/2$. Replacing a and n using equations (10) and (7), shows that,

$$\dot{d(t)} = \frac{3p_b\dot{a_0}}{\rho_b a_0}\,d(t). \tag{15}$$

The energy conservation equation is derived directly from the junction conditions (8) and (9) as shown by [6],

$$\dot{\rho}_b = -3(\rho_b + p_b)\frac{\dot{a_0}}{a_0}. \tag{16}$$

By using this to solve the differential equation (15), it gives us the desired expression for $d(t)$,

$$d(t) = \frac{2F}{\rho_b a_0^3}, \tag{17}$$

where F is an integration constant which, when non-zero dictates to what extent the Z_2 symmetry is broken. Combining this with equation (6) and doing the usual replacements (first found by [15]) to obtain the standard $H^2 \propto \rho$ relation at late times: $\rho_b = \rho + \rho_\lambda$ and $\kappa^2\rho_B/6 + \kappa^4\rho_\lambda^2/36 = 0$, (where ρ_λ is the brane tension and ρ is the physical brane energy density) results in the Friedmann equation for a brane without the Z_2 symmetry,

$$\left(\frac{\dot{a_0}}{a_0}\right)^2 = \frac{\kappa^4\rho_\lambda}{18}\rho + \frac{\kappa^4}{36}\rho^2 - \frac{k}{a_0^2} + \frac{\mathcal{C}}{a_0^4} + \frac{F^2}{(\rho+\rho_\lambda)^2 a_0^8}. \tag{18}$$

So the absence of the Z_2 symmetry gives rise to an extra term in the Friedmann equation. For a radiation dominated Universe where $\rho = \gamma/a_0^4$, the extra term behaves as F^2/γ^2 as $\rho \to \infty$ and $(F\rho/\gamma\rho_\lambda)^2$ as $\rho \to 0$. Note that equation (18) is the same as that found by [14], but differs from the Friedmann equations found by [11, 12]. Therefore all the following analysis and results of this section also apply to the setup described in [14]. In order to obtain standard cosmology at late times we need to make the identification,

$$\frac{\kappa^4\rho_\lambda}{18} = \frac{8\pi G_4}{3} = \frac{1}{3\widetilde{M}_4^2}, \tag{19}$$

where we have used the reduced 4D Planck mass defined by $\widetilde{M}_4^2 = M_4^2/8\pi$ and will use the 5D reduced Planck mass defined by $\widetilde{M}_5^3 = M_5^3/8\pi$. This implies that the brane tension, $\rho_\lambda = 6\widetilde{M}_5^6/\widetilde{M}_4^2$. Using this, the fact that $\kappa^2 = 1/\widetilde{M}_5^3$ and also defining the dimensionless constants $f = F\widetilde{M}_4^2/\gamma\widetilde{M}_5^3$, $c = 3\widetilde{M}_4^2\mathcal{C}/\gamma$ and assuming $k = 0$, allows us to write (18) in terms of f, c, $\widetilde{M}_4$, $\widetilde{M}_5$ (and ρ_λ),

4

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{1+c}{3\widetilde{M}_4^2}\rho + \frac{1}{(6\widetilde{M}_5^3)^2}\left[\rho^2 + \frac{f^2\rho_\lambda^2\rho^2}{(\rho+\rho_\lambda)^2}\right]. \tag{20}$$

This shows that the expansion of the universe is initially dominated by the ρ^2 term, while at late times the standard cosmology phase with the usual $H^2 \propto \rho$ behaviour is obtained. If $f < \sqrt{8(1+c)}$ the third, 'f'-term of (20) is always less significant than the other terms, and the resulting cosmology is similar to a brane cosmology with a Z_2 symmetry.

If $f > 3 + 2c$ there will be a period between the ρ^2 and ρ driven phases when the 'f'-term is dominant. It is reasonable to suggest that f is significantly greater than c, since as will be demonstrated later in this section, nucleosynthesis constrains c to be $\ll 0.2$. Now in order to get an explicit solution for $a_0(t)$ so that we can ascertain the effect of the new f term, we are forced to make several approximations. First we use,

$$\frac{\rho^2\rho_\lambda^2}{(\rho+\rho_\lambda)^2} = \begin{cases} \rho^2 & \rho < \frac{1}{4}\rho_\lambda \\ \frac{1}{4}\rho_\lambda\rho & \frac{1}{4}\rho_\lambda < \rho < 4\rho_\lambda \\ \rho_\lambda^2 & \rho > 4\rho_\lambda \end{cases}, \tag{21}$$

to approximate the 'f'-term. Now it is possible to solve for $a_0(t)$ in equation (20) which demonstrates that the time dependance of $a_0(t)$ in each of the three phases takes the form:

$$a_0(t) \propto \begin{cases} \gamma\left(\frac{4(1+c)}{3\widetilde{M}_4^2}t^2 + \frac{2\sqrt{1+f^2}}{3\widetilde{M}_5^3}t\right)^{\frac{1}{4}} & \rho < \frac{1}{4}\rho_\lambda \\ \gamma\left(\frac{16(1+c)+f^2}{12\widetilde{M}_4^2}t^2 + \frac{2}{3\widetilde{M}_5^3}t\right)^{\frac{1}{4}} & \frac{1}{4}\rho_\lambda < \rho < 4\rho_\lambda \\ \gamma\left(\frac{(1+c)}{\rho_\lambda}\left[\cosh\frac{2\rho_\lambda f}{3\widetilde{M}_5^3}t - 1\right] + \frac{1}{\rho_\lambda}\sinh\frac{2\rho_\lambda f}{3\widetilde{M}_5^3}t\right)^{\frac{1}{4}} & \rho > 4\rho_\lambda \end{cases}, \tag{22}$$

Evaluating the exact constants that replace the proportional signs in (22) is rather cumbersome, so in order to obtain an order of magnitude approximation of the effects of the 'f'-term we instead ignore all subdominant terms in each phase of the universe, assume that $f \gg 1$ and then solve for $a_0(t)$ again using (20). The resulting evolution of the universe then divides into 5 phases as described below. In the following it is useful to define the time scale $t_{\rm F} = \widetilde{M}_4^2/(4\widetilde{M}_5^3 f)$,

- PHASE 1 ($0 < t < t_{\rm F}$): Initially (20) is dominated by the ρ^2 term. This continues until $t = t_{\rm F}$ when $\rho = f\rho_\lambda$, after which the 'f'-term becomes dominant. Until that happens,

$$a_0(t) = \left[\frac{2\gamma}{3\widetilde{M}_5^3}\right]^{1/4} t^{1/4} \tag{23}$$

- PHASE 2a ($t_{\rm F} < t < t_{\rm F} + t_{\rm I}$), with $t_{\rm I} = t_{\rm F}\ln(f/4)$: For large ρ the 'f' term can be approximated by a constant. During this period the universe expands exponentially,

as in inflation,

$$a_0(t) = \left[\frac{\widetilde{M}_4^2 \gamma}{6\widetilde{M}_5^6 fe}\right]^{1/4} e^{Ht} , \quad H = \frac{\widetilde{M}_5^3}{\widetilde{M}_4^2} f. \tag{24}$$

- PHASE 2b $(t_\mathrm{I} + t_\mathrm{F} < t < t_\mathrm{I} + 7t_\mathrm{F})$: For $\rho \sim \rho_\lambda$ the 'f'-term starts to decrease, and is approximately proportional to $f^2 \rho_\lambda \rho$. During this phase,

$$a_0(t) = \left[\frac{\gamma f^2}{6\widetilde{M}_4^2}\right]^{1/4} (t - t_\mathrm{I} + t_\mathrm{F})^{1/2}. \tag{25}$$

- PHASE 2c $(t_\mathrm{I} + 7t_\mathrm{F} < t < t_\mathrm{B} = t_\mathrm{I} + (3 + f^2/2)t_\mathrm{F})$: For small ρ the 'f'-term is approximately $f^2 \rho^2$. It ceases to be the dominant term in (20) when $\rho = 2\rho_\lambda/f^2$. Until then,

$$a_0(t) = \left[\frac{2\gamma f}{3\widetilde{M}_5^3}\right]^{1/4} (t - t_\mathrm{I} - 3t_\mathrm{F})^{1/4}. \tag{26}$$

- PHASE 3 $(t > t_\mathrm{B})$: Finally, at late times the ρ term dominates (20) to give the standard cosmology,

$$a_0(t) = \left[\frac{4\gamma(1 + c)}{3\widetilde{M}_4^2}\right]^{1/4} (t - t_\mathrm{B} + f^2 t_\mathrm{F})^{1/2}. \tag{27}$$

The above solution for $a_0(t)$ presents a very different picture of the evolution of our universe: it has the unconventional early ρ^2 behaviour as seen in most brane world models, but now this is broken up by a period of exponential expansion. During this extra phase the scale factor increases by a factor of $f^{1/4}$. Like inflation this could help to solve the flatness problem, however unlike inflation it is not followed by reheating and so cannot help with the horizon and monopole problems. Eventually, as expected, the standard cosmology is obtained. The above approximate solution (equations (23–27)), suggests that there will be no exponential expansion unless $f \gtrsim 4 + 2c$. If f is very large $(f \gtrsim \widetilde{M}_4^2/\widetilde{M}_5^2)$ then $f\rho_\lambda > \widetilde{M}_5^4$ and Phase 1, and some of the succeeding phases will be above the 5D Planck scale, and may not actually occur.

