Opportunity Cost Bidding by Wind Generators in Forward Markets: Analytical Results

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Abstract—Wind generation must trade in forward electricity markets based on imperfect forecasts of its output and real-time prices. When the real-time price differs for generators that are short and long, the optimal forward strategy must be based on the opportunity costs of charges and payments in real time rather than a central estimate of wind output. We present analytical results for wind's optimal forward strategy. In the risk-neutral case, the optimal strategy is determined by the distribution of real-time available wind capacity, and the expected real-time prices conditioned on the forward price and wind out-turn; our approach is simpler and more computationally efficient than formulations requiring specification of full joint distributions or a large set of scenarios. Informative closed-form examples are derived for particular specifications of the wind-price dependence structure. In the usual case of uncertain forward prices, the optimal bidding strategy generally consists of a bid curve for wind power, rather than a fixed quantity bid. A discussion of the risk-averse problem is also provided. An analytical result is available for aversion to production volume risk; however, we doubt whether wind owners should be risk-averse with respect to the income from a single settlement period, given the large number of such periods in a year.

Index Terms—Power generation economics, Wind power generation, Risk analysis.

I. INTRODUCTION

THE penetration of wind generation is increasing in power systems worldwide. In contrast to conventional plant, whose availability is mainly a matter of mechanical availability, the availability of wind generation capacity is primarily determined by the weather.

As a consequence, the statistical properties of wind capacity availability are very different from conventional plant, both on planning and operating timescales. On a planning timescale, this manifests itself as a probability distribution for available capacity at some distant time in the future. On an operating timescale, which will be considered here, the relevant probability distribution is that for wind out-turn conditional on the information available when decisions are taken; a wind generation owner cannot contract a level of output in a forward market and be sure that this precise amount will be delivered.

When generators contract to sell power in forward markets, imbalances between the forward-contracted volume and actual output must be rectified in the real-time market. In some systems, there is a single real-time price seen by generators irrespective of whether they are short or long with respect to their forward-contracted volumes [1]. In others, there are different real-time prices for generators that are short or long. Examples include the Great Britain [2], Scandinavian [3] and Iberian [4] markets. Forward market bidding by wind generation is thus a problem of optimisation under uncertainty regarding both the production out-turn and the real-time price, as the forward market bid must be made before precise information on these real-time quantities is available.

Optimal trading strategies for wind generation have received comparatively little attention in the literature. [5] introduced the concept of optimising the forward market bid based on imbalance costs, in both risk-neutral and risk-averse formulations. [6] then introduced a closed form expression for the optimal contract volume in the forward market given the forward and real-time prices; the same optimisation approach was used in [7], which compares strategies based on point prediction and probabilistic wind forecasts, and also discusses risk-aversion. [3] presented a stochastic mixed integer linear program (LP) formulation, based on representing the uncertainty in wind out-turn using a finite number of discrete scenarios; this has been extended to coordinated operation of wind and hydro generation in [8]. This LP approach was extended from a two-market system (forward and real-time) to a three-market (day-ahead, adjustment and real-time) system in [4]; in addition, a more efficient continuous LP model was presented, in which risk-averse preferences could be modelled. Other relevant work includes a detailed discussion of utility metrics for risk-averse traders [9], assessment of the cost of wind forecast errors [10] and real-time prices [11], advanced wind forecasting methods [12], operational strategies coordinating wind generation with reserves, storage and hydro (e.g., [13]–[15]), and designing power markets to minimise imbalance costs to wind generators [16]. Relevant market modelling results have been presented in [17] (the effect of wind forecasts on forward market prices), and [18] (the statistical relationship between real-time and forward prices).

This paper presents closed-form methods for optimising a wind generation owner's forward market bid in a two-settlement (forward and real-time) system. These require either evaluation of a single formula, or numerical solution of a specified integral or equation. New results are presented for uncertain real-time prices that are correlated with the forward price and real time (RT) wind output, and also for risk-averse bidding strategies; for the simpler case of uncorrelated wind out-turn and real-time prices, the more general derivation clarifies that the expected (in the mathematical sense) real-

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time prices should be used as the price forecast in the relevant expressions. The benefits of analytical methods over previous approaches requiring solution of LP models are:

- *Transparency of results.* In closed form expressions and direct formulas, it is much easier to identify what parameters drive the results obtained.
- *Computational efficiency.* The expressions presented here may be evaluated more quickly than LP models.
- *Reduced data requirements.* The methods in this paper require only specification of expectation values for real-time prices conditioned on the forward price and wind out-turn, not full joint distributions or a scenario tree.

An analytical approach is particularly valuable when the optimal bidding strategy model must be embedded in a larger model, perhaps of the entire power market, or of generation investment decisions. In contrast, it is highly undesirable to have to embed a large stochastic LP model for wind's strategy in such a whole-market model. This paper provides a computationally simpler alternative.

