An experimental investigation of auctions and bargaining in procurement

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Abstract

In reverse auctions, buyers often retain the right to bargain further concessions from the winners. The optimal form of such procurement is an English auction followed by an auctioneer's option to engage in ultimatum bargaining with the winners. We study behavior and performance in this procurement format using a laboratory experiment. Sellers closely follow the equilibrium strategy of exiting the auction at their costs and then accepting strictly profitable offers. Buyers generally exercise their option to bargain according to their equilibrium strategy, but their take-it-or-leave-it offers vary positively with auction prices when they should be invariant. We explain this deviation by modeling buyers' subjective posteriors regarding the winners' costs as distortions of the Bayesian posteriors, calculated using a formulation similar to a commonly used probability weighting function. We further test the robustness of the experimental results and the subjective posterior explanation with three additional experimental treatments.

JEL classifications: C34; C92; D03; D44

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1 Introduction

Auctions that set a benchmark price and award exclusive rights to negotiate final purchase terms are commonly used in many areas of commerce such as procurement in supply chains (Tunca and Wu, 2009; Elmaghraby, 2007), corporate mergers and acquisitions (Hege, Slovin, Lovo, and Sushka, 2009), and even in professional sports when teams exchange the rights of players.[1] This practice is particularly common in government procurement. For example, we learned in interviews with the procurement center of the Hunan Province in China that in 2012 they attempted to procure over 9000 orthopedic related objects, with over 12000 unique suppliers participating in the process. Whenever there were at least three qualified suppliers they conducted a reverse auction. If the auction price didn't meet their basis price, then they further negotiated with the auction winning supplier. Sometimes bargaining resulted in an improved price, and other times in no purchase at all. This case exhibits two key characteristics of effective auction-bargaining practice: further negotiations are often chosen, and sometimes trade doesn't occur even when there are potential benefits to both parties.

There is a strong theoretical basis for using the auction-bargaining mechanism. A seminal article by Bulow and Klemperer (1996) shows that conducting a reverse (forward) English auction with the auctioneer retaining the right to make a take-it-or-leave-it offer to the auction winner implements the optimal mechanism - Myerson (1981) - to sell (purchase) an object. On the other hand, they also establish the auctioneer's expected welfare increases by foregoing this auction-bargaining mechanism and instead conducting an English auction with an additional serious bidder. Unfortunately, identifying and validating additional serious bidders is often cost prohibitive and difficult, particularly in the procurement setting (Wan and Beil, 2009; Wan, Beil, and Katok, 2012). Consequently, we feel evaluating behavior and performance in Bulow and Klemperer's auction-bargaining mechanism within a procurement setting is an important task.

The Nash equilibrium implementing the optimal mechanism, despite having a simple structure, relies upon some behaviorally questionable assumptions. A seller's strategy is to exit the auction when her costs exceeds the auction price, and accept any subsequent profitable ultimatum offer. This opposes what is observed in ultimatum game experiments where responders commonly reject profitable offers that are less than equitable even when experienced (Cooper and Dutcher, 2011), or there is incomplete information about the amount

[1]For example the Nippon Professional Baseball League and Korean Baseball Organization have a posting system, that allows a player to ask his current team to conduct an auction granting a period of exclusive negotiating rights to a Major League Baseball team; about one player per year leaves the Nippon League through this process. An even more relevant practice - in which no compensation occurs in absence of a final contract - is trade of players between teams in the National Basketball Association or National Football League conditional upon the player and new team agreeing to a contract extension.

of potential gains from exchange (Croson, 1996; Harstad and Nagel, 2004).[2] In our experiment, sellers generally exit the auction at their cost, and they rarely reject profitable take-it-or-leave-it offers.

The buyer's equilibrium strategy is characterized by a price threshold that is conditional upon his willingness-to-pay. He accepts an auction outcome when the price is below his threshold; otherwise, he makes a take-it-or-leave-it offer equal to the threshold. This strategy has the uncanny flavor of an ex post reserve price, and that is not a coincidence. Both the auction-bargaining mechanism and the English auction with an ex ante reserve price implement the optimal direct mechanism. As a consequence, the buyer's threshold in the auction-bargaining mechanism is the optimal reserve price in the English auction, and therefore independent of the realized auction price. But this appears paradoxical; the buyer only commits to a threshold after the auction, and hence gaining additional information about the winning seller's cost. We show the price invariance of the buyer's threshold crucially relies upon the buyer forming correct Bayesian posterior beliefs regarding the winner's cost.

This result must seem paradoxical to the subjects as well. While the buyer's strategy accurately predicts when auction outcomes are rejected in favor of further bargaining, subsequent take-it-or-leave-it offers have a strong positive relationship with the auction price - contradicting the prediction of price invariance. This results in reducing the buyer's surplus by approximately 7.5%. We also find substantial individual heterogeneity in this auction price sensitivity.

We find an explanation for this behavior by generalizing how a buyer formulates his posterior belief regarding the auction winner's cost conditional upon the auction price. We propose the subjective posterior is formed by applying the two-parameter Prelec (1998) probability weighting function to transform the Bayesian posterior. This allows the subjective posterior to flexibly reflect differing types of perceived affiliation between the auction price and winner's cost. Structural estimates of this model demonstrate its ability to capture both the observed wide varying pattern of buyer behavior and the general property of the positive relationship between auction price and bargaining offer. We further show that simply allowing for risk aversion does not lead to any relationship between auction price and bargaining offer, and anticipated regret only leads to a negative relationship. Thus, of these three alternatives, our subjective posterior model is the only plausible explanation.

We also conduct additional treatments to challenge the robustness our explanation and findings. First, we enrich the feedback that auction losing sellers receive by informing them of the actions taken in negotiation phase. However, this does not give impetus to any increased adherence to their equilibrium strategy in the auction phase nor inspire any rejections of

[2] An exception is found in Salmon and Wilson (2008) which we later discuss in detail.

profitable offers. Second, we challenge the distorted posterior explanation with the alternative hypothesis that buyer's don't hold accurate beliefs about the seller's strategies. In response, we run a treatment where buyers play against computerized sellers automated to follow the equilibrium strategy. We also inform buyers about the nature of these sellers. The buyer's offers are still sensitive to the price and the estimated posteriors, on average, exhibit stronger affiliation between the winner's and loser's costs.

Finally we explore whether distorted 'subjective posteriors' are an easily corrected bias. We augment the computerized seller treatment by giving buyers a calculator that provides detailed information on the probability of acceptance for alternative offers and expected payoffs. We even obligate them to inspect this information for any take-it-or-leave-it offer they make. The results are startling in that buyer's don't make offers more in line with the Bayesian benchmark, but rather they show a greater willingness to bargain and make offers reflecting even more distorted posterior beliefs. This has practical implications for management practice; distorted subject posteriors can't be corrected by simply providing expert decision support systems.

2 Basic theory

We present an alternative derivation of the Nash equilibrium of the auction-bargaining mechanism that elucidates how the buyer dynamically processes information influences his strategy.[3] Consider a buyer who desires an indivisible object. His value for this object is the random variable v with the absolutely continuous distribution function $H(v)$ on the interval $[\underline{v}, \overline{v}]$, $\underline{v} > 0$, and associated probability distribution function $h(v)$. Only the buyer knows his realized valuation. There are N possible sellers, indexed by i. Upon selling an object, seller i incurs a unit cost of c_i. Each seller's unit cost is an independent random variable with the common absolutely continuous distribution function $F(c)$ on the interval $[0, \overline{c}]$ and associated probability distribution function $f(c)$. We also assume that $c + \frac{F(c)}{f(c)}$ is strictly increasing on the support of F. Only a seller knows her realized cost, which she learns prior to any strategic interaction. $H(v)$ and $F(c)$ are also independent, so a seller's realized cost reveals no additional information about the buyer's value.

Here are the specific rules of the auction-bargaining mechanism. First, the sellers compete in a reverse English clock auction in which price starts at $\overline{c}$ and all N sellers in the auction. Price falls with time, and a seller can irreversibly exit the auction at any time. The auctions closes, and setting the auction price p, when either $N-1$ sellers have exited or price reaches zero. Ties in either being the $N-1$ seller to exit, or winning at a price of zero are settled

[3]Note that both Bulow and Klemperer (1996) and Myerson (1981) consider the strategically equivalent case of an individual selling an indivisible unit of a good to N possible buyers.

randomly. Any seller who does not win the auction receives a payoff of zero. The buyer is informed of the auction price and chooses to either accept the auction price, resulting in payoffs of $v - p$ for the buyer and $p - c_i$ for the winner, or makes a take-it-or-leave-it offer o to the winner. In the case of the latter, the winning seller either accepts the offer, resulting in payoffs of $v - o$ and $o - c_i$, or rejects the offer and all parties receive a zero payoff.

A seller's strategy has two parts: a function that maps from possible unit costs to auction exit prices; and a function that maps from possible counter-offer information sets to reject and accept decisions. The buyer's strategy is a function that maps from possible value and auction price pairs to possible counter offers join with accepting the auction outcome. We show the strategy profile in which each seller exits the auction at her unit cost and accepts any profitable take-it-or-leave-it offer, and the buyer accepts all auction prices below some threshold level - conditional on v - otherwise makes an optimal counter offer is a Nash equilibrium.