For $\sqrt{8(1 + c)} < f < 3 + 2c$ there will be a short period of 'f'-term domination. This occurs after the time when the ρ-term of (20) starts to dominate the ρ^2-term. At this stage the 'f'-term is no longer approximately constant, and so there is no exponential expansion.

Nucleosynthesis provides constraints on both c and f. If c is nonzero then it gives rise to a term of the form $\mathcal{C}/a_0^4$ in the Friedman equation (18) which at the time of nucleosynthesis behaves like additional relativistic degrees of freedom. The energy density at this time is given as $\rho(t_N) = g_* \frac{\pi^2}{30} T_N^4$, where t_N and T_N are the time and temperature of Nucleosynthesis, and g_*, the number of effective relativistic degrees of freedom is strongly constrained by the observed abundances of light elements which show that any deviation

from g_* given by Δg_* satisfies: $\Delta g_* < 2$. This implies the following constraint for $\mathcal{C}$, as pointed out by [13],

$$\frac{\mathcal{C}}{a_0^4(t_N)} \ll \frac{\pi^2}{30} \frac{\Delta g_* T_N^4}{3\widetilde{M}_4^2}. \tag{28}$$

Using the fact that $\rho(t_N) = \gamma/a_0^4(t_N) = g_* \frac{\pi^2}{30} T_N^4$, leads to,

$$\frac{\mathcal{C}}{\gamma} \ll \frac{\Delta g_*}{3\widetilde{M}_4^2 g_*}, \tag{29}$$

which implies that

$$c \ll \frac{\Delta g_*}{g_*} \simeq 0.2, \tag{30}$$

where we have taken the standard values $g_* = 10.75$ and $\Delta g_* < 2$. Hence it is reasonable to suggest that f could play a cosmologically significant role but that c cannot. Note that this constraint on c is independant of the approximations used to derive equations (22) and (23–27) and comes purely from our knowledge of the restrictions on extra relativistic degrees of freedom at the time of nucleosynthesis.

We can also obtain rough restrictions on f in terms of $\widetilde{M}_5$ by demanding that standard cosmology is in place by the time of nucleosynthesis. This is equivalent to requiring that phase 2 is over well before nucleosynthesis begins, which implies that,

$$\frac{1+c}{3\widetilde{M}_4^2}\rho(t_N) \gg \frac{1}{(6\widetilde{M}_5^3)^2}f^2\rho(t_N)^2. \tag{31}$$

At this time the universe is radiation dominated and $\rho(t_N)$ can be written in terms of the temperature at nucleosynthesis, $T_N \approx 1\text{MeV}$, and the number of effective relativistic degrees of freedom, g_*,

$$\rho(t_N) = g_* \frac{\pi^2}{30} T_N^4. \tag{32}$$

Substituting this into (31), assuming c to be negligible due to (30) and switching to the standard 4D and 5D Planck masses, gives the following relation between the 5D Planck mass M_5 and the Z_2 symmetry breaking parameter f,

$$M_5 \gg \left(\frac{g_* \pi^3}{45} T_N^4 M_4^2 f^2\right)^{1/6} \approx 30 f^{1/3}\text{TeV}. \tag{33}$$

For the case where $f = 0$, only phase 1 and 3 exist and demanding just that phase 3 is over well before t_N gives, from (20),

$$\frac{1+c}{3\widetilde{M}_4^2}\rho(t_N) \gg \frac{1}{(6\widetilde{M}_5^3)^2}\rho(t_N)^2. \tag{34}$$

which leads as above to:

$$M_5 \gg 30\text{TeV}. \tag{35}$$

This is a fairly weak bound on M_5. For the infinite extra dimension scenario, experiments testing gravity at small distances have already demonstrated using the corrections to Newton's gravity law calculated by Randall and Sundrum [5] that $M_5 > 10^5\text{TeV}$. This experimental constraint however, is not applicable to compactified scenarios. Supposing that we do live in an infinite 5th dimension and that M_5 has a value that is just outside our experimental reach, we can then use (33) to constrain f; $f \ll 10^{11}$. In this case, the period of exponential expansion (Phase 2a) would only last for $t_{\text{I}} = M_4^2/(4M_5^3 f)\ln f \simeq 10^7\text{TeV}^{-1}$ and the scale factor would increase during this time by a factor of around 500. Increasing M_5 relaxes the bound on f and would appear to lead to more inflation. However, if M_5 is too large then some, or all, of the resulting expansion occurs at energies higher than the 5D Planck scale. Consequently the maximum inflation occurs for $M_5 = 5 \times 10^6$ TeV and $f = 10^{17}$, which leads to an expansion of only 10^4. Unfortunately, this is not cosmologically relevant. These order of magnitude expansion rates due to a non-zero f are dependant on the approximations made to generate equations (23-27), however an exact treatment would likely lead to the same conclusion: that the cosmological effect of f is negligible.