Section II describes the simplest case of a single real-time price. Section III then describes how multiple real-time prices are used in some systems, and presents the optimal bid strategy in this case. Sections IV-VI illustrate how this may be applied, including cases with both independent and correlated wind output and real-time prices, and also known or uncertain forward price. In the usual case of uncertain forward prices, the optimal bidding strategy generally consists of a bid curve for wind power, rather than a fixed quantity bid. Finally Section VII generalises some of the results to the case of aversion to the risk of low real-time wind output. Throughout, examples are based on British data; the methods may also be applied to related market designs such as in Scandinavia [3].

II. OPTIMAL VOLUME: SINGLE REAL-TIME PRICE

A. Forward Trading Based On Opportunity Cost

Like any other form of generation, a wind generator may contract in the forward market to deliver power in real-time, whether this be through a bilateral trading or pool system. It might then deviate from its forward-contracted position, with any resulting system imbalance being rectified in the real-time market. In contrast to conventional generation, however, wind in real-time will almost always generate at precisely its maximum available capacity due to its very low marginal cost of production. Exceptions may include curtailment on system security grounds at very high penetrations, or where there is limited transmission capacity. This paper assumes competitive markets in which individual wind owners are price-takers, and wind always generates at available capacity.

Optimising wind's strategy in the forward market is a non-trivial problem based on uncertainty in real-time wind out-turn and power prices. The *opportunity* cost of a forward contract in terms of foregone revenue in the real-time market in the real-time market is then relevant, rather than the negligible internal marginal cost of supply. Opportunity costs are relevant in many aspects of power systems operations and bidding [19], [20], such as:

- Bidding in ancillary services markets based on revenue foregone in the energy market.
- Bidding in one geographical location's market based on revenue foregone by not selling in other areas.
- Arbitrage between forward and real-time markets (called "virtual bidding" in U.S. markets).
- Hydro power with a finite water resource; bids would reflect revenue sacrificed at other times.
- Finite pollution permits for fossil fuel units.
- For peaking units, if a maximum number of starts per year under maintenance contracts.

In all these situations, selling in one product market at one time or place means that selling in other markets may not be possible, and the cost of selling in the former should in part reflect foregone revenues in the latter.

B. Single Real-Time Price

The simplest case of the forward contracting problem considers a wind operator selecting a contract volume q with forward price π_F (often called the system marginal price, SMP, or market clearing price, MCP), with a single real-time (RT) price[1] Π applied to all imbalances. Π and production out-turn W (conditioned on the available information at time of contracting) are random variables with joint probability density function $f_{\Pi,W}(\pi, w)$. The expected total income is then

$$\pi_F q + \int \int \pi(w - q) f_{\Pi,W}(\pi, w) \mathrm{d}\pi \mathrm{d}w. \tag{1}$$

Differentiating with respect to q to find the optimal bid $\hat{q}$ (assuming no bounds on $\hat{q}$):

$$\pi_F = \int \int \pi f_{\Pi,W}(\pi, w) \mathrm{d}\pi \mathrm{d}w = E[\Pi]. \tag{2}$$

This is a standard result (assuming risk-neutral behaviour by market players) that arbitrage between the RT and forward markets means that the forward price is the expected RT price.

Similar undramatic results may be obtained from various generalisations of the single real-time price problem, such as a market where a number of identical wind owners bid simultaneously, and where the real-time price is obtained endogenously within the model via a supply curve for conventional generation. These generalisations will not be discussed further in this paper, because the final result is not very exciting, while the necessary algebra can become quite complex.

III. QUANTITY BIDDING: DIFFERENT SHORT AND LONG PRICES

In this section, we consider the problem of the optimal quantity to schedule in the forward market, in the face of a known forward price and uncertain balancing prices that differ depending on whether the generator is short or long in the real-time market. In Section V, we generalise this to bidding in a forward market in which the forward price is unknown; in general, this can result in a wind generator submitting a sloped bid curve rather than a fixed quantity.

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[1]Throughout, the convention that lower case letters (e.g., w) denote numbers and capital letters (e.g., W) denote random variables; the probability density function for W is $f_W(w)$.

A. Introduction: GB Market Design

In some power markets the prices felt by generators or trades in the real-time market differ depending on whether they are short or long. Examples of this setup include the British [2], Nordic [3] and Iberian [4] markets. If the wind generator is short/long in real-time (delivers more/less than the forward-contracted volume), then this imbalance will be rectified at the short price π_S / long price π_L; these may respectively be greater/less than the forward market price π_F. This differentiation is justified on the grounds that if a generator is in imbalance then the System Operator (SO) has to take actions to make good the imbalance of power, and that the cost of such actions should then be charged to those who caused them. It should be emphasised that in many (perhaps even a majority of) power markets worldwide, wind is not penalised in this way for being short or long; the discussion in Section II-B of a single real-time price then applies [1].