Consider the optimality of the buyer's strategy conditional upon those of the sellers. The buyer's conditional payoff function is

$$\pi(o|v,p) = \max\{v - p, \max(v - o)G(o|p)\}. \tag{1}$$

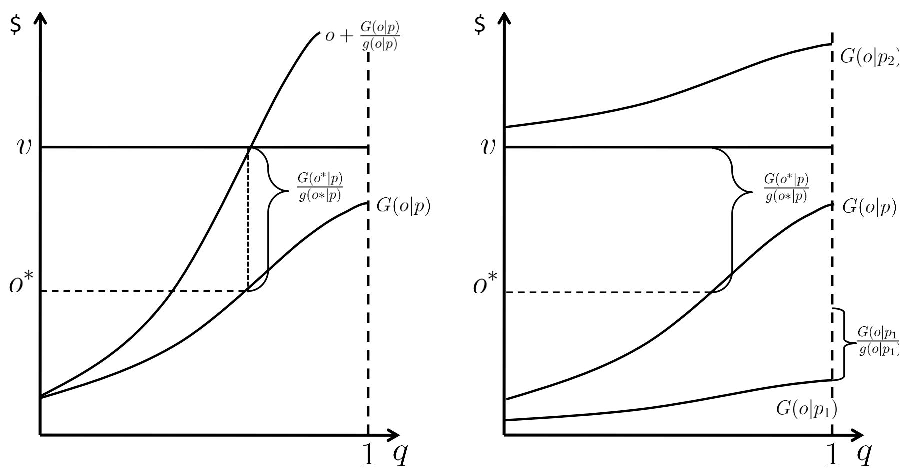
$G(o|p)$ is the buyer's subjective probability distribution of the auction winner's cost which need be not derived according to Bayes rule, but $o + \frac{G(o|p)}{g(o|p)}$ must be strictly increasing. The first order condition for an interior maximum of the second argument of (1) implies,

$$v - o^* = \frac{G(o^*|p)}{g(o^*|p)}. \tag{2}$$

We provide an economic interpretation of Equation (2) from the theory of a price setting monopsonist; a direct analog to the classic interpretations of optimal forward auctions with monopoly theory (Bulow and Roberts, 1989; Bulow and Klemperer, 1996). In this case the buyer is a monopsonist, the probability of purchase is analogous to market quantity, q, and the offer o is analogous to market price set by the monopsonist. The buyer's marginal and average values (of additional probability of successfully purchasing) are the same constant v. Now the market supply function is the buyer's subjective posterior distribution of the winner's cost, i.e $q = G(o|p)$. And the marginal cost (of additional probability of successfully purchasing) is $MC(G(o|p)) = o + \frac{G(o|p)}{g(o|p)}$. If there is an optimal interior quantity $G(o^*|p)$, we know it is found where $MR = MC$, which is simply Equation (2).

The relationship between v, market supply, and marginal cost are depicted in the left hand graph of Figure 1. In the right hand of this Figure, we depict an interior solution, and two types of corner solutions. In one corner solution, marginal cost never exceeds v in which case $o^* = G^{-1}(1|p_1)$. In the other corner solution, the buyer believes with probability zero that the winner's cost is below his value, see $G(o|p_2)$, and thus setting $o^* = v$ is an optimal offer.[4] In all three cases, the buyer accepts the auction outcome if $v - p \geq (v - o^*)G(o^*|p)$.

Figure 1: Optimal offer: the left graph illustrates an interior solution. The right graph illustrates an interior solution for auction price p, a corner solution for p_1 where marginal cost never crosses v and $o^* = G^{-1}(1|p_1)$, and another corner solution for p_2 where $G^{-1}(0|p_2) > v$ and $o^* = v$.



Thinking of $G(o|p)$ as a state dependent supply function, where the state is the realized auction price, the optimal state pricing rule is found by rewriting Equation (2) as

$$o^* = \frac{v}{1 + \dfrac{1}{E_{G,o}(o^*)}}, \text{ where } E_{G,o}(o) = \frac{o}{G(o|p)} g(o|p). \tag{3}$$

In other words, $E_{G,o}(o)$ is the price elasticity of the supply function, and Equation (3) is the standard monopsony inverse elasticity pricing rule.

When $G(o|p)$ is calculated according to Bayes' rule this elasticity function is the same across all states, and o^* is state invariant for interior solutions. To see this, first consider the following proposition.

Proposition 1. *Let c_i, $i = 1, \dots N$, be independent realizations from the distribution F, with ordering $c_1 \leq c_2 \leq \cdots \leq c_N$, then the conditional distribution of c_1 given c_2, is the same distribution as F truncated at c_2.*

Proof: This is Theorem 2.7 of David (1981).

[4] In this case the optimal offer is set valued, $o^* = [0, G^{-1}(0|p_2)]$.

In equilibrium, the auction price equals the realized second lowest cost, and according to this proposition the Bayesian posterior is

$$G(o|p) = \frac{F(o)}{F(p)}. \tag{4}$$

Thus, the state dependent supply functions have a multiplicative relationship. For example for two different auction outcomes, p' and p'', their Bayesian posteriors are related as follows, $G(o|p') = \frac{F(p'')}{F(p')}G(o|p'')$. One of the properties of an elasticity measure is that it's invariant to multiplicative scaling of the relationship. Accordingly, one can show $E_{G,o}(o) = E_{F,o}(o)$ and the optimal monopsony pricing rule of Equation (3) is invariant of the price, i.e.,

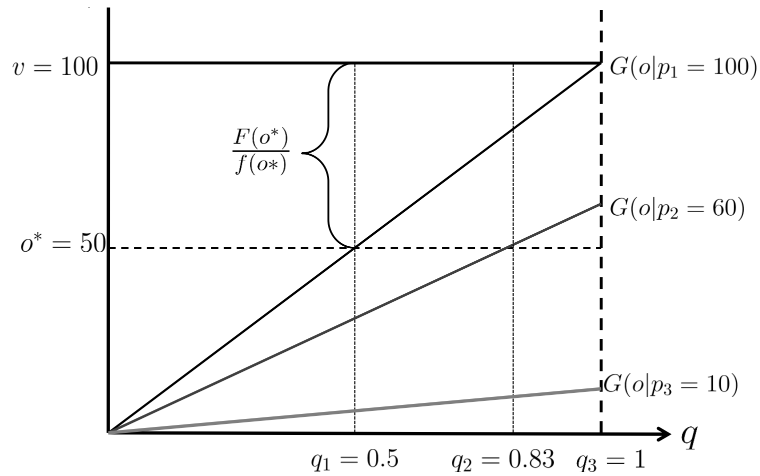
$$o^* = \frac{v}{1 + \dfrac{1}{E_{F,o}(o^*)}}. \tag{5}$$

In summary, the Bayesian posterior is simply a scalar proportion of the prior. Correspondingly, the elasticity of the posterior does not change and neither does the optimal offer, as indicated by Equation (5). However, the probability that the optimal offer is accepted by the auction winner does change and is inversely proportional to the prior evaluated at the auction price as indicated by Equation (4).

Briefly, the seller's strategy is an optimal response given the buyer's strategy because she can never strictly increase her payoff by accepting an offer above her cost, nor by rejecting one above her cost. Correspondingly, exiting the auction at her cost is optimal by standard arguments (For example, see Krishna (2009) page 15.) for exiting at cost in typical private cost English auctions.

Consider the following example which is also the setting we use in our experiment. The distribution of the buyer's value $H(v)$ is the uniform distribution on $[50, 150]$. There are two sellers and $F(c)$ is the uniform distribution on $[0, 100]$. Notice, that $F(c)$ is a linear function so its elasticity is always 1, as is the elasticity of any posterior. For a realized value v, by Equation (3), $o^* = v/2$. So while the optimal threshold does not depend upon price, the post auction probability of a purchase when engaging in take-it-or-leave-it bargaining does. Suppose $v = 100$ and consider three scenarios: $p_1 = 100$, $p_2 = 60$, and $p_3 = 10$. Figure 2 depicts that the optimal offer is the same in the first two scenarios, but the probability of purchase differs, and the optimal decision is to accept the auction in the last scenario.

Figure 2: The optimal offer for $v = 100$ and $F(c) = \frac{c}{100}$. For $p > 50$ the optimal counter offer is 50 and does not vary with the price, but the corresponding probability of purchasing, $G(50|p)$ does. For $p \leq 50$ the buyer accepts the auction outcome.



3 Experimental design and hypotheses

3.1 Experimental design

Our experiment consists of the following session flow. First, we recruit 18 subjects to participate in a two hour experimental session. We randomly designate 6 subjects as buyers and 12 subjects as sellers; these designations are fixed for the session. The session consists of 2 practice periods and 30 rounds for which subjects receive compensation based upon their decisions. Every period we randomly form 6 trios consisting of 2 sellers and 1 buyer. Participants are informed of the random rematching protocol and that all costs and values are redrawn each period.

Each trio plays the previous example of the auction-bargaining mechanism, and we induce common knowledge by publicly reading and displaying instructions at the start of the experiment.[5] Each period starts with each buyer and seller learning their respective value and cost. The two sellers in a trio participate in a descending English clock auction - without knowing the buyer's realized value. The initial auction price is \$100 and decrements by \$1 every 0.7 second. When a seller exits the auction, the auction concludes with winner and price determination as previously indicated.[6] Next, the buyer is informed of his auction price and presented the choice to either accept the auction outcome or make a take-it-or-leave-it

[5]Instructions are available upon request from the authors.

[6]We never observe a case where both sellers don't exit and the auction closes at a price of zero.

offer to the auction winning seller. If he accepts the auction outcome, all trio members are informed of their payoffs and the period concludes. If the buyer instead engages in bargaining, the auction winner is presented with the counteroffer and decides whether to accept or reject. In either case, payoffs are reported and the period ends.

We conducted 8 sessions at the Finance and Economics Experimental Laboratory (FEEL) at Xiamen University.[7] This gives us a total of 144 subjects (48 buyers and 96 sellers) each with 30 observations. Subjects on average earned 70 RMB for their participation. We recruited subjects through the ORSEE system (Greiner, 2004), all of whom were undergraduate and graduate students enrolled in Xiamen University, and none had previous experience in this study. The experimental software was programmed in Z-tree (Fischbacher, 2007).

3.2 Hypotheses

Our ex ante hypotheses consists of one regarding economic performance and three regarding buyer and seller behavior. The main result of the Bulow and Klemperer (1996) study is that while the auction-bargaining offers the auctioneer more value than a simple N-bidder English auction, it offers the auctioneer less value than adding another serious bidder[8] and conducting a $N + 1$ bidder English auction.

Hypothesis 1. *Buyer profit is greater than expected profit in a two-bidder English auction and less than a three-bidder English auction.*[9]

Next, the Seller's Nash equilibrium strategy provides two additional hypothesize. This first is about seller's behavior in the auction phase of the auction-bargaining mechanism.

