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# Total differential cross sections for $\mathrm{Ar}-\mathrm{CH}_{4}$ from an ab initio potential 

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#### Abstract

Total differential cross sections for the $\mathrm{Ar}-\mathrm{CH}_{4}$ scattering complex at $E_{\mathrm{CM}}=90.1 \mathrm{meV}$ were obtained from converged close-coupling calculations based on a recent ab initio potential computed by symmetry-adapted perturbation theory (SAPT). Agreement with experiment is good, which demonstrates the accuracy of the SAPT potential. © 1998 American Institute of Physics. [S0021-9606(98)00612-6]


## I. INTRODUCTION

During the last few decades, a number of experimental and theoretical studies have been devoted to the rotationally elastic and inelastic scattering of $\mathrm{Ar}-\mathrm{CH}_{4}$. In their pioneering work ${ }^{1-3}$ Secrest and coworkers applied both the closecoupling (CC) and coupled-states (CS) formalisms in the calculations of differential cross sections (DCS). Buck et al. ${ }^{4}$ measured total differential cross sections in a crossed beam experiment at $E_{\mathrm{CM}}=90.1 \mathrm{meV}$. These results were used to derive an empirical potential of the Morse-spline-van der Waals type including the leading anisotropic contribution with tensor order $l=3$. Time-of-flight (TOF) spectra were reported ${ }^{5}$ and compared with CS calculations that used the potential from Ref. 4. Total differential cross sections for $\mathrm{Ar}-\mathrm{CH}_{4}$ were also measured at slightly different collision energies. ${ }^{6}$ Liuti et al. ${ }^{7}$ reported total integral cross sections for rare gas-methane scattering as functions of the velocity. These data were used to further improve the isotropic term in the potential of Ref. 4.

Recently, we calculated an $a b$ initio intermolecular potential energy surface for $\mathrm{Ar}-\mathrm{CH}_{4}$ using symmetry-adapted perturbation theory (SAPT). ${ }^{8}$ After performing an analytical fit to the computed points, the long range dispersion and induction coefficients in the fit were replaced by coefficients calculated in a larger basis set. See Ref. 9 for a more detailed description of this asymptotic scaling procedure. We applied this $a b$ initio potential in CC calculations of state-to-state integral cross sections for $A, E$, and $T$ states of methane rotationally excited by collisions with argon. The results were generally in good agreement with experimental data. ${ }^{10}$ The $a b$ initio SAPT potential was also used to generate second virial coefficients for the $\mathrm{Ar}-\mathrm{CH}_{4}$ complex. ${ }^{11}$ Virial co-
efficients computed from the asymptotically scaled potential were found to agree well with experiment, substantially better than those calculated from the original non-scaled potential.

In a previous work, ${ }^{12}$ total differential cross sections and TOF spectra for $\mathrm{He}-\mathrm{C}_{2} \mathrm{H}_{2}$ computed from a SAPT potential ${ }^{13}$ were found to be in excellent agreement with experiment. In the current paper, we present total differential cross sections for $\mathrm{Ar}-\mathrm{CH}_{4}$, computed at the collision energy $E_{\mathrm{CM}}=90.1 \mathrm{meV}$, from converged CC scattering calculations. We compare the results obtained from the asymptotically scaled and the non-scaled SAPT potentials with the experimental data from Ref. 4.