1) Great Britain (GB) Market Design:

This section describes how prices are set in the Great Britain power market; the situation in Scandinavia is very similar (in terms of divergence of short and long prices in real-time; see Section II of [3].) The forward market in GB formally operates by bilateral trading (Chapter 10 of [21]); there is no centralised pool or power exchange. Trades between generators and demands are reported to the system operator (SO) by Gate Closure, which is one hour ahead of real-time. There are however independent power exchanges, and the SO publishes a forward price index as a time series for each half-hour settlement period, based on data from these exchanges.

In real-time, the market might either be long (total forward-contracted generation greater than out-turn demand) or short (contracted generation less than out-turn demand). Each period's short price (paid by trades which are short, called the system buy price in GB) and long price (paid to trades which are long, called in GB the system sell price) are defined as follows [22] (forward prices are based on market index data):

- Market short
  - Short price: average price of offers to increase output accepted by SO.
  - Long price: the forward price.
- Market long
  - SBP: the forward price.
  - SSP: average price of bids to reduce output accepted by SO (typically offers will be to *pay* the SO, as a reduction in output represents a saving on fuel costs).

Imbalances in the opposite direction to the market are thus settled at the forward price. The given justification for this is that such imbalances are helpful to the market, and should therefore not be penalised relative to the income from a perfectly balanced contract.

2) Great Britain Historic data:

The historic short, long, and forward market index prices from 2008 are displayed in Fig. 1. In order to reveal trends as demand varies, the half hour periods are ranked in order of increasing demand, and a 101-point moving average is
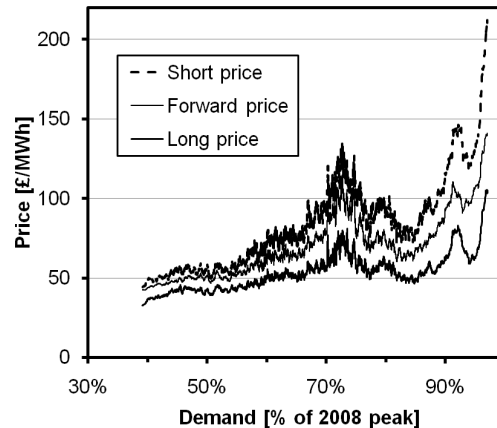

Fig. 1. Smoothed short and long prices, and forward Market Index Price (MIP) from the GB market in 2008. The raw price data is taken from [23], and demand data from [24].

then taken[2]. As each time series includes the data from all half hour periods (not differentiating between hours when the market was long or short), this graph presents the data as seen at the time of the forward market when players do not yet know whether the market will be long or short. As expected, the overall trend is for all the prices to increase as demand increases. However, there are clear local maxima in the price-demand curves at around 72% and 92% of peak demand, and a less prominent one at 79%.

B. Quantity Bidding: Risk-Neutral Case

1) Model Definition:

If the wind owner is willing to make decisions on the basis of its expected income, then it is termed risk-neutral. If it demands a risk premium due to the possibility of its income being less than the expected value then it would be termed risk-averse [25]. It is assumed here that the wind generator is indeed risk-neutral; due to the large number of time periods over which annual profit is measured, provided there is no systematic bias in the probability distributions used, any random fluctuations away from the mean will cancel to a good approximation.

In this risk-neutral case (and indeed if the wind generator is assumed averse to quantity risk but neutral to price risk), it is necessary to specify only expectation values for the real-time prices conditional on wind out-turn, rather than a full joint probability distribution. Working in terms of expected values for prices does not sacrifice any generality, save for these issues around risk-aversion.

The forward price is assumed to be known precisely at the time the forward contract volume is chosen (bid curves for the more general case of uncertain forward price will be discussed in Section V.) The wind owner must therefore decide on its trading strategy in the forward market, given the uncertainty in its output in real-time and the real-time prices. The generation out-turn W and the real-time prices (short price Π_S, and

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[2]Points are not plotted for the highest and lowest 50 demands, as the moving average would then be unbalanced.

long price Π_L), conditioned on the information available when trading in the forward market, are then modelled as random variables. In GB these expected prices must take into account that, if the wind owner is in imbalance in the opposite direction to the market, then in real time the owner pays or is paid the forward price. It is convenient to define the following expected short and long penalties

$$E[\Delta_S|w,\pi_F] = E[\Pi_S|w,\pi_F] - \pi_F \qquad (3)$$
$$E[\Delta_L|w,\pi_F] = \pi_F - E[\Pi_L|w,\pi_F], \qquad (4)$$

where $E[\Pi_{(S,L)}|w,\pi_F]$ are the expected real-time prices conditioned on wind out-turn and forward price.