Hypothesis 2. *Sellers exit the auction at their realized costs.*

We have a strong prior for confirming this hypothesis due to previous experimental results on independent private value forward (Coppinger, Smith, and Titus, 1980) and private cost reverse (Shachat and Wei, 2012) English auctions that show close adherence to theoretical predictions regarding expected price and bidders' strategy. However, there is uncertainty regarding how the bargaining phase plays out in the experiment and how this change in feedback affects the saliency of the seller optimal action in the auction. For example, a rational

[7]This facility is designed for the purpose of conducting economic experiments and has privacy carrels, a private payment and sign-in area, and a separate monitor room from which the experimenter conducts the experiment.

[8]In our design, sometimes a seller fails to satisfy the serious bidder criteria $c_i \leq v$. However, our design does satisfy the more lax sufficient condition that the losing supplier's expected cost is no more than the buyer's value. See Bulow and Klemperer (1996) page 185 footnote 15 for more detail.

[9]In this paper, we use the theoretical predictions of the two and three bidder English auctions benchmarks to test this hypothesis rather than run separate control treatments.

seller may "overstay" in the auction if she has a "joy of winning" the auction component in her utility function or she believes with probability one there will be a counteroffer and an opportunity to reject it. Alternatively, she may fail to make a rational choice because of difficulty perceiving the optimal strategy, as often is found in the sealed bid second price auctions.

Our second hypothesis about sellers concerns behavior when a seller confronts a take-it-or-leave-it offer.

Hypothesis 3. *Sellers don't reject profitable offers, and reject non-profitable ones.*

This hypothesis derived from sequential rationality has a stronger alternative hypothesis than it would appear at first glance. The bargaining phase of the game is strategically equivalent to an ultimatum game in which the buyer and seller only respectively know the upper and lower ends of the "pie" interval to be shared. The very large literature on Ultimatum Game experiments, starting with Guth, Schmittberger, and Schwarze (1982), has shown that a non-negligible proportion of responders reject minimally profitable offers. In particular, this still holds true in studies that introduce asymmetric information about the pie size (Croson, 1996; Huck, 1999; Harstad and Nagel, 2004). However, a glaring counterexample are the results of Salmon and Wilson (2008) who study a two unit experimental English auction where the first unit is sold to the auction winner and a take-it-or-leave-it offer is made to the last exiting bidder for the second unit. In this study, approximately only 4% of profitable offers were rejected. Our study is similar in that we imbed the ultimatum game as an after stage to the auction to determine possible responder participation. However, our setting differs as we have two-sided incomplete information and the responder does not have incentives to misrepresent her type (cost).

Our final hypothesis, developed in the previous section, regards the buyer's behavior.

Hypothesis 4. *Buyers follow their optimal strategy according to Bayesian posteriors.*

4 Results

Our experimental data set consists of 1440 plays of the auction-bargaining mechanism, each including the losing seller's exit price and the buyer's decision of whether to bargain. Buyers chose to bargain 963 times, and the winning seller rejected the take-it-or-leave-it offer 280 of these times. We start by comparing observed economic performance versus theoretical benchmarks for buyer profit, seller profit, and welfare improving trades. Then we examine the extent subjects follow their Nash equilibrium strategies.

4.1 Market performance

We start by presenting various summary statistics of our auction-bargaining with 2 sellers experiments, and the corresponding theoretical benchmarks from English auctions with 2 and 3 sellers. Table 1 presents these statistics and theoretical benchmarks. Note when calculating the theoretical benchmarks of the English auctions for expected buyer and seller profit, and potential social surplus, $\max\{0, v - \min\{c_i\}\}$, we use the realizations of the buyers' values and sellers' cost rather than the distributions they are drawn from.

Table 1: Realized economic performance versus theoretical benchnmarks

Performance Measure	A-B Mechanism experiment	Theoretical Prediction A-B Mechanism equilibrium	English auct. 2 bidders	English auct. 3 bidders
Average buyer profit	40.66	43.92	36.83	51.64
Standard deviation	32.54			
Average seller profit	21.91	16.84	25.23	21.11
Standard deviation	20.92			
% of Periods with trade	80.56	79.58	95.14[a]	97.57[a]

[a] For the English auctions, the reported value is the percentage of auctions for which $v \geq \min\{c_i\}$.

A first observation is that the main theorem of Bulow and Klemperer (1996) holds.

Result 1. *Using the auction-bargaining mechanism, buyer profit is greater than expected profit in a two-bidder English auction and less than a three-bidder English auction. Hypothesis 1 is confirmed by the t-tests of the third and fourth row of Table 2.*

While the qualitative prediction regarding the buyer's welfare is confirmed. Table 2 shows that we reject the theoretical predictions of average buyer and seller profit. Either the buyers or sellers are not uniformly following their respective equilibrium strategies. We show it is the buyers, not the sellers, who are deviating.

4.2 Seller behavior

Do sellers exit the auction when price reaches their costs? Consider Figure 3, which plots the 1440 exit prices for the auction losing sellers versus their respective costs. There is a clear concentration of observations along the forty-five degree line. To provide quantitative evidence that sellers exit at price equals costs we conduct an ordinary least squares regression, finding an intercept of 3 and a slope coefficient of 0.95 (with an adjusted R^2 statistic of 0.94). If we suppress the intercept term, then the slope coefficient is 0.99. These results

Table 2: Hypothesis tests comparing observed profit and theoretical benchmarks

Player role	Null hypothesis	Alternative hypothesis	t-statistic[a]	p-value
Buyer	AB = AB equilibrium[b]	AB $\neq$ AB equilibrium	−3.80	0.00
	AB = EA 2 bidders[c]	AB > EA 2 bidders	4.47	0.00
	AB = EA 3 bidders	AB < EA 3 bidders	−12.78	0.00
Auction Winning Seller	AB = AB equilibrium	AB $\neq$ AB equilibrium	9.18	0.00
	AB = EA 2 bidders	AB > EA 2 bidders	−6.02	0.00
	AB = EA 3 bidders	AB > EA 3 bidders	1.45	0.93

[a] The t-stat is calculated $\frac{\bar{\mu}-\mu_0}{\hat{\sigma}_\mu/\sqrt{n}}$, where $\bar{\mu}$ is the sample average, μ_0 is the theoretical prediction, the number of observations is $n = 1440$, and $\hat{\sigma}_\mu$ is the sample standard deviation. The test statistic has a student-t distribution with $n-1$ degrees of freedom.
[b] AB is the Auction-Bargaining mechanism.
[c] EA is the English auction.

are consistent with those previously found in English auctions in both forward (Coppinger, Smith, and Titus, 1980) and reverse (Shachat and Wei, 2012) contexts. Admittedly, there is some evidence that the potential bargaining phase leads to some overstaying in the auction for high costs, as seen in Figure 3, and noisy adherence to the equilibrium strategy. For example, the percentage of absolute deviations of bids from cost less than \$1, \$2, and \$3 are respectively 56.5%, 75.0%, and 82.6%. With these minor caveats, we state our next result.

Result 2. *Sellers tend to exit the auction at their realized costs, confirming Hypothesis 2.*

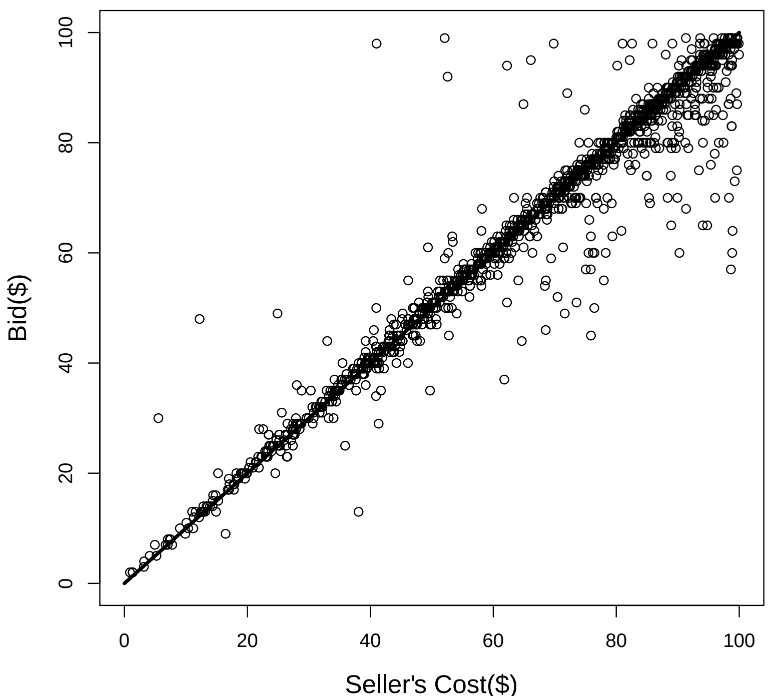
Turning our attention to seller behavior in the bargaining phase, Figure 4 plots the 963 take-it-or-leave-it offers versus the winning seller's cost. The forty-five degree line separates profitable and non-profitable offers. First note that sellers reject all 274 non-profitable offers, but only 6 out of the 689 offers that exceed the seller's cost. Clearly, the types of reciprocal behavior and other regarding preferences found pervasive in Ultimatum Bargaining experiments are not a factor here. These results also demonstrate the robustness of the findings in Salmon and Wilson (2008).