We have therefore derived constraints on c and f, showing both that c must be negligible in size: $c \ll 0.2$ and that although f can be large, its cosmological effect is insignificant in that it causes inflation that results in an increase in $a_0(t)$ only of order 10^4.

Note that, from equations (11), (12), (17) and (31) the effect of the Z_2 breaking term decreases with increasing time such that the universe reverts to standard cosmology. This suggests that brane world scenarios where the physical Universe is on a brane without this symmetry are not viable after nucleosynthesis. The proposal made in [16], which described a setup that would solve the hierarchy problem despite our physical universe existing on a positive tension brane, is unfortunately found to be unrealistic due to this reason.

3 Global Solutions

After having examined the phenomenological effects resulting from the Friedmann equation of a non-Z_2 symmetric brane world, we will now solve the 5D Einstein equations in the bulk and hence derive the corresponding global solutions for $a(t,y)$ and $n(t,y)$ for the non-Z_2 symmetric brane in an infinite extra dimension. We derive this solution for the general case first and then for a specific cosmologically realistic brane. To do this we can adapt the previously known general global solution for a brane with tension ρ_b and negative 5D bulk cosmological constant ρ_B to the non-Z_2 symmetric case.

We know from the (0,0) component of Einstein's equations that the new non-symmetric solution will have a form similar to the symmetric case as derived in [13];

$$a^2(t,y) = a_0^2 \left(A(t)\cosh \mu y + B(t)\sinh \mu|y| + C(t) \right), \tag{36}$$

where $\mu = \sqrt{-2\rho_B/(3\widetilde{M}_5^3)}$. The requirement $a^2(t,0) = a_0^2$ trivially implies that $A(t) + C(t) = 1$ for all t. Now using equation (11) it can be seen that,

$$a'(t,0^\pm) \;=\; \{a'_{sym}(t,0^\pm)\} \;+\; \frac{d(t)}{2}, \tag{37}$$

$$\Rightarrow \;\; B(t) \;=\; \left\{ -\frac{\rho_b}{\sqrt{-6\widetilde{M}_5^3\rho_B}} \right\} \;\pm\; \frac{d(t)}{a_0\mu}. \tag{38}$$

Here the $\pm$ in the expression for $B(t)$ corresponds to the solution on either side of the brane and we use $\{\ldots\}$ to denote the solution found in the Z_2 symmetric case. $C(t)$ is found from the differential equation for $a^2(t,y)$ which is derived from the Einstein equations, (see [13]),

$$C(t) = \frac{3\widetilde{M}_5^3(\dot{a}_0^2 + k)}{\rho_B a_0^2}. \tag{39}$$

Rewriting this using the 'new' Friedmann equation (18) gives,

$$C(t) = \left\{ \frac{1}{2}\left(1 + \frac{\rho_b^2}{6\widetilde{M}_5^3\rho_B}\right) + \frac{3\widetilde{M}_5^3\mathcal{C}}{\rho_B a_0^4} \right\} + \frac{3\widetilde{M}_5^3 F^2}{\rho_B \rho_b^2 a_0^8}, \tag{40}$$

and therefore $A(t)$ is trivially given by,

$$A(t) = \left\{ \frac{1}{2}\left(1 - \frac{\rho_b^2}{6\widetilde{M}_5^3\rho_B}\right) - \frac{3\widetilde{M}_5^3\mathcal{C}}{\rho_B a_0^4} \right\} - \frac{3\widetilde{M}_5^3 F^2}{\rho_B \rho_b^2 a_0^8}. \tag{41}$$

Using (37) and (17) leads to,

$$B(t) = \left\{ -\frac{\rho_b}{\sqrt{-6\widetilde{M}_5^3\rho_B}} \right\} \pm \sqrt{\frac{6\widetilde{M}_5^3}{-\rho_B}} \frac{F}{\rho_b a_0^4}. \tag{42}$$

Again the $\pm$ signs in the expression for $B(t)$ give the two different solutions on either side of the brane. The solution for $n(t,y)$ in the non-symmetric case is found from the above solution for $a(t,y)$ by using equation (7) as before. It is easily seen that setting F to zero in the above solutions recovers the Z_2 symmetric situation.