2) Derivation of Optimal Forward Volume:

The expected net revenue, which the risk-neutral wind owner seeks to maximise, is then

$$
\begin{aligned}
E[R] &= \pi_F q - \int_0^q E[\Pi_S|\pi_F,w](q-w)f(w)\mathrm{d}w \\
&\quad + \int_q^{w^+} E[\Pi_L|\pi_F,w](w-q)f(w)\mathrm{d}w \qquad (5) \\
&= \pi_F E[W|\pi_F] - \int_0^q E[\Delta_S|\pi_F,w](q-w)f(w)\mathrm{d}w \\
&\quad - \int_q^{w^+} E[\Delta_L|\pi_F,w](w-q)f(w)\mathrm{d}w, \qquad (6)
\end{aligned}
$$

where $f(w)$ is the estimated probability density function for real-time output. This differs from expressions in [6], [7] in that it does not assume independence between real-time prices and wind out-turn, and from the equivalent expression in [3] in that it uses continuous probability distributions rather than discrete scenarios. This latter difference enables us to derive closed-form solutions as described below, this type of derivation not being possible in the scenario approach. Differentiating, the optimal volume $\hat{q}$ is

$$\int_0^{\hat{q}} \mathrm{d}w E[\Delta_S|\pi_F,w]f(w) = \int_{\hat{q}}^{w^+} \mathrm{d}w E[\Delta_L|\pi_F,w]f(w). \qquad (7)$$

The key features of this expression are as follows:

- In order to evaluate the optimal forward contract volume, it is sufficient to specify just the expected real-time prices/penalties, conditioned on the forward price and wind out-turn. This is much more straightforward than specifying a full joint distribution for prices and wind out-turn, or specifying a large set of discrete scenarios.
- (7) may be solved using direct numerical methods (numerical integration and equation solving). It is not necessary to use a mathematical optimisation algorithm, which is required in LP formulations such as [3], [4].

The imbalance prices clearly also depend on the total real-time imbalance volume. The model accounts implicitly for this dependence through the conditional probability distributions for prices; this reflects the fact that the market imbalance volume does not affect the wind-owner's behaviour directly, but rather through its effect on real-time prices.
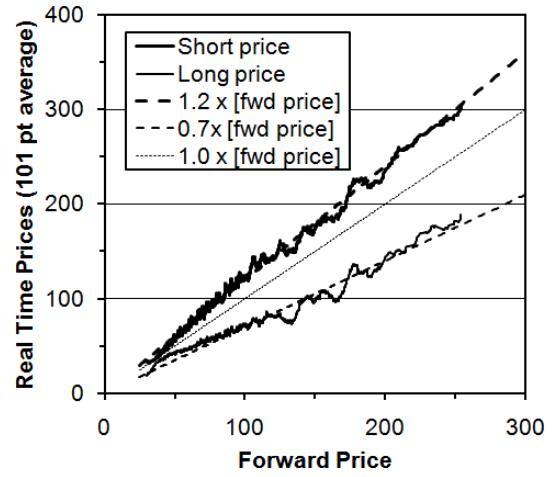

Fig. 2. Expected real-time prices as a function of forward price in Great Britain.

IV. APPLICATION I: UNCORRELATED WIND OUT-TURN AND REAL-TIME PRICES

A. Expression for Optimal Forward Contract Volume

If the short and long penalties are independent of the wind out-turn, then the expression for the optimal forward contract quantity $\hat{q}(\pi_F)$ simplifies to

$$p(W \le \hat{q}(\pi_F)) = \frac{E[\Delta_L|\pi_F]}{E[\Delta_S|\pi_F] + E[\Delta_L|\pi_F]}, \qquad (8)$$

where $E[\Delta_{(S,L)}|\pi_F]$ is the expected value of $\Delta_{(S,L)}$ given forward price π_F. This expression fits with intuition; for instance, if the expected short price is very high, then the optimal forward volume is small, reducing exposure to this high short price.

This expression is similar to those in [6], [7]. The derivation here makes it explicit that expected penalties given the forward price should be used, clarifying the statement in [7] that real-time price 'forecasts or estimates' must be used.

The formula for the optimal volume is completely closed-form, with the benefits of transparency and minimal computational effort that this brings. However, with a substantial installed wind capacity and consequent large forecast errors in MW terms, independence of real-time prices and wind out-turn might not be a good approximation; in this case the more generally applicable expressions in Section VI are of greater relevance.