Result 3. *Sellers rarely reject profitable offers, and always reject non-profitable ones, confirming Hypothesis 3.*

4.3 Buyer behavior

Our assessment of how the buyers' choices agree with their equilibrium strategy starts with a visualization of the buyer's decisions in Figure 5. In this figure the x-axis is the buyer's

Figure 3: Auction exit price versus realized cost

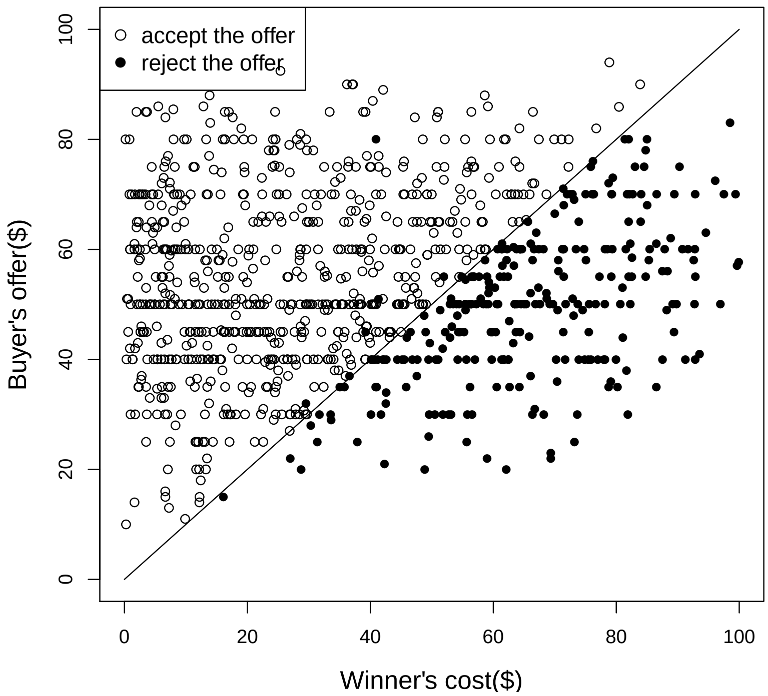


value and the y-axis is the value of the auction price and take-it-or-leave-it offer. A triangle marks an accepted auction price. A closed circle marks a rejected auction price, and the connected open circle is the corresponding take-it-or-leave-it offer. The upper line flowing from southwest to northeast is where auction price (and counter offer) equals the buyer value. The lower line is where auction price (and counter offer) equals one-half the buyer value, i.e. the theoretical prediction. We refer to this as the optimal offer line (OOL). If buyers adhere perfectly to the equilibrium strategy, all auction prices below the OOL are accepted and those above are rejected and countered with an offer on the OOL. Note, we are only plotting a randomly selected one-fourth of the data to avoid over-cluttering.

The figure suggests mixed evidence regarding how theory does track the data. Buyers reject 82% of the auction prices above the OOL, and accept 74% of those below the OOL; roughly matching the theoretical predictions. However, we can recognize there are three types of behavior which deviate the theoretical prediction. First, we observe that when some high auction prices are rejected, the subsequent counteroffer is greater than the theoretical optimal offer. Second, some auction prices above the OOL are accepted. Third, some counteroffers are below the OOL, with some associated with rejected auction prices below the OOL. We explore whether these deviations are unbiased noisy adoption of equilibrium strategies, or are they reflecting alternative structural behavior.

A natural way to test the veracity of the buyer's equilibrium strategy, modulo unbiased

Figure 4: Plot of take-it-or-leave-it offers versus the auction winning seller's cost: an open circles marks an accepted offer and a solid circle marks a rejected offer.



error terms, is to nest it in the following Tobit regression model. We assume that a buyers optimal offer is a linear function of his value and the auction price plus some normally distributed error term. We observe his optimal offer whenever is less than the auction price, otherwise it is censored at the auction price. Formally, the Tobit model is

$$o_{it} = \begin{cases} \alpha + \beta p_{it} + \gamma v_{it} + \epsilon_{it}, & \text{if } \alpha + \beta p_{it} + \gamma v_{it} + \epsilon_{it} < p_{it} \\ p_{it}, & \text{if } \alpha + \beta p_{it} + \gamma v_{it} + \epsilon_{it} \geq p_{it} \end{cases}, \tag{6}$$

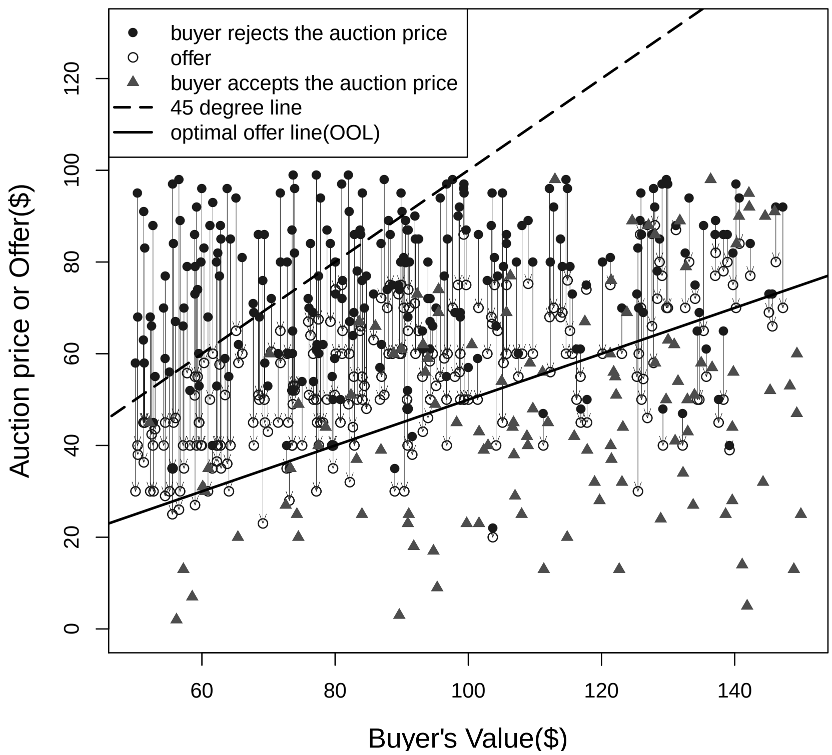
where ϵ_{it} is a normally distributed error term, $N(0, \sigma_\epsilon^2)$.

If buyers follow the optimal strategy, given in Equation (1), then we should find $\alpha = \beta = 0$, and $\gamma = 0.5$. We report the maximum likelihood estimates of Equation (6) in Table 3.[10]

The Tobit regression results in Table 3 reveal that the theory does a remarkably good job of predicting when a buyer chooses to bargain, but fails to capture important aspects of what determines the size of take-it-or-leave-it offers. First, we expect a buyer to bargain when $\frac{\alpha+\gamma}{1-\beta} v < p$. We find that α is not significant and then by substituting parameter estimates for γ and β, we estimate the condition for choosing to bargain condition is $\frac{0.34}{1-0.35} v = 0.52v < p$;

[10] Out of concern that buyers use one rule to determine when to bargain and another to determine the counter offer, we attempt to estimate a sample selection model. However, the correlation coefficient ρ always converges to one - leading to a singular Information matrix. We interpret this as strong evidence that the selection and offer models are the same.

Figure 5: Buyer choices conditional on realized value: accepted auction prices and reject auction prices with counteroffer. A solid triangle is an accepted auction price. A solid circle is a rejected auction price, and the connected open circle is the subsequent counteroffer. The upper dashed line is the 45 degree line indicating where auction prices equal buyer values. The solid line is the theoretical optimal offer line. In equilibrium, we should see all prices above this line rejected with counter offers on the line, and all prices below the line accepted.



almost exactly what the theory predicts. However, inconsistent with the theoretical prediction is the significant positive relationship price has on the take-it-or-leave-it offer amount. We seek to explain this systematic deviation from the theory.

First, we identify significant individual heterogeneity in the buyers' sensitivities to price. We estimate an individual linear Tobit model for each buyer and report the sum of individual Log-likelihood values, in the last row of Table 3. A likelihood ratio test soundly rejects the pooled model in favor of the individual model. In Figure 6 we present scatter plots of the estimated individual coefficients for price and value. We classify three types of individual estimates: open squares mark the cases that we can't reject that α and β are jointly zero via an F-test, open triangles mark the estimated coefficients for the 5 subjects whose estimated α is significantly different from zero, and closed circles mark the estimated coefficients for all others.

These scatter plots of individual estimated pairs β and γ demonstrate that individuals

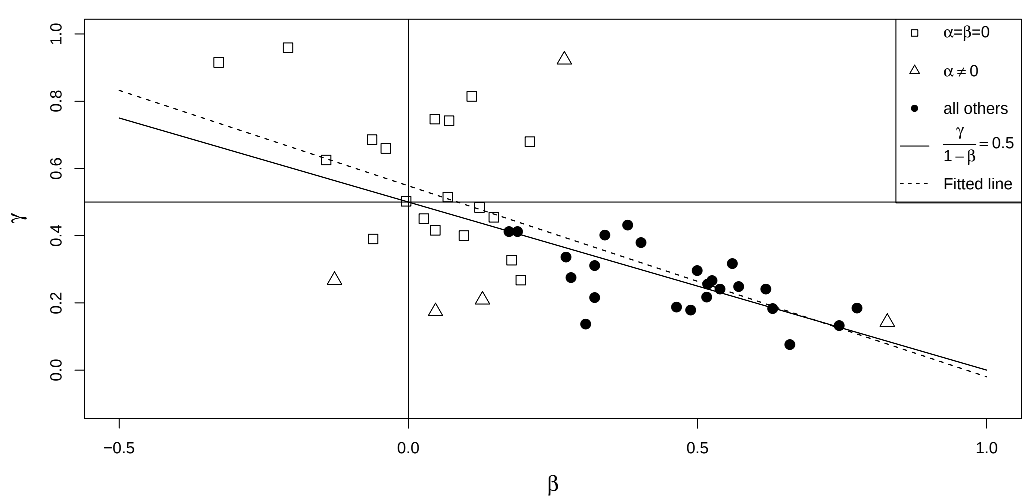
Table 3: MLE of the linear Tobit model for the pooled data

Variable	Estimate	Standard Error[a]
Constant	-0.24	3.23
Price	0.35^{***}	0.05
Value	0.34^{***}	0.02
σ_ϵ	12.68^{***}	0.73
Log Likelihood	Pooled	-4070
Log Likelihood	Individual	-3444

[a] Standard errors are clustered by session.
*** Coefficient is significant at the 1% level.

generally agree on when to bargain, but differ in how strong counter offers depend upon the auction price. We provide a vertical reference line at $\beta = 0$ and a horizontal reference line at $\gamma = 0.5$. The line with a slope of negative one-half passing through $(0, 0.5)$ represents the pairs of parameter values corresponding to the rule of negotiating when $p > 0.5v$. South-eastern movements along this line, away from $(0, 0.5)$, indicate counteroffers with an increasing positive relationship with auction price. We separate individuals into those whose offers depend upon price, identifying them with solid circles, and those whose offers don't, identifying them with open squares. The distribution of estimated parameter pairs are clustered around the theoretical line, as suggested by the close agreement between this and the fitted line, for rejection of the auction price but spanning a large range of price sensitivity of counter offers. A natural question is whether these dependencies of take-it-or-leave-it offers on auction prices are consistent with less restrictive assumptions on subject preferences or formation of beliefs.

