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Collaborative Learning with Multi-Touch Technology: Developing Adaptive Expertise

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Abstract

Developing fluency and flexibility in mathematics is a key goal of upper primary schooling, however, while fluency can be developed with practice, designing activities that support the development of flexibility is more difficult. Drawing on concepts of adaptive expertise, we developed a task for a multi-touch classroom, NumberNet, that aimed to support both fluency and flexibility. Results from a quasi-experimental study of 86 students (44 using NumberNet, 42 using a paper-based comparison activity) indicated that all students increased in fluency after completing these activities, while students who used NumberNet also increased in flexibility. Video analysis of the NumberNet groups indicate that the opportunity to collaborate, and learn from other groups expressions, may have supported this increase in flexibility. The final phase of the task suggests future possibilities for engaging students in mathematical discourse to further support the development of mathematical adaptive expertise.

Keywords: Adaptive Expertise, Collaborative learning; Group work; Computer Supported Collaborative Learning, Mathematics learning.

1. Introduction

Developing fluency and flexibility with mathematical constructs and skills is a key goal of primary education in the UK, aiming to provide students with a solid basis to understand more complex mathematics concepts in later years. However, while fluency with the application of standard procedures can be attained through sustained practice (Doyle, 1983), developing flexibility is more complex (Greeno, 1991). In this paper, we describe a tool, NumberNet, that uses computer-supported collaborative learning activities to foster mathematical flexibility and reasoning, through a series of small group and whole class activities, contrasting its use with standard classroom activities to explore whether collaborative engagement in mathematics practice can support the development of flexibility.

Mathematics education in the primary years aims to teach students basic numbers and calculations and to prepare them to learn more complex mathematics by developing an understanding of arithmetic and numerical principles. In order to achieve these two goals, students need to become adept at applying standard procedures to anticipated problems, and also understand the range of possible procedures and strategies they can use when they encounter novel problems (Baroody, 2003). While developing ‘number-sense’, both flexibility and accuracy are seen as desirable outcomes for students learning mathematics, and are behaviours that are seen in adult mathematicians, our understanding of how a deep conceptual understanding of mathematics develops, and its relationship to mathematical practice, is not complete (DeHaene, 2011).

Preparing students to engage in more complex mathematics requires that we consider what mathematical expertise looks like. Researchers differentiate between two types of experts: routine experts, who can expertly apply formulae or procedures, although they lack a deep understanding of the structure of the discipline, and adaptive experts, who can flexibly approach novel problems and apply a range of solutions. Initially described by Hatano and Inagaki (1986) to differentiate between application of procedural and conceptual knowledge, the concept of adaptive expertise has become a challenge to those developing educational activities which support students in understanding the complexities of mathematics (De Smedt, Torbeyns, Stassens, Ghesquière & Verschaffel, 2010).

Conceived as the application of conceptual understanding of a discipline, adaptive expertise has been described as being beyond routine expertise, developing once routine expertise has been established (Salomon & Perkins, 1989), or as a different form of expertise. Schwartz, Bransford and Sears (2005) hypothesized that adaptive expertise, rather than being further along the expertise continuum than routine expertise, was a form of expertise that brought a dimension of innovation to routine expertise. This framework places innovation and efficiency as orthogonal constructs, and proposes that adaptive expertise emerges when learners balance efficient use of procedures with an innovative approach to problems. Thus, preparing students to be adaptive experts requires that they have opportunities to practice the application of procedures, and that they encounter situations within which they need to innovate and identify new solutions (Inagaki, Hatano & Morita, 1998). The concept of adaptive expertise as the balance between innovation and efficiency in problem solving aligns with the goals of primary mathematics, where fluency of efficiency with mathematical procedures needs to develop alongside a flexible, more innovative approach to problem solving.

Research on developing adaptive expertise in mathematics finds that primary-aged students can be supported in developing an understanding of mathematical concepts through exploration. Markovits and Sowder (1994) designed a three-month long curriculum for seventh-grade students, that focused on number magnitude, mental computation and computational estimation. The instruction provided opportunities to explore the relationships between numbers and a range of operators. When compared to students using a traditional curriculum, the students in the experimental condition were more likely to choose solutions to problems that indicated number-sense, differences that were still identified in a post-test six months after the instructional period had ended. Due to the nature of the instruction, as well as its relative brevity, the authors conclude that the students in the experimental condition were unlikely to have learned new procedures during the instruction, but rather, the experimental condition encouraged the development of a deeper conceptual understanding of the content they had already acquired, which allowed them to solve novel problems.

Similarly, Martin & Schwartz (2005), in studies teaching fractions to nine- and ten-year-olds, found that using relatively unstructured manipulatives (e.g. tiles) rather than well-structured manipulatives (e.g. pie pieces), resulted in better transfer to new problems. Giving students the ability to reconfigure the manipulatives meant that it took longer for the students to grasp the concepts initially, but supported a deeper understanding of the concepts, which they could then apply in novel situations. This suggests that rather than focusing on the most efficient way to teach, students should be given opportunities to make sense of the concepts, in order to prepare them for more complex problem solving.

While cognitive psychology has begun to unpick the nature of how to support the development of adaptive expertise in the individual learner, the concept of adaptive expertise, as defined by Hatano and Inagaki (1986) is inherently situated within the environment in which it is developed and used. The process of moving from novice to expert was described by Hatano and Inagaki as “novices become adaptive experts – performing procedural skills efficiently, but also understanding the meaning and nature of their object” (1986, pp. 262-623), indicating that adaptive expertise cannot be separated from the context in which it is applied. Although cognitive approaches describe the move to adaptive expertise as one that requires deep conceptual understanding, it is clear that this conceptual understanding must be rooted in an understanding of the practices of the discipline. Thus, understanding the development of mathematical adaptive expertise also requires an understanding of the environment within which the learning of mathematics occurs (Hatano & Oura, 2003; Verschaffel et al., 2009).