B. Examples

1) Wind Owner in Great Britain:

Expected real-time prices conditioned on the forward price for Great Britain in 2008 are displayed in Fig. 2, the data used being the same as in Fig. 1. These expected RT prices are calculated by ordering half hour periods by forward price and taking a 101 point moving average. This carries an implicit assumption that the expected RT prices depend only on the forward price, with no seasonal and diurnal effects; comparison with plots considering one season only shows that this is reasonable for an illustrative example such as here. A
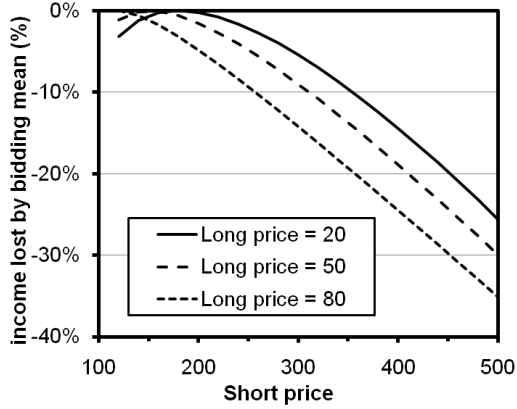
Fig. 3. Percentage decrease in expected revenue from forward-contracting expected out-turn, compared to revenue from the optimal strategy presented here. The out-turn wind load factor conditioned on the information available at time of forward contracting follows the $N(0.5, 0.125)$ distribution.

full analysis of the historic price data from GB is beyond the scope of this paper, and will be addressed in future research.

Over a wide range of forward prices the expected real-time, short and long prices are approximated well by $1.2\pi_F$ and $0.7\pi_F$ respectively, giving $E[\Delta_S|\pi_F] \simeq 0.2\pi_F$ and $E[\Delta_L|\pi_F] \simeq 0.3\pi_F$ (the highest and lowest forward prices require a more detailed statistical treatment.) Fig. 2 also shows the line RT price = forward price (i.e., no penalties); the vertical distance between that line and the lines $1.2\pi_F$ and $0.7\pi_F$ is equal to the penalty for being short and long respectively. Over this range, the ratio of the short and long penalties remain constant, and the optimal contract volume is given by $p(W \leq \hat{q}) = 0.6$. The actual MW optimal volume depends on the probability distribution for wind out-turn.

This penalty structure has not remained constant over the years. In the early years of the present market structure, the short penalty was typically higher. For instance, in 2004, an equivalent analysis would give an expected short penalty of approximately $0.4\pi_F$, with the expected long penalty again around $0.3\pi_F$.

2) Benefits of Optimal Strategy:

The benefits of following this optimal strategy, instead of forward-contracting expected out-turn, are demonstrated in Fig. 3. The wind load factor here is normally distributed with mean 0.5 and standard deviation (SD) 0.125. This distribution is consistent with the day-ahead forecast accuracy used in [3], and is similar to four-hour ahead data from GB. The forecast SD in GB is, however, about half the size at Gate Closure 1 hour ahead.

The forward price is assumed to be 100 units, with expected long prices down to a factor of 5 below this, and expected short prices up to a factor of 5 above. It may be seen that for small expected penalties (differences between forward and expected real-time prices), the benefits of the optimal strategy are not very great. With a moderate short price, optimising under a very low long price does not provide benefits of more than a few percent over the alternative of contracting expected out-turn volume. However, if the expected short price is very

high, then the benefits of the optimal strategy can be very substantial indeed. This greater potential effect of the short price is simply due to the expected long price usually being constrained to values above zero, the most common exception being when wind is the marginal generator.

V. FORWARD MARKET BID CURVES

A. Uncertain Forward Price in a Pool of Exchange

If the forward market price is known with certainty at the time the bidding strategy is devised, then the above approach of contracting a fixed volume is sufficient. Also, as seen in (8), if the ratio of expected short and long penalties is independent of forward price, then the same forward contract volume will be optimal irrespective of the forward price.

In general, however, the optimal forward contract volume can depend on the out-turn forward price (and the resulting expected penalties). If a bid must be submitted to a pool or power exchange, then at the time the bid is devised the forward price cannot be known with certainty; account should be taken of this when devising the form of the bid. The result is that, in general, a sloped bid curve is optimal, in which the forward quantity bid is a function of the forward price. This curve reflects the opportunity cost of selling in the forward market rather than the real-time market.

B. Dependence of Optimal Quantity on Forward Price

The optimal bid quantity (8) may be rearranged as

$$p(W \leq \hat{q}(\pi_F)) = \left(1 + \frac{E[\Delta_S|\pi_F]}{E[\Delta_L|\pi_F]}\right)^{-1}. \qquad (9)$$

Thus, for a given forward price, the optimal quantity is determined by the ratio of the expected short and long penalties. As the forward price increases, the ratio $E[\Delta_S|\pi_F]/E[\Delta_L|\pi_F]$ may increase, stay constant, or decrease. The optimal forward volume then has the following relationship with that ratio:

- If the ratio *increases*, then the optimal forward volume decreases;
- If the ratio *is constant*, then the optimal forward volume is constant;
- If the ratio *decreases*, then the optimal forward volume increases.

Fig. 4 shows examples for each of these, with $E[\Delta_S|\pi_F] = 0.2\pi_F$, and $E[\Pi_S|\pi_F] = 0.3\pi_F + x$ where $x = 0$ (i.e., ratio constant), 10 (i.e., ratio increasing) and -10 (i.e., ratio decreasing).