Figure 6: Individual estimated price and value coefficients from the linear Tobit model



5 Alternative models

In this section we explain buyer behavior by supposing posterior beliefs regarding the auction winner's cost are distortions of the Bayes rule determined posteriors. We parameterize these distortions using a well known probability weighting function that allows an interpretation of subjects perceiving various kinds of affiliation between the lowest and second lowest realized costs. We also show the observed patterns of when buyers choose to bargain and the positive relationship between take-it-or-leave-it offers and auction price can't be explained by models solely assuming risk aversion or anticipated regret, models that have previous success explaining empirical auction behavior.

5.1 A model of transformed Bayesian posteriors

We previously showed that the invariance of the buyer's optimal offer depends on the use of Bayes' rule to formulate the posterior distribution of the auction winning seller's cost. We now generalize from Bayesian updating by using a two-parameter transformation of Bayesian posteriors. Namely, the buyer's subjective posterior of the winner seller's cost is

$$G(o|p) = \begin{cases} 0 & \text{if } o = 0 \\ e^{-\mu\left(-\ln\left(\frac{F(o)}{F(p)}\right)\right)^{\lambda}} & \text{if } 0 < o \leq p \end{cases}, \tag{7}$$

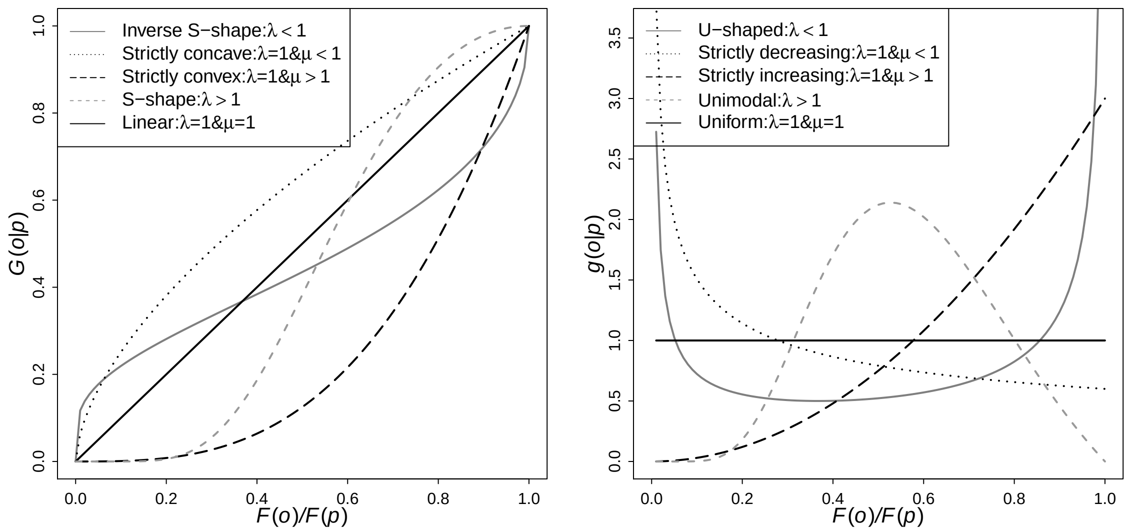
where μ and $\lambda > 0$.

Readers may recognize that Equation (7) is the Prelec (1998) form of a probability weighting function. Probability weighting is a component of Prospect theory (Kahneman and Tversky, 1979) capturing the empirical regularity that individuals' valuation of a risky prospect is more sensitive to changes in probabilities close to zero and one than at more central quantiles. This common pattern of valuation results in an inverted S-shape probability weighting function. Since we are modeling distorted posteriors we adopt the Prelec form which allows various forms of biases. Using our uniform prior, for example, the unbiased transformation of the Bayesian posterior is the linear $G(o|p)$ presented in Figure 7. Now consider the short-dashed S-shaped $G(o|p)$ in the same figure. In this case, the bias reflects a single-peaked posterior PDF with a modal belief that the winner's cost is about 50% of the auction price; essentially perceiving a non-existent affiliation between the auction price and the winner's cost. A stronger bias towards a positive affiliation in costs is found in the convex shaped $G(o|p)$, while a concave $G(o|p)$ suggests a bias in favor of low costs.[11] In this

[11] When $\lambda = 1$ this model is behavioral indistinguishable from a model of a buyer whose expected utility function is $u(x) = x^{\frac{1}{\mu}}$. One can establish this by making the preference invariant transformation $Z\left((v-o)\left(\frac{F(o)}{F(p)}\right)^{\mu}\right) = (v-o)^{\frac{1}{\mu}}\frac{F(o)}{F(p)}$, where $Z(y) = y^{\frac{1}{\mu}}$. We refer the reader to Goeree, Holt, and Pal-

case the subjective posterior puts increasing probability mass nearer the auction price. Let's finally consider the inverted S-shaped $G(o|p)$ which implies increasing posterior beliefs on costs close to zero and the price. This suggests a rather unusual u-shaped posterior PDF.

Figure 7: Alternative shapes for transformed posterior distribution function, $G(o|p)$, and density function, $g(o|p)$



Now suppose a buyer formulates his posterior distribution of the auction winner's cost according to Equation (7). The next proposition describes when there is a unique optimal offer, and under what conditions an interior optimal offer is strictly increasing in price.

Proposition 2. *Assuming $F(c)$ is the uniform distribution and $G(o|p)$ is calculated according to Equation (7), then*

(i) if $\lambda \geq 1$ and $f'(o) \leq 0$, then $o + \frac{G(o|p)}{g(o|p)}$ is strictly increasing and there is a unique o^*;

(ii) if $\lambda > 1$, then an interior optimal offer o^* is strictly increasing in the auction price p.

Proof: See appendix.

A buyer's Nash equilibrium strategy, when following our two parameter subjective posterior model, directly leads to the following non-linear Tobit regression model;

$$o_{it} = \begin{cases} o^*_{it}(\mu, \lambda | p_{it}, v_{it}) + \epsilon_{it}, & \text{if } o^*_{it}(\mu, \lambda | p_{it}, v_{it}) + \epsilon_{it} < p_{it} \\ p_{it}, & \text{if } o^*_{it}(\mu, \lambda | p_{it}, v_{it}) + \epsilon_{it} \geq p_{it} \end{cases}$$

frey (2002) p.265 for further discussions.

where ϵ_{it} is a normally distributed error term, $N(0, \sigma_\epsilon^2)$, and $o_{it}^*(\mu, \lambda | p_{it}, v_{it})$ comes from the first order condition $v - o^* = \frac{G(o^*|p)}{g(o^*|p)}$.

The likelihood function of the non-linear Tobit model is,

$$\max_{\mu,\lambda} L(\mu, \lambda, \sigma^2 | o, v, p) = \prod_{i=1}^{48} \prod_{t=1}^{30} \prod_{p_{it} \leq o_{it}} \Pr(o_{it}^* \geq p_{it}) \prod_{p_{it} > o_{it}} f(o_{it} | o_{it}^* < p_{it}) \Pr(o_{it}^* < p_{it}).$$

Table 4 reports the maximum likelihood estimation results of this nonlinear Tobit model. The estimates of $\mu = 2.39$ and $\lambda = 1.35$ imply that $G(o|p)$ is s-shaped and there is a positive relationship between the offers and auction prices.[12] A Wald test rejects the Bayesian model, $\mu = \lambda = 1$, in favor of this subjective prior model with a p-value of less than 0.001. Once again there is significant heterogeneity across individuals as a full fixed coefficient model can't be rejected with a likelihood ratio test (see the last row of Table 4 for the Log-likelihood value of the fixed coefficient regression.)

The fixed coefficient model exhibits a diversity of behavioral rules which we now connect to the individual estimates of the structural parameters μ and λ. First, we present in Figure 8 a scatter plot of each buyer's joint estimate of (μ_i, λ_i). We classify each buyer's subjective posterior PDF according to the following joint hypothesis tests.[13]

1. Uniform - we fail to reject $\mu = \lambda = 1$.
2. Unimodal - we reject $\lambda = 1$ in favor of $\lambda > 1$.
3. U-shaped - we reject $\lambda = 1$ in favor of $\lambda < 1$.
4. Strictly increasing - we fail to reject $\lambda = 1$ and we reject $\mu = 1$ in favor of $\mu > 1$.
5. Strictly decreasing - we fail to reject $\lambda = 1$ and we reject $\mu = 1$ in favor of $\mu < 1$.

Let's consider four subjects with differing shaped subjective posterior PDF's, and compare their behavior in the experiment. First, consider Buyer A whose parameter estimates are marked "A" in Figure 8 and close to the Bayesian type at (1,1). Buyer A's estimated $g(o|p)$ and his choice data are presented in the first row of plots of Figure 9. Here we see that his subjective posterior transformation is nearly one-to-one with the Bayesian posterior, and correspondingly his experimental choices of when to bargain and consequent take-it-or-leave-it offers closely agree with the theory. Buyer B, in contrast, has a unimodal subjective PDF consistent with a disproportionately high perception that the auction winner's cost be between 20-80% of the auction price. This leads Buyer B to reject every auction outcome

[12]Finding probability weighting functions that are not inverted s-shaped is common in strategic decision tasks; for example bidders in first price sealed bid auctions have been estimated to have convex shaped functions (Goeree, Holt, and Palfrey, 2002; Ratan, 2012) and s-shaped in normal form games (Goeree, Holt, and Palfrey, 2003).