We are interested in these solutions for a cosmologically realistic brane, so we make the same substitutions as were made to generate (18) and also assume a radiation dominated Universe. Setting $\rho = \gamma/a_0^4$ where γ is a constant, we obtain expressions for $A(t)$, $B(t)$ and $C(t)$ corresponding to a brane with a viable cosmology,

$$A(t) \;=\; 1 + \chi + \frac{1}{2}\chi^2 + c\chi + \frac{f^2\chi^2}{2\left(1+\chi\right)^2}, \tag{43}$$

$$B(t) \;=\; -(1+\chi) \pm \frac{f\chi}{(1+\chi)}, \tag{44}$$

$$C(t) \;=\; -\chi - \frac{1}{2}\chi^2 - c\chi - \frac{f^2\chi^2}{2\left(1+\chi\right)^2}. \tag{45}$$

9

Where we have defined the dimensionless variable $\chi = \widetilde{M}_4^2 \rho / 6 \widetilde{M}_5^6$. Equation (43) shows that the metric off the brane is greatly altered by a large value of f, in fact the 'f'-terms dominate during the period when $f \gtrsim \chi \gtrsim 1/f$. During this time, $a(t,y)$ vanishes on the side of the brane corresponding to the negative sign in equation (38), at position y_0 which is given by:

$$y_0 \simeq \frac{1}{\mu} \ln \left[\frac{(f+2)\chi + 2}{(f-2)\chi - 2} \right], \tag{46}$$

and $n(t,y)$ will only become small on the opposite side at position y_0 given by:

$$y_0 \simeq \frac{1}{\mu} \left| \frac{4(\chi^2 - 1)}{f\chi(\chi - 3)} \right|, \tag{47}$$

although the exact position of the zeros will depend upon the negleglected terms in (43). After this time, $a(t,y)$ no longer vanishes, however $n(t,y)$ vanishes roughly as for the symmetric case where y_0 is now given by:

$$y_0 \simeq \frac{1}{\mu} \ln \left[1 + \frac{2\chi + \sqrt{2\chi(2\chi + c)}}{\chi(3\chi + 2c)} \right]. \tag{48}$$

Calculation of the Ricci tensor and scalar show that they are both finite at the points where $a(t,y)$ and $n(t,y)$ vanish, which implies that these points just correspond to coordinate singularities. A similar result has been obtained in [17] for the symmetric case. Since there are no obvious problems with the global solutions, the only restrictions on f and c are those obtained in the previous section, which are applicable to all scenarios, not just the infinite extra dimensional one considered here.

We have seen how in a cosmological context, the various new features that are presented by a non-Z_2 symmetric brane world as opposed to an ordinary 4D Universe, are not necessarily dangerous and providing that the basic parameters M_5, c and f satisfy some fairly relaxed constraints, then the phenomenological predictions of the scenario proposed in this paper are still in agreement with current experimental observation. This has been shown to be true, however only for the cosmological case. While it is comforting to know that the Friedmann equation for a homogeneous, isotropic brane world gives sensible late time cosmology, the extra features such as the electric part of the 5D Weyl tensor and the extra degrees of freedom in the metric due to dropping the Z_2 symmetry condition, could cause major problems when pertubations to the background metric are considered. Although several papers have been produced addressing the issue (see [18, 19, 20, 21]), the complete analysis remains to be done. Only a fully rigorous treatment would be able to show whether or not the Weyl tensor or the lack of a Z_2 symmetry would lead to pertubations growing in a disasterous manner and whether a possible choice of initial conditions could help prevent this.

4 Conclusions

We have derived the Friedmann equation and bulk solutions in a brane world scenario where there is no Z_2 symmetry of the metric across the brane. The bulk solutions were shown to be well behaved. Relaxing the Z_2 symmetry introduces an extra term that behaves as an effective cosmological constant at early times. Approximate solutions of the Friedmann equation on the brane were found with this extra term. If the asymmetry is sufficient this will introduce a period of exponential expansion during the early *brany* evolution.

Demanding that standard cosmology is in place by the time of nucleosynthesis constrains the Z_2 symmetry breaking parameter. This limits the amount of exponential expansion of the scale factor at early times to be of order 10^4, which is not cosmologically significant. Thus this would only be a small mitigating factor in the flatness problem.

We also note that the effects of the Z_2 breaking term decrease with time. This is essential to ensure that the standard cosmology is recovered at late times. This suggests that the scenarios without this symmetry at late times, such as [16] are not viable.

Acknowledgements

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