C. Derivation of Bid Curves

A bid curve for a pool or power exchange may be derived by swapping the axes on a plot of optimal quantity versus forward price; the bid curve indicates what price is required for the generator to supply a given quantity in the forward market. For the cases where the ratio of expected short to long penalties is constant with price (i.e., a fixed optimal volume independent
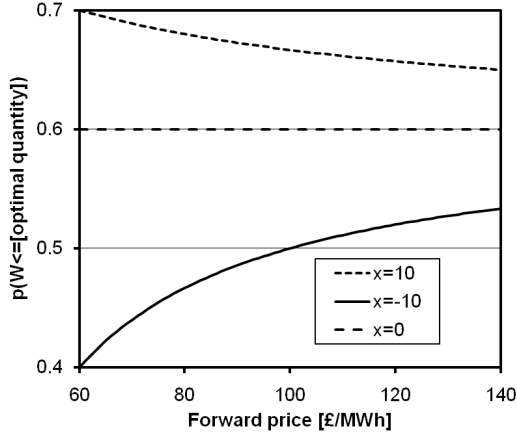
Fig. 4. Optimal bid volume as a function of forward price for expected short price $E[\Delta_S|\pi_F] = 0.2\pi_F$ and expected long price $E[\Delta_L|\pi_F] = 0.3\pi_F + x$, where $x = (-10, 0, 10)$ /MWh.

of forward price, $x = 0$ in Fig. 4), or decreasing (implying optimal volume increasing with forward price, $x = -10$), the bid curve will have a typical non-decreasing form.

The case where the ratio of expected short to long penalty increases as the forward price increases, however, implies a bid curve where price required decreases as quantity increases ($x = 10$). This could not occur where the bid strategy reflects internal short run marginal costs of supply, as then the generator would never wish to decrease its supply quantity as price increases. Indeed, in some markets only non-decreasing bid curves are permitted, as a protection against exercise of market power. For the situation here where the forward market strategy is not based on internal supply costs, the bid curve can in principle be negative sloping for certain penalty structures, as demonstrated in (8) and Fig. 4. It should be further noted that the forward contract volume does not usually affect wind's behaviour in real time, when it will generate at the maximum available output level.

VI. APPLICATION II: CORRELATED WIND OUT-TURN AND REAL-TIME PRICES

This section presents two applications of the model developed in Section III-B, in which the wind out-turn and real-time prices are not independent. Both provide closed-form results for the optimal bid volume, which give insight into drivers of the optimal bidding behaviour.

A. Expected Penalties Based on Thermal Supply Function

Suppose that the GB wind owner's portfolio is well-distributed across the country, so that if its output is w then the total GB wind output is Nw for some constant N. If there were a single real-time price in a perfectly competitive market, this would then be $\pi_R = \pi_T(d - Nw)$, where $\pi_T(q)$ is the marginal cost of supply by thermal generation at quantity q.

As described above, the GB market does not have a single real-time price. However, one can gain insight into how wind out-turn affects the optimal bid strategy by assuming

a dependence of the short and long penalties on the wind out-turn based on the competitive (price-taker) case.

The thermal supply quantity resulting from the expected wind out-turn is $q = d - N\mu_W$. Making a quadratic approximation to the thermal supply function about this point, we obtain:

$$
\begin{aligned}
\pi_T(q) \quad \simeq \quad & \pi_T(d - N\mu_W) + c_1(q - (d - N\mu_W)) \\
& + c_2(q - (d - N\mu_W))^2.
\end{aligned} \tag{10}
$$

For a convex thermal supply function, the constants c_1 and c_2 are positive. Assuming the same form for the dependence of the short and long prices on the wind out-turn,

$$
\begin{aligned}
E[\Delta_S|\pi_F, w] \quad = \quad & E[\Delta_S|\pi_F, w = \mu_W] - c_1 N(w - \mu_W) \\
& + c_2 N^2(w - \mu_W)^2 \tag{11} \\
E[\Delta_L|\pi_F, w] \quad = \quad & E[\Delta_L|\pi_F, w = \mu_W] + c_1 N(w - \mu_W) \\
& - c_2 N^2(w - \mu_W)^2. \tag{12}
\end{aligned}
$$

For simplicity, this assumes that if the wind generator is well below (above) its expected real time output then the market is definitely short (long); for the illustrative example here this assumption is reasonable at high wind penetrations.

The optimal bid quantity $\hat{q}$ is then given by

$$
p(W \leq \hat{q}) = \frac{E[\Delta_S|\pi_F, W = \mu_W] - c_2\sigma_W^2}{E[\Delta_S|\pi_F, W = \mu_W] + E[\Delta_L|\pi_F, W = \mu_W]}. \tag{13}
$$

Comparison with the expression for uncorrelated real-time prices and wind out-turn (8) shows that the '$-c_2\sigma_W^2$' term acts as a correction to the numerator of the expression for $p(W \leq \hat{q})$, decreasing the optimal forward contract volume if the thermal supply curve is convex (i.e., positive second derivative). This fits with intuition, as with a convex thermal supply curve the real-time prices increase more when wind is short than they decrease when wind is long. Hence wind should tend more towards being long to avoid this enhanced short penalty. This substantial correction term appears despite the assumption that the wind owner's behaviour in the forward market does not affect the forward price.