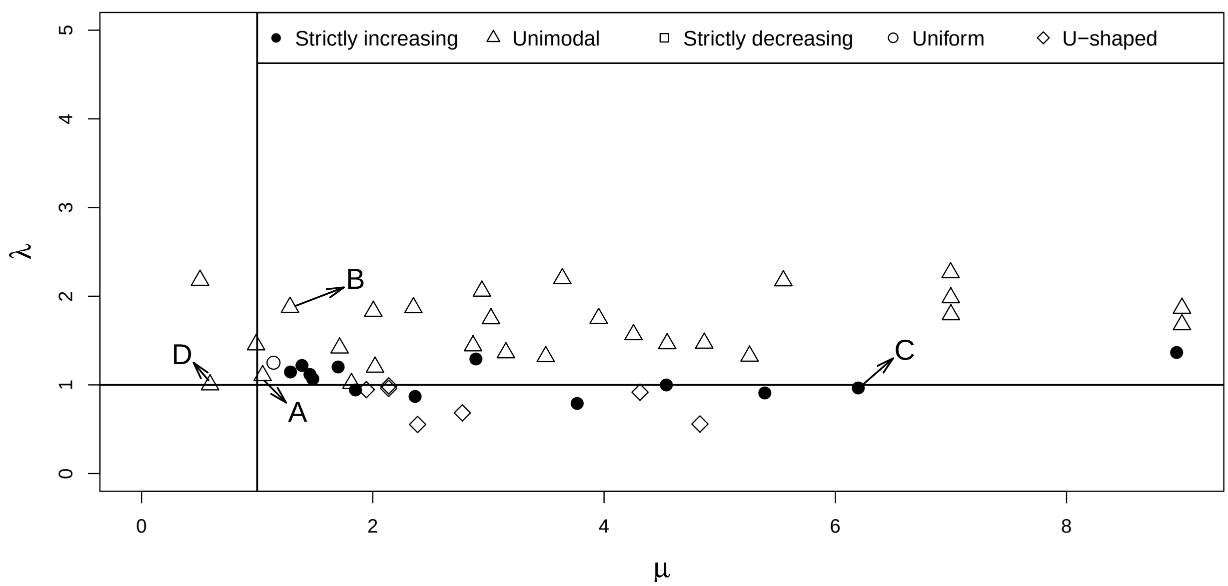
[13]We choose a test size of 0.05 in each case.

Table 4: MLE of the non-linear Tobit model with the two parameter subjective posterior model

Parameter	Estimate	Standard Error
μ	2.39***	0.002
λ	1.35***	0.002
σ_ϵ	11.31***	0.248
Log Likelihood	Pooled	-4070[a]
Log Likelihood	Individual	-3613

*** Coefficient is significant at the 1% level.
[a] This is not a typo, this is the same value for the maximized likelihood as we find for the linear Tobit model. However, the linear Tobit model has one more parameter than the nonlinear Tobit model.

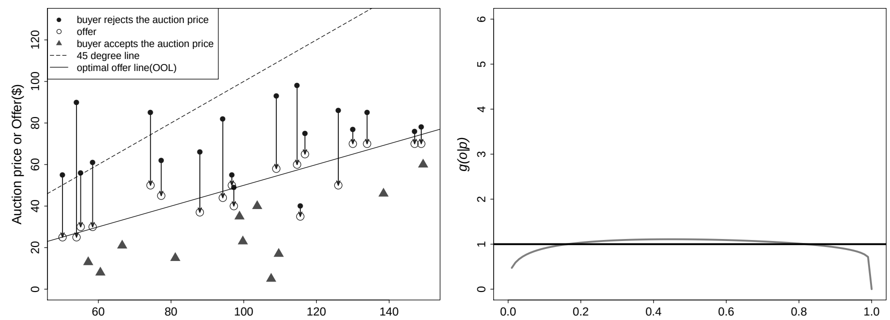
Figure 8: Individual estimates of parameters μ and λ from the subjective posterior model



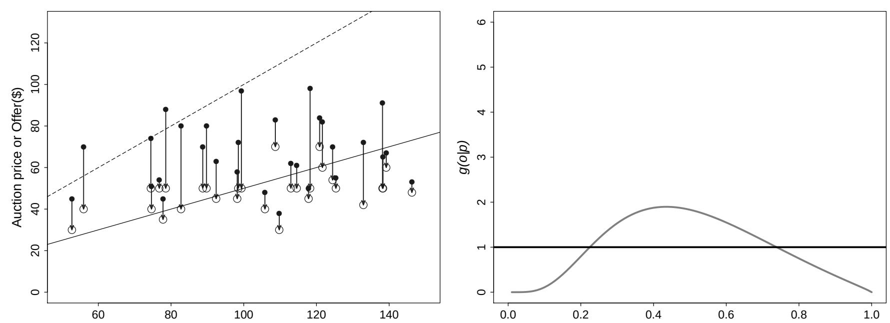
and make aggressive counter offers. Buyer C's posterior reflects a different belief of affiliation; the strictly increasing posterior PDF reflects a belief the winner's cost is very likely close to the auction price.[14] Consequently, the buyer seldom rejects the auction outcome and demands very small price reductions in the few cases he does. On the other end of the optimism spectrum, is Buyer D whose strictly decreasing posterior PDF exhibits a strong negative bias on the winners cost, and leads to a high rejection rate and very aggressive counteroffers.

[14]The reader may notice that we estimate seven buyers having inverted s-shaped $G(o|p)$; however, in each of these cases the function is essentially convex with only a slight overweighing of a small interval of low probabilities.

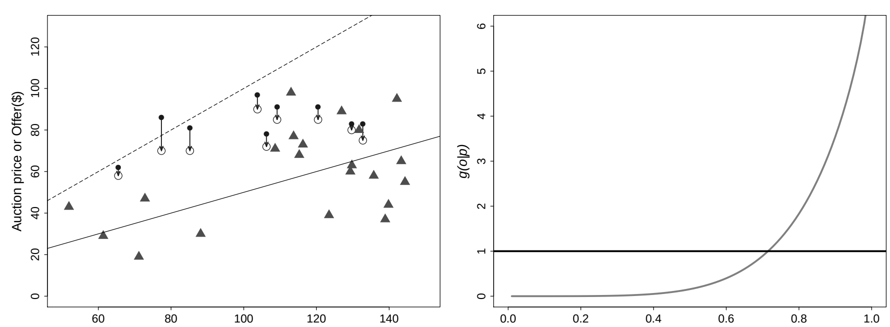
Figure 9: Four different estimated $g(o|p)$ and the corresponding buyer subject's behavior



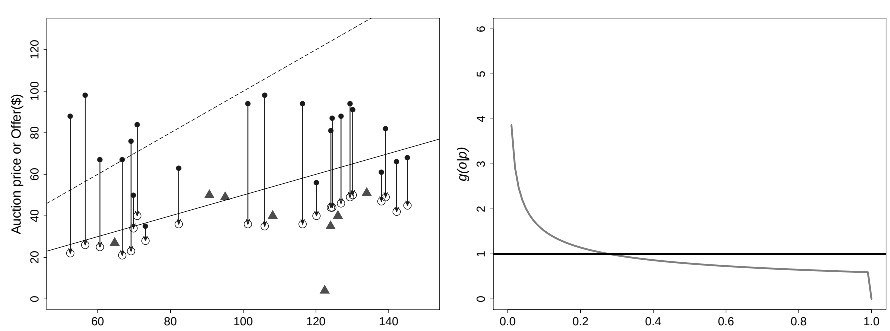
(a) Buyer A $(\mu, \lambda) = (1.05, 1.10)$



(b) Buyer B $(\mu, \lambda) = (1.28, 1.88)$



(c) Buyer C $(\mu, \lambda) = (6.20, 0.96)$



(d) Buyer D $(\mu, \lambda) = (0.59, 1.00)$

5.2 Risk aversion and anticipated regret: two (non)explanations

Two behavioral models successfully used to explain bidder deviations from Nash equilibrium strategies are risk aversion (Cox, Roberson, and Smith, 1982; Cox, Smith, and Walker, 1988) and anticipated regret (Engelbrecht-Wiggans and Katok, 2007, 2009; Filiz-Ozbay and Ozbay, 2007). However, neither model can explain the positive relationship between auction prices and take-it-or-leave-it offers. Risk aversion can only influence the location of the optimal offer line. While in the case of anticipated loss regret, there is no impact on the buyer's strategy when the auction price exceeds his value and there is a negative relationship between take-it-or-leave-it offers and auction prices when his value exceeds the auction price.

First consider the case where the buyer is strictly risk averse but forms Bayesian posteriors. The following proposition shows that for any v the optimal take-it-or-leave-it offer exceeds the risk neutral offer and does not vary with price.

Proposition 3. *Assume the buyer's expected utility function satisfies $u(0) = 0$, $u'(x) > 0$, and $u''(x) < 0$, then*

(i) the optimal offer o^* is greater than the optimal offer for the risk neutral case.

(ii) For an interior optimal offer o^*, $\frac{\partial o^*}{\partial p} = 0$.

Proof: See appendix.

Consider an example in which a buyer in our experiment has the expected utility function $u(x) = x^r$, characterized by the constant coefficient of relative risk aversion $1 - r$. For a strictly positive r, the buyer's Nash equilibrium strategy is to accept all auction prices below the threshold $o^*(v; r)$ and to counter offer this threshold otherwise. It is straight forward to show that $o^*(v; r) = \frac{v}{1+r}$. In Figure 10, the left hand side plot shows the hypothetical behavior of a buyer for whom $r = 0.5$. Clearly the offers don't depend upon the price and OOL is steeper than the risk neutral case.