## II. COMPUTATIONAL DETAILS

The CC equations for atom-spherical top scattering were given in Refs. 8,14. The monomer wave functions for methane are expressed as linear combinations of symmetric top functions (normalized rotation matrix elements). The coefficients in these combinations are obtained by constructing and diagonalizing for each angular momentum $j$ a rotational Hamiltonian matrix that includes the tetrahedral centrifugal distortion. ${ }^{8,14}$ The following values were assigned to the rotational constant $b$, the centrifugal distortion constant $d_{j}$, and the tetrahedral centrifugal distortion constant $d_{t}$ : $b=5.2410356 \mathrm{~cm}^{-1}, \quad d_{j}=1.10864 \times 10^{-4} \mathrm{~cm}^{-1}$, and $d_{t}=4.425 \times 10^{-6} \mathrm{~cm}^{-1} .{ }^{15}$ The CC equations were solved applying the modified log-derivative-Airy integrator. ${ }^{16}$ The log-derivative propagator was used from $R_{\min }=4.5$ bohr to $R_{\text {mid }}=15$ bohr with a constant step size corresponding to 5 steps per half wavelength for the open channel of highest kinetic energy in the asymptotic region. From $R_{\text {mid }}$ to
$R_{\max }=100$ bohr the Airy propagator was used. Partial wave contributions with increasing values of the total angular momentum $J$ were included until both the elastic and inelastic state-to-state cross sections were converged within $0.001 \AA$. With the asymptotically scaled potential, the highest values of $J$ included for the $A, E$, and $T$ symmetries were 309,310 , and 297 , respectively. If the lengths of the partial wave expansions were smaller, the elastic contributions showed unphysical oscillations, especially at large angles. The angular basis contained all channels with monomer angular momentum $j$ up to 14 inclusive. However, for the $E$ and $T$ symmetries the components with $J \geqslant 190$ and 182, respectively, were calculated in a basis including only channels with $j \leqslant 11$. The total number of channels for the bases with $j \leqslant 14$ and $j \leqslant 11$ were $\approx 385$ and 190 , respectively, for the $E$ symmetry, and $\approx 560$ and 290 , respectively, for the $T$ symmetry. Thus, decreasing the maximum value of the monomer angular momentum $j$ from 14 to 11 caused the sizes of the basis sets to be reduced by approximately a factor of two. For the $A$ symmetry, the total number of channels was $\approx 90$. We tested that the deviations caused by the reduction of the monomer basis at large $J$ were negligible. We performed additional calculations for the $A$ states with the non-scaled potential and the same set of parameters. The highest value of $J$ included was 307 in this case. The $\mathrm{Ar}-\mathrm{CH}_{4}$ interaction potential, which is a function of the distance $R$ between the $\mathrm{CH}_{4}$ center-of-mass (c.m.) and the Ar atom and of the polar angles $(\theta, \phi)$ of the vector $\mathbf{R}$, was expanded in real combinations of spherical harmonics $Y_{m}^{l}(\theta, \phi)$ with $l \leqslant 18$. The expansion coefficients were computed by means of 25-point Gauss-Legendre and Gauss-Chebyshev quadratures in $\theta$ and $\phi$, respectively. The reduced mass of $\mathrm{Ar}-\mathrm{CH}_{4}$ is 11.441478 $\mathrm{amu} .{ }^{17}$ The integration parameters, the size of the monomer basis, the number of partial waves, and the length of the expansion of the potential were chosen such that the state-to-state cross sections were converged within $1 \%$ at worst. All scattering calculations were performed using the MOLSCAT system of codes. ${ }^{18}$ Differential cross sections in the c.m. frame were generated from the calculated S-matrix elements using the DCS program. ${ }^{19}$ This program was extended such that it can be applied to atom-spherical top systems. The c.m. scattering angle $\Theta_{\mathrm{CM}}$ was varied from $0^{\circ}$ to $180^{\circ}$ with steps of $0.5^{\circ}$. It was assumed that initially the methane molecules are in the lowest rotational state permitted for each of the symmetry types, i.e., $j^{\prime \prime}=0,1$, and 2 for $A, T$, and $E$ symmetries, respectively.

In order to compare the $a b$ initio total differential cross sections, summed over all elastic and inelastic channels, with experiment, the calculated values were transformed from the c.m. frame to the laboratory-fixed (lab) frame and averaged over the velocity and angular distributions of the methane and argon beams. Furthermore, corrections were made for the energy response and physical size of the detector. The averaging procedure is described in detail elsewhere. ${ }^{20}$ The key parameters are the full width at half maximum of the c.m. angular and velocity distributions, $\delta=1.46^{\circ}$ and $\Delta g / g$ $=0.13$, respectively. They are calculated using a MonteCarlo procedure with the quantities mentioned above as input information. The actual calculations are then performed


FIG. 1. State-to-state inelastic (a) and elastic, total inelastic, and total (b) differential cross sections for the $A$ symmetries of $\mathrm{Ar}-\mathrm{CH}_{4}$ calculated with the asymptotically scaled SAPT potential as functions of the c.m. angle. The molecular rotational quantum numbers of the final states are indicated.
on a $12 \times 12$ point grid in the two variables. The energy dependence of the cross sections is explicitly taken into account with the help of a scaling law ${ }^{21}$ which has been checked against exact calculations.