In competitive markets, it is unusual to assume that one player knows in advance that all competitors will make the same decision, and furthermore knows what that decision will be. However, in this case the wind owners do not really make a decision regarding their output, rather they all inevitably generate at the maximum available capacity due to having a negligible short-run marginal cost of supply in real time. The correction $-c_2\sigma_W^2$ above follows from an assumption that the day-ahead error in forecasting the wind owner's out-turn load factor follows the same probability distribution as the error in forecasting the overall GB wind out-turn load factor. However, in fully realistic cases where this statistical relationship is less strong, this correction term will be reduced somewhat.

B. Dependence Structure From a Multivariate Normal Distribution

This example demonstrates a more flexible approach to modelling the dependence structure, while still providing a

direct method for obtaining the optimal forward contract quantity through numerical solution of an equation.

The wind out-turn is assumed to follow a normal distribution, with probability density function

$$f_W(w) = \frac{1}{\sigma_W \sqrt{2\pi}} \exp\left[-\frac{(w - \mu_W)^2}{2\sigma_W^2}\right]. \quad (14)$$

The expected short penalty given wind out-turn w is then

$$E[\Delta_S | W = w] = \mu_S + \left(\frac{\rho_S \sigma_S}{\sigma_W}\right)(w - \mu^W). \quad (15)$$

A similar expression holds for the long penalty in terms of parameters ρ_L, μ_L and σ_L. In a multivariate normal distribution, these would be the correlation coefficient between wind out-turn and prices, and the mean and standard deviation of the prices, but this assumption of the distribution's precise form is not required for the results which follow. The specification of parameters in (15) must account for the possibility of the real-time price being set equal to the forward price if the wind owner is in imbalance in the opposite direction to the market.

Substituting in (7), assuming that the probability of output near maximum or minimum is negligible, the following equation for the optimal forward contract volume $\hat{q}$ is obtained:

$$(\mu_S + \mu_L)p(W \le \hat{q}) =$$
$$\mu_L + \frac{(\rho_L \sigma_L + \rho_S \sigma_S)}{\sqrt{2\pi}} \exp\left[-\frac{(\hat{q} - \mu_W)^2}{2\sigma_W^2}\right] \quad (16)$$

Once again, this may be regarded as adding correction terms to the 'uncorrelated' case (8). It does not provide a completely closed-form expression for the optimal quantity, but this may be evaluated by direct numerical solution of the equation. It is possible to analyse the effects of the relationship between the wind out-turn and prices as follows:

- If the short penalty is negatively correlated with the wind out-turn (which is likely to be the case) then, compared to the independent case, the wind owner reduces the forward contract volume at a given price. This behaviour mitigates the consequences of an increased short price when the wind out-turn is low.
- If the long penalty is positively correlated with the wind out-turn (which is likely to be the case) then, compared to the independent case, the wind owner increases the forward contract volume at a given price. This behaviour mitigates the consequences of a decreased long price when the wind out-turn is high.

As a result, the behaviour of wind relative to the independent case is determined by the relative volatilities of the short and long prices, as well as by the degree to which they are correlated with the wind out-turn.

VII. RISK-AVERSE TRADING BEHAVIOUR

A. VaR and CVaR

The previous results have all been for risk-neutral behaviour, where the wind owner seeks to maximise expected revenue, without any consideration of variance of revenues or of how far the revenue can decrease below that expected value. We now relax that assumption and consider risk-averse behaviour. The most common measures used to analyse risk-averse trading strategies are Value at Risk (VaR) and Conditional Value at Risk (CVaR) [26].

The VaR for the revenue R at the α confidence level is defined as the revenue r_α such that $p(R \le r_\alpha) = 1 - \alpha$. The CVaR for the revenue R at the α confidence level is then defined as the expected revenue conditioned on the revenue being below r_α, i.e., in mathematical notation the CVaR is $E[R | R \le r_\alpha]$.

CVaR is more commonly used as the objective function when optimising risk-averse strategies, because unlike VaR it exhibits appropriate mathematical behaviour under general conditions ('coherence') [26], and can be included within LP formulations [27]. CVaR also allows a smooth transition from risk-neutral behaviour (i.e., $\alpha = 0$) to increasing degrees of risk aversion by varying the parameter α.

B. Risk-averse Forward Trading Strategy

Our approach of deriving closed-form expressions may be extended to risk-averse behaviour, provided that the long and short real-time prices can be treated using expected values. The wind owner would then be assumed to be averse to quantity risk but not price risk. Aversion to price risk requires evaluation of various double integrals of the joint probability distribution for prices and wind out-turn. While this full risk-averse problem is numerically tractable, the degree of transparency seen in other results from this paper is not available.