Now we show that in a model of anticipated regret, there is no relationship between the optimal offer and auction price when this price exceeds the buyer's value and the relationship is negative when the auction price is below the buyer's value. In an anticipated regret model the decision maker's utility function includes disutility for the failure to capture all ex post realized potential gains by ex ante decisions. For buyers in our experiment both loss and a win regrets are possible. A loss regret occurs when the buyer opts to bargain rather than accept a profitable auction price and then his take-it-or-leave-it offer is rejected. A win regret occurs when an offer is accepted, and the buyer realizes a lower offer could have been accepted. In this case the magnitude of the win regret is unknown, unlike the case of loss regret. Here we follow the suggestion of Davis, Katok, and Kwasnica (2011) and set the win regret equal to the accepted offer less the expected winner seller's cost conditional on this bargaining outcome; namely $\frac{o}{2}$.[15]

Let's consider an explicit model in which win and loss regrets result in decrements to utility by proportional penalties ω_w and ω_l respectively. In this case we can express the expected utility of an offer o as

$$E[u(o|v,p;\omega_w,\omega_l)] = \begin{cases} \left(v - o\left(1 + \frac{\omega_w}{2}\right)\right)\frac{o}{p} & \text{if } p \geq v \\ \left(v - o\left(1 + \frac{\omega_w}{2}\right)\right)\frac{o}{p} - \omega_l\left(v-p\right)\left(1 - \frac{o}{p}\right) & \text{if } p < v. \end{cases} \tag{8}$$

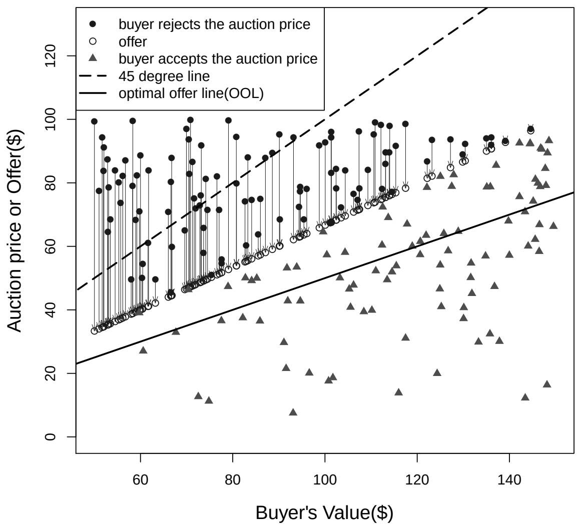
Conditional upon bargaining, the optimal take-it-or-leave-it offer is

$$o^*(v,p,;\omega_w,\omega_l) = \begin{cases} \dfrac{v}{2\left(1+\frac{\omega_w}{2}\right)} & \text{if } p \geq v \\ \dfrac{v + \omega_l\left(v-p\right)}{2\left(1+\frac{\omega_w}{2}\right)} & \text{if } p < v. \end{cases} \tag{9}$$

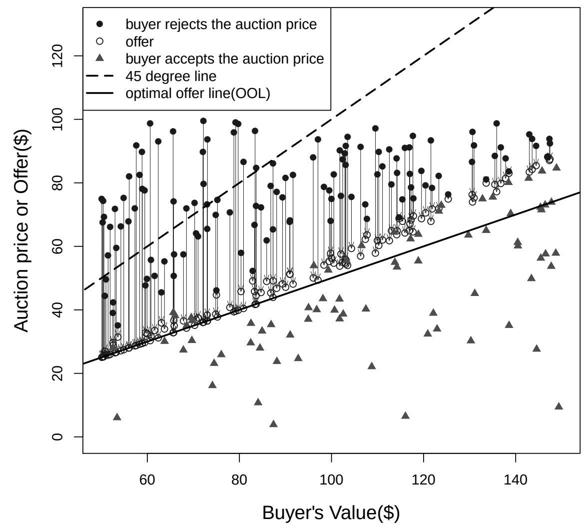
From Equation (9) when the auction price exceeds the buyer's value, his optimal offer does not depend upon the auction price. When price is below his value, then there exists a *negative* relationship which is the opposite of what we observe. When deciding whether to bargain, the buyer evaluates whether $v - p$ exceeds his expected utility (8) evaluated at his optimal offer given in Equation (9). We present the hypothetical decisions of a buyer for whom $\omega_w = 0$ and $\omega_l = 0.5$ on the right hand side of Figure 10.

[15]We proceed assuming there is no winner's regret for accepting the auction outcome. Doing so only influences the decision whether to bargain - not the actual amount of the optimal offer conditional on bargaining.

Figure 10: Buyer behavior under risk aversion and anticipated regret: some hypothetical decisions for randomly selected scenarios



(a) Risk averse buyer where $r = 0.5$



(b) Anticipated regret buyer where $(\omega_w, \omega_l) = (0, 0.5)$

6 Experimental treatment testing robustness

After reporting our main experimental results and subsequent ex post behavioral explanations, it's natural to question the robustness of these results to the information subjects are given and alternative explanations of buyer behavior. We report on three additional experimental treatments to address theses concerns. First, we examine whether sellers learn to adhere more closely to the equilibrium strategy in the auction phase or learn to be more "imperfect" in the bargaining phase when given more extensive feedback on the procurement outcome. Second, we test whether our estimated transformed subjective posteriors are actually non-equilibrium beliefs by having buyers play against automated equilibrium playing sellers - and informing buyers of this. Finally, we investigate whether distorted posteriors can be corrected by providing buyers with an expert decision support system which is based on the true probabilities an offer will be accepted.

6.1 Effects of enriching sellers' feedback

Procurement processes in practice generally provide a richer set of feedback to auction losing sellers than we do. The Nash equilibrium doesn't change if we inform the auction loser of the subsequent actions taken in the bargaining phase; however, this additional information could affect seller behavior through more rapid learning, or invoking social preference norms.

Accordingly we conduct two sessions, using the same pairings, values and costs from two sessions of our original treatment, but now inform the auction losing seller about buyers decision to bargain, the amount of any take-it-or-leave-it offer, and the winning sellers response. We call this the Enriched Feedback (EF) treatment.

We find no significant differences in the sellers' auction or bargaining behaviors. We pool the auction data from the first two sessions of the original treatment with the two EF sessions and then estimate the bid function component of the seller's strategy. Table 5 reports the OLS estimates in Panel A. First, we note that an F-tests fails to reject the absence of treatment effects in both the slope and constant terms. Further the slope, as in the original treatment, is very close to one. Given the lack of a treatment effect, we find with enriched feedback sellers still don't exactly bid their cost. Panel B of Table 5 shows as slight increase in the number of absolute bid deviations from cost of less than one dollar, but also a slight increase in the number of deviations greater than three dollars. With respect to the Bargaining Phase, there is essentially no treatment effect. Sellers rejected only 3 out of 188 profitable offers in the EF treatment and 0 out of 169 profitable offers in the corresponding two sessions of the original treatment.

Table 5: Comparison of sellers' behavior in the auction phase for the Original and Enriched Feedback treatments

Panel A: OLS estimation of the sellers' auction bid functions

Variable	Estimate	Standard Error
Constant	2.79^{***}	0.79
d_o[a]	-1.19	1.13
Cost	0.97^{***}	0.01
$d_o\times$Cost	0.01	0.02
Adjusted-R^2	0.95	
H_0: Coefficients of d_o and $d_o\times$Cost are jointly zero	F-stat $= 2.63$[b]	p-value $= 0.07$

*** Coefficient is significant at the 1% level.
[a] d_O is the dummy variable for the Original treatment
[b] The distribution of the test statistic is $F_{2,716}(x)$.

Panel B: comparison of cumulative absolute bid deviations from realized costs

Treatment	$\lvert b-c\rvert \leq 1$	$\lvert b-c\rvert \leq 2$	$\lvert b-c\rvert \leq 3$
Enriched Feedback	66.39%	74.17%	78.61%
Original - first 2 sessions	60.83%	78.89%	87.50%
Original - all 8 sessions	56.53%	74.93%	82.64%

6.2 Robustness of the transformed Bayesian posterior model

An alternative to our transformed posterior model is that subjects' posteriors are Bayesian but that their beliefs regarding the sellers' strategies are incorrect.[16] To test this alternative hypothesis we conduct the Automated Seller (AS) treatment. In this treatment, all subjects are buyers who face computerized sellers following their Nash equilibrium strategy. We inform buyers that sellers are following this strategy.[17] We conduct 3 sessions, each with 16 subjects, of this treatment. If buyers deviate from their equilibrium strategy in the original treatment because of disequilibrium beliefs, then buyer behavior should now conform to theoretical predictions. As we show shortly, they do not.

In our final treatment, we test whether transformed posteriors are a simple to correct bias. In the Automated Seller with Calculator (ASC) treatment, we augment the AS treatment by providing buyers with a calculator both when deciding whether to bargain and when choosing the offer size. When the buyer enters a potential offer, the calculator returns six items: profit if the buyer accepts the auction outcome, probability the offer will be accepted, profit if the offer is accepted, probability the offer will be rejected, profit if the offer is rejected (always zero), and the expected profit from making the offer. The calculator also has a history window showing all previously evaluated offers of the current period, and corresponding output. Further, before a buyer can submit a take-it-or-leave-it offer he is required to successfully enter the offer in the calculator. This ensures the buyer has seen the correct probability of his offer being accepted and the expected payoff. Despite providing - and in fact obligating buyers to use - an expert decision support system, the positive relationship between price and offers strengthens.

We first show how the two automated seller treatments alter the relationship between auction prices and buyers' offers. Table 6 presents the linear Tobit estimates of the buyers' offers from the pooled Original, AS, and ASC treatments. While a likelihood ratio test rejects this pooled regression in favor of a full fixed coefficient model, we still believe this is an informative starting point for discussions. When switching from human (Original) to automated sellers (AS) the sensitivity of offers with respect to price and value doesn't change. When we provide buyers with calculators the results are striking in that buyers' offers have an even stronger positive relationship with the auction prices.

[16]For an discussion on the extensive use of rational models with disequilibrium beliefs please see Crawford, Costa-Gomes, and Iriberri (2013).

[17]We use the same sequences of realized costs and values as the buyers faced in our original treatment.