## III. RESULTS AND DISCUSSION

Figures 1(a) and 1(b) depict the state-to-state, total inelastic, and total differential cross sections for the $A$ symmetries of $\mathrm{Ar}-\mathrm{CH}_{4}$ as functions of the c.m. angle $\Theta_{\mathrm{CM}}$. The same results for the $T$ species are presented in Figs. 2(a) and 2(b) and for the $E$ states in Figs. 3(a) and 3(b). Only cross sections for the transitions with $j^{\prime} \leqslant 7$ are shown ( $j^{\prime}$ denotes here the rotational quantum number of the final state). Cross sections with higher $j^{\prime}$ are not reported since their contribution to the total differential cross sections is very small. An inspection of these figures shows that at small angles the elastic component dominates the total differential cross sections. At larger angles the inelastic transitions become increasingly important, especially for the $A$ and $T$ symmetries. For these symmetries the elastic term has a deep minimum


FIG. 2. State-to-state inelastic (a) and elastic, total inelastic, and total (b) differential cross sections for the $T$ symmetries of $\mathrm{Ar}-\mathrm{CH}_{4}$ calculated with the asymptotically scaled SAPT potential as functions of the c.m. angle. The molecular rotational quantum numbers of the final states are indicated.
around $\Theta_{\mathrm{CM}}=60^{\circ}$. It is interesting to note that although various state-to-state differential cross sections show very different dependence on the c.m. deflection angle, the total cross sections (i.e., the cross sections summed over the elastic and all inelastic channels) are almost identical.

The theoretical total differential cross sections for the $A$ states of $\mathrm{Ar}-\mathrm{CH}_{4}$, transformed to the lab frame and averaged over the experimental conditions, are reported as a function of the laboratory angle in Fig. 4. It was essential that different inelastic contributions were transformed individually, since they have different final velocities. The use of the purely elastic transformation would substantially underestimate the theoretical differential cross sections at large lab angles. The experimental cross sections from Ref. 4 are displayed in this figure as well. In order to remove the strong angular dependence of the elastic differential cross section the data are multiplied by $\Theta^{7 / 3}$. Although the state-to-state differential cross sections for the $E$ and $T$ states of $\mathrm{Ar}-\mathrm{CH}_{4}$ are different from those of the $A$ species (cf. Figs. 1-3), the total differential cross sections in the lab frame were found to


FIG. 3. State-to-state inelastic (a) and elastic, total inelastic, and total (b) differential cross sections for the $E$ symmetries of $\mathrm{Ar}-\mathrm{CH}_{4}$ calculated with the asymptotically scaled SAPT potential as functions of the c.m. angle. The molecular rotational quantum numbers of the final states are indicated.
be almost identical. Therefore, these results are not shown. The position of the maximum of the total differential cross sections, the rainbow angle, is very well reproduced, but the slope of the theoretical curve beyond the rainbow angle is somewhat different from experiment. These two features of the total differential cross sections are mainly sensitive to the well depth of the potential, so the results presented in Fig. 4 suggest that the well depth of the $a b$ initio potential is underestimated, but only by a very minor amount. Furthermore, the positions and amplitudes of the diffraction oscillations, which modulate the maximum of the curve, agree well with experiment. This result indicates that both the onset of the repulsive wall ( $R_{0}$ ) and the anisotropy of the minimum distance $R_{m}(\Theta)$ of the potential are realistic. On the other hand, the coarse structure amplitude of the curve, which represents the rainbow maximum, slightly deviates from the experimental data. Test calculations of the averaging process show that the rainbow amplitude is not influenced by small changes of the resolution.

A possible reason for such a behavior could be too weak


FIG. 4. Total differential cross sections (multiplied by $\Theta^{7 / 3}$ ) computed from the asymptotically scaled SAPT potential (solid line) and the non-scaled SAPT potential (dashed line) as functions of the lab angle. Circles denote experimental data from Ref. 4, the error bars represent $\pm 1$ standard deviation of the mean.
a damping of the rainbow amplitude due to the anisotropy of the well depth. However, a careful inspection of the total differential cross sections in the rainbow region shows that they are completely dominated by the elastic contributions. This is also reflected in the potential terms in the attractive part near the inflection point which is the part of the potential that is most sensitively probed in rainbow scattering. Here $V_{0}$ is much larger than the anisotropic $V_{3}$ and $V_{4}$ terms. ${ }^{8}$ Therefore we attribute the small discrepancy in the rainbow amplitude to a small error in the shape of the potential near the inflection point: the rainbow impact parameter is slightly too large, which influences the rainbow amplitude. ${ }^{22}$ Indeed the best fit isotropic potentials of Refs. 4, 7 exhibit smaller rainbow impact parameters and thus slightly smaller amplitudes.

For comparison, we have plotted in Fig. 5 the total differential cross sections for the $A$ states of $\mathrm{Ar}-\mathrm{CH}_{4}$, calculated from the semiempirical potential $V_{0}^{c}+V_{3}^{c}$ reported by Buck et al. ${ }^{4}$ This figure is not identical with Fig. 8 of Ref. 4, which was obtained from the same potential, because in the present work we applied the close-coupling instead of the coupled-states formalism and did not retransform the results to the c.m. frame. The agreement of the present curve with experiment is worse between $5^{\circ}$ and $6^{\circ}$ (lab angles), the same up to $20^{\circ}$, and better for larger angles. The ab initio results of Fig. 4 agree somewhat better with experiment than the semiempirical results for angles smaller than $12^{\circ}$, somewhat worse between $12^{\circ}$ and $23^{\circ}$, and are comparable for larger angles. On the average, agreement with the experimental results is equally good for the SAPT potential and for the semiempirical potential of Ref. 4 that was mainly fitted to these data.

Also reported in Fig. 4 are the theoretical total differential cross sections calculated from the non-scaled SAPT potential. An inspection of the figure shows that the agreement


FIG. 5. Total differential cross sections (multiplied by $\Theta^{7 / 3}$ ) computed from the semiempirical $V_{0}^{c}+V_{3}^{c}$ potential of Ref. 4 as functions of the lab angle. Circles denote experimental data from Ref. 4, the error bars represent $\pm 1$ standard deviation of the mean.
with experiment ${ }^{4}$ is considerably worse. The predicted rainbow angle is too small, indicating that the well of this potential is too shallow. A change in the well depth of the potential also affects the onset of the repulsive wall. Consequently, the oscillations on the curve for the non-scaled potential in Fig. 4 are slightly out of phase with respect to the experimental data. A similar result concerning the well depth was found from the analysis of the second virial coefficients calculated with the asymptotically scaled and the non-scaled SAPT potentials. ${ }^{11}$

We conclude that the asymptotically scaled SAPT potential reported in Ref. 8 is able to reproduce well the total differential cross sections for $\mathrm{Ar}-\mathrm{CH}_{4}$. The only deviation from experiment is in the rainbow amplitudes suggesting that there are small inaccuracies in the shape of the potential near the inflection point. The well depth of the potential, however, is very realistic, since the position of the rainbow angle is accurately reproduced. Comparison with the results obtained from the semiempirical potential of Ref. 4, which was fitted to the measured differential cross sections, shows that the results from the SAPT potential agree about equally well with these experimental data. Work on the application of this $a b$ initio potential to generate the infrared spectrum of the $\mathrm{Ar}-\mathrm{CH}_{4}$ complex in the region of the $\nu_{3}$ mode is in progress. Comparison with the experimental spectrum ${ }^{23-25}$ will provide a further check of the accuracy of the potential.

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