In 1988, Hatano described conditions under which the deep conceptual knowledge necessary for adaptive expertise was developed. Recognizing that the process of “constructing, elaborating or revising” a model (p. 57) is essential for the development of adaptive expertise, he noted the importance of motivation to engage in this process. This motivation comes from being surprised by incorrect predictions, perplexed by competing ideas or becoming aware of a lack of coordination between pieces of information. Hatano indicates that students must encounter novel problems, be encouraged to seek comprehension and be free of immediate drives for external reinforcement, which hinders the ability to focus on the complexity of problems. Additionally, Hatano notes the importance of dialogue between learners, which introduces more instances of surprise, perplexity and disco-ordination. These conditions describe the importance of environmental supports that contribute to the development of a adaptive expertise.

Yackel and Cobb (1996) used the term sociomathematical norms to describe what counts as appropriate mathematical discourse, which regulate the forms of mathematical argumentation and opportunities to engage with mathematical concepts in a particular classroom. Working with second and third grade teachers, they explored the development of these norms in classrooms committed to inquiry-based mathematics teaching. The authors report that providing the students with opportunities to make sense of the arguments of their peers, drawing on the classroom norms to reach higher levels of mathematical reasoning, supported increased sophistication and flexibility in their use of mathematical constructs. This emphasises the importance of the learning context and opportunities to engage in discussion about mathematics as important elements in the development of mathematical adaptive expertise.

The context within which mathematical adaptive expertise develops was described in detail by Boaler, studying a project-based mathematics class. Boaler, (1998, 2000) argues for the importance of understanding not only how to teach the procedures that students need to learn, but focusing on the mathematical practices that they develop while they are learning. She argues that the use of collaborative problem-based learning allowed students to develop a rich understanding of the discipline of mathematics, and become engaged in the practices of mathematics, as well as the procedures. It is in understanding these practices, and applying and adapting mathematical procedures, that the students were prepared for standardized tests and also for the adaptation of mathematical knowledge to real-life situations, which can be identified as adaptive expertise.

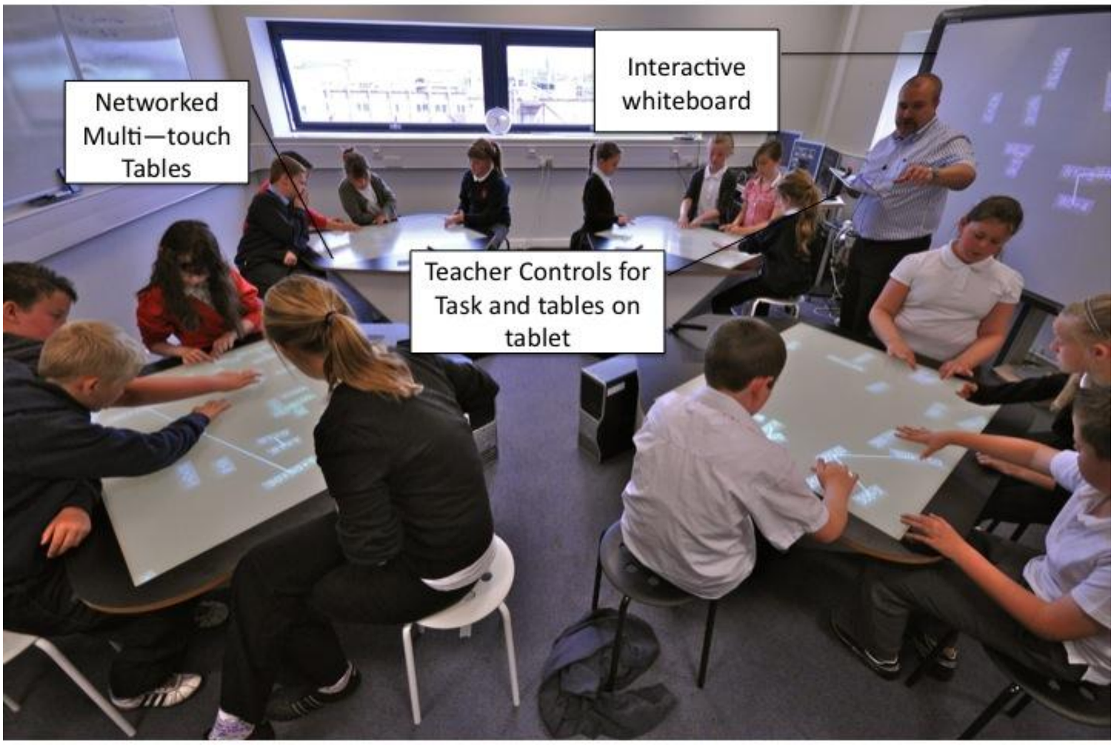
There is a long history of using collaboration to support the learning of mathematics, (e.g. Barron, 2003; Esmonde, 2009; Slavin & Lake, 2008; Webb &

Farivar, 1994). Many of these studies indicate that the process of collaboration can effectively support mathematical problem solving, and that this learning can be transferred to new tasks. As noted by Hatano (1988), the motivation to engage deeply with content, engaging in the types of learning that lead to adaptive expertise, can come from situations where the learner is required to reconsider their own conceptions of the material. Research on collaborative groups suggests that they can provide an opportunity for this type of engagement with content, as students encounter the ideas and questions of members of their group, forcing them to reconsider their own understanding, or consider the content in a deeper or more complex manner. However, for the most part, these studies focus on the workings of single groups of learners (c.f. Tolmie et al., 2010), with little opportunity for groups to learn from other groups within the same classroom, despite the recognition of the centrality of the classroom discourse and interactions in developing mathematical knowledge (Greeno, 1991).

By drawing on these cognitive and socio-cultural perspectives of the development of adaptive expertise, and our understanding of the value of collaborative learning to engage students more deeply in conceptual discussions, we hypothesized that engaging students in collaborative mathematical activities in a classroom setting would support the development of a flexible approach to mathematical calculations. The existing empirical evidence indicating that flexibility is an important and distinctive feature of being good at mathematics or having true mathematical expertise is described as ‘scarce’ (Verschaffel, Luwel, Torbeyns & Van Dooren, 2011) .Drawing on the affordances of the project’s multi-touch classroom (see Fig 1), we designed an activity to support within and between group learning, seeking to promote both the application of known procedures and the invention of novel calculations within collaborative groups.

The development of multi-touch surfaces has created an opportunity to embed computer-supported collaborative learning seamlessly into classrooms (Dillenbourg & Evans, 2011; Higgins et al, 2011). In the project's multi-touch classroom, four networked multi-touch student tables are controlled by a tablet, and can be projected to the classroom's multi-touch interactive whiteboard. As the tables are networked, the content from the tables can be passed between tables, which is under teacher control for this activity.

Figure 1: The Multi-touch Classroom



Research on collaborative learning using multi-touch tables is still in its infancy, although findings indicate that the use of multi-touch can promote more task-focused conversation, more equitable participation (Harris et al., 2009) and joint attention (Higgins et al, 2012). Using this type of technology to support within group collaborative learning, while leveraging the networking capabilities to create opportunities for interaction between groups and within the entire classroom, provides an opportunity to alter the way collaborative learning could be used in classrooms (Higgins et al, 2011).

NumberNet was designed to use the affordances of multi-touch to engage in a collaborative activity that would help students become more flexible in their use of mathematics, and also allow each group to learn from the other groups in the classroom. The final stage of the activity was designed to create an opportunity to engage in socio-mathematical discourse within the classroom.

Developed from a mathematics classroom task to “make up some questions” for a target answer, as recommended by the non-statutory guidance in the UK’s National Curriculum (DES, 1989, p. D7; see Fig. 4 for an example of this activity). The “make up some questions” task is typically assigned as an individual activity, where students are given a target number and asked to create as many expressions equivalent to that number as they can. This task is often used in primary classrooms in the UK as a warm-up activity, and provides the teacher with a snap-shot assessment of the students’ current capabilities.

Building on this task, NumberNet was designed so that the teacher assigns different target numbers to each table, asking groups at each table to create unique expressions for the target number and then rotates the target numbers, along with the correct expressions, to the next table (See Fig. 2 for a screen shot of NumberNet). By receiving a new number, for which the previous table has already created some expressions, the group now has a harder task, as the previous group may have started with the easiest expressions. However, this also provides the opportunity to learn from the expressions created by the previous group. Once each group in the classroom has an opportunity to work on all four of the numbers, the final mode of NumberNet can be activated. In this mode, the correct calculations remain on the tables (for the single target number displayed), and links can be created between the calculations. These links allow the students to impose a web-like structure on the calculations that they, and the groups at other tables, have created. The teacher can project any of the table displays to the interactive white-board, allowing all groups to examine the networks created by other groups, to facilitate a whole-class discussion around the structures of the number networks (See Figures 4a and 4b).

Figure 2: NumberNet Screen Shot

1.2. The Present Study

We investigated the impact of NumberNet on mathematical fluency and flexibility, comparing outcomes between students who used NumberNet and students who completed the individual “make up some questions” task. The goal was to explore whether the increased fluency and flexibility seen when using NumberNet (Hatch et al, 2011), was due to the effect of practicing creating expressions, or due to features within NumberNet. We also explored the processes through which learning with NumberNet might have occurred, looking at opportunities for within and between group learning in the NumberNet task. The research questions that are addressed in this paper are 1) do students who use NumberNet differ in either fluency or flexibility when compared with students undertaking a comparison activity and 2) what were the processes through which NumberNet might have influenced fluency or flexibility.

As noted in Strijbos and Fischer, (2007), there is an increased need to use mixed methods in research on collaborative learning, bringing together perspectives that explore the cognitive outcomes of a collaborative learning activity, and perspectives that explore the processes of knowledge creation during the learning activity. This seeks to bridge the acquisition and participation metaphors of learning (Sfard, 1998) by considering outcomes in relation to the interaction processes, and participation in social practices (Cobb & Bowers, 1999). In this paper, we used pre and post-tests to examine individual cognitive outcomes to compare NumberNet with the traditional “make up some questions tasks.” We also explored whether NumberNet supported the collaborative knowledge creation process through the use of video analysis and case studies of the groups in the NumberNet conditions so as to provide a bridge between the experimental data and inter-subjective perspectives generated during the course of the activities (Suthers, 2006): an important methodological dimension in computer-supported collaborative learning.

1.2.1. Hypotheses. In this study, we explored the following hypotheses: (1) using NumberNet increases mathematical flexibility and fluency when compared to the comparison, individual “make us some questions” task (hypothesis one) and (2) within the NumberNet sample, there will be identifiable learning opportunities, including within-group interaction and exploration of content created by another group (hypothesis two).

2. Methods

2.1 Design

This study was designed as a mixed-methods, quasi-experimental pre-post design. By using insights from both qualitative and quantitative approaches we explore "workable solutions" (Johnson & Onwuegbuzie, 2004, p.16) for the design of learning tasks for the development of mathematical adaptive expertise.

2.2 Participants

Data were collected from 91 children, with complete data from 86 students, in the penultimate year in primary school from four schools in England (Mean Age = 10 years, 2 months, SD = 4 months). Of these, students from two schools participated in the experimental condition, coming into the Multi-touch lab to use NumberNet, while students from the other two schools acted as the comparison group. The schools served similar populations, and were randomly assigned to condition, although attention was paid to pairing of schools, so that each condition had one school with a lower number of free lunch-eligible children (about 10%) and one school with a higher number (25-30%), and one school with a lower percentage of students attaining proficiency (75%) and one with a higher percentage (85-95%).

All students from the experimental schools were invited to participate in this research study. Thirty students from school one and 16 students from school two participated in the lab-based data collection. Forty-four of these students were present for the pre- and post-tests.

All the students who were present in the comparison schools on the first day of data collection were invited to participate in the study. Of those, forty-two of these students were present for the pre- and post-tests. Informed consent was collected from the students' parents or guardians.

2.3 Study Procedure

Participants in both conditions received an individual paper-based pre-test in their classroom one week before the intervention. The intervention then either took place in the Multi-touch classroom (see 2.3.1 NumberNet Protocol) or in the students' own classroom (see 2.3.2 Comparison Activity Protocol), after which the classes completed a distracter task and then the individual, paper-based post-test. Two members of the research team, one a former teacher, visited the schools and conducted the pre-tests and the comparison activity, distracter task and post-test. The NumberNet classes were led by another member of the research team, who was also a former teacher.

During the whole-class discussion in the NumberNet session, the teacher based the discussion around the patterns that the groups had found on the tables, projecting the table content to the interactive whiteboard. For the comparison activity, the teacher asked the students to identify any patterns that they had created in their expressions, to elaborate patterns from other students' expressions, or provide examples of different types of expressions. By design, this activity differed between the two conditions, however, as the comparison activity intervention was conducted after the NumberNet intervention, the teacher for the comparison activity had observed (via live video stream in another room) the NumberNet discussions, and attempted to replicate the discussion as closely as possible without the benefit of the shared display or collaborative activity to identify patterns before the discussion.

Table 1: Summary of Procedures

	NumberNet		Comparison Activity	
	Time and Content	Setting	Time and Content	Setting
Pre-test	1 number; 2 minutes	Individual	1 number; 2 minutes	Individual
Intervention	4 numbers 2 minutes each; Rotated between tables	Group	4 numbers; 2 minutes each; Distributed in turn	Individual
Discussion	5 minutes	Whole class	5 minutes	Whole Class
Distracter task	Logic Problem on MTT	Group	Logic Problem on paper	Group
Post-test	1 number 2 minutes	Individual	1 number 2 minutes	Individual

2.3.1 NumberNet protocol. One week after the pre-test, students visited the Multi-touch lab classroom. They completed activities to become familiar with the multi-touch tables, and then began using NumberNet. The teacher explained the procedure to the students and allowed them to practice with the target number of 100, checking every child knew how to create, send and correct their calculations.

Each table was assigned a different number, and given about two minutes to create as many calculations as they could for that number. The target numbers for this stage were 61, 150, 230 and 84. These numbers were selected as they represented a prime number, a three-digit number that is divisible by 2 and 3, a three-digit number that is divisible by 5, and a two-digit number with multiple factors.

After the first two minutes, the teacher turned off the number-pads, hid the correct calculations and gave the students a minute to review any incorrect calculations that were left on their screen. The teacher then rotated the numbers and correct calculations, gave the students some time (increasing as the number of calculations increased with each rotation) to review the correct calculations created by the previous group(s), and returned their number pads, before students spent another two minutes creating new calculations.

Once a full rotation of all four numbers was completed, the students were given time to create networks of similar calculations. The teacher then selected one screen to project to the interactive whiteboard and led the class in a discussion of the patterns that the groups had created.

The students then left the lab for a short break, before returning and completing a distracter task which did not contain any numbers or require numerical calculations. Finally, the students completed the individual post-test on paper.

2.3.2 Comparison protocol. One week after the pre-test, two members of the research team visited the comparison schools to conduct the comparison intervention. This intervention consisted of five numbers (the same as the NumberNet condition) which were completed individually by each child. The task was displayed in the same manner as the pre- and post-tests, and the protocol was the same, with two minutes per number.

After the students completed the activities, the researchers led them in a five-minute long discussion about the strategies they had used to complete the activity. The goal of this was to provide an opportunity for the students to consider their choice in using either patterns or unique expression strings to create as many expressions as possible during the activity. The teacher asked students to share with the class any patterns that they had created, asked the class to consider any elaborations on the patterns, and also asked for different types of expressions created. The students then completed a paper-based version of the distracter task in groups. Finally, the students completed the post-test.

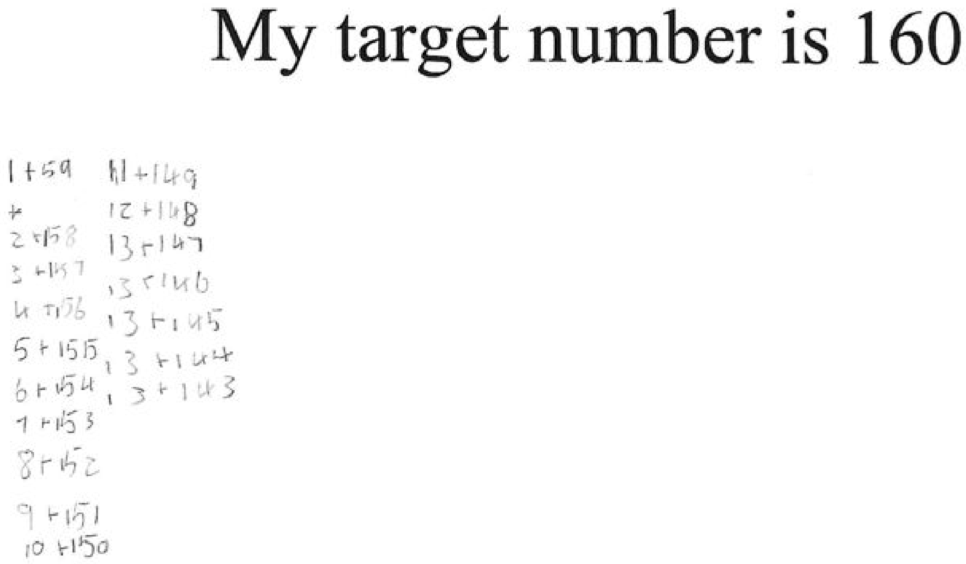
2.3.3 Distracter task. In both conditions, students worked in groups to complete a distracter task between the intervention and post-test. The distracter task was a logic mystery, which students completed in groups. As described elsewhere (Higgins et al, 2012), mysteries are designed with a question and number of clues, and groups of students need to work through the clues to solve the mystery. This particular mystery, Dinner Disasters, does not contain any numbers, but requires the students to use the information in the clues to construct an answer to the question “What should Mike have for dinner?”. The task was completed in the traditional paper-based form in the comparison condition, and on the multi-touch tables in the NumberNet condition.

2.4 Measures

The measures consisted of paper-based versions of the ‘make up some questions’ task, conducted a week before the intervention and 30 minutes after the intervention.

The test consisted of a single sheet of paper, with space for the student’s name on one side and the statement: My target number is x, with x being replaced with one of three possible numbers (120, 180 and 240) on the other side (see Fig. 3).

Figure 3: Paper based pre-test (and comparison activity)



During the pre-test, a number was randomly given to each child, but ensuring no two children next to each other had the same number. The children were then assigned one of the other numbers for the post-test.

For the pre-test, an example sheet with “My target number is 100” was shown to the class, and they were asked to give examples of how to create 100; in all cases, the classes were prompted to give one addition and one multiplication example. The tests were distributed to each student. They were then given brief instructions (all students were familiar with the task from prior use in their mathematics class), and told they had two minutes to write as many calculations as possible. After two minutes, the students were asked to put down their pencils and their tests were collected.

The calculations created on each test were recorded, and each participant received a score for:

1. Number of correct expressions created.
2. Maximum number of operators used in a single expression (e.g. a count of the number of operators used in the longest expression; this is not a measure of the unique operators used, but a measure of the length of the mathematical expression)
3. Number of unique calculation strings.

Drawing on the research on adaptive expertise, we argue that fluency with mathematical expressions can be aligned with routine expertise, where students have a simple understanding of how to apply some mathematical principles. However, a more flexible approach indicates developing adaptive expertise, with students who use range of operators, or operators in more complex combinations, showing higher levels of awareness of the mathematical constructs underlying the task. Thus, the pre and post test measures were designed to allow for an assessment of the fluency (routine expertise) of the students, and their flexibility (adaptive expertise).

The first measure, Number of Correct Expressions, was designed to be a measure of fluency with mathematical concepts. This measure was used as a way of assessing how efficient the students were, as one aspect of adaptive expertise. The second measure, Maximum Number of Operators, which assessed the maximum number of operators in a single expression was designed to assess the students' general mathematical flexibility, examining how innovative the students were in creating expressions with a range of operators. The third measure, Number of Unique

Strings, which also assessed flexibility and innovation, was designed to create a score of the range of unique strings of expressions that the students created, with the assumption that students who created a range of unique strings had a more innovative approach to the task than students who created multiple expressions using the same pattern.

A unique calculation string was identified using the rules below:

1. Repeated use of single calculations
   - 120+0; 119+1; 118+2…
2. Repeated use of multiple operators
   - 100+10+10; 50+50+10+10
3. Use of addition/subtraction of 0
   - 120 +0; 120-0
4. Use of multiplication/division by 1
   - 120/1; 120*1
5. Multiplication by factors
   - 60*2; 30*4; 15*8
6. Repeated pairs of calculations
   - 150-30, 90+30; 140-20; 100+20;
7. Commutative calculations
   - 60*2; 2*60; 30*4; 4*30…
8. Multiplication or division by factor of 10
   - 12*10; 1.2*100;
9. Random calculations that do not belong to any apparent chain

2.5 Interaction Analysis

The Multi-touch classroom is equipped with video, audio and screen capture equipment, to allow for the recording of the groups' interactions during the task. The recordings were synced and transcribed verbatim. Viewing the videos, screen capture and transcripts simultaneously, members of the research team identified and classified a range of learning opportunities, which were summarized into codes as shown in table 2.

The types of learning opportunities were determined by building on the idea that having the opportunity to collaborate, construct and revise ideas and experience surprise, perplexity or disco-ordination can lead students to develop a more complex understanding of a discipline (Hatano, 1988). Initially the data from two group were viewed to identify interactions that provided opportunities to engage in the content, and the coding scheme was created and applied to another two groups to finalize the codes. This final coding scheme was once more applied to all groups. The codes identifying strategy from another group, and finding patterns both provided opportunities for surprise, perplexity and disco-ordination. While strategizing and discussion expressions allowed for within-group collaboration and help seeking and correcting provided opportunities for elaboration of ideas within the group.

The codes were applied to the twelve groups in the NumberNet condition by one author, a second coder applied to the codes to three groups, with 83% agreement on codes (Cohen's Kappa = .623), with all disagreement occurring between the classification of discussing expressions and help seeking.

Table 2: Interaction Codes

Code	Description
Strategizing within group	The group strategizes about how they are going to do the task.
Identifying strategy from another group	Groups identify an expression coming from another group AND discuss or attempt to copy it.
Correcting	Members of a group correct another member and help them to fix the mistake.
Identifying patterns	Members of the group identify patterns in the expressions, or explicitly discuss the patterns they are making.
Discussing expressions	Group members discuss how to make an expression, or what is wrong or interesting about one.
Help Seeking	Participant asks for help from their group members.

3. Results

3.1 Quantitative Results

Quantitative analysis on the pre and post-test data was conducted in order to explore whether there were differences between students who used NumberNet and students in the comparison condition in the fluency and flexibility of the expressions they created. Analysis was conducted on the data to examine whether there were differences between the NumberNet and comparison conditions in the number, accuracy and complexity of calculations created. Table 3 shows the mean and standard deviation for both the NumberNet and comparison groups for each of the three measures.

Table 3: Means (SD) for measures across conditions

	NumberNet n=44		Comparison n=42	
	Pre	Post	Pre	Post
Number of correct calculations	7.23	9.87	7.53	11
	(3.65)	(6.05)	(5.12)	(6.85)
Max operators in a single calculation	1.45	2.02	1.42	1.52
	(.97)	(1.68)	(.69)	(1.19)
Number of unique strings	2.8	3.11	3.09	2.52
	(.77)	(1.1)	(1.15)	(1.23)

A multi-variate repeated measures Analysis of Variance was conducted to examine differences in performance on the pre- and post-test between the comparison and experimental groups. Time of task (pre- and post) was the within-subjects factor, condition (experimental or comparison) was the between-subjects factor, and total correct calculations, maximum number of operators in a single calculation and number of unique strings were the dependant measures.

Results indicated that the effect of time was significant for correct calculations, $F(1, 84) = 31.01$ $p <.001$, $\eta_p^2 = .27$, and for maximum number of operators in a calculation, $F(1, 84) = 4.469$, $p = .037$, $\eta_p^2 = .051$. The effect of time was not significant for unique strings, $F(1, 84) = .858$, $p = .357$, $\eta_p^2 = .01$.

Results indicated that the time by condition interaction was not significant for correct calculations, $F(1, 84) =.186$, $p = .667$, $\eta_p^2 = .002$, or for maximum number of operators in a single calculation, $F(1, 84) = 2.036$, $p = .157$, $\eta_p^2 = .024$, indicating that there was no difference between the NumberNet and traditional conditions across time in the number of calculations created.

However, the time by condition effect was significant for number of unique strings, $F(1, 84) = 11.63$, $p = .001$, $\eta_p^2 = .122$, with participants in the experimental condition creating more unique strings of calculations at post-test than participants in the comparison condition. This is an equivalent effect size to a standardized mean difference (Cohen's d) of 0.74 (Cohen, 1988) with a Standard Error (SE) of 0.22 (Lipsey & Wilson, 2001: 207).

3.2 Interaction Analysis

Interaction analysis was used to explore whether there were processes within the NumberNet activity that could explain changes in the flexibility seen in the post-test data (research question 2).The coding scheme described in table 2 was applied to each of the 12 groups in the NumberNet condition. Groups varied in both the types and frequency of learning opportunities. This is reflected in the range of outcomes seen in the section 3.1 and table 4, which shows the change from pre to post for each participant in the NumberNet condition and the total number of learning opportunities per group.

Table 4: Change from pre to post and Total Learning Opportunities.

			Change from Pre to Post		
	Total Learning Opportunities	Participant	Total correct	Max operators	Unique strings
Class 1 Blue	1	b609	2	0	0
		b610	5	4	-1
		b611	1	0	0
Class 1 Red	9	Jack	13	0	-2
		Chelsea	1	0	1
		Adam	-14	0	0
Class 1 Green	13	g605	3	0	0
		g606	7	0	-1
		g607	8	1	0
		g608	3	-5	0
Class 1 Yellow	3	y601	1	1	2
		y602	-2	1	0
		y603		No pretest	
		y604	-2	0	2
Class 2 Blue	5	b622	7	0	-1
		b623	5	-1	0
		b624	-6	0	2
		b625	0	1	2
Class 2 Red	8	r626	3	0	2
		r627	-3	1	0
		r628	9	0	1
		r629	3	0	0
Class 2 Green	7	John	5	0	-1
		Robbie	0	1	1
		Paul	-4	9	-1
		Megan		No pretest	
Class 2 Yellow	7	y615	9	2	0
		y616	-4	0	0
		y617	12	0	0
		y618	6	0	3
Class 3 Blue	5	b638	10	-2	-1
		b639	2	0	1
		b640	2	0	1
		b641	2	0	2

Class 3 Red	4	r642	-1	3	0
		r643	13	0	-1
		r644	-2	2	1
		r645	4	1	1
Class 3 Green	4	g634	-2	2	1
		g635	14	0	-2
		g636	-1	2	0
		g637	-1	0	0
Class 3 Yellow	9	Andrew	4	-1	0
		Nathan	3	1	1
		Lucy	4	3	0
		Becca	3	1	2

Of the groups where there were few learning opportunities identified, the participants tended to show less change in their total correct and unique strings from pre to post, when compared with groups where more opportunities to learn were identified. In the following sections, vignettes from three groups will be used to explore the types of learning opportunities in more detail.

3.2.1: Class 3 Yellow. Nine learning opportunities were identified in the Class 3 Yellow group and all students in that group increased in the number of expressions created at post-test. Additionally, three group members, Nathan, Lucy and Becca, used more operators in their expressions at post-test, and Nathan and Becca also increased in the range of unique expression strings they created.

Discussing expressions. The third number that this group received was 150, followed by their final number, 61. As the group prepare to work on 61, Nathan describes a calculation that he had been going to use for 150, which then continues into a discussion of how to adapt it to calculate 61.

Vignette 1:

Nathan: you know what I was gonna do for 150, I was gonna do 40, add 10 add 50 add 50

Andrew: 40 add 10 is 50

Nathan: plus 11

Andrew: add 40 add 10

Becca: I've done, I've done, Andrew, I've done 50 add 10 add 1!

Nathan: 40 plus 11

Andrew: just do 40 plus 11

[Nathan types in 40+11 and sends it to the table]

Having not fully adapted Nathan's strategy of using multiple operators, the group end up with one incorrect expression on their table (40+11), which prompts a discussion about whether it is actually wrong, or how they could have made it correctly.

Vignette 2:

Becca: 40 and 11 isn't 61

Nathan: exactly, it is

Becca: it's not, 11

Andrew: 40 plus 10 plus 11

Becca: 40, 11, 30, 40, 50 *[counting aloud]*

Andrew: Lucy did that one

Becca: Plus 10, and then plus 11!

Nathan: You're so wrong [*to Becca*]

Andrew: I know, it’s 40 plus 21!

Help seeking. While working on creating expressions to make 150, Becca asks for help, receiving some feedback, and finally, a direct answer to her question from Andrew.

Vignette 3:

Becca: What’s 200 take away what?

Becca: 200 take away what?

Nathan: 200 take away….99

[six unrelated turns]

Becca: what can you take away from 200?

Andrew: 50

3.2.2: Class 2 Green. Class 2 Green group showed high levels of discussion throughout the task, engaging in conversations about the different expressions they were creating. However, the change data suggests mixed learning outcome, particularly for Paul. This student appears to have reduced in fluency, but increased in flexibility during the task, creating four multi-operator expressions at post-test, in contrast to the nine single-operator expressions he created at pre-test. At post-test Robbie made the same number of expressions, but used one more unique string, and one more operator in his expressions. John increased only in the number of correct expressions created, using one less unique expression at post-test than at pre-test. Megan was not present for the pre-test, so no change data for her can be computed.

Discussing expressions. In vignette four, while working on their third target number, 61, John recognized that Megan is using the commutative property of addition to create more expressions quickly, coping her example to replicate his own.

Vignette 4:

Megan: There's one [*sends 1+30+30*]

John: 1 add 30 add 30 - that's just that one the other way around! [*points to 30+30+1*]

Megan: I know!

Robbie: I know!

John: Well so, I'll be able to do…. [*enters 31+30; having created 30+31 already*]

A second discussion begins towards the end of the same target number, 61, when John makes the statement that all their group's expressions will be right this time. Paul demonstrates his most recent one to the group, who quickly identify an error and strategy to fix it.

Vignette 5:

John: I bet you all ours are right

Paul: Mine's right!

John: What is it Paul?

Paul: Ten add ten add ten add ten add ten add ten add ten and take away ten.

Megan: 70 take away 10.

Paul: Yep..70 take away 10.

John: 70 take away 10 is 60.

Megan: It's sixty… it's 61 [*pointing to target number*]

John: Eh, you put it wrong!!

Megan: You’ve got it wrong!

John: Take it off

Teacher: OK, I’m going to stop you again *[teacher freezes the tables and hides the number-pads*]

Megan: 30, 40, 50, 60, 70, 60 – why didn’t you say take-away 9! [*reading the expressions aloud to the group*]

3.2.3: Class 1 Red. The Class 1 Red group had mixed outcomes at post-test. Chelsea created one more expression, and one more unique string at post-test, while Jack created 13 more expressions, but two less unique expressions. Adam created two correct expressions, and then ten expressions that calculated 200, rather than his target number of 240 during the post-test.

Strategizing. Vignette 6 comes from the beginning of the task, when the students were working on their first target number, 61. Chelsea asked whether anyone else has done the calculation she is planning to do, which leads to an agreement about which operators each child will use.

Vignette 6:

Chelsea: Is any of you doing 30 add 31?

Adam: I'm doing all the take aways! [*i.e. subtraction calculations*]

Jack: I'm doing take aways as well.

Chelsea: I'll do add.

Adam: I'm doing take aways.

Identifying strategy from another group. Jack, Adam and Chelsea continued to work on each new number as it was rotated to their table, but were only using single-operator calculations. However when the group received their fourth and final number, 150, from the previous table Jack identifies a calculation that used two operators.

Vignette 7:

Jack: Who done... Who's green? Jiminy… That's quite smart! [*the calculations have a coloured border indicating the table where they were created, so Jack is asking which is the green table, and so who was responsible for the calculation*]

Adam: Oh look at that! 10 times 10 that equals 100, add 50! Now that's clever, whoever did that! I'm doing that…

Vignette 8:

Once the teacher turns on the number pads, Jack goes on to adapt the calculations he has seen, creating the calculations 10*10+51-1, and drawing Adam’s attention to it:

Jack: Haha! Adam, look at the size of that!

Adam: Oh yes, did it... 1... 5...

Jack: ‘Cause 10 times 10 is 100, add 51 is 151 and take away 1 is 150... bingo!

Adam: Bingo!

[All three students at this table go on to work on multiple operator expressions]

Creating patterns. In the final stage of the task, the teacher asked the students to look for patterns, and then turned on the linking tool, whereby students can connect calculations to each other to indicate an association.

Vignette 9:

Teacher: Can you see if you can find any that are similar?

Jack: There... I've found two.

Teacher: See if you can find a pattern and organise it so I can see the pattern.

Adam: Oh I've got a pattern! [*bringing together all the subtraction calculations*]

Adam: Oh where's number 3? Aw where's number 3? [*noticing that 153-3 is missing from his set*]

Teacher: They may not all be there.

Vignette 10:

Teacher [*to whole class*]: Right I'm going to stop you again for a second and if you can look at the interactive whiteboard over in the corner [*see Figure 4a for the projected screen*]. This group have started to identify some patterns... Adam you can see has been working on finding all of the ones here 150 minus 0, minus 1, minus 2 - there isn't a minus 3 - minus 4, minus 5, so you've found all of the ones that are subtraction calculations starting with 150?

Adam: [*nods in agreement*]

Teacher: Can you look at your tables and see if you can find any other patterns or connections between them?

This final stage is designed to engage the students in mathematical discourse, and while it is the type of activity that will develop within a classroom over time, the sixth vignette shows the group as they began to identify patterns. In Figure 4b, their final connections can be seen, where the students have connected the calculations that use only subtraction, only addition, and only multiplication, and have grouped the calculations with multiple operators together.

In the ninth vignette, the group started to work on identifying patterns, while in the tenth vignette, the teacher projected the contents of the table for the whole class to see, drawing the students' attention to the fact that although they have identified all the subtraction calculations, they have also grouped a particular string of calculations, those that start with 150-0, and go from there using 150 as the starting number to add to, while subtracting from the other side of the operator.

At the end of the tenth vignette, the teacher instructs the class to try to make networks, along the lines of Adam's, the final product of which can be seen in Figure 4b.

Figure 4a: Screen shot of finding patterns

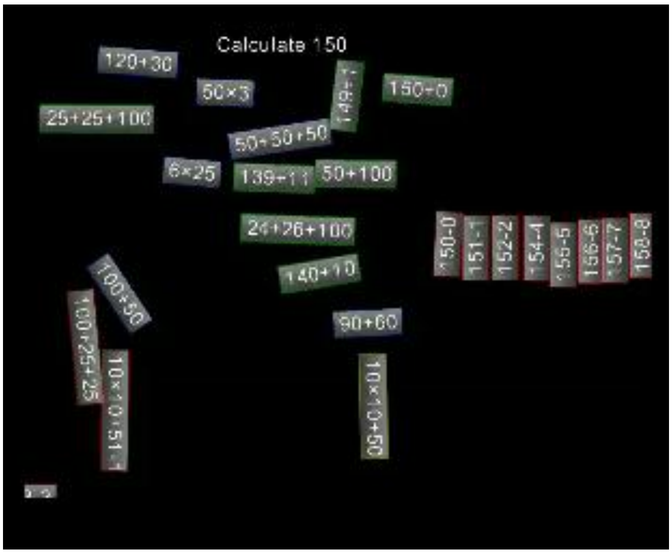
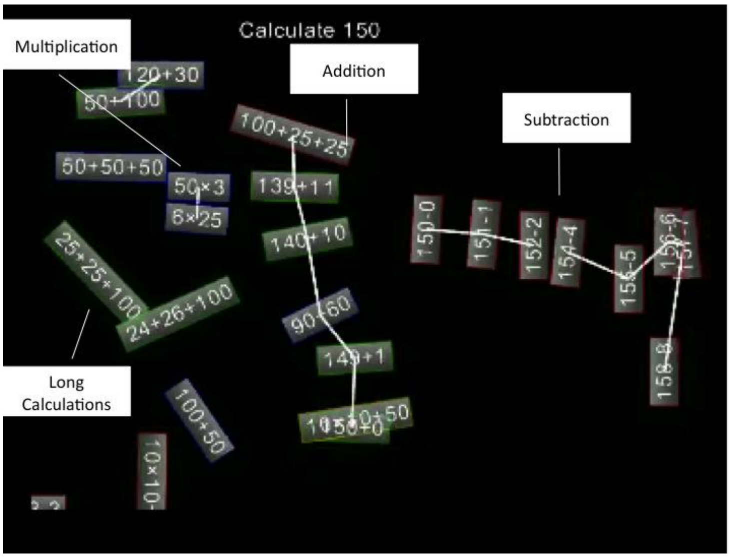


Figure 4b: Final network of patterns



The ten vignettes from the three case study groups illustrate a range of learning opportunities that were possible during the NumberNet activity, that are not present when this task is conducted in the traditionally individual manner in classrooms. As summarized in table 5, these interactions provide possible opportunities to be innovative with this task.

Table 5: Summary of Vignettes

Vignette	Interaction Code	Innovation possibility
1 & 2	Discussing Expressions	Recognizing and using multiple operators in one expression
3	Help Seeking	Gaining direct help on an expression
4	Discussing Expressions	Recognizing and using commutative nature of addition.
5	Discussing Expressions	Correcting misconceptions; engaging in mathematical discourse.
6	Strategizing	Recognizing different strategies.
7 & 8	Identifying strategy	Recognizing and using multiple

	from other group	operators in one expression
9	Finding patterns	Recognizing that the students used patterns to create a string of expressions.
10	Finding patterns	Engaging in mathematical discourse with the whole class about patterns.

4. Discussion

In this paper, we set out to examine whether NumberNet supported the development of mathematical adaptive expertise, and specifically aspects of fluency and flexibility, when compared to a similar, individual task. Our research questions were 1) do students who use NumberNet differ in either fluency or flexibility when compared with students undertaking a comparison activity and 2) what were the processes through which NumberNet might have influenced fluency or flexibility. The results indicated that all students, regardless of condition, became more fluent through practice at the ‘make up some questions’ task, with both conditions showing an increase in the number of correct calculations created from pre- to post-test. The results also identified a significant time by condition interaction in the number of unique strings created, indicating that the students in the NumberNet condition created more unique strings at post-test than at pre-test, while students in the comparison condition decreased in the number of unique strings of calculations that they created at post-test.

From these results, it appears that both conditions support the development of routine expertise, and the individual paper-based version of the ‘make up some questions’ task appears to be as useful as NumberNet in supporting the development of fluency and speed with simple calculations. However, students from the NumberNet condition produced a wider range of calculation strings in the post-test, indicating that they were approaching the task with more flexibility and possibly developing a more complex number-sense or greater adaptive expertise in mathematics.

The analysis of the interactions during the NumberNet activities sheds some light onto the possible processes through which mathematical flexibility may increase when using a collaborative classroom activity, which was the focus of our second research question. Six possible learning opportunities were identified and the twelve groups were coded for evidence of these. As is common in collaborative learning research, the groups varied in the amount and types of interaction that they engaged in, with a range from one to thirteen possible learning opportunities within the groups.

The vignettes further illustrate the possible learning opportunities that occurred during NumberNet, where the structure of the activity and the opportunities to interact provide the environment in which students may be surprised, perplexed or experience disco-ordination, a situation in which the student may then be motivated to develop adaptive expertise (Hatano, 1988). This analysis provides evidence of the possible processes through which differences in the flexibility measures at post-test between the experimental and comparison conditions may have come about. And although they do not provide direct evidence of the cause of the change due to the brevity of the study and the complexity of collaborative interaction, they are indicative of the potential of classroom collaboration for fostering adaptive expertise.

There were a number of limitations to this initial study of NumberNet, including the brevity of the study and the relatively short time between intervention and post-test, the lab-classroom context of the NumberNet activity, the use of research staff rather than the students' own teachers to conduct the intervention activities and the nesting of students within groups and schools. While the use of experimental and lab procedures was necessary to conduct comparisons across the two activities, future work in more standard classroom environments will further our understanding of the role of collaboration in supporting the development of adaptive expertise. Additionally, the data recording equipment in the lab is discrete, allowing students to proceed without being distracted by reminders of being recorded. By using research staff, we lost the opportunity to explore the final stage of NumberNet more fully, where a teacher more familiar with the students might elaborate further the possible mathematical discussions that the final stage prompts. It is expected that with repeated use by a teacher who is familiar with the tool, this stage of the activity could support rich discussions about the structures and patterns in the expressions the students create. Additionally, using two different members of research staff between conditions may have introduced an additional source of error, although attempts were made to keep the interactions of the teacher and students similar during the expression-creation phase, and replicate the patterns discussion as closely as possible given the different contexts. Finally, as with all research on collaboration, students are part of groups who, in this case, were drawn from the same school, which leads to concerns about the non-independence of the individual data. Thus, the quantitative results should be interpreted with caution and in relation to the qualitative results that provide a richer understanding of the differences that emerged at post-test.

Furthermore, NumberNet was created to include a range of additional features that were not explored in the present study. These include the facility for the teacher to use the tablet to restrict the keys on the number pads, so that students can only use certain operators or numbers, and to monitor the calculations made or adapted by each student.

Our findings support the value of implementing collaborative and whole-class learning activities (Tolmie et al. 2011), designed to support adaptive expertise which therefore provide opportunities for students to be innovative as well as efficient (Verschaffel et al. 2009). We add to the empirical literature in this area with the quantitative analysis of children’s learning in mathematics linked to analysis of the learning processes observed Verschaffel et al. (2011). In a similar manner to the study by Markovits and Sowder (1994), the results indicate that NumberNet provided an opportunity for the students to engage in innovative mathematical activities, and recognise how existing knowledge can be used in a flexible manner, rather than teaching the students novel content. Our findings confirm the importance of practice for developing fluency and routine expertise, while indicating that having the opportunity to collaborate over the creation of mathematical expressions may foster deeper engagement with the concepts and lead to increased flexibility and adaptive expertise. Future work will explore which aspects of the tool – the within-group collaboration, sharing of strategies between groups, and the final networking task – influence the development of number-sense and how to adapt these to more complex mathematical tasks. The results also point to the importance of exploring the role of interaction at the small group and whole class level when designing activities aimed at supporting students in the development of both adaptive and routine expertise, indicating the importance of drawing on both cognitive and socio-cultural understanding of how learning occurs when designing such tasks.

Acknowledgements

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