Table 6: Linear Tobit estimates for the Original, AS, and ASC treatments: pooled data

Variable	Estimate[a]	Standard Error[a]
Constant	-0.45	3.10
d_{AS}	-5.41^{*}	3.16
d_{ASC}	-4.49	3.31
Price	0.35^{***}	0.04
$d_{\mathrm{AS}}\times$Price	0.03	0.04
$d_{\mathrm{ASC}}\times$Price	0.09^{***}	0.05
Value	0.34^{***}	0.01
$d_{\mathrm{AS}}\times$Value	-0.03^{**}	0.01
$d_{\mathrm{ASC}}\times$Value	-0.06^{***}	0.02
σ_{ϵ}	12.34^{***}	0.26
Log Likelihood Pooled	-13451	
Log Likelihood Individual	-11518	
Likelihood Ratio Test Stat.	3864[b]	p-value = 0.00

[a] Standard errors are clustered by session.

[b] ***, **, and * indicate significance at the levels of 1%, 5%, and 10% respectively.

[c] The null of the specification test is the pooled model and the alternative is the individual model. The test statistic is $LR = -2(\text{LL Pooled} - \text{LL Individual})$ and is distributed χ^2_{566}.

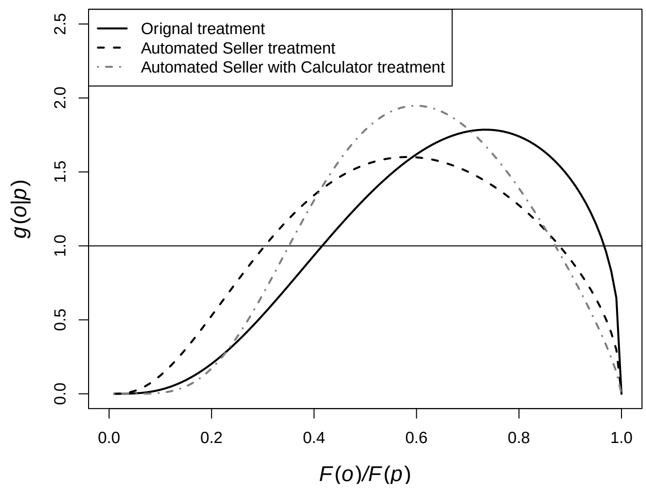
Table 7: Non-linear Tobit estimates of the subjective posterior model for the Original, AS, and ASC treatments: pooled data[a]

Parameter	Original Treatment	AS Treatment	ASC Treatment
μ	2.39	1.68	2.24
λ	1.35	1.46	1.71
σ_{ϵ}	11.31	11.80	11.35
Log Likelihood Pooled	-4070	-4769	-4592
Log Likelihood Individual	-3613	-4359	-4012

[a] All coefficients are significant at the 1% level. Likelihood ratio tests reject the pooled model in favor of the individual model at a 1% level of significance for all 3 models.

In both the AS and ASC treatments, buyers exhibit subject posterior density functions that are unimodal and whose modes occur at lower quantiles. Table 7 presents the non-linear Tobit estimates of the subjective posterior model for the Original, AS, and ASC treatments.

Figure 11: Estimated subjective posteriors for the Original, AS, and ASC treatments



The three estimated subjective posterior densities are plotted in Figure 11. For AS and ASC, the estimated posteriors reflect modes occurring at lower quantiles and lower density in the top three deciles than for the Original treatment posterior. This reflects behaviorally in a greater willingness to bargain and more aggressive offers because the perceived affiliation between the winner's cost and the auction price is stronger and the expected winner's cost is a smaller percentage of the price.

7 Concluding remarks

In this study we examine behavior in reverse English auctions, followed by a buyer option to engage in ultimatum bargaining. As Bulow and Klemperer (1996) showed, a Nash equilibrium of this game implements the optimal mechanism for procurement. We find strong support for this equilibrium modulo buyer's ultimatum offers having a positive relationship with auction prices. This turns out to have *economic* significance as the average buyer's surplus in our experiment was about 7.5% less than would have been earned with optimal offers. For the procurement official using reverse auctions, our results provide justification for the practice of engaging in post auction negotiations when qualifying additional suppliers is not feasible. This justification is two-fold: expected buyer surplus increases and it appears sellers do not harbor strong social utility concerns when being forced to negotiate post auction. However, the behavioral issue of decision makers failing to use the information revealed in the auction in an optimal Bayesian manner leads to unrealized potential benefits.

The bad news for procurement organizations is that our results reveal the suboptimal processing of auction information is difficult to correct. As we show, simply providing accurate decision support systems does not affect surplus improving behavior. This suggests either the subjective posterior model is a preference primitive and organizations need to consider how to construct mechanisms to account for this, or it is a deep-seated bias that requires more intensive training to correct.

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A Appendix

Proposition 2: Assuming $F(c)$ is the uniform distribution and $G(o|p)$ is calculated according to Equation (7), then

(i) if $\lambda \geq 1$ and $f'(o) \leq 0$, then $o + \frac{G(o|p)}{g(o|p)}$ is strictly increasing and there is a unique o^*;
(ii) if $\lambda > 1$, then an interior optimal offer o^* is strictly increasing in the auction price p.

Proof: Start by noting the following

$$g(o|p) = f(o)\frac{\mu\lambda}{F(o)}\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda-1} G(o|p),$$

and

$$\begin{aligned} g'(o|p) =& f'(o)\frac{\mu\lambda}{F(o)}\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda-1} G(o|p) - f^2(o)\frac{\mu\lambda}{F^2(o)}\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda-1} G(o|p) \\ &- (\lambda-1)f^2(o)\frac{\mu\lambda}{F^2(o)}\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda-2} G(o|p) + f^2(o)\frac{\mu^2\lambda^2}{F^2(o)}\left(-\ln\frac{F(o)}{F(p)}\right)^{2\lambda-2} G(o|p). \end{aligned}$$

Now the derivative we are interested in is,

$$\frac{d(o + \frac{G(o|p)}{g(o|p)})}{do} = 1 + \frac{g^2(o|p) - G(o|p)g'(o|p)}{g^2(o|p)} = 2 - \frac{G(o|p)g'(o|p)}{g^2(o|p)}.$$

Simplifying we get,

$$\frac{d(o + \frac{G(o|p)}{g(o|p)})}{do} = 1 + \frac{1}{\mu\lambda\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda-1}} + \frac{\lambda-1}{\mu\lambda\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda}} - \frac{f'(o)}{\mu\lambda f^2(o)\frac{1}{F(o)}\left(-\ln\frac{F(o)}{F(p)}\right)^{\lambda-1}}. \quad (10)$$

By inspection of Equation (10) we can that our assumption that $\lambda \geq 1$ and $f'(o) \leq 0$ ensures this expression is positive.

Next, we substitute for $G(o|P)$ and $g(o|P)$ into Equation (2), the F.O.C for an interior optimal take-it-or-leave-it offer, we have

$$\mu\lambda \frac{p}{o^*} \left(-\ln \left(\frac{o^*}{p} \right) \right)^{\lambda-1} = \frac{p}{v - o^*}.$$

Take ln to both sides,

$$\ln(\mu\lambda) - \ln(o^*) + (\lambda - 1) \ln \left(-\ln \frac{o^*}{p} \right) + \ln (v - o^*) = 0.$$

We differentiate with respect to p at the optimal solution to obtain the following:

$$\frac{\lambda - 1}{\ln(p) - \ln(o^*)} \left(\frac{1}{p} - \frac{1}{o^*} \frac{\partial o^*}{\partial p} \right) - \frac{1}{o^*} \frac{\partial o^*}{\partial p} - \frac{1}{v - o^*} \frac{\partial o^*}{\partial p} = 0. \tag{11}$$

Rearranging terms we obtain

$$\frac{\partial o^*}{\partial p} = \frac{(\lambda - 1)(v - o^*) o^*}{\left(v \ln \left(\frac{p}{o^*} \right) + (\lambda - 1)(v - o^*) \right) p}. \tag{12}$$

Obviously, the optimal offer should be smaller than the value and larger than the auction price. Hence, when $\lambda > 1$, $\frac{\partial o^*}{\partial p}$ is strictly positive and the optimal offer is strictly increasing in the auction price. Also note that when $\lambda = 1$ there is no relationship between optimal offer o^* and auction price p; not surprising as the model becomes observationally equivalent to assuming the buyer is risk averse/loving utility. Finally, when $\lambda < 1$, $\frac{\partial o^*}{\partial p}$ is ambiguous.

Proposition 3: Assume the buyer's expected utility function satisfies $u(0) = 0$, $u'(x) > 0$, and $u''(x) < 0$, then

(i) the optimal offer o^* is greater than the optimal offer for the risk neutral case;
(ii) For an interior optimal offer o^*, $\frac{\partial o^*}{\partial p} = 0$.

Proof: The first order condition for maximization can be expressed

$$u(v - o^*) \frac{f(o^*)}{F(p)} - u'(v - o^*) \frac{F(o^*)}{F(p)} = 0.$$

Rearranging terms yields

$$\frac{u(v - o^*)}{u'(v - o^*)} = \frac{F(o^*)}{f(o^*)}. \tag{13}$$

Let $z(x) = \frac{u(v-x)}{u'(v-x)}$, then

$$z'(x) = \frac{u(v - x)u''(v - x)}{u'(v - x)^2} - 1.$$

By the strict concavity of $u(x)$, $z'(x) < -1$.

When $o^* = 0$ the left hand side of (13) is strictly positive and the right hand side is zero. Further, as the left hand side is strictly decreasing and the right hand side increasing, they will intersect at most one time on the domain $[0, p]$. Since the slope of the left hand side is less than -1 it is decreasing faster than the risk neutral case. Also the utility at $u(0) = 0$ the same as in the risk neutral case, so $u(v) > v$. Therefore it will intersect the right hand side higher at a higher offer level than the risk neutral case. Thus, the optimal offer is higher than the risk neutral case.

Finally, the independence of the optimal offer from the auction price is clear from first order condition expressed as (13).

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