Given the large number of time periods over which a trading strategy will be used, in practice the degree of risk aversion that wind owners demonstrate in any single period may be limited; in the long run, a large degree of cancellation is to be expected between the consequences of below-average and above-average wind out-turns for single periods. A case might be made that aversion to price-risk is equally important as quantity risk, due to the possibility of extreme spikes in the short price. Given that the GB market as a whole is typically long, further investigation of whether (and in what way) price risk aversion is exhibited would be valuable. However, the greatest long-run risk could well be systematic error in the estimated wind distribution, so that the calculated optimal quantity has a bias towards being either above or below the true optimum. In this case, quantity risk aversion could be more important.

Based on this limited picture of aversion to wind out-turn risk, assuming a known forward price π_F and real-time prices independent of wind out-turn, the CVaR at the α confidence level is

$$\begin{aligned} \text{CVaR}_\alpha &= \pi_F q - \frac{E[\Pi_S | \pi_F]}{1 - \alpha} \int_0^q \mathrm{d}w(q - w)f(w) \\ &\quad + \frac{E[\Pi_L | \pi_F]}{1 - \alpha} \int_q^{w_\alpha} \mathrm{d}w(w - q)f(w), \quad (17) \end{aligned}$$

where $p(W \le w_\alpha) = 1 - \alpha$. It is also assumed that the optimal forward quantity is less than w_α (which is reasonable for small α). Differentiating to obtain the optimal bid volume $\hat{q}$:

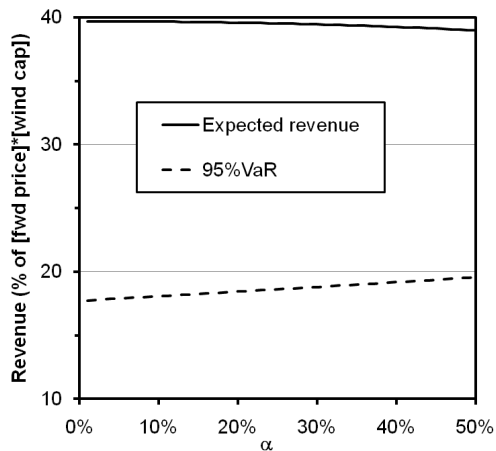$$p(W \le \hat{q}) = (1 - \alpha)\frac{E[\Delta_L | \pi_F]}{E[\Delta_S | \pi_F] + E[\Delta_L | \pi_F]}. \quad (18)$$

Fig. 5. Expected revenue, and 95%-VaR, for varying risk aversion parameter α. The revenue is measured in units of [forward price] multiplied by [installed wind capacity].

The optimal forward contracted volume decreases as the degree of risk aversion increases. This is as anticipated since, under risk aversion, the increased expected utility resulting from long payments does not compensate for the loss of expected utility resulting from penalties for being short. That is, poorer outcomes are weighted more highly than good outcomes under risk-aversion.

C. Example

This section presents an illustrative example, with parameters based on wind and price data from GB. The short and long prices are assumed to be respectively 120% and 70% of the forward price, based on the data in Fig. 2. The out-turn wind load factor is assumed to have a normal distribution with mean 0.42 and standard deviation 0.12. These parameters are based on the variation in wind out-turn for those hours in 2008 during which the 4 hour ahead load factor was between 40% and 50%.

Results using the wind-volume-risk-averse strategy (18) are shown in Fig. 5. When the risk aversion parameter α is below 50%, a small decrease (less than 2%) in the expected revenue is seen. As a measure of the positive effect of the risk-averse strategy, VaR at the 95% confidence level is also plotted (VaR is used instead of CVaR here because of its more intuitive interpretation.) Over the same range of α, the gain in the 95%-VaR is slightly larger in absolute terms (and considerably larger in relative terms) than the decrease in expected revenue; a similar result has been noted in [4]. However, as discussed in Section VII-B, it is doubtful whether looking at individual periods in isolation is the correct approach to considering risk aversion in this application.

VIII. Conclusions

We have presented new analytical expressions for determining the optimal forward market strategy for wind generators, including cases where the real-time prices and real-time production are not independent, and where the wind owner is averse to production volume risk. Examples based on data from Great Britain have been presented. The methods are also directly applicable to other markets such as those described in [3], [4].

The new methods presented here require specification only of the expected real-time prices given the forward price and wind out-turn; this is simpler than specifying a full joint distribution or a representative finite set of scenarios. The new analytical approach is also much more computationally efficient than previous linear programming formulations. Moreover, in the risk-neutral case the new approach does not sacrifice any generality in the optimisation problem for a single period. When the forward price is unknown, the result is in general a sloped bid curve, reflecting the opportunity cost of trading in the forward as opposed to real-time